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General treatment of second-class currents in field theory*

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The general problem of introducing second-class currents into renormalizable field theory is considered. (a) A suitable definition of first- and second-class currents is given for arbitrary integer isospin. (b) The general construction of these currents from fermion and boson fields in renormalizable theories is given. Second-class currents constructed from fermion fields require the existence of two distinct isomultiplets of fermion fields with all of the same quantum numbers except isospin. (c) It is shown that these second-class currents can emerge from gauge theories of the weak interactions. (d) However, the requirement that these currents contribute to β decay places severe restrictions on the field theory of the strong interactions. Only two classes of theories allow nonzero β-decay matrix elements: those which include strongly coupled scalar or pseudoscalar fields, and gauge theories for which the weak and strong gauge groups do not commute. In the latter case the physical hadrons cannot be singlets if the gauge group is not broken. Other topics considered include second-class neutral currents, the divergences of second-class currents, mechanisms such as isospin breaking that might mimic the effects of second-class currents, the phenomenology of first- and second-class currents of arbitrary isospin, the role of time-reversal violation in the weak interactions, and the role of anomalous phases associated with P, C, and T transformations.

I. INTRODUCTION

Two recent experiments[1,2] have suggested the existence of large second-class current[3] effects in β decay. These effects (a) are much larger than would be expected from electromagnetic corrections to first-class currents, and (b) cannot be due entirely to nuclear physics complications. Therefore, if the results of Calaprice *et al.*[1] and Sugimoto *et al.*[2] are verified, it will almost certainly be necessary to introduce explicit second-class currents into the weak Lagrangian (alternate possibilities are discussed below).

The purpose of this article is to explore the issue of how and whether second-class currents can be introduced into renormalizable field theory. The following questions are considered: (a) How are second-class currents constructed? (b) Can these currents originate in gauge theories of the weak interactions? (c) What are the conditions that must be satisfied by the field theory of the strong interactions in order for these currents to contribute to β decay?

By renormalizable field theory we mean theories involving fundamental fields of spin 0, $\frac{1}{2}$, and 1 only. Spin-1 fields must be gauge bosons, and currents must be of the form that can emerge from weak-interaction gauge theories.

The results are disquieting. We find that there are only two classes of field theories which allow second-class current effects in β decay: (1) theories involving strongly interacting elementary scalar or pseudoscalar fields (these could be in addition to quark and gauge fields), and (2) gauge theories of the strong interactions under which the physical hadron states are not singlets and for which the weak and strong gauge groups do not commute. (The first restriction could be relaxed if the strong gauge symmetry is dynamically broken.)

Both types of theory are in strong conflict with the now popular standard model.[4] In the standard model the strong interactions are given entirely by an unbroken gauge theory of quarks and gluons which commutes with the weak gauge group and under which the physical hadrons are singlets. (The addition of spin-0 particles to the strong gauge theory endangers[5] asymptotic freedom and leads[6] to parity violation to order α.) Therefore, second-class currents, if really present, will drastically disrupt our picture of particle physics.

The plan and detailed summary of this paper are as follows. Section II is mainly devoted to background material. We define first- and second-class currents for arbitrary integer isospin. Included are discussions of the relation of the G-parity and charge-symmetry definitions of second-class currents, and the possibility of time-reversal violation in the weak interactions. The phenomenological implications of first- and second-class currents for arbitrary integer isospin are reviewed, and second-class neutral currents are discussed. Most of these results have appeared previously in the literature, at least implicitly. A brief review is also given of experimental results and of phenomenological models.

Section III is devoted to currents constructed from fermion fields with nonderivative couplings.[7] We show that the only possible second-class currents in this class are generalizations of models that have been proposed by Weinberg,[3] by Maiani,[8] and by Holstein and Treiman.[9] The simplest example in this class (that of Holstein and Treiman) involves the second-class charge-raising current

$$J_\mu \equiv \bar{\psi}_p^a \Gamma_\mu \psi_n^b - \bar{\psi}_p^b \Gamma_\mu \psi_n^a, \tag{1.1}$$

where ψ^a and ψ^b refer to two distinct isodoublets of fermion fields (e.g. quarks) and $\Gamma_\mu = \gamma_\mu$ or $\gamma_\mu \gamma^5$. We show that second-class fermion currents must always involve two distinct isomultiplets of fermion fields arranged in an antisymmetric fashion similar to (1.1). The next major point concerns whether currents such as (1.1) can emerge from weak-interaction gauge theories. The conclusion, illustrated by several examples, is that these currents can easily be generated in gauge theories.

Currents such as (1.1) present obvious phenomenological difficulties, such as probably leading to a doubling of the number of physical hadrons.[10] In the last part of Sec. III we show that the situation is in fact much worse: Currents generalized from (1.1) do not even contribute to β decay for a wide class of field theories of the strong interactions (including any generalization of the standard model). Basically, the issue is that the matrix element of a current such as (1.1) between states which are members of the same isomultiplet will vanish unless ψ^a and ψ^b carry all of the same quantum numbers except isospin. This in turn requires that there be mixing between the fields, which can come about either by strongly coupled spin-0 fields or by spin-1 gauge fields. In the latter case the mixing can only occur if the current is a nonsinglet under the strong gauge group; hence, the weak and strong gauge groups would not commute, and the external hadron states would have to be nonsinglets under the strong gauge group. This possibility therefore creates enormous difficulties, but it does eliminate the need for extra quarks and for spin-0 fields. The various possibilities are all discussed and illustrated in detail, as are such complications as spontaneous and dynamical symmetry breaking. The divergences of second-class currents are also discussed.

Section IV deals with currents constructed from strongly coupled elementary boson fields. Here the requirement of renormalizability implies that only currents constructed from one or two spin-0 fields need be considered (currents involving gauge fields are higher order in the weak coupling). Both vector and axial-vector second-class currents can be constructed from products of two spin-0 fields. As in the fermion case, two distinct isomultiplets are required. Second-class currents can also be constructed from the derivative of a single isomultiplet of spin-0 fields, but the contribution of these currents to β decay is small. Again, all of the currents can emerge from weak-interaction gauge theories. The matrix elements of these boson currents present little difficulty beyond the requirement that the spin-0 fields be strongly coupled to hadrons. The advantage to boson currents is that they eliminate the need to introduce extra quarks.

Section V is mainly devoted to a short discussion of alternate mechanisms that might mimic the effects of second-class currents in renormalizable theories. The principal possibilities are (1) the effect of isospin-violating corrections to first-class current matrix elements, (2) a failure of the conserved vector current (CVC) hypothesis, and (3) the effects of Higgs scalar fields. It will be argued that none of these mechanisms provide a comfortable alternative to second-class currents.

Throughout this paper we assume that the strong interactions are exactly invariant under the discrete asymmetries P, C, and T and under isospin. (The effect of a small isospin-violating component of the strong interactions is considered along with electromagnetic corrections in Sec. V.)

II. GENERAL PROPERTIES OF SECOND-CLASS CURRENTS

A. Definitions

Let $J^I_{m\mu}$ represent a vector or axial-vector current of isospin I and z component m. (We consider only the case in which I is an integer.)

For currents which have definite transformation properties under the discrete symmetries P, C, and T we have

$$\begin{aligned} PJ^I_{m\mu}(x)P^{-1} &= \eta_P J^{I\mu}_m(x_r),\\ TJ_{m\mu}(x)T^{-1} &= \eta_T J^{I\mu}_m(x_t),\\ CJ^I_{m\mu}(x)C^{-1} &= -\eta_C J^I_{m\mu}(x)^\dagger, \end{aligned} \tag{2.1}$$

where η_P, η_T, and η_C are phase factors, $x_r = (t, -\vec{x})$, $x_T = (-t, \vec{x})$, and our metric is $g_{00} = -g_{11} = -g_{22} = -g_{33} = 1$. For vector currents $\eta_P = +1$ and for axial-vector currents $\eta_P = -1$. According to the TCP theorem[11] we can always choose $\eta_P \eta_C \eta_T = +1$ for any current which couples to a spin-1 field.

