"Radiation" and "vacuum polarization" near a black hole*

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A previously reported two-dimensional model calculation indicates that the outward flux from a collapsing body does not become infinite at the horizon. The contrary interpretation rests on a physically irrelevant distinction between "radiation" (or "particles") and "static vacuum polarization." Under our interpretation, the Hawking thermal radiation originates mostly *outside* the collapsing body.

Given the classical background metric of a body collapsing to form a black hole, quantum field theory predicts a thermal radiation flux at infinity at late times.[1] Speculation and controversy abound concerning the physical situation near the body and near the horizon. At least three schools of thought have emerged.

According to the first, the Hawking radiation must originate in the collapsing matter. This agrees with one's intuition that the static geometry outside the body should not create particles just because the metric is time-dependent elsewhere. Moreover, this picture seems, at least at first sight, to be supported by explicit calculations.[2-4]

The objection to this first view is that it requires the emission of quanta of larger and larger energy (as measured, say, in a frame comoving with the surface of the body) in order to compensate the red-shift experienced by the radiation in passing to infinity. In other words, the steady flux seen for an infinite time by an asymptotic observer can be traced back to a finite interval of proper time along a surface enclosing the body and crossing the horizon at a finite affine distance from the body; consequently, the energy flux and density must become infinite as the horizon is approached along such a surface. But there is nothing peculiar about the local geometry at the horizon; the horizon is distinguished only by its global relationship to a singularity in the future. Therefore, it is not plausible that the local physics becomes singular there.

It has been argued that the singularity on the horizon (if there is one) may be due to the inadequate physics of the model; if the reaction of the created radiation on the metric via the gravitational field equations were taken into account, the infinity (and perhaps the horizon itself) might disappear. This remark, however, would not resolve the evident mathematical paradox of singular behavior in a system governed by a smooth differential equation (the matter field equation, with coefficients determined by the metric) and a smooth initial condition (space initially empty of matter).

Those rejecting the picture just described divide into two camps. The opinion of Hawking[5] is that a precise description of the physics in a small region of high curvature is impossible, because wave frequencies are well defined only over nearly flat space-time regions of some finite extent. At best, one can make approximate, operational, highly observer-dependent statements. In this context it is argued that an observer crossing the horizon near the body would "see" very little radiation (cf. Refs. 12 and 13).

The other point of view is that, although one cannot define *particles* in regions of strong curvature, it should still be possible to give a detailed, observer-independent physical interpretation of the quantum field theory there. In particular, such local functions of the field as the energy-momentum tensor, $T_{\mu\nu}(x)$, should have meaning as quantum-theoretical observables.

The latter approach has been pursued with success for two-dimensional scalar field models,[6,7] and progress is being made in four dimensions. The principal problem is to extract the finite, physically meaningful energy-momentum tensor (or other observable) from the divergent formal expression for that quantity. In two dimensions this can be done relatively unambiguously, and the theory yields a vacuum expectation value, $\langle T_{\mu\nu}(x)\rangle$, of gratifying physical plausibility. In particular, $\langle T_{\mu\nu}\rangle$, for a model collapsing black hole (Ref. 6), has the following properties: (1) The Hawking flux at infinity at late times is reproduced. (2) The tensor is covariantly conserved: $\langle \nabla_\mu T^\mu_\nu\rangle = 0$. (3) The tensor is a finite, smooth function of space and time near the horizon.

Here we wish to elaborate on the third property, since the calculational results of Refs. 6, 4, and 3 can be misconstrued to support the conclusion that an infinite flux of radiation emerges from the collapsing body.

The main conclusion of Ref. 6 (see also Ref. 13) is that the vacuum stress outside a two-dimensional collapsing body of "mass" M at late times is

$$\langle T_{\mu\nu}\rangle = T^S_{\mu\nu} + (24\pi)^{-1} w_\mu w_\nu \,, \tag{1}$$

where $T^S_{\mu\nu}$ is the value which $\langle T_{\mu\nu}\rangle$ would take outside a static body of mass M (and hence radius greater than $2M$), and w_μ is an outward-directed null vector satisfying

$$\nabla_\mu w^\mu = 0, \quad w^\mu \nabla_\mu w_\nu = 0 \,. \tag{2}$$

Each term of Eq. (1) separately satisfies the covariant conservation law, $\nabla_\mu T^\mu_\nu = 0$.

