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Determination of charmed-hadron masses and electromagnetic mass differences in a broken-SU(8) quark model

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Assuming that hadron mass differences result from intrinsic quark mass differences and two-body spin-spin interactions together with the Coulomb and magnetic interactions in an SU(8) model, we determine the charmed-hadron mass spectrum. A complete set of mass relations is derived for the baryon electromagnetic mass differences. In addition to the prediction of charmed-baryon mass differences a modified SU(6) relation is found to be well satisfied. $D^+ - D^0$ and $D^{*+} - D^{*0}$ are estimated to be about 5.2 MeV and 3.7 MeV, respectively.

Recently, there has been strong evidence that charmed mesons have indeed been found.[1,2] Several candidates can be well interpreted as charmed baryons.[3,4] They have narrow widths and their masses fit into the ranges predicted by the De Rújula-Georgi-Glashow (DGG) model.[5] Because of their importance in the kinematics of productions and decays, the electromagnetic mass differences of charmed hadrons have drawn considerable attention. Lane and Weinberg[6] and Fritzsch[7] independently used Dashen's theorem to relate the Coulomb part of $K^+ - K^0$ to $\pi^+ - \pi^0$ mass difference and obtained $D^+ - D^0 \sim 7$ MeV, which is substantially lower than the value of 15 MeV estimated by De Rújula, Georgi, and Glashow using a simple quark model.[8] In addition to the basic problem of relating partial conservation of axial-vector current (PCAC) to the quark model and the question of the applicability of the nonrelativistic quark model to the pion, these approaches do not seem to give a consistent picture of electromagnetic mass splittings (EMS) when applied to the low-lying hadrons in general.

It is the purpose of this paper to present a complete and consistent calculation of hadronic mass splittings in which pion masses are not used as the determining factors. We shall show that the differences of the intrinsic constituent-quark masses m_i together with a spin-spin interaction term $(\vec{s}_i/m_i)\cdot(\vec{s}_j/m_j)$, as suggested by the DGG model, is sufficient to give an extremely good description of the S-wave spectrum. While this quark mass dependence of the spin-spin interaction term is known to be essential for the understanding of the strong spectrum, the fact that it also contributes an important factor to the nonphoton part of the EMS has never been recognized in any previous calculation. We shall show that this additional nonphoton contribution and the magnetic interaction are absolutely essential for a complete and consistent picture of the hadronic EMS. Our prediction on the EMS of charmed hadrons should therefore be more reliable than any previous predictions.

We assume that hadrons are supermultiplets of SU(8) and singlets in color. The latter symmetry is exact. In the following we shall adopt the SU(8) tensor notations to facilitate a convenient presentation, but otherwise our approach is completely equivalent to the conventional quark model. The quark q and antiquark $\bar{q}$ bound states Φ belong to the $\underline{63}\oplus\underline{1}$ representation which decomposes into $\underline{15}+\underline{1}$ of 0^-, ϕ, and $\underline{15}$ of 1^-, V (Ref. 9):

$$\Phi_\alpha^\beta = \frac{1}{\sqrt{2}}(\phi_A^B \delta_a^b + \vec{V}_A^B \cdot \vec{\sigma}_a^b),$$

where $\alpha=(A,a)$, $\beta=(B,b)$, $A,B=1,\ldots,4$, and $a,b=1,2$ are SU(8), SU(4), and SU(2) spin indices, respectively. The low-lying baryons are bound states of three quarks, belonging to the $\underline{120}$-dimensional representation of SU(8), completely antisymmetric in their color indices (suppressed) and symmetric under the SU(8) indices. It decomposes under SU(4)$\otimes$SU(2) into $\underline{20}$ of $\frac{3}{2}^+$, χ and $\underline{20'}$ of $\frac{1}{2}^+$, Ψ:

