PHYSICAL REVIEW D VOLUME 15, NUMBER 9 1 MAY 1977

Causality and the proton-neutron mass difference

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(Received 8 September 1976)

The Cottingham formula for the electromagnetic proton-neutron mass difference is simplified by imposing certain causality conditions. The Born contribution to the mass difference is calculated using the modified formula and is found to be insignificant. The divergent part and the deep-inelastic contribution are discussed.

I. INTRODUCTION

The problem of the calculation of the proton-neutron electromagnetic mass difference is a classical problem in particle physics, one that has frustrated many efforts over the past one or two decades. These efforts were mostly based on the Cottingham formula[1] for the self-energy of a hadron in first-order electromagnetic interaction. Major stumbling blocks in these attempts include ultraviolet divergence, unknown subtraction constants and fixed-pole parameters in Compton amplitudes, and the wrong sign of the substantial Born contribution. To remedy these, or some of them, and to make headway in computation, several authors made *ad hoc* assumptions[2] that lacked clear physical justification. However, on the fundamental level most of these difficulties (which might well be related) remain unresolved and it is not even clear that the electromagnetic interaction is alone responsible for most of the proton-neutron mass difference. It is therefore gratifying and significant in such a situation to observe that some general principle could be brought to bear on the problem. We discuss in this paper some restrictions that causality imposes on the Cottingham formula for the proton-neutron mass difference and explore their consequences.

The relevance of causality considerations to the problem of electromagnetic mass differences has previously been noted by other authors.[3] Our work is based on the observation that certain causality conditions, first noted by Meyer and Suura,[4,5] can be used to considerably simplify the original Cottingham formula. In particular we find that of the two invariant Compton amplitudes only one survives in the causality-modified mass-difference formula. Fortunately this turns out to be the well-behaved and experimentally better-known amplitude t_2. The modified formula may be employed to reestimate the Born and the high-energy contributions.

In Sec. II we obtain the causality-modified Cottingham formula by imposing the causality conditions on the original formula. The proof that the invariant amplitudes t_1 and t_2 are causal is given in the Appendix. In this section we also point out that the conditions required for the validity of the causality sum rules that we use are the same as those of Refs. 4 and 5.

In Sec. III we employ the modified formula to estimate the Born contribution. The result is that the Born contribution is of the wrong sign but is insignificantly small; its magnitude is 5% of the observed mass difference.

The divergent part of the formula is discussed in Sec. IV under the assumption of Bjorken scaling in electroproduction. It is found that, for large q^2, only the logarithmic divergence survives. Its elimination may be effected by imposing the sum rule

$$\int_0^1 [F_2^p(\omega) - F_2^n(\omega)]d\omega = 0. \tag{1.1}$$

The compatibility of this requirement with experiment is discussed. Two other alternatives for the elimination of the logarithmic divergence, namely a cutoff and mathematical regularization, are also considered. A full summary of the results of the paper is given in Sec. V.

II. COTTINGHAM'S FORMULA AND CAUSALITY

To lowest order in the electromagnetic interaction the proton-neutron mass difference, $\Delta m \equiv m_p - m_n$, is given by Cottingham's formula[1]

$$\Delta m = \frac{i}{4\pi} \int \frac{M(p,q)}{q^2 + i\epsilon} d^4q, \tag{2.1}$$

where $M = g^{\mu\nu}M_{\mu\nu}$, $M_{\mu\nu}$ being the covariant gauge-invariant Compton-scattering amplitude

$$M_{\mu\nu} = t_1(\nu, q^2) L_{\mu\nu}^{(1)}(p,q) + t_2(\nu, q^2) L_{\mu\nu}^{(2)}(p,q), \tag{2.2}$$

with

$$L_{\mu\nu}^{(1)}(p,q) = q_\mu q_\nu - q^2 g_{\mu\nu}, \tag{2.3}$$

$$L_{\mu\nu}^{(2)}(p,q) = \nu(p_\mu q_\nu + q_\mu p_\nu) - q^2 p_\mu p_\nu - \nu^2 g_{\mu\nu}. \tag{2.4}$$