$T^S_{\mu\nu}$ is expressed in the usual Schwarzschild coordinates (t, r) in Eqs. (5) of Ref. 6. [See also Eq. (8) of the present paper.] It is a function only of r, with $T^S_{tr} = 0$. *Except for an overall negative sign*, it could be the stress tensor of a cloud of classical matter in its rest frame. In the Schwarzschild null coordinates

$$u = t - r^*, \quad v = t + r^*, \\ r^* = r + 2M \ln(r/2M - 1) \,, \tag{3}$$

one has

$$w_v = 0, \quad w_u = (32M^2)^{-1/2} \,; \tag{4}$$

the gist of the conservation conditions (2) is that w_u is independent of v. Thus the term $T^R_{\mu\nu} = (24\pi)^{-1} w_\mu w_\nu$ classically might represent massless radiation coming from inside the body.

It is tempting to interpret $T^R_{\mu\nu}$ as the energy-momentum tensor of the Hawking radiation caused by collapse, and $T^S_{\mu\nu}$ as a vacuum "polarization" associated with the static curvature of space, which would be present even if the body had never collapsed, but had maintained a constant radius (necessarily smaller than the r coordinate of the point where $\langle T_{\mu\nu}\rangle$ is being evaluated). We argue that this language is *not* appropriate for a description of the local physics of the model, especially near the horizon.

First, note that if the body initially had a large radius (compared to $2M$) and a low density, then at early times $\langle T_{\mu\nu}\rangle$ would have been rather small everywhere in space. Points with coordinates $r \simeq 2M$ would have been inside the body then, where the Schwarzschild form of $T^S_{\mu\nu}$ does not apply. From this point of view the large "vacuum polarization" near the black hole at late times is "caused" by the collapse of the body just as much as the Hawking flux is.

Second, each term in Eq. (1) becomes infinite as the point x approaches the horizon, *but the sum is finite*. (These statements refer to the components of the tensors with respect to a suitable local frame.) One way to see this is to transform to the Kruskal coordinate frame; one finds (Ref. 13) that

$$\langle T_{UU}\rangle = (768\pi M^2)^{-1} V^2 r^{-2} e^{-r/M} \left(\frac{3M^2}{r^2} + \frac{M}{r} + \frac{1}{4}\right), \\ \langle T_{VV}\rangle = (6\pi)^{-1} M^2 V^{-2} \left(\frac{3M^2}{2r^4} - \frac{M}{r^3}\right), \\ \langle T_{UV}\rangle = -(12\pi)^{-1} M^2 r^{-4} e^{-r/2M} \,, \tag{5}$$

where

$$ds^2 = 2Mr^{-1} e^{-r/2M}\, dU\, dV \,, \\ UV = -16M^2 (r/2M - 1) e^{r/2M} \,. \tag{6}$$

The (future) horizon is the line $U = 0$. (Although $\langle T_{VV}\rangle$ appears to blow up on the past horizon, $V = 0$, the formula is irrelevant there since that line would be inside the body.)