$$B_{\{\alpha\beta\gamma\}} = \chi_{\{ABC\},\{abc\}} + \tfrac{1}{3}\left[(\sigma_2)_{ab}\Psi_{[AB]C,c} + (\sigma_2)_{bc}\Psi_{[BC]A,a} + (\sigma_2)_{ca}\Psi_{[CA]B,b}\right],$$

where $\{\ \}$ indicates symmetrization and $[\ ]$ antisymmetrization, and Ψ is constrained by $\Psi_{[AB]C} + \Psi_{[BC]A} + \Psi_{[CA]B} = 0$. Ψ is further decomposed into SU(3) multiplets according to

$$\Psi_{[AB]C} = \begin{cases} \dfrac{1}{\sqrt{2}}\epsilon_{ABD}\psi_C^{(8)D}, & A,B,C\neq 4 \\[2mm] \dfrac{1}{\sqrt{3}}\epsilon_{ABD}\psi^{(\bar{3})D}, & A,B\neq 4,\ C=4 \\[2mm] \dfrac{1}{\sqrt{2}}\psi_{\{BC\}}^{(6)} - \dfrac{1}{2\sqrt{3}}\epsilon_{BCD}\psi^{(\bar{3})D}, & B,C\neq 4,\ A=4 \\[2mm] \dfrac{1}{\sqrt{2}}\psi_A^{(3)}, & A\neq 4,\ B=C=4, \end{cases}$$

where the superscript (N) denotes the representation of SU(3).

In terms of the diagonal quark mass matrix

$$\mathfrak{M} = \begin{bmatrix} m_u & & & \\ & m_d & & \\ & & m_s & \\ & & & m_c \end{bmatrix},$$

charge matrix

$$Q = \frac{1}{3}\begin{bmatrix} 2 & & & \\ & -1 & & \\ & & -1 & \\ & & & 2 \end{bmatrix},$$

and spin matrix $\vec{\sigma}$, the masses of hadrons can be satisfactorily described by the matrix elements of the mass operators in the following form:

$$\langle 63|M|63\rangle = \mu_M \operatorname{Tr}\Phi^2 + 2\operatorname{Tr}\Phi^2\mathfrak{M} - S_M m_u^2 \operatorname{Tr}\Phi\mathfrak{M}^{-1}\vec{\sigma}\cdot\Phi\mathfrak{M}^{-1}\vec{\sigma} + \alpha[C_M \operatorname{Tr}\Phi Q\Phi Q + D_M \operatorname{Tr}\Phi Q\mathfrak{M}^{-1}\vec{\sigma}\cdot\Phi Q\mathfrak{M}^{-1}\vec{\sigma}]$$
$$+X[\text{terms of the form } \operatorname{Tr}\Phi(\ )\operatorname{Tr}\Phi(\ )], \qquad (1)$$

$$\langle 120|M|120\rangle = \mu_B \bar{B}B + 3\bar{B}\mathfrak{M}_1 B + 3S_B m_u^2 \bar{B}(\mathfrak{M}^{-1}\vec{\sigma})_1\cdot(\mathfrak{M}^{-1}\vec{\sigma})_2 B$$
$$+3\alpha[C_B \bar{B}Q_1Q_2B - D_B m_u^2 \bar{B}(Q\mathfrak{M}^{-1}\vec{\sigma})_1\cdot(Q\mathfrak{M}^{-1}\vec{\sigma})_2 B], \qquad (2)$$

where the subscript n of a matrix denotes that the matrix multiplication is taken with respect to the nth index of the baryon tensor ($\bar{B}O_1B = \bar{B}^{\alpha\beta\gamma}O_{\alpha}^{\alpha'}B_{\alpha'\beta\gamma}$). The SU(8)-symmetric terms, μ_M and μ_B, are expected to be small. The second terms represent the constituent quark masses. The SU(4) symmetry of the strong two-body spin-spin interaction is broken by the inequality of the intrinsic quark masses in the same manner as the magnetic dipole interaction. The photon contributions to the elctromagnetic mass differences consist of the Coulomb parts C_M, C_B and the magnetic parts D_M, D_B. Finally, the quark-antiquark annihilation effects are summarized in X[terms of the form $\operatorname{Tr}\Phi(\ )\operatorname{Tr}\Phi(\ )$] relevant only for the $S=0$ and $Q=0$ meson states. Because of the lack of understanding of the annihilation channels we shall avoid any use of these terms in this paper.