Under an isospin mirror transformation (a rotation by π around the y axis), J transforms as

$$R^\dagger J^I_{m\mu} R = (-1)^{I+m} J^I_{-m\mu}, \tag{2.2}$$

where $R \equiv \exp(-i\pi I_y)$. The distinction between first- and second-class currents involves the relation between $J^I_{-m\mu}$ and $J^{I\dagger}_{m\mu}$, which also carries isospin $I_z = -m$. We will define the currents $J^+_{m\mu}$ and $J^-_{m\mu}$ (the isospin label I is suppressed for clarity) to be first or second class, respectively, according to[12]

$$R^\dagger J^{\pm}_{m\mu} R = \pm(-1)^I \eta_T^* J^{\pm\dagger}_{m\mu}. \quad (2.3)$$

Definition (2.3) agrees with that of Pais[13] but differs from that of Bég and Bernstein[14] by a factor $(-1)^{I+1}$. The reason for incorporating the η_T^* in (2.3) will become clear below. For the most interesting case of $I=1$, (2.3) becomes[15]

$$R^\dagger J^{\pm}_{m\mu} R = \mp \eta_T^* J^{\pm\dagger}_{m\mu}\,. \quad (2.4)$$

Of course, the ordinary isovector current

$$j_\mu \equiv \bar\psi_p \gamma_\mu (1+\gamma^5)\psi_n \quad (2.5)$$

is first class with $\eta_T = +1$ (ψ_p and ψ_n can represent either quark or nucleon fields).

An alternative definition of first- and second-class currents is

$$G J^{\pm}_{m\mu} G^{-1} = \mp(-1)^I \eta_P J^{\pm}_{m\mu}\,, \quad (2.6)$$

where the G-parity operator is defined as $G = CR^\dagger = R^\dagger C$. It is easy to verify that (2.3) and (2.6) are equivalent as long as the TCP theorem holds. In the special case of $I=1$, (2.6) reads

$$\begin{aligned} G V^{\pm}_{m\mu} G^{-1} &= \pm V^{\pm}_{m\mu}\,, \\ G A^{\pm}_{m\mu} G^{-1} &= \mp A^{\pm}_{m\mu}\,, \end{aligned} \quad (2.7)$$

where V and A represent vector and axial-vector currents, respectively. Equations (2.7) are Weinberg's original definition[3] of first- and second-class currents.

It is worth digressing briefly at this point to consider time-reversal violation in the weak interactions. Suppose the term in the Lagrangian representing the coupling of a current $J^{\pm}_{m\mu}$ to an intermediate-boson field W_μ is

$$\mathcal{L} = g W^\mu J^{\pm}_{m\mu}\,, \quad (2.8)$$

where g is real. Then if $\eta_T \neq 1$, the coupling in (2.8) will violate time-reversal invariance, provided of course that W^μ also couples to other currents [e.g. the ordinary lepton currents or the hadron current in (2.5)] with $\eta_T = +1$. In this case one can define a new current

$$J'^{\pm}_{m\mu} \equiv e^{i\theta_T/2} J^{\pm}_{m\mu}\,, \quad (2.9)$$

where $\exp(i\theta_T) \equiv \eta_T$, so that the time-reversal phase of J' is $\eta'_T = +1$. Thus, for example, if $\eta_T = -1$ we have $J'_\mu = iJ_\mu$. In the following sections it will often prove convenient to perform this type of phase transformation so that the time-reversal phase is $+1$. In terms of J' the Lagrangian becomes

$$\mathcal{L} = g e^{-\theta_T/2} W^\mu J'^{\pm}_{m\mu}\,, \quad (2.10)$$

so the time-reversal violation is manifested by the explicit factor of $\exp(-i\theta_T/2)$. The first- or second-class nature of $J^{\pm}_{m\mu}$ is not changed by an overall phase transformation because of the presence of η_T^* in (2.3).

B. Implications

The first- or second-class nature of a weak current defines the relation[3] between mirror pairs of processes such as $\Sigma^\pm \to \Lambda e^\pm \nu(\bar\nu)$, which for charged $\Delta S = 0$ transitions are governed by the matrix elements of $J^\dagger_{m=1}$ and $J_{m=1}$, respectively [of course, explicit phase factors such as in (2.10) must also be conjugated]. From the definition (2.3) it is apparent that the relation is

$$\langle \beta | J^{\pm}_{m\mu} | \alpha \rangle = \pm(-1)^I \eta_T^* \langle \tilde\beta | J^{\pm\dagger}_{m\mu} | \tilde\alpha \rangle\,, \quad (2.11)$$

where

$$|\tilde\alpha\rangle \equiv R^\dagger |\alpha\rangle \quad (2.12)$$

is the isospin mirror of $|\alpha\rangle$.

The tightest restriction occurs when the states α and β are members of the same isomultiplet with isospin I_α. In this case we can use the Wigner-Eckart theorem to write

$$\langle m_f s_f K_f | J^{\pm}_{m\mu} | m_i s_i K_i \rangle = \frac{\langle I_\alpha m_i I m | I_\alpha m_f \rangle}{(2I_\alpha+1)^{1/2}} \times \langle s_f K_f \| J^{\pm}_\mu \| s_i K_i \rangle, \quad (2.13)$$

where m, s, and K refer to the z components of isospin, the spin projection, and the momentum of the state, $\langle I_\alpha m_i I m | I_\alpha m_f \rangle$ is a Clebsch-Gordan coefficient, and $\langle s_f K_f \| J^{\pm}_\mu \| s_i K_i \rangle$ is the reduced matrix element. Application of (2.11) through (2.13) leads to

$$\langle s_f K_f \| J^{\pm}_\mu \| s_i K_i \rangle = \pm \eta_T^* \langle s_i K_i \| J^{\pm}_\mu \| s_f K_f \rangle^*\,, \quad (2.14)$$

which is true for any I and I_α. Equation (2.14) is equivalent to a result first written down for general isospin by Bég and Bernstein.[14]

The most interesting applications of (2.14) occur when α and β carry ordinary spin $\frac{1}{2}$ or spin 0, but arbitrary isospin. In the spin-$\frac{1}{2}$ case we can write for the reduced matrix element

$$\langle s_f K_f \| J^{\pm}_\mu \| s_i K_i \rangle = \bar u_f \left(\gamma_\mu f_1^{\pm} + i\sigma_{\mu\nu} \frac{q^\nu}{2m} f_2^{\pm} + \frac{q_\mu}{2m} f_3^{\pm} + \gamma_\mu \gamma^5 g_1^{\pm} + i\sigma_{\mu\nu}\frac{q^\nu}{2m}\gamma^5 g_2^{\pm} + \frac{q_\mu}{2m}\gamma^5 g_3^{\pm} \right) u_i\,, \quad (2.15)$$

where $q_\mu = (K_i - K_f)_\mu$. Application of (2.14) to (2.15), combined with the time-reversal transformation (2.1), yields

$$f_1^- = f_2^- = g_1^- = g_3^- = f_3^+ = g_2^+ = 0\,, \tag{2.16}$$

while the remaining form factors have the phase $(\eta_T^*)^{1/2}$. Hence, the form factors f_1, f_2, g_1, and g_3 are associated only with first-class currents, while f_3 and g_2 are associated with second-class currents. We reiterate that this holds for any I and I_α and any value for η_T. Of course, small violations of these results can be attributed to isospin breaking.

Only vector currents have nonzero matrix elements between spin-0 states in the same isomultiplet. We define the form factors $f_i^\mp$ in this case by

$$\langle K_f \| V_\mu^\pm \| K_i \rangle = f_1^\pm (K_f + K_i)_\mu + f_2^\pm (K_f - K_i)_\mu\,. \tag{2.17}$$

The combination of (2.14) and (2.1) implies

$$f_1^- = f_2^+ = 0\,. \tag{2.18}$$

An additional restriction results if the external mesons are self-conjugate (i.e., the charge conjugate of the state is a member of the same isomultiplet), as is the case for pion or η states. Then the external states will have definite G parity and the matrix element will vanish unless the current has even G parity. Comparing Eqs. (2.6) and (2.18) we find that only f_2^- is nonzero when I is even, while only f_1^+ is nonzero for I odd.