We shall give in more detail an alternative, more instructive analysis of the terms in $\langle T_{\mu\nu}\rangle$. Consider the components of these tensors relative to an orthonormal tetrad aligned with the Schwarzschild coordinate system. Let

$$z = r - 2M \,. \tag{7}$$

We shall ignore in this discussion the trace component, $\langle T_{uv}\rangle = T^S_{uv} \propto R g_{uv}$, since its tensor transformation properties are trivial and it is clearly well behaved on the horizon. Also, to save writing we drop an overall factor of $(192\pi)^{-1}$. Using a caret to remind us of all these conventions, we obtain from

$$T^S_{uu} = T^S_{vv} = (24\pi)^{-1} \left(\frac{3M^2}{2r^4} - \frac{M}{r^3}\right) \tag{8}$$

(as given in Ref. 6) that the traceless part of the static vacuum polarization is

$$\hat{T}^S_{tt} = f(z) < 0, \quad \hat{T}^S_{tr} = 0 \,, \tag{9}$$

where $f(z) \simeq -\frac{1}{4} M z^{-3}$ as $z \to \infty$ and

$$f(z) \simeq -\frac{1}{Mz} - \frac{1}{2M^2} + \frac{3z}{2M^3} + \cdots \text{ as } z \to 0 \,. \tag{10}$$

The "radiation" tensor is

$$\hat{T}^R_{tt} = -\hat{T}^R_{tr} = \frac{1}{2Mz} + \frac{1}{4M^2} \tag{11}$$

and hence dominates at large z. The sum of contributions (9) and (11) blows up at the horizon ($z = 0$), but only because the Schwarzschild coordinate system becomes singular there. The Killing vector determining the time axis of our tetrad becomes null on the horizon. If an observer near the horizon moves nearly at the speed of light, he naturally sees whatever matter is there as extremely blue-shifted. To reach a sensible result we must apply a local Lorentz transformation to the tetrad in a z-dependent way. Such a transformation is most simply prescribed on the components in the null directions: We let

$$\hat{T}_{u'u'} = (Az)^{-1}\hat{T}_{uu}, \quad \hat{T}_{v'v'} = Az\,\hat{T}_{vv}, \tag{12}$$

where the primes indicate that the basis vectors are not aligned with the original coordinate axes. In this new frame we have

$$\left.\begin{matrix}\hat{T}^S_{t't'} \\ -\hat{T}^S_{t'r'}\end{matrix}\right\} = -\frac{1}{2MAz^2} - \frac{1}{4M^2Az} + \frac{3}{4M^3A} \mp \frac{A}{2M} + O(z)\,, \tag{13}$$

$$\hat{T}^R_{t't'} = -\hat{T}^R_{t'r'} = \frac{1}{2MAz^2} + \frac{1}{4M^2Az}\,. \tag{14}$$

The singularities of both terms at $z=0$ have become worse than before; one may say that the observer sees the putative outgoing radiation blueshifted because he is plunging into it, and, similarly, that he is now in motion relative to the vacuum polarization. However, these singularities cancel in Eq. (1):

$$\left.\begin{matrix}\langle\hat{T}_{t't'}\rangle \\ -\langle\hat{T}_{t'r'}\rangle\end{matrix}\right\} = \frac{3}{4M^3A} \mp \frac{A}{2M} + O(z)\,. \tag{15}$$

Therefore, the total stress tensor is well behaved at the horizon, as anticipated from the smoothness of the dynamical problem which should define it. This is not to deny that the tensor (15) has unusual properties from a classical point of view. Since $|\langle T_{t't'}\rangle| < |\langle T_{t'r'}\rangle|$, there is no "rest frame" where $\langle T_{t'r'}\rangle = 0$; instead, one can choose A so that $\langle T_{t't'}\rangle = 0$. It should be kept in mind that $\langle T_{\mu\nu}\rangle$ does not represent an actual flow of matter, but rather gives probabilistic information about the outcomes of certain idealized experiments.