Ignoring the electromagnetic mass differences we can express the ordinary hadron masses in terms of six parameters[10]:

$$\begin{aligned} m &= m_u = m_d = (\Lambda - N)(\Sigma^* - \Sigma)/(\Delta - N - \Sigma^* + \Sigma) \\ &= 335.7 \text{ MeV}, \\ m_s &= m(\Delta - N)/(\Sigma^* - \Sigma) = 512.5 \text{ MeV}, \\ S_B &= \tfrac{1}{6}(\Delta - N) = 48.8 \text{ MeV}, \\ \mu_B &= \tfrac{1}{2}(\Delta + N) - 3m = 78.2 \text{ MeV}, \\ S_M &= \tfrac{1}{4}(K^* - K)m_s/m = 151.9 \text{ MeV}, \\ \mu_M &= \tfrac{1}{4}(3K^* + K) - m - m_s = -53.7 \text{ MeV}. \end{aligned} \qquad (3)$$

The charmed-quark mass $m_c = 1674$ MeV needed for the charmed-hadron masses is obtained from the equation $D - K = m_c + \frac{3}{4}(K^* - K)(1 - m_s/m_c)$. Our value of quark mass m agrees excellently with the value determined from the proton magnetic moment ($p/\mu_p = 335.9$ MeV). The ratio $m/m_s = 0.655$ is also consistent with the estimates from the Λ magnetic moment ($3\mu_\Lambda m/p = 0.72 \pm 0.07$), SU(3) σ model [$f_\pi/(2f_K - f_\pi) = 0.64$],[11] and the radiative meson decay rates (~0.7).[11,12] The equivalence of the constituent-quark masses in mesons and baryons implies the mass relation

$$K^* - \rho = \Lambda - N - \tfrac{1}{4}(K^* - K)\left(\frac{\Delta - N}{\Sigma^* - \Sigma} - 1\right). \qquad (4)$$

The ρ mass determined by this equation is 770 MeV.

The hadron masses calculated from this set of parameters are listed in Table I. The agreement with the known values is about 10 MeV, except for π where the relativistic effect is expected to be important. The masses of the three possible candidates of charmed baryons C_0, C_1, and C_1^* are within the range of the predicted values.[13,14] The mixing of S and A is small as expected. Based on these comparisons we estimate the uncertainty of the predicted charmed-hadron masses to be 20–30 MeV.

If the electromagnetic part of the mass matrices (1) shares the same success as the strong part, the electromagnetic mass differences can be predicted to within a fraction of 1 MeV. The eighteen charmed and uncharmed baryon EMS can be expressed in terms of three unknown parameters $m_d - m_u$, C_B, and D_B. There are fifteen mass relations from which other baryon EMS can be calculated from $p - n$, $\Sigma^+ - \Sigma^0$, and $\Sigma^0 - \Sigma^-$ (Ref. 15):

$$\Delta^{++}+\Delta^{0}-2\Delta^{+}=\Delta^{+}+\Delta^{-}-2\Delta^{0}=\Sigma^{*+}+\Sigma^{*-}-2\Sigma^{*0}=C_1^{++}+C_1^{0}-2C_1^{+}=C_1^{*++}+C_1^{*0}-2C_1^{*+}$$
$$=\Sigma^{+}+\Sigma^{-}-2\Sigma^{0}=1.78\ \text{MeV}=\alpha(C_B-D_B)\,, \tag{5}$$