It has been emphasized recently[16,17] that second-class neutral ($m = 0$) currents cannot be easily coupled to the weak interactions. The problem in this case is that from (2.2) and (2.3)

$$J_{0\mu}^\pm = \pm \eta_T^* J_{0\mu}^{\pm\dagger}\,. \tag{2.19}$$

If $J_{0\mu}^\pm$ is to be coupled to a Hermitian intermediate boson with a real coupling constant, as in (2.8), then Hermiticity requires $\eta_T = -1$ for a neutral second-class current. Thus, if W^μ also couples to ordinary neutral lepton currents with $\eta_T = +1$, such as $\bar\psi_\nu \gamma_\mu (1+\gamma^5)\psi_\nu$, the second-class current must be accompanied by T violation.[18]

It is also interesting to note that for neutral currents, the form-factor conditions (2.16) and (2.18) can be obtained directly from the Hermiticity condition (2.19) without utilizing isospin arguments. That is, (2.16) and (2.18) remain true even in the presence of electromagnetic corrections.

C. Experiments and models

According to (2.11) the interference between first- and second-class currents (e.g. with $I = 1$ and $\eta_T = 1$) can lead to differences in the ft values for mirror pairs of decays, such as $^{12}B \to {}^{12}C e^- \bar\nu$ and $^{12}N \to {}^{12}C e^+ \nu$. Unfortunately, differences can also be generated[19,20] by meson-exchange currents, off-mass-shell effects, recoil corrections, etc. As of now there is no definitive evidence for second-class currents from the ft values.[20]

Detailed measurements of angular correlations and asymmetries are sensitive[21] tests of the existence of time-reversal-invariant second-class axial-vector currents and are relatively free of nuclear physics ambiguities. Calaprice *et al.*[1] have recently studied the positron asymmetry in the analog decay $^{19}Ne \to {}^{19}F e^+ \nu$. Sugimoto *et al.*[2] have made similar measurements for the mirror pair $^{12}B \to {}^{12}C$ and $^{12}N \to {}^{12}C$. Both experiments indicate the presence of a second-class axial-vector current term comparable in size to the weak-magnetism term. Alternatively, the second-class current could be dispensed with in each case if the weak-magnetism form factors are nearly twice as large as predicted by the conserved-vector-current (CVC) hypothesis.

On the other hand, experiments by Tribble and Garvey[22] and by Wilkinson and Alburger[23] on the mass-8 system show no sign of second-class effects.[24]

These experiments have been analyzed[1,2,25] in terms of nuclear form factors[26]; hence, the need for second-class currents is independent of the validity of the impulse approximation. It is interesting, however, to quote the values found by Holstein and Treiman[9] for the induced tensor form factor $g_2(0)$ in the impulse approximation (assuming CVC): $g_2(0) = (8 \pm 3) g_A$ for ^{19}Ne, where $g_A \equiv g_1(0) = 1.23$; $g_2(0) = (3.5 \pm 1) g_A$ for the mass-12 system; $g_2(0) = (0.5 \pm 0.8) g_A$ for the mass-8 system. Delorme and Rho[19] have emphasized that the impulse approximation is not reliable for these types of effects. Whether corrections to the impulse approximation can account for the discrepancy between the values of $g_2(0)$ obtained from different decays has not been settled.[25]

To complicate matters, Calaprice and Holstein[27] have recently reanalyzed the famous Lee, Mo, and Wu experiment[28] which tested CVC. Their analysis suggests that CVC is violated, presumably by a second-class vector current contribution to weak magnetism (which would not show up in the impulse approximation). We comment on CVC violation in Sec. V.

The experimental situation is therefore very confused, and further experiments are urgently necessary. A number of phenomenological models[29] of second-class currents have been proposed. Most are generalizations of Lipkin's meson-exchange current[30] $A_\mu = \omega_\mu \pi$. The most detailed model for particle reactions is that of Chen *et al.*[17] Another possibility involves fermion fields with derivative couplings,[31] such as

$$A_\mu = \partial^\nu(\bar{\psi}_p \sigma_{\mu\nu}\gamma^5\psi_n)\,. \tag{2.20}$$

Kubodera, Delorme, and Rho[19] have proposed a detailed model for incorporating nuclear physics effects which utilizes both mechanisms. Renormalizable models are described in the next section.

III. RENORMALIZABLE CURRENTS CONSTRUCTED FROM FERMION FIELDS

We now consider renormalizable currents constructed from elementary spin-$\frac{1}{2}$ fields. Currents constructed from spin-0 and spin-1 fields will be discussed in the next section. Section III A deals with how first- and second-class currents are constructed from the strongly interacting fields of the theory. Section III B is concerned with whether these currents can emerge from gauge theories of the weak interactions, and Sec. III C treats the constraints placed on the field theory of the strong interactions by the requirement that the matrix elements of the second-class currents be nonvanishing. The divergences of second-class currents are discussed in Sec. III D.

A. Construction of currents

If one arranges the strongly interacting spin-$\frac{1}{2}$ fields of a theory into an n-component column vector ψ, then a renormalizable spin-1 current must be of the form

$$J_\mu = \bar{\psi}\gamma_\mu(T_1 + \gamma^5 T_2)\psi\,, \tag{3.1}$$

where T_1 and T_2 are $n\times n$ matrices in the space of internal indices of the fermion fields. The assumption that the strong interactions are invariant under isospin guarantees that the fermion fields can be grouped into irreducible representations. We choose the phases of an isomultiplet of (adjoint) fields so that $\bar{\psi}^a_m$ transforms as a spherical tensor operator with isospin I_a and z component m. Hence, the conjugate fields

$$\psi^a(m) \equiv c_\psi(-1)^m \psi^a_{-m} \tag{3.2}$$

also transform as a tensor operator with $I_z = +m$, where c_ψ is 1 or i when I_a is an integer or half-integer, respectively, and

$$\psi^a_{-m} \equiv (\bar{\psi}^a_{-m}\gamma_0)^\dagger\,. \tag{3.3}$$

It is now straightforward to construct currents of definite integral isospin I from two isomultiplets $\bar{\psi}^a$ and $\bar{\psi}^b$ (which may or may not be distinct). The two possibilities are

$$O^I_{m\mu} \equiv \sum_{m_a m_b} \langle Im\,|\,I_a m_a I_b m_b\rangle c_b(-1)^{m_b}\bar{\psi}^a_{m_a}\Gamma_\mu \psi^b_{-m_b} \tag{3.4}$$

and

$$\hat{O}^I_{m\mu} \equiv \sum_{m_a m_b} \langle Im\,|\,I_b m_b I_a m_a\rangle c_a(-1)^{m_a}\bar{\psi}^b_{m_b}\Gamma_\mu \psi^a_{-m_a}\,, \tag{3.5}$$

where $\Gamma_\mu = \gamma_\mu$ or $\gamma_\mu\gamma_5$. In the case that $\bar{\psi}^a$ and $\bar{\psi}^b$ represent the same isomultiplet ($a=b$) then $O^I_{m\mu} = \hat{O}^I_{m\mu}$. Under Hermitian conjugation, O and $\hat{O}$ are taken into each other:

$$O^{I\dagger}_{m\mu} = (-1)^m \hat{O}^I_{-m\mu} \tag{3.6}$$

and similarly for $\hat{O}^\dagger$. Hence, first- and second-class currents will be linear combinations of O and $\hat{O}$. Before making the construction, let us note the P, C, and T transformation properties of O and $\hat{O}$,

$$\begin{aligned} PO^I_{m\mu}P^{-1} &= \eta^*_{Pa}\eta_{Pb}\eta_{P\Gamma}O^{I\mu}_m\,,\\ CO^I_{m\mu}C^{-1} &= -\eta^*_{Ca}\eta_{Cb}\eta_{C\Gamma}O^{I\dagger}_{m\mu}\,,\\ TO^I_{m\mu}T^{-1} &= \eta^*_{Ta}\eta_{Tb}O^{I\mu}_m\,, \end{aligned} \tag{3.7}$$

where η_{Pa}, η_{Ca}, and η_{Ta} are the intrinsic phases of the fields, $\eta_{P\Gamma} = \eta_{C\Gamma} = \pm 1$ for $\Gamma_\mu = \gamma_\mu$ or $\gamma_\mu\gamma_5$, respectively, and where the appropriate transformations of the space-time coordinates are implied. Similar equations hold for $\hat{O}$, except that $\eta^*_{Pa}\eta_{Pa}$ is replaced by $\eta_{Pa}\eta^*_{Pb}$, etc.