We are taking $p^2 = 1$ and $\nu = pq$. Thus Cottingham's formula reads

$$\Delta m = \frac{1}{4\pi i}\int \frac{1}{q^2+i\epsilon}[3q^2t_1(\nu,q^2)+(q^2+2\nu^2)t_2(\nu,q^2)]d^4q. \tag{2.5}$$

It is shown in the Appendix that the amplitudes $t_i(\nu,q^2)$ satisfy the causal Jost-Lehmann-Dyson (JLD)[6] representation

$$t_i(\nu,q^2)=\frac{1}{2\pi}\int\frac{\psi_i(u,s)d^4u\,ds}{s-(q-u)^2-i\epsilon},\quad i=1,2, \tag{2.6}$$

where $\psi_i(u,s)$ are the JLD spectral functions in the causal representations of the structure functions $V_i(\nu,q^2)$ occurring in the decomposition of the commutator of the electromagnetic currents [see Eqs. (A2), (A9), and (A11)]. From the representation (2.6) and condition (A8) of the Appendix one obtains the causality sum rules

$$\int_{-\infty}^{\infty} t_i(\nu,q^2)dq_0=0,\quad i=1,2. \tag{2.7}$$

Thus under the conditions allowing the derivation of these sum rules, Cottingham's formula (2.5) reduces to

$$\Delta m=\frac{1}{2\pi i}\int\frac{\nu^2t_2(\nu,q^2)}{q^2+i\epsilon}\,d^4q. \tag{2.8}$$

After performing the rotation in the integration contour in Eq. (2.8) and effecting the angular integration the formula reads

$$\Delta m=2\int_0^\infty\frac{dq^2}{q^2}\int_0^{(q^2)^{1/2}}d\nu\,\nu^2(q^2-\nu^2)^{1/2}\times t_2(i\nu,-q^2). \tag{2.9}$$

This demonstrates that, under certain conditions, the use of causality in Cottingham's formula eliminates all dependence upon the amplitude $t_1(\nu,q^2)$. These conditions, given by Eq. (A8) in the Appendix, are the same as those required for the derivation of causality sum rules for the electromagnetic structure functions,[4] namely

$$\int V_i(\nu,q^2)dq_0=0,\qquad \int q_0V_i(\nu,q^2)dq_0=c_i, \tag{2.10}$$

where c_i are constant. Further, (2.10) are equivalent[7] to the causality sum rules of Leutwyler and Stern[5] based on canonical light-cone behavior of the electromagnetic currents. The agreement with experiment found in Ref. 5 indicates that the underlying assumptions on asymptotic behavior are physically sound.

We also remark that the causal representation of the amplitude M was previously employed by Cottingham and Gibb[3] in discussing the proton-neutron mass difference. These authors also use the retarded product in the mass-difference formula. However, they do not note the consequences of the causality of the invariant amplitudes t_i.

In the following section we calculate the Born contribution Δm_B to the mass difference employing the formula (2.9).

III. THE BORN CONTRIBUTION

We write

$$t_2^B=\frac{4}{\pi}\frac{q^2f_2(q^2)}{4\nu^2-q^4}. \tag{3.1}$$

This gives

$$\Delta m_B=\frac{-4}{\pi}\int_0^\infty dq^2\int_0^{(q^2)^{1/2}}d\nu\frac{\nu^2(q^2-\nu^2)^{1/2}f_2(-q^2)}{q^4+4\nu^2}, \tag{3.2}$$

leading to

$$\Delta m_B=-\tfrac{1}{8}\int_0^\infty q^2\{2+q^2-[q^2(q^2+4)]^{1/2}\}\times f_2(-q^2)dq^2. \tag{3.3}$$

The form factor $f_2(q^2)$ is related to the electric and magnetic form factors G_E and G_M by

$$f_2(q^2)=\frac{e^2}{(2\pi)^2}\frac{q^2G_M{}^2(q^2)+4G_E{}^2(q^2)}{q^2(q^2+4)}. \tag{3.4}$$

From (3.4) one observes that the integrand in Eq. (3.3) possesses a pole at $q^2=4$, unless

$$G_E(-4)=G_M(-4), \tag{3.5}$$

a condition which is clearly violated by the experimental data.[8] Thus the contribution of the neighborhood of the pole at $q^2=4$ to Δm_B requires careful handling, since it is sensitive to small variations in the slope of the integrand at $q^2=4$.