$$C_1^{*+}-C_1^{*0}-(C_1^{+}-C_1^{0})=\tfrac{1}{2}(\chi_u^{*++}-\chi_d^{*+})-\tfrac{1}{2}(\chi_u^{++}-\chi_d^{+})=(S^{*+}-S^{*0})-(S^{+}-S^{0})$$
$$=\Xi^{*0}-\Xi^{*-}-\Xi^{0}+\Xi^{-}-\tfrac{3}{2}(S^{+}-S^{0})+\tfrac{3}{2}(A^{+}-A^{0})=-\frac{3m}{2m+m_c}(C_1^{+}-C_1^{0}-p+n)$$
$$=3\,\frac{m}{m_c}\left[S_B\left(\frac{m_d-m_u}{m}\right)-\tfrac{2}{3}\,\alpha D_B\right], \tag{6}$$

$$C_1^{++}-C_1^{0}-(\chi_u^{++}-\chi_d^{+})=\Delta^{+}-\Delta^{0}=(m_u-m_d)\left(1-2\frac{S_B}{m}\right)+\tfrac{1}{3}(\Sigma^{+}+\Sigma^{-}-2\Sigma^{0})$$
$$=\Sigma^{*+}-\Sigma^{*-}-(\Xi^{*0}-\Xi^{*-})=\Sigma^{+}-\Sigma^{-}-(\Xi^{0}-\Xi^{-})=p-n$$
$$(-0.8\pm1.2)\ \text{MeV}=\ (-1.6\pm0.6)\ \text{MeV}\ =-1.29\ \text{MeV}\,, \tag{7}$$

$$\frac{3m}{m_s-m}(C_1^{+}-C_1^{0}-S^{+}+S^{0})=\Sigma^{*0}-\Sigma^{*-}-(\Sigma^{0}-\Sigma^{-})=\tfrac{1}{2}(\Xi^{*0}-\Xi^{*-}-\Xi^{0}+\Xi^{-})=-\frac{3m}{m_s+2m}(\Sigma^{+}-\Sigma^{0}-p+n)$$
$$(1.55\pm0.3)\ \text{MeV}=(1.54\pm0.07)\ \text{MeV}$$
$$=3\,\frac{m}{m_s}\left[S_B\left(\frac{m_d-m_u}{m}\right)+\tfrac{1}{3}\,\alpha D_B\right]. \tag{8}$$

Among the ordinary baryons, the Coleman-Glashow relation and a successful SU(6) relation [Eq. (7)] remain unaltered.[16] However, as a result of the modification by the ratio m/m_s, the remaining SU(6) relation [Eq. (8)] is now even better satisfied. The complete SU(8) limit can be obtained by letting $m_u=m_d=m_s=m_c$ in Eqs. (5)–(8).[17] Since the SU(8) symmetry is badly broken this limit is unlikely to be interesting. In Refs. 5 and 7 the Coulomb interaction is taken to be the only contribution to the EMS other than the quark mass differences m_d-m_u. This is equivalent to setting $S_B=D_B=0$ in Eqs. (5)–(8). Under such a condition it is clear that the last equality of Eq. (8) is not satisfied. Furthermore m_d-m_u determined from Eq. (7) would be 1.9 MeV rather than 4.5 MeV as claimed in Ref. 7, and is therefore inconsistent with their estimate 4.8 MeV from the meson EMS.

The charmed-baryon EMS are listed in Table II. Interestingly, their values seem to fall in the middle between the predictions of the SU(8) limit and those of Ref. 6. Particularly worth noting is the smallness of their magnitudes.

Equations (5)–(8) also contain relations from which the three unknown parameters can be determined. $m_d-m_u=2.68$ MeV is substantially smaller than the estimates of Refs. 6, 7, and 8. The Coulomb part $\alpha C_B=2.96$ MeV corresponds to $C_B=\langle 1/r\rangle=404$ MeV and the magnetic-dipole interaction energy $\alpha D_B=1.18$ MeV. It is gratifying that both C_B and D_B are positive as they must be in the quark model.