Define the angle θ_T by

$$e^{i\theta_T} = \eta^*_{Ta}\eta_{Tb}\,, \tag{3.8}$$

so that the currents $\exp(i\theta_T/2)O^I_{m\mu}$ and $\exp(-i\theta_T/2)\hat{O}^I_{m\mu}$ both possess the time-reversal phase $\eta_T = 1$ [Eq. (2.1)]. Finally, we define the new currents $J^{\pm I}_{m\mu}$ to be

$$J^{\pm I}_{m\mu} \equiv e^{i\theta_T/2}O^I_{m\mu} \pm e^{-i\theta_T/2}\hat{O}^I_{m\mu}\,. \tag{3.9}$$

It is easily verified that $J^\pm$ are first and second class, respectively, according to the definition (2.3). Both J^+ and J^- transform under time reversal according to (2.1) with $\eta_T = +1$; the form factors associated with their matrix elements are therefore real. (As discussed in the preceding section, time-reversal violation can be introduced into the weak interactions by multiplying $J^\pm$ by an overall complex factor. This does not alter the first- or second-class nature of the current.) In most cases, the time-reversal phases satisfy $\eta^*_{Ta}\eta_{Tb} = \pm 1$. Then,

$$J^{\pm I}_{m\mu} = O^I_{m\mu} \pm \hat{O}^I_{m\mu} \tag{3.10}$$

for $\eta^*_{Ta}\eta_{Tb} = +1$, while

$$J^{\pm I}_{m\mu} = iO^I_{m\mu} \mp i\hat{O}^I_{m\mu} \tag{3.11}$$

for $\eta^*_{Ta}\eta_{Tb} = -1$.

For those theories in which $\eta^*_{Pa}\eta_{Pb}$ is real the currents $J^{\pm I}_{m\mu}$ have definite P and C (and therefore

G parity) transformation properties. From (3.7) one can verify that $J^{\pm I}_{m\mu}$ satisfy the alternate G-parity definition of first- and second-class currents given in (2.6).

In the special case that the two isomultiplets of fields are the same ($a=b$) we have $\eta^*_{Ta}\eta_{Tb}=1$ and $O^I_{m\mu}=\hat{O}^I_{m\mu}$. Hence, from (3.10) we see that $J^-=0$ in this case. Therefore, two distinct isomultiplets of fields are required to construct a second-class current of any isospin.

Several examples are now in order. The simplest case is $I=I_a=I_b=0$, with $\eta^*_{Ta}\eta_{Tb}=+1$. Then

$$J^{\pm}_{\mu}=\bar{\psi}^a\Gamma_\mu\psi^b\pm\bar{\psi}^b\Gamma_\mu\psi^a. \tag{3.12}$$

The simplest isovector charge-raising current ($I=m=1$) is constructed from two isodoublets ($I_a=I_b=\frac{1}{2}$). Then

$$O^1_{1\mu}=-\bar{\psi}^a_p\Gamma_\mu\psi^b_n,\quad \hat{O}^1_{1\mu}=-\bar{\psi}^b_p\Gamma_\mu\psi^a_n, \tag{3.13}$$

where $\bar{\psi}^a_p$ and $\bar{\psi}^a_n$ are the $m_a=\pm\frac{1}{2}$ components of the isodoublet. If the a and b fields have the same time reversal phase,

$$J^{\pm}_{1\mu}=-(\bar{\psi}^a_p\Gamma_\mu\psi^b_n\pm\bar{\psi}^b_p\Gamma_\mu\psi^a_n), \tag{3.14}$$

which is essentially the example given recently by Holstein and Treiman.[9] On the other hand, for $\eta^*_{Ta}\eta_{Tb}=-1$ we have

$$J^{\pm}_{1\mu}=-i(\bar{\psi}^a_p\Gamma_\mu\psi^b_n\mp\bar{\psi}^b_p\Gamma_\mu\psi^a_n). \tag{3.15}$$

This example was first given by Maiani,[8] except he omitted the factor i so that J^- would violate CP (and T) invariance.[32] A final example, due to Weinberg,[3] involves $I_a=1$, $I_b=0$. We denote the isovector fields by $\bar{\psi}_\Sigma$ and the isoscalar field by $\bar{\psi}_\Lambda$. Then, for $\eta^*_{Ta}\eta_{Tb}=+1$, the currents are

$$J^{\pm}_{1\mu}=\bar{\psi}_{\Sigma^+}\Gamma_\mu\psi_\Lambda\mp\bar{\psi}_\Lambda\Gamma_\mu\psi_{\Sigma^-}. \tag{3.16}$$

B. Origin in weak gauge theories

It is straightforward to devise weak-interaction gauge theory models which generate both first- and second-class currents. A few examples, based on the SU(2) × U(1) group,[33] should illustrate the procedure.

The idea is to utilize the fact that the weak interactions violate isospin, so that the representation content of the fields under the weak SU(2) group and the strong isospin group need not be the same.

Let us first consider four examples of time-reversal-invariant weak-interaction models.

(a) Let $\bar{\psi}^a$ and $\bar{\psi}^b$ represent two isodoublets of fields with $\eta^*_{Ta}\eta_{Tb}=+1$. Let $\bar{\sigma}$, a doublet of field operators under the *weak* SU(2) group, be related to $\bar{\psi}^a$ and $\bar{\psi}^b$ by

$$\bar{\sigma}\equiv\begin{pmatrix}\bar{\sigma}_p\\ \bar{\sigma}_n\end{pmatrix}=\begin{pmatrix}\bar{\psi}^a_p\\ \bar{\psi}^a_n\cos\theta+\bar{\psi}^b_n\sin\theta\end{pmatrix}. \tag{3.17}$$

Finally, take $\bar{\psi}^b_p$ and $-\bar{\psi}^a_n\sin\theta+\bar{\psi}^b_n\cos\theta$ to be singlets under the weak SU(2) (one can restrict all of these considerations to the left-handed projections of the fields). The weak charge-raising current associated with σ is then

$$\begin{aligned}j_{+\mu}&=-\bar{\sigma}_p\Gamma_\mu\sigma_n\\&=-\bar{\psi}^a_p\Gamma_\mu\psi^a_n\cos\theta-\tfrac{1}{2}\sin\theta(\bar{\psi}^a_p\Gamma_\mu\psi^b_n+\bar{\psi}^b_p\Gamma_\mu\psi^a_n)\\&\quad-\tfrac{1}{2}\sin\theta(\bar{\psi}^a_p\Gamma_\mu\psi^b_n-\bar{\psi}^b_p\Gamma_\mu\psi^a_n),\end{aligned} \tag{3.18}$$

which is a sum of first- and second-class isovector currents. The neutral weak current is

$$\begin{aligned}j_{0\mu}&=\frac{1}{\sqrt{2}}(\bar{\sigma}_p\Gamma_\mu\sigma_p-\bar{\sigma}_n\Gamma_\mu\sigma_n)\\&=\frac{1}{\sqrt{2}}[\bar{\psi}^a_p\Gamma_\mu\psi^a_p-\bar{\psi}^a_n\Gamma_\mu\psi^a_n\cos^2\theta-\bar{\psi}^b_n\Gamma_\mu\psi^b_n\sin^2\theta\\&\quad-\sin\theta\cos\theta(\bar{\psi}^a_n\Gamma_\mu\psi^b_n+\bar{\psi}^b_n\Gamma_\mu\psi^a_n)],\end{aligned} \tag{3.19}$$

which is a linear combination of first-class $I=0$ and $I=1$ currents. The absence of second-class currents illustrates the remarks in the last section that time-reversal-invariant neutral second-class currents cannot couple to the same gauge field as first-class currents.