We attempted a numerical evaluation of Δm_B using the experimental fits of Blatnik and Zovko,[9] taking $G_E^n\equiv 0$. With $M_p=1$, the contributions beyond $q^2=4.7$ for the proton, and $q^2=0.75$ for the neutron, were negligible. In the case of the proton the integrand in (3.3) starts with -2 at $q^2=0$, vanishes about $q^2=0.6$, reaches a local maximum of 0.04 at $q^2=1.2$, followed by a broad minimum of value 1.5×10^{-2} at $q^2=2.8$ preceding the pole at $q^2=4$. The neutron integrand is a small positive hump with a maximum value of 6.5×10^{-2} at $q^2=0.2$. The numerical integration is carried to $q^2=3.9$ and beyond $q^2=4.1$ leaving the interval $3.9\leqslant q^2\leqslant 4.1$ as the neighborhood of the pole. The contribution of this neighborhood depends upon the slope of the residue of the integrand at $q^2=4$. From the fits it appears that the residue is approximately a linear function over the pole neighborhood with a slope of -5×10^{-3}.

The numerical estimate that we obtain for the integral in (3.3) is

$$\int_0^\infty q^2\{2+q^2-[q^2(q^2+4)]^{1/2}\}\,[f_2^p(-q^2)-f_2^n(-q^2)]dq^2 \simeq -0.21, \quad (3.6)$$

leading to

$$\Delta m_B \simeq 0.06\ \text{MeV}. \quad (3.7)$$

Although this estimate of the Born contribution to the mass difference still maintains the conventiona' wrong sign, it is significantly different from the value of about 1 MeV typically obtained for Δm_B using the original Cottingham formula. The result that we obtain implies that the Born term gives an insignificant contribution to the mass difference.

IV. THE DIVERGENT PART

To discuss the asymptotic behavior of the integrand in Eq. (2.9) with the purpose of detecting possible divergence, we write an unsubtracted fixed-q^2 dispersion relation for $t_2(\nu, q^2)$, assuming the usual Regge behavior for this amplitude. In this respect it is fortunate that the use of causality enabled us to reduce the formula (2.5) to (2.8) eliminating the invariant amplitude t_1, since inclusion of this amplitude would have introduced possible subtraction constants.

We thus write

$$t_2(\nu, q^2)=\frac{\mu\omega}{\pi q^4}\int_{-1}^{1}\frac{F_2(\omega', q^2)}{\omega-\omega'}d\omega', \quad (4.1)$$

where $\mu=e^2/(2\pi)^2$, $\omega=-q^2/2\nu$, and $F_2(\omega, q^2)=\nu W_2(\nu, q^2)$. Equation (2.9) then gives

$$\Delta m=\frac{2\mu}{\pi}\int_0^\infty\frac{dq^2}{q^4}\int_0^{(q^2)^{1/2}}d\nu\,\nu^2(q^2-\nu^2)^{1/2} \times\int_{-1}^{1}d\omega'\frac{F_2(\omega', -q^2)}{q^2-2i\nu\omega'}. \quad (4.2)$$

When the integral over ν is performed, one gets

$$\Delta m=\frac{\mu}{2}\int_0^\infty dq^2\int_{-1}^{1}d\omega\frac{F_2(\omega, -q^2)}{[(q^2+4\omega^2)^{1/2}+(q^2)^{1/2}]^2}. \quad (4.3)$$

If one now assumes Bjorken scaling for the deep-inelastic structure functions νW_2^p, νW_2^n

$$F_2(\omega, -q^2)\sim F_2(\omega), \quad (4.4)$$

it is evident that Δm is logarithmically divergent unless

$$\int_{-1}^{1}[F_2^p(\omega)-F_2^n(\omega)]d\omega=0. \quad (4.5)$$

This sum rule is therefore a necessary consequence of scaling and the considerations leading to Eq. (4.3). Experimental estimates of the integrals in this sum rule are given in the literature. Gilman[10] quotes

$$\int_{0.1}^{1}F_2^p(\omega)d\omega\simeq 0.14\pm0.02, \qquad \int_{0.1}^{1}F_2^n(\omega)d\omega\simeq 0.10\pm0.02. \quad (4.6)$$