The calculation of the meson electromagnetic mass differences usually relies on the estimate of the photon contribution from $\pi^+-\pi^0$. In the quark model the small mass of the pion can invalidate the nonrelativistic assumption. The possibility of some contribution from the annihilation (into one photon and gluons) terms may not be discounted. Another approach is to estimate the photon contribution from the pion via Dashen's theorem $m_{K^+}{}^2-m_{K^0}{}^2=m_{\pi^+}{}^2-m_{\pi^0}{}^2$.[18] However, the electrostatic self-energy deduced from Dashen's theorem is not the Coulomb energy of the bound-state quarks. The author of Ref. 7 did not make this distinction. In addition, he had to assume that π, K, and D mesons either all satisfied PCAC or nonrelativistic dynamics. Neither approach is reasonable. To extract the Coulomb contribution the authors of Ref. 6 had to assume that K mesons behave both as relativistic PCAC particles and nonrelativistic bound states. This and the other arguments in Ref. 6 which are possible are by no means general and rigorous.

Since we have already determined m_d-m_u from the baryons we can use K^+-K^0 instead of $\pi^+-\pi^0$ to estimate the photon contribution:

$$K^+-K^0=m_u-m_d+3\left(\frac{m_u-m_d}{m_s}\right)S_M+\tfrac{1}{3}\alpha C_M$$
$$+\frac{m}{m_s}\alpha D_M$$
$$=-4.0\pm0.1\ \text{MeV} \tag{9}$$

implies

$$\alpha C_M+3\,\frac{m}{m_s}\alpha D_M=3.7\pm0.4\ \text{MeV}\,. \tag{10}$$

The positivity of the Coulomb interaction energy C_M and the magnetic interaction energy D_M imposes severe constraints on[19]

TABLE I. Predicted and observed masses (in MeV) of baryons and mesons. Input values are denoted by parentheses. Square brackets indicate that only $\Sigma^* - \Sigma$ is used for input.

	Predicted	Observed		Predicted	Observed
N	(939)	939	C_0	2277	2260 ± 10 (Ref. 14)
Σ	[1183]	1193	C_1	2433	2426 ± 12(?) (Ref. 13)
Λ	(1116)	1116	S	2600	
Ξ	1332	1318	T	2773	
			A	2505	
			χ	3725	
			χ_s	3915	
			(SA)	−6	
Δ	(1232)	1232	C_1^*	2492	2500(?) (Ref. 14)
Σ^*	[1375]	1385	S^*	2649	
Ξ^*	1524	1533	T^*	2811	
Ω	1678	1672	χ^*	3783	
			χ_s^*	3953	
			Θ	5106	
π	162	138	D	(1865)	1865 (Ref. 1)
K	(496)	496	F	2073	
ρ	770	770	D^*	1987	2000 (Ref. 2)
K^*	(894)	894	F^*	2153	

$$D^+ - D^0 = m_d - m_u + 3\left(\frac{m_d - m_u}{m_c}\right) S_M + \frac{2}{3}\alpha C_M + 2\,\frac{m}{m_c}\alpha D_M\,, \tag{11}$$

$$D^{*+} - D^{*0} = m_d - m_u - \left(\frac{m_d - m_u}{m_c}\right) S_M + \frac{2}{3}\alpha C_M - \frac{2}{3}\,\frac{m}{m_c}\alpha D_M\,, \tag{12}$$

such that 4.2 MeV $< D^+ - D^0 <$ 6.2 MeV and 1 MeV $< D^+ - D^0 - (D^{*+} - D^{*0}) <$ 2 MeV. The values $D^{*+} - D^{*0} = D^+ - D^0 = 15$ MeV estimated by De Rújula, Georgi, and Glashow are much higher than our upper limit.[8]