One can write

$$\langle\hat{T}_{\mu\nu}\rangle = \hat{T}^{\text{out}}_{\mu\nu} + \hat{T}^{\text{in}}_{\mu\nu} + O(z)\,, \tag{16}$$

where

$$\hat{T}^{\text{out}}_{t't'} = -\hat{T}^{\text{out}}_{t'r'} > 0 \quad (\hat{T}^{\text{out}}_{uu} > 0, \quad \hat{T}^{\text{out}}_{vv} = 0) \tag{17}$$

and

$$\hat{T}^{\text{in}}_{t't'} = +\hat{T}^{\text{in}}_{t'r'} < 0 \quad (\hat{T}^{\text{in}}_{uu} = 0, \quad \hat{T}^{\text{in}}_{vv} < 0)\,. \tag{18}$$

These parts can be picturesquely regarded as an outward beam of ordinary radiation and an inward beam of negative-energy radiation, respectively. The outward beam is of finite strength, but at infinity (i.e., late on $\mathscr{I}^+$) it becomes the Hawking radiation. The inward beam, integrated over the horizon outside the body, yields a flux of negative energy into the black hole. In the scenario where the body is originally so rarefied that $\langle T_{\mu\nu}\rangle$ can be neglected for all practical purposes at early times, one can say that the two beams are "created" later in the region outside the body. One might even speak of the creation of particles in pairs.[8] Although these expressions are only metaphorical, they contain more truth than the alternative description in which all of the Hawking radiation is traced back to the body. More prosaically put, Eq. (16) is a more useful decomposition than Eq. (1) if z is small.

Taking the late-time limit to obtain Eq. (1) involved neglecting terms which can make a finite, nonvanishing addition to $\hat{T}_{u'u'}$. This term depends on the details of the collapse model and can fairly be interpreted as radiation produced directly by the time-dependent geometry in the collapsing body.

The third argument is that the tendency to regard the decomposition in Eq. (1) as having particular physical significance is most likely based on a misconception that $T^S_{\mu\nu}$ is intrinsically associated with the local curvature of the Schwarzschild space-time, while the other term represents "real matter" which just happens to be moving against that background. On the contrary, the traceless part of $T^S_{\mu\nu}$ is not a local function of the geometry alone, but depends on the quantum state chosen. The tensor given in Eqs. (8)–(10) is inseparably linked with the use of the Schwarzschild timelike Killing vector to define "positive frequency" and hence a vacuum state. In the case of a static body (with Schwarzschild geometry on the exterior only) this procedure provides a smooth initial condition to fix the state of the quantum field. However, when a past horizon exists (as in the full Schwarzschild-Kruskal manifold), this initial condition[9] becomes singular because the Killing vector is null on the horizon. If we call this vacuum state Ψ, then $\langle\Psi|\,T_{\mu\nu}|\Psi\rangle$ equals $T^S_{\mu\nu}$ in the two-dimensional model. Such a state represents a physically implausible initial configuration of the field system. It is precisely analogous to a state constructed in a piece of flat space-time with a Lorentz-transformation generator as the basic timelike Killing vector.[10] A natural geometrical definition of "vacuum" initial conditions on the past horizon, unprejudiced by reference to the coordinate system which is singular there, reproduces the conventional vacuum in flat space, while in Kruskal space it yields a state which is nonsingular away from the past horizon and contains a Hawking flux at future null infinity.[11] Returning to the case of a collapsing body, one sees that the initial vacuum is a nonsingular state everywhere, but that if one subtracts from $\langle T_{\mu\nu}\rangle$ the expectation value of $T_{\mu\nu}$ in the singular state Ψ, as evidently intended in Ref. 3, then the remainder has a singularity at the horizon, which constitutes the alleged infinite flux of particles or radiation. If one attributes no fundamental physical significance to $\langle\Psi| T_{\mu\nu}|\Psi\rangle$, then this subtraction is pointless.

So long as $T_{\mu\nu}$ is the only observable considered, there is no way to distinguish the two terms in Eq. (1) as far as local physics is concerned. Presumably, however, there are experiments whose outcomes are not described by the energy-momentum tensor operator alone. In particular, if one can give some independent meaning to the concept of "particles" in regions of strong curvature, for instance by studying the interaction of the field with model detectors,[12] then the question of the origin of the particles making up the Hawking flux might be reopened. However, Unruh argues that such particles cannot come entirely from the body, but must mostly be produced outside it near the horizon.[13] His arguments are entirely independent of the ansatz used in Ref. 6 to define the renormalized stress tensor (1).

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