The experimental errors for the omitted region $0\leq\omega\leq0.1$ are in fact much larger and the estimate (4.6) does not exclude the sum rule (4.5). Altarelli[11] quotes

$$\int_0^1 F_2^p(\omega)d\omega=0.17\pm0.01, \qquad \int F_2^n(\omega)d\omega=0.11\pm0.02, \quad (4.7)$$

which are in disagreement with (4.5). However, we observe that the proton data[11] indicate an error in the integral which is of order 25%. We therefore believe that realistic estimates of the errors would render these values 0.17 ± 0.04 and 0.11 ± 0.03, so that (4.5) remains an experimental possibility. Bodek *et al.*[12] extract values for the difference $F_2^p(\omega)-F_2^n(\omega)$ from the deuterium data using an impulse approximation and the assumption $R_p=R_n=0.18$. Their results give a positive definite value to the integral in (4.5). The assumptions that go into their numerical analysis, however, leave considerable room for differences with their calculations. Broadhurst *et al.*[13] also quote values close to those in (4.7), namely 0.16 ± 0.02, 0.12 ± 0.02, which just admit the sum rule (4.5).

To estimate the inelastic contribution to the mass difference, assuming the validity of the sum rule (4.5), the divergent part of the integrand in (4.3) may now be subtracted. This gives

$$\Delta m_{\text{inel}}=\frac{\mu}{2}\int_{-1}^{1}\kappa(\lambda, \omega)F_2(\omega)d\omega, \quad (4.8)$$

where

$$\kappa(\lambda, \omega)=\int_\lambda^\infty\left[\frac{1}{(q^2+4\omega^2)^{1/2}+(q^2)^{1/2}}-\frac{1}{4q^2}\right]dq^2, \quad (4.9)$$

and λ is the value of q^2 at which Bjorken scaling sets in. Integration of (4.9) yields

$$\kappa(\lambda,\omega)=\frac{1}{8}(2\ln 2-1)$$

$$-\frac{1}{16\omega^4}\{\lambda^2+4\omega^2\lambda-(\lambda+2\omega^2)[\lambda(\lambda+4\omega^2)]^{1/2}\}$$

$$-\frac{1}{4}\ln\left[\left(1+\frac{4\omega^2}{\lambda}\right)^{1/2}+2\frac{\omega^2}{\lambda}+1\right]. \qquad (4.10)$$

However, we note that any constant term in $\kappa(\lambda,\omega)$ does not contribute to the mass difference since the sum rule (4.5) holds. This enables us to write the effective function in (4.8) as

$$-\frac{1}{16\omega^2}\{\lambda^2+4\omega^2\lambda-(\lambda+2\omega^2)[\lambda(\lambda+4\omega^2)]^{1/2}\} -\tfrac{1}{4}\ln[(\lambda^2+4\omega^2\lambda)^{1/2}+2\omega^2+\lambda], \qquad (4.11)$$

which, in particular, removes the divergence at $\lambda=0$. If we now take $\lambda=0$, the inelastic contribution to the mass difference becomes

$$\Delta m_{\text{inel}}=-\frac{\mu}{2}\int_0^1 F_2(\omega)\ln\omega\, d\omega. \qquad (4.12)$$

Since $F_2^p(0)=F_2(0)$, $F_2(\omega)$ vanishes at $\omega=0$, and this integral is convergent.

In using Eq. (4.12) one must be careful since it is closely linked to the sum rule (4.5). In particular, any parameterization for $F_2(\omega)$ that does not satisfy (4.5) should not be inserted into (4.12).

Although we have argued that the present data does not exclude the sum rule (4.5), we have found it difficult to parameterize the existing data so that this sum rule is exactly satisfied. We are not therefore able to give a numerical estimate for Δm_{inel} from (4.12) using the available experimental results. It appears that the validity of the sum rule (4.5) requires that $F_2^p(\omega)-F_2^n(\omega)$, which is[14] apparently positive for $\omega>0.2$ becomes sharply negative for $\omega\lesssim 0.2$ before it increases to zero at $\omega=0$. If such behavior is not observed, the sum rule (4.5) must be discarded.