(b) In the second example, we let

$$\bar{\tau}\equiv\begin{pmatrix}\bar{\tau}_p\\ \bar{\tau}_n\end{pmatrix}=\begin{pmatrix}\bar{\psi}^b_p\\ -\bar{\psi}^a_n\sin\theta+\bar{\psi}^b_n\cos\theta\end{pmatrix} \tag{3.20}$$

and $\bar{\sigma}$ both transform as doublets. The weak charged current is now

$$\begin{aligned}j_{+\mu}&=-\bar{\sigma}_p\Gamma_\mu\sigma_n-\bar{\tau}_p\Gamma_\mu\tau_n\\&=-(\bar{\psi}^a_p\Gamma_\mu\psi^a_n+\bar{\psi}^b_p\Gamma_\mu\psi^b_n)\cos\theta\\&\quad-(\bar{\psi}^a_p\Gamma_\mu\psi^b_n-\bar{\psi}^b_p\Gamma_\mu\psi^a_n)\sin\theta;\end{aligned} \tag{3.21}$$

the second term is second class. The neutral current is

$$\begin{aligned}j_{0\mu}&=\frac{1}{\sqrt{2}}(\bar{\psi}^a_p\Gamma_\mu\psi^a_p-\bar{\psi}^a_n\Gamma_\mu\psi^a_n)\\&\quad+\frac{1}{\sqrt{2}}(\bar{\psi}^b_p\Gamma_\mu\psi^b_p-\bar{\psi}^b_n\Gamma_\mu\psi^b_n),\end{aligned} \tag{3.22}$$

which is a first-class isovector current.

(c) Suppose $\bar{\psi}_{\Sigma^+}$, $\bar{\psi}_{\Sigma^0}$, and $\bar{\psi}_{\Sigma^-}$ constitute an isotriplet and let $\bar{\psi}_\Lambda$ be an isosinglet. Suppose that under the weak interactions

$$\bar{\rho}=\begin{pmatrix}\bar{\rho}_+\\ \bar{\rho}_0\\ \bar{\rho}_-\end{pmatrix}=\begin{pmatrix}\bar{\psi}_{\Sigma^+}\\ \bar{\psi}_{\Sigma^0}\cos\theta+\bar{\psi}_{\Lambda}\sin\theta\\ \bar{\psi}_{\Sigma^-}\end{pmatrix} \tag{3.23}$$

transforms as a triplet, while $-\bar{\psi}_{\Sigma^0}\sin\theta+\bar{\psi}_\Lambda\cos\theta$ transforms as a weak singlet. The charged weak current is then

$$j_{+\mu}=\frac{1}{\sqrt{2}}(\bar{\rho}_+\Gamma_\mu\rho_0+\bar{\rho}_0\Gamma_\mu\rho_-)$$

$$=\frac{1}{\sqrt{2}}(\bar{\psi}_{\Sigma^+}\Gamma_\mu\psi_{\Sigma^0}+\bar{\psi}_{\Sigma^0}\Gamma_\mu\psi_{\Sigma^-})\cos\theta$$

$$+\frac{1}{\sqrt{2}}(\bar{\psi}_{\Sigma^+}\Gamma_\mu\psi_\Lambda+\bar{\psi}_\Lambda\Gamma_\mu\psi_{\Sigma^-})\sin\theta. \tag{3.24}$$

The first term is a first-class isovector and the second term is a second-class isovector [Eq. (3.16)].

(d) One can also adopt more intricate schemes, such as incorporating $\bar{\psi}_\Sigma$ and $\bar{\psi}_\Lambda$ into weak doublets. That is, let

$$\bar{\rho}=\begin{pmatrix}\bar{\psi}_{\Sigma^+}\\ \bar{\psi}_{\Sigma^0}\end{pmatrix} \tag{3.25}$$

and

$$\bar{\sigma}=\begin{pmatrix}\bar{\psi}_{\Lambda}\\ \bar{\psi}_{\Sigma^-}\end{pmatrix} \tag{3.26}$$

transform as weak doublets. The weak current generated is

$$j_{+\mu}=-\bar{\psi}_{\Sigma^+}\Gamma_\mu\psi_{\Sigma^0}-\tfrac{1}{2}(\bar{\psi}_\Lambda\Gamma_\mu\psi_{\Sigma^-}+\bar{\psi}_{\Sigma^+}\Gamma_\mu\psi_\Lambda)$$

$$-\tfrac{1}{2}(\bar{\psi}_\Lambda\Gamma_\mu\psi_{\Sigma^-}-\bar{\psi}_{\Sigma^+}\Gamma\psi_\Lambda). \tag{3.27}$$

The first term is a linear combination of $I=1$ and $I=2$ first-class currents, the second term is a second-class isovector, and the third term is a first-class isovector.

We have thus far assumed that the fields have the same intrinsic time-reversal phase. One can reconsider the examples in the case in which $\eta_{Ta}^*\eta_{Tb}=-1$. In example (a), the second and third terms in the charged current (3.18) would now be second and first class, respectively, and these terms would violate time-reversal invariance in the weak interactions because they lack a factor of i [see (3.15)]. The last term in the current (3.19) would be a time-reversal–violating second-class neutral current, which is in accord with the remarks in the preceding section.

Finally, let us consider a final example in which the strong isodoublets are arranged into weak multiplets as follows:

$$\bar{\sigma}\equiv\begin{pmatrix}\bar{\sigma}_p\\ \bar{\sigma}_n\end{pmatrix}=\begin{pmatrix}\bar{\psi}_p^a\\ \bar{\psi}_n^a\cos\theta+i\bar{\psi}_n^b\sin\theta\end{pmatrix} \tag{3.28}$$

is a weak doublet, while $\bar{\psi}_p^b$ and $i\bar{\psi}_n^a\sin\theta+\bar{\psi}_n^b\cos\theta$ are weak singlets (note the factors of i). In this case the weak charged current is

$$j_{+\mu}=-\bar{\psi}_p^a\Gamma_\mu\psi_n^a\cos\theta+i\frac{\sin\theta}{2}(\bar{\psi}_p^a\Gamma_\mu\psi_n^b+\bar{\psi}_p^b\Gamma_\mu\psi_n^a)$$

$$+i\frac{\sin\theta}{2}(\bar{\psi}_p^a\Gamma_\mu\psi_n^b-\bar{\psi}_p^b\Gamma_\mu\psi_n^a). \tag{3.29}$$

The first term is first class and even under time reversal. For $\eta_{Ta}^*\eta_{Tb}=+1$, the second and third terms are first and second class, respectively, and violate T invariance. For $\eta_{Ta}^*\eta_{Tb}=-1$, they are second and first class, respectively, and are time-reversal-invariant.

The a-b interference term in the neutral current is

$$-\frac{i}{\sqrt{2}}\sin\theta\cos\theta(\bar{\psi}_n^b\Gamma_\mu\psi_n^a-\bar{\psi}_n^a\Gamma_\mu\psi_n^b), \tag{3.30}$$

which is a linear combination of $I=0$ and $I=1$. For $\eta_{Ta}^*\eta_{Tb}=+1$ it is second class and T violating while for $\eta_{Ta}^*\eta_{Tb}=-1$ it is first class and invariant under T [cf. Eq. (2.25)].

It should be clear from these examples that by choosing representations judiciously, one generate both first- and second-class currents, of any isospin, and currents that are either invariant or not invariant under time reversal.

C. Matrix elements of currents

We have seen that it is relatively easy to construct second-class currents and that these can emerge from gauge theories of the weak interactions. As we shall now see, however, the β-decay matrix elements of these currents are zero for a wide class of field theories of the strong interactions.

Consider the matrix element $M_\mu^\pm$ of the first- or second-class currents $J_\mu^\pm$ between states α and β which are members of the same isomultiplet:

$$M_\mu^\pm\equiv\langle\beta|J_\mu^\pm|\alpha\rangle. \tag{3.31}$$

For example, $M_\mu^\pm$ might represent the matrix element of an isovector current between neutron and proton states, between ^{19}Ne and ^{19}F, or between other nuclear states in the impulse approximation. However, most of the considerations are easily generalized to other transitions that do not involve the creation of new quantum numbers, such as the hadronic matrix elements relevant to β decay (beyond the impulse approximation), to $\Sigma^\pm\to\Lambda e^\pm\nu(\bar{\nu})$, or to $\nu N \to \mu + N +$ pions. Also, the arguments are independent of whether the physical states are bound states or "elementary particle" states. Isospin indices will generally be suppressed.