As suggested in Ref. 6 a rough estimate of C_M and D_M can be obtained from Schnitzer's ground-state wave function of a linear potential $V(r) = ar$ (Ref. 20):

$$C_M = \langle 1/r\rangle = \left(\frac{32\mu a}{3\pi^2}\right)^{1/3} = 350\text{–}400\ \text{MeV}\,, \tag{13}$$

$$D_M = \frac{2\pi}{3m^2}|\psi(0)|^2 = \frac{\mu a}{3m^2} = 122\text{–}185\ \text{MeV}\,, \tag{14}$$

where μ is the reduced mass and a is estimated to be 0.2–0.3 GeV^2. These values yield $K^0 - K^+ = 4.2$–4.6 MeV, $D^+ - D^0 = 5.8$–6.5 MeV, and $D^{*+} - D^{*0} = 4.1$–4.2 MeV in reasonably good agreement with our estimates. It further suggest that $C_M/D_M = C_B/D_B = 2.52$ may be used in conjunction with Eq. (10)

TABLE II. Predicted electromagnetic mass differences (in MeV) of charmed particles from the present model, the SU(8) model of Itoh *et al.* (Ref. 15), and the model of Lane and Weinberg (Ref. 6).

	Present work	SU(8)	Lane and Weinberg
$C_1^{++} - C_1^+$	1.1	4.1	−2
$C_1^+ - C_1^0$	−0.7	2.3	−4
$S^+ - S^0$	−1.0	2.3	−4
$A^+ - A^0$	−3.2	−2.5	−4
$\chi_u^{++} - \chi_d^+$	1.6	7.7	
$C_1^{*++} - C_1^{*+}$	0.8	0.5	
$C_1^{*+} - C_1^{*0}$	−1.0	−1.3	
$S^{*+} - S^{*0}$	−1.2	−1.3	
$\chi_u^{*++} - \chi_d^{*+}$	1.1	0.5	
$D^+ - D^0$	5.2	13.2	6.7
$D^{*+} - D^{*0}$	3.7	9.0	6.7

to determine C_M and D_M. With $\alpha C_M = 2.1$ MeV and $\alpha D_M = 0.83$ MeV we obtain $D^+ - D^0 = 5.2$ MeV and $D^{*+} - D^{*0} = 3.7$ MeV in comparison with 6.7 MeV, 8 MeV, and 15 MeV from Refs. 6, 7, and 8, respectively. The mass of D^+ is reported to be 1876 ± 15 MeV (Ref. 2) and D^0 1865 MeV (Ref. 1). Hopefully, future experiments will reduce the uncertainty of D^+.

Equations (13) and (14) can also be used to estimate the effect of various reduced masses on C_M and D_M. The net result would increase $D^+ - D^0$ and $D^{*+} - D^{*0}$ by less than 0.3 MeV since the C_M term only contributes 1.4 MeV and the D_M term is greatly suppressed by the charmed-quark mass. Therefore the error introduced by assuming universal C_M and D_M is insignificant.

$K^{*0} - K^{*+} = 1.34 \pm 0.10$ MeV is found to be too small to compare with the value 4.1 ± 0.6 MeV given by the Particle Data Group.[21] The discrepancy in the K^* mass splitting is also encountered in the DGG model.[8] We adopt their interpretation that the experimental determination of the K^* mass difference is in error.

In conclusion we have formulated a broken-SU(8) quark model by incorporating the basic idea that the internal-symmetry-breaking effects originate entirely from the inequalities of the constituent quark masses. The two-body spin-spin interaction $(\vec{s}_i/m_i) \cdot (\vec{s}_j/m_j)$ is shown to be sufficient to give a good description of the hadron spectrum. Our main contribution is to recognize the fact that when $m_u \neq m_d$ this spin-spin interaction term also gives an essential contribution to the nonphoton part of the electromagnetic mass differences. This mechanism has not been properly taken into account in all previous calculations. We have analyzed consistently the complete low-lying hadronic spectrum and electromagnetic mass differences from the mass matrices (1) and (2). The overall agreement with the observed values and the consistency with other independent estimates on various parameters give credence to our predictions on the charmed-particle masses and mass differences. If charmed hadrons are in fact as abundant as expected[3] these predictions can very well be tested in the near future.

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