When the sum rule (4.5) is definitely precluded by experiment, two alternatives suggest themselves:

(i) A cutoff in the integral over q^2, imposed by other interactions, which removes the logarithmic divergence and the necessity for the sum rule. Then (4.3) yields

$$\Delta m_{\text{inel}}=\frac{\mu}{4}\ln\Lambda\int_0^1 F_2(\omega)d\omega, \qquad (4.13)$$

where the cutoff Λ is undetermined.

(ii) The integral over q^2 in (4.3) may be regularized in the sense of generalized functions[15] so that it is rendered finite by analytic continuation. In the context of the present problem, this procedure was first proposed by Suzuki.[16] In our case it yields

$$\text{reg}\int_0^\infty \frac{dq^2}{[(q^2+4\omega^2)^{1/2}+(q^2)^{1/2}]^2}=-\frac{1}{8}, \qquad (4.14)$$

using[15]

$$\text{reg}\int_0^\infty x^\alpha dx=0,\quad \forall\alpha. \qquad (4.15)$$

Condition (4.5) is then unnecessary and (4.3) gives

$$\Delta m_{\text{inel}}=\frac{-\mu}{8}\int_0^1 F_2(\omega)d\omega. \qquad (4.16)$$

It is evident from Eqs. (4.12), (4.13), and (4.16), that these alternatives lead to quite different results, on which we give further remarks in our next section.

V. SUMMARY AND DISCUSSION

We have shown that use of the causality of the electromagnetic commutator reduces the Cottingham formula for the proton-neutron mass difference to the form given in (2.8). In this form Δm is independent of the invariant amplitude t_1. This feature is welcome, since it is this amplitude which requires a subtraction in its dispersive representation and on which experimental information is scant. In fact, several authors[2,17] have made *ad hoc* assumptions neglecting it in order to arrive at definite conclusions. These conclusions do not necessarily agree with ours, since the part of M that, by causality, drops out of our calculation is $3t_1+t_2$ and not just t_1.

Using our formula (2.8), we estimated the contribution of the Born term to the mass difference. We found this contribution to be insignificant: about 5% in magnitude, but of the wrong sign, a conclusion markedly different from the conventional calculation in which causality is neglected. The idea is that causal amplitudes satisfy sum rules like (2.7) in which the Born contributions are exactly balanced by the non-Born contributions. This feature indicates that the expectation of a satisfactory result from the Born term by itself is rather unfounded.

It is known that when Bjorken scaling is used in the original mass-difference formula one has, in general, both a quadratic and a logarithmic divergence. On the other hand, in the causality-modified formula (2.8) only the logarithmic divergence survives, so that one has eliminated the quadratic divergence without introducing extraneous assumptions.

Elimination of the logarithmic divergence may be effected by imposing the sum rule (4.5). This sum rule is not excluded by the existing experimental data, although it is difficult to reach a definite conclusion. However, we note that its validity requires that $F_2^p(\omega) - F_2^n(\omega)$, which is presumably positive over most of its range,[14] becomes sharply negative in the small-ω region.

If one assumes the validity of the sum rule (4.5), the contribution $\Delta m_{\rm inel}$ of the deep-inelastic region to the mass difference may be estimated as in (4.12). As noted above, (4.5) demands that $F_2^p(\omega) - F_2^n(\omega)$ vanishes for at least one value of ω. If this occurs at just one point, $\omega = \omega_0$ say, it is easy to see that (4.12) gives a *negative* contribution. For condition (4.5) and the existing data would then imply that

$$\int_0^1 F_2(\omega)\ln\omega\, d\omega = \int_0^{\omega_0} F_2(\omega)\ln\frac{\omega}{\omega_0}d\omega + \int_{\omega_0}^1 F_2(\omega)\ln\frac{\omega}{\omega_0}d\omega > 0. \tag{5.1}$$

The magnitude of this contribution can only be estimated when one has a reasonable parameterization of the data that exactly satisfies (4.5).