The basic issue is that since the states α and β in (3.31) are members of the same isomultiplet, they must have all of the same quantum numbers (except I_z). Therefore, $M_\mu^\pm$ will be nonzero only in those theories for which $J_\mu^\pm$ carries no conserved quantum numbers other than isospin. However, second-class currents must be constructed from the bilinear combinations $\bar\psi^a\Gamma_\mu\psi^b$ and $\bar\psi^b\Gamma_\mu\psi^a$, where a and b are different.[34] Therefore, $M_\mu^\pm$ will vanish unless the fields ψ^a and ψ^b carry all of the same quantum numbers except isospin (they can, of course, have different mass).

The interaction term in a renormalizable field theory of the strong interactions involving the fields ψ^a, ψ^b, and mesons will be of the form

$$\mathcal{L}_I = :\sum_i (\bar\psi^a\Gamma_1^i\phi^i\psi^a + \bar\psi^b\Gamma_2^i\phi^i\psi^b + \bar\psi^a\Gamma_3^i\phi^i\psi^b + \bar\psi^b\Gamma_4^i\phi^i\psi^a): + \mathcal{L}_m, \qquad (3.32)$$

where the index i runs over all the spin-0 and spin-1 boson fields of the theory, the matrices Γ_j, $j = 1, \ldots, 4$ carry Dirac, isospin, and Lorentz indices, and $\mathcal{L}_m$ includes the couplings among the mesons. A necessary condition for the fields ψ^a and ψ^b to carry the same quantum numbers is that there be mixing between them; that is, it is necessary that some of the off-diagonal couplings Γ_3 and Γ_4, as well as some of the diagonal couplings, Γ_1 and Γ_2, be nonzero. If, on the other hand, there were no off-diagonal couplings, then there would be conserved quantum numbers N_a and N_b associated with the a and b fields and M_μ^- would vanish.

For example, suppose there is only one scalar field ϕ in the theory. Then (3.32) becomes

$$\mathcal{L}_I = :g_1\bar\psi^a\psi^a\phi + g_2\bar\psi^b\psi^b\phi + g_3(\bar\psi^a\psi^b + \bar\psi^b\psi^a)\phi: + \mathcal{L}_m. \qquad (3.33)$$

For $g_3 \neq 0$ there are nonzero diagrams contributing to the matrix elements of $\bar\psi^a\Gamma_\mu\psi^b$ and $\bar\psi^b\Gamma_\mu\psi^a$ between the elementary particle states associated with the a field, such as those shown in Fig. 1(a). Similarly, suppose α is a bound state of three quarks (for example, two a quarks and one b quark). Then the matrix element $\langle\alpha|\bar\psi^a\Gamma_\mu\psi^b|\alpha\rangle$ will be nonzero if $g_3 \neq 0$. A simple diagram is shown in Fig. 1(b). Similar statements apply to $\bar\psi^b\Gamma_\mu\psi^a$.

On the other hand, if there were no mixing ($g_3 = 0$) then the only nonzero matrix elements of $\bar\psi^a\Gamma_\mu\psi^b$ would be those for which the initial state has one more b quark and one fewer a quark than the final state. Hence $M_\mu^- = 0$ since states in the same isomultiplet must have the same quark content

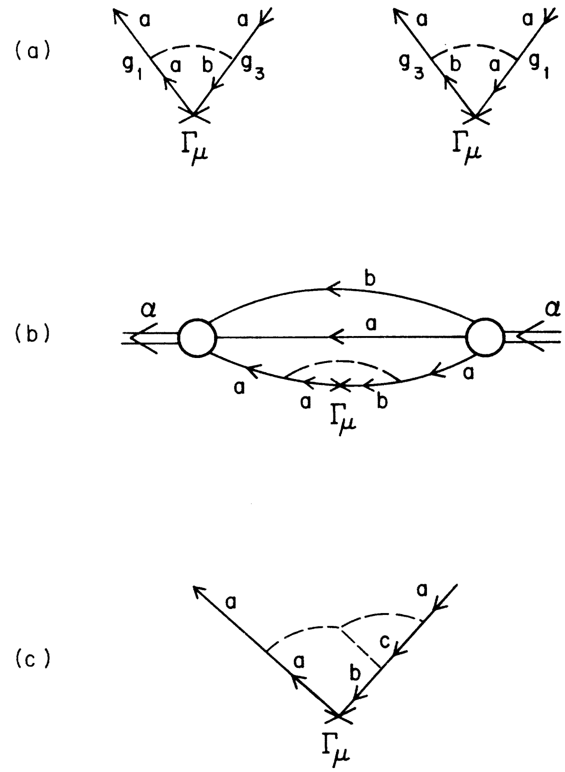

FIG. 1. (a) Diagrams contributing to $\langle a|\bar\psi^a\Gamma_\mu\psi^b|a\rangle$ and $\langle a|\bar\psi^b\Gamma_\mu\psi^a|a\rangle$. (b) A diagram contributing to $\langle\alpha|\bar\psi^a\Gamma_\mu\psi^b|\alpha\rangle$, where α is a bound state of three quarks. (c) A more complicated contribution to $\langle a|\bar\psi^a\Gamma_\mu\psi^b|a\rangle$.

(except for the quark-isospin indices).

Only strong-interaction theories possessing this kind of mixing can have $M_\mu^- \neq 0$. Of course, the form of the mixing can be more complicated than in (3.32) and (3.33). Figure 1(c), for example, illustrates a scheme in which the coupling between ψ^a and ψ^b proceeds through a third isomultiplet ψ^c.

Before proceeding, we mention that the mixing effects implied by Lagrangians such as (3.32) and (3.33) are real. The mixing can only be rotated away in the special case that the fermion mass matrix commutes with the matrices that characterize the Yukawa couplings. Furthermore, there is no reason to require that the mixing be a small perturbation. In fact, the mixing must be large in order to obtain a large value for M_μ^-.

Let us now catalog the various known renormalizable theories to determine those for which this mixing is possible. We shall attempt to be systematic and not restrict ourselves to theories which are realistic or currently popular.

(a) Yukawa theories. There is no difficulty in obtaining $M_\mu^- \neq 0$ in theories involving fundamental scalar or pseudoscalar fields. The Lagrangian in

(3.33) and Fig. 1 is a simple example (pseudoscalar fields work just as well). Of course, such theories involve other serious problems, such as not being asymptotically free.[5] Also, if ψ^a and ψ^b are quark fields, such a theory would probably lead to a doubling of the number of hadrons. (The two types of hadrons would have the same quantum numbers but different masses.)

An old-fashioned (nonquark) example in this class, which at least avoids doubling the number of hadrons, is a theory involving an octet of baryon fields which couple to an octet of pseudoscalar-meson fields. Then the matrix element

$$M_\mu^- = \langle p | \overline{\psi}_{\Sigma^+} \Gamma_\mu \psi_\Lambda + \overline{\psi}_\Lambda \Gamma_\mu \psi_{\Sigma^-} | n \rangle \qquad (3.34)$$

of the Weinberg current (3.16) between proton and neutron states is nonzero. The lowest-order diagrams are shown in Fig. 2(a). It is amusing that the two diagrams in Fig. 2(a) cancel in the SU(3) limit, as can be verified by explicit computation. In fact, one can show by SU(3) arguments that the sum of all diagrams involving the exchange of mesons across the weak vertex, as in Fig. 2(b), also vanishes in the SU(3) limit. However, diagrams such as shown in Fig. 2(c) do not cancel because of the momentum dependence of the strong vertex function.