A physically motivated cutoff in the integral over q^2 in (4.3) is an alternative if (4.5) is rejected by experiment. The deep-inelastic contribution, given by (4.13), is then clearly positive. Other, largely *ad hoc* assumptions[2,17] have previously produced a similar result with a negative sign. Our considerations, however, clearly lead to the positive value in (4.13). A positive result for the deep-inelastic contribution has also been obtained by Leutwyler and Gasser[18] who say that a possible tadpole contribution may reverse the sign. To us it appears that a large positive contribution like (4.13) is undesirable, and we tend to regard this alternative an unlikely possibility.

If (4.5) is experimentally invalidated, another alternative is regularization of the divergent integral by analytic continuation.[16] In contrast to the cutoff prescription, regularization invariably introduces a reversal of sign.[16] For the case under consideration this leads to an inelastic contribution, (4.16), with the correct sign. Its magnitude, however, which is insignificantly small as given by (4.16), depends on the energy at which Bjorken scaling sets in. We have taken $\lambda = 0$, thus obtaining (4.16). But λ may be adjusted to a value, albeit unreasonably high, such that $\Delta m_{\rm inel}$ reproduces the observed mass difference. This arbitrariness and the lack of a natural parameter for analytic continuation render regularization an unattractive alternative.

ACKNOWLEDGMENT

We would like to thank Dr. Irshadulla Khan for discussions on regularization and generalized functions.

APPENDIX: CAUSALITY OF THE AMPLITUDES

We define the physical Compton amplitude $M_{\mu\nu}$ as the covariant gauge-invariant part of the retarded product $R_{\mu\nu}$:

$$R_{\mu\nu} = i\int e^{iqx}\theta(x_0)\langle p|[J_\mu(x), J_\nu(0)]|p\rangle d^4x. \tag{A1}$$

This part may be explicitly exhibited using

$$\int e^{iqx}\langle p|[J_\mu(x),J_\nu(0)]|p\rangle d^4x = V_1(\nu,q^2)L_{\mu\nu}^{(1)}(p,q) + V_2(\nu,q^2)L_{\mu\nu}^{(2)}(p,q) \tag{A2}$$

and

$$\theta(x_0) = \frac{1}{2\pi i}\int\frac{e^{i\lambda x_0}}{\lambda - i\epsilon}d\lambda. \tag{A3}$$

One obtains[19]

$$\begin{aligned} R_{\mu\nu} = {} & t_1(\nu,q^2)L_{\mu\nu}^{(1)}(p,q) \\ & + t_2(\nu,q^2)L_{\mu\nu}^{(2)}(p,q) + S_1(g_{\mu 0}g_{\nu 0} - g_{\mu\nu}) \\ & + S_2[p_0(p_\mu g_{\nu 0} + p_\nu g_{\mu 0}) - p_\mu p_\nu - p_0{}^2 g_{\mu\nu}], \end{aligned} \tag{A4}$$

where

$$t_i(\nu,q^2) = \frac{1}{2\pi}\int\frac{V_i(q_0', q)}{q_0' - q_0 - i\epsilon}dq_0' \tag{A5}$$

and

$$S_i = \frac{1}{2\pi}\int q_0 V_i(\nu,q^2)dq_0. \tag{A6}$$

In obtaining (A4) use was made of the causality conditions[4]

$$\int V_i(\nu,q^2)dq_0 = 0, \tag{A7}$$

valid under the assumption

$$\lim_{s\to\infty} s\psi_i(u,s) = 0, \tag{A8}$$

where $\psi_i(u,s)$ are the JLD spectral functions of $V_i(\nu,q^2)$

$$V_i(\nu,q^2) = \int\epsilon(q_0 - u_0)\delta[(q-u)^2 - s]\,\psi_i(u,s)d^4u\,ds. \tag{A9}$$

Causality also implies that S_i, defined by (A6), are constant. Thus

$$M_{\mu\nu} = t_1(\nu,q^2)L_{\mu\nu}^{(1)}(p,q) + t_2(\nu,q^2)L_{\mu\nu}^{(2)}(p,q), \tag{A10}$$

and Eqs. (A5) and (A9) yield

$$t_i(\nu,q^2) = \frac{1}{2\pi}\int\frac{\psi_i(u,s)}{s - (q-u)^2 - i\epsilon}d^4u\,ds, \tag{A11}$$

so that $t_i(\nu,q^2)$ are causal.

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