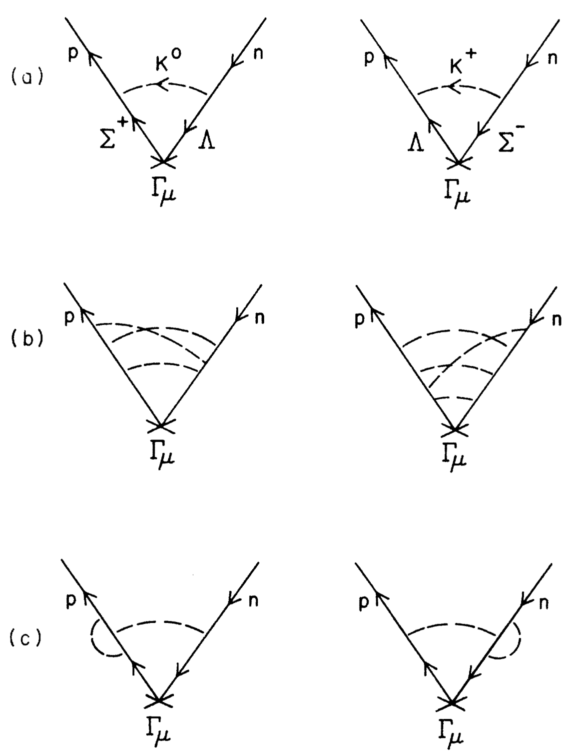

FIG. 2. (a) The lowest-order diagrams contributing to (3.34). (b) The class of diagrams which cancel in the SU(3) limit. (c) The type of diagram which survives in the SU(3) limit.

(b) Gauge theories. Suppose the strong interactions are given by a gauge theory with gauge group G_s. Also, let the weak-interaction gauge group be G_w. Three distinctions must now be made: Is G_s broken (spontaneously, or dynamically)? Do G_s and G_w commute? Does G_s commute with the strong isospin group I? We consider first the popular view that G_s commutes with both G_w and I.

1. $[G_s, G_w] = [G_s, I] = 0$. In this case the overall gauge theory G is a direct product of G_s and G_w. The weak current is therefore a singlet under G_s, and a second-class weak current involves terms such as

$$J_\mu^i \sim \sum_i \overline{\psi}^{ai} \Gamma_\mu \psi^{bi} + \cdots , \qquad (3.35)$$

where isospin indices are still suppressed and the omitted terms are those for which a and b are interchanged. The index i is the strong gauge index. The strong gauge couplings do not mix the a and b fields. Hence, $M_\mu^- = 0$ for this class of theories.

One could, however, obtain $M_\mu^- \neq 0$ in hybrid theories in which the gauge couplings are supplemented by Yukawa couplings. The simplest example would be that in which the new spin-0 field is a singlet under G_s. In this case one would add extra terms to the Lagrangian similar to those in Eq. (3.33) except that terms such as $\overline{\psi}^a \psi^b$ represent a contraction of the gauge indices. The problem with such hybrid theories is that they would probably not be asymptotically free[5] and would lead to parity violation[6] to order α. One might hope that these problems and the problem of doubling the number of hadrons might be lessened by making the Yukawa couplings very weak or by making one of the quarks very massive. This does not work, however. Explicit calculations at the one-loop level of Fig. 1(a) indicate that the Yukawa couplings would have to be of order unity and the quarks reasonably light in order to account for the observed second-class effects.[1,2]

Finally, neither spontaneous[35] (with Higgs bosons) nor dynamical[36] (without Higgs bosons) symmetry breaking appreciably alters the situation for this class of theories. Spontaneous symmetry breaking will not induce mixing between ψ^a and ψ^b unless the Higgs bosons themselves possess strong off-diagonal Yukawa couplings (which reproduces the hybrid theory above). Dynamical symmetry breaking could not lead to mixing unless the solutions to the theory possess off-diagonal couplings that were not present in the original Lagrangian. This possibility is fascinating but far outside the realm of conventional thinking.

2. $[G_s, G_w] \neq 0$ *but* $[G_s, I] = 0$. In this case, in which the weak current is a nonsinglet under G_s, one eliminates the need for extra quarks and for spin-0 fields: The a and b labels can be strong gauge group indices and the gauge fields provide the mixing. However, enormous theoretical and phenomenological difficulties are introduced. The theoretical problem is that the noncommuting nature of G_s and G_w spoils renormalizability. That is, the coupling of the weak and strong gauge fields W_μ^i and B_μ^j to fermions is

$$\mathcal{L}_I = g_w \sum_i \bar{\psi} T_w^i \not{W}^i \psi + g_s \sum_j \bar{\psi} T_s^j \not{B}^j \psi \,, \tag{3.36}$$

where the fermion fields are arranged in a column vector ψ, and the weak and strong representation matrices T_w^i and T_s^j do not commute. Hence, the weak-coupling terms are not invariant under G_s, and vice versa, and renormalizability is spoiled. It might be possible to save renormalizability if both G_s and G_w are subgroups of a larger gauge group G, but it is not clear whether G could be spontaneously broken down to an effective G_s and G_w theory.

Let us brush aside this difficulty and consider the more immediate phenomenological problems. First, suppose G_s is not spontaneously or dynamically broken. Suppressing isospin indices, a second-class current will be of the form

$$J_\mu^- = \bar{\psi}^a \Gamma_\mu \psi^b - \bar{\psi}^b \Gamma_\mu \psi^a \,, \tag{3.37}$$

where a and b are unequal G_s indices.[37] (One could also let J_μ^- be a nonsinglet under G_s with ψ^a and ψ^b in different G_s multiplets, but this only compounds the difficulties.) Then, in order that $M_\mu^- \neq 0$, we require that (a) the external hadron states be nonsinglets under G_s, and (b) J_μ^- not carry conserved quantum numbers. For example, if G_s is the SU(3) color group[38] the physical hadrons would have to be color nonsinglets. Since G_s has been assumed to be unbroken, this would imply the existence of (unobserved) classes of degenerate new hadrons carrying color quantum numbers.

Furthermore, J_μ^- could not carry conserved quantum numbers. For example, if the quarks were in a color triplet with ψ^a and ψ^b being red and white quarks, respectively, then J_μ^- in (3.37) would be a color-changing current and M_μ^- would vanish. Of course, J_μ^- would have nonzero matrix elements between degenerate hadron states with different colors, but these states would not be in the same isomultiplet. The distinction in (2.16) between first- and second-class form factors would be lost and the matrix element would not interfere with the ordinary (color-conserving) β-decay amplitude. The only way for J_μ^- to conserve color would be to put the quarks into higher-dimensional representations. For example, one could arrange the quarks in a color octet, with the Σ_c^0 and Λ_c members carrying color isospin 1 and 0, respectively, and z component 0. Then

$$J_\mu^- = \bar{\psi}^{\Sigma_c^0} \Gamma_\mu \psi^{\Lambda_c} - \bar{\psi}^{\Lambda_c} \Gamma_\mu \psi^{\Sigma_c^0} \tag{3.38}$$

would be a color nonsinglet but would not carry conserved color quantum numbers.

Now consider the case in which G_s is spontaneously or dynamically broken. Because of the symmetry breaking, the matrix element of J_μ^- in (3.38) can be nonzero even when the external states start out as G_s singlets. Hence, the problem of introducing new classes of physical hadrons is avoided. It is still necessary, however, that J_μ^- not carry conserved quantum numbers.

To illustrate this for the SU(3) color theory, let M_μ^c represent the matrix element of the color-carrying current (3.37) in the case that ψ^a and ψ^b are the red and white members of a color triplet. Also, let M_μ^N be the matrix element of the color-neutral current in (3.38). If the symmetry breaking is dynamical,[36] the gauge bosons will acquire mass and the invariance of the couplings will be broken. Hence M_μ^N will be nonzero. In order to have M_μ^c nonzero, however, it would also be necessary for the dynamical breaking to induce new effective couplings which do not conserve the color quantum numbers. For spontaneous symmetry breaking[35] (i.e., involving Higgs bosons) the Higgs vacuum expectation values can violate quantum-number conservation. To have $M_\mu^c \neq 0$, however, one must make sure that the vacuum symmetry breaking cannot be rotated into a color-neutral direction. This could be implemented by introducing two or more octets of Higgs bosons with vacuum expectation values in different directions in color space.[39]

3. $[G_s, I] \neq 0$. The last class of theories is that in which $[G_s, I] \neq 0$. In order that both I and G_s be unbroken I must then be a subset of G_s, and I is therefore a local symmetry. (If G_s is broken, I must be maintained as at least a global symmetry.) One could, if desired, identify the ρ-meson fields as the gauge bosons associated with I.

In this case the isospin-carrying weak currents are nonsinglets under G_s, so $[G_w, G_s] \neq 0$. (The associated difficulties were discussed previously.) Of course, most physical hadrons are also nonsinglets.

It is still necessary that J_μ^- be constructed from two distinct isomultiplets ψ^a and ψ^b that carry all the same quantum numbers other than isospin.

If G_s is larger than I, then ψ^a and ψ^b could be in the same multiplet with respect to the larger G_s

group, and the gauge bosons can provide the necessary mixing. For example, if G_s were a local version of SU(3) in which the ρ, ϕ, and K^* mesons are gauge bosons, then the Weinberg current in (3.34) would have $M_\mu^- \neq 0$. The discussion following (3.34) applies just as well to this case except that the mesons in Fig. 2 are now vector mesons.

If ψ^a and ψ^b are in different G_s multiplets (for example, if $G_s = I$ and ψ^a and ψ^b are both isodoublets), then the gauge fields do not mix ψ^a and ψ^b. Hence $M_\mu^- = 0$ unless additional Yukawa couplings are introduced explicitly to mix ψ^a and ψ^b.

This completes the catalog of renormalizable theories. We summarize the discussion by stating that all of the theories which allow nonzero matrix elements for second-class currents either have enormous difficulties of their own or are in violent disagreement with current theoretical ideas.

D. The divergence of second-class currents

The question of whether $\partial^\mu J_\mu^-$ vanishes is of relevance to nuclear transitions.[40] The divergence of the current in (3.9) will in general not be zero. The fermion mass terms in the Lagrangian yield nonzero divergences proportional to $m_a - m_b$ for vector currents and $m_a + m_b$ for axial-vector currents. In addition, the Yukawa or gauge couplings required to mix the fields also contribute to the divergences of these currents.

For example, consider the case of the second-class isoscalar current

$$J_\mu^- = \bar{\psi}^a \Gamma_\mu \psi^b - \bar{\psi}^b \Gamma_\mu \psi^a \tag{3.39}$$

and the strong Lagrangian given in (3.33). If $\Gamma_\mu = \gamma_\mu$ we have

$$\begin{aligned}\partial^\mu J_\mu^- = {} & i(m_a - m_b)(\bar{\psi}^a\psi^b + \bar{\psi}^b\psi^a) \\ & + 2ig_3(\bar{\psi}^a\psi^a - \bar{\psi}^b\psi^b)\phi \\ & - i(g_1 - g_2)(\bar{\psi}^a\psi^b + \bar{\psi}^b\psi^a)\phi \,.\end{aligned} \tag{3.40}$$

In the axial-vector case $\Gamma_\mu = \gamma_\mu\gamma^5$ the divergence is

$$\begin{aligned}\partial^\mu J_\mu^- = {} & i(m_a + m_b)(\bar{\psi}^a\gamma^5\psi^b - \bar{\psi}^b\gamma^5\psi^a) \\ & - i(g_1 + g_2)(\bar{\psi}^a\gamma^5\psi^b - \bar{\psi}^b\gamma^5\psi^a)\phi \,.\end{aligned} \tag{3.41}$$

In both cases we see that the same terms needed to give mixing effects (the mixing can be rotated away if the masses are equal or zero) lead to nonzero divergences. This result, which is true in general, has a simple interpretation: If the divergence of the current is zero, the current is associated with a symmetry of the strong interactions. The current then carries conserved quantum numbers, and its matrix element between states in the same isomultiplet is zero.

IV. CURRENTS CONSTRUCTED FROM STRONGLY COUPLED BOSON FIELDS

It was shown in Sec. III that the only strong-interaction theories which allow nonzero β-decay matrix elements for second-class currents constructed from fermion fields are (a) those involving strongly coupled spin-0 fields, and (b) gauge theories for which the weak and strong gauge groups do not commute. The theoretical and phenomenological difficulties of the second type of theory are severe. Quark versions of the first type of theory suffer not only from the presence of the spin-0 fields but also from the necessity of introducing new isomultiplets of quarks (and presumably of new physical hadrons). Fortunately, one can at least eliminate the need for extra quarks in this class of theories by constructing the second-class currents from the spin-0 fields.

Within the context of weak-interaction gauge theories one encounters several types of currents constructed from boson fields. (By a current we mean any combination of fields that couples to a gauge boson.) The three- and four-point couplings between the weak gauge bosons lead to currents constructed from two or three gauge fields. When spin-0 fields are present in the weak Lagrangian, their gauge-invariant kinetic energy terms lead to currents constructed from one or two spin-0 fields and from one or two spin-0 fields in combination with a gauge field. (The single meson currents occur if the spin-0 field has a nonzero vacuum expectation value.)

The matrix elements for ordinary weak processes of currents involving weak gauge fields are of higher order in the weak and electromagnetic coupling constants. Similarly, if the weak and strong gauge groups are subgroups of a larger gauge group G, there will be currents involving the new gauge fields in G. However, these new gauge fields would be expected to couple very weakly to ordinary hadrons. Hence, we will ignore all currents involving gauge fields and restrict our attention to currents constructed from one or two spin-0 fields. These fields must couple strongly to hadrons to be of relevance to β decay. We note in passing that the phenomenological current[30] $A_\mu = \omega_\mu \pi$ will not arise in any known renormalizable theory. (We assume, of course, that ω_μ is not a weak gauge boson.)

The two meson currents arising from gauge theories are of the form

$$J_\mu = \phi^\dagger i \overleftrightarrow{\partial}_\mu T \phi \,, \tag{4.1}$$

where the meson fields are arranged in a column vector ϕ, T is a matrix in this space of meson indices, and $\overleftrightarrow{\partial}_\mu \equiv \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu$. Currents of definite integer isospin are constructed in analogy with the fermion case. Let $\phi_m^{a\dagger}$ represent an isomultiplet of meson fields of isospin I_a. From two isomultiplets of fields construct the currents

$$O^I_{m\mu} \equiv i \sum_{m_a m_b} \langle Im | I_a m_a I_b m_b \rangle C_b (-1)^{m_b} \phi^{a\dagger}_{m_a} \overleftrightarrow{\partial}_\mu \phi^b_{-m_b} \tag{4.2}$$

and

$$\hat{O}^I_{m\mu} = i \sum_{m_a m_b} \langle Im | I_b m_b I_a m_a \rangle C_a (-1)^{m_a} \phi^{b\dagger}_{m_b} \overleftrightarrow{\partial}_\mu \phi^a_{-m_a} . \tag{4.3}$$

The transformation of these currents under P, C, and T, and the construction of first- and second-class currents is exactly the same as in the fermion case and will not be repeated. Equations (3.6) through (3.11) and the accompanying discussion apply equally to the present case. [In (3.7) one replaces $\eta_{P\Gamma}$ and $\eta_{C\Gamma}$ by one.]

The only additional complication occurs in the case that ϕ^a and ϕ^b are both self-conjugate (ϕ^a and $\phi^{a\dagger}$ are members of the same isomultiplet). It then follows from (4.2) and (4.3) that

$$O^I_{m\mu} = -(-1)^{I-I_a-I_b} \hat{O}^I_{m\mu} , \tag{4.4}$$

implying an obvious vanishing of some of the currents in (3.9).

When $a=b$, $\eta^*_{Ta}\eta_{Tb}=1$ and $O^I_{m\mu}=\hat{O}^I_{m\mu}$ so that $J^-_\mu=0$. That is, two distinct isomultiplets of fields are again required to form a second-class current. Finally, if $\phi^a=\phi^b$ is self-conjugate, one has

$$O^I_{m\mu} = \hat{O}^I_{m\mu} = -(-1)^{I-2I_a} O^I_{m\mu} , \tag{4.5}$$

so that $J^+_\mu=0$ if $I-2I_a$ is even.

It should also be mentioned that self-conjugate scalar fields usually have $\eta_T=+1$ while self-conjugate pseudoscalar fields usually have $\eta_T=-1$.

For example, consider a self-conjugate isovector pseudoscalar field π ($\eta_{P\pi}=\eta_{T\pi}=-1$), and a self-conjugate isoscalar field σ. From (4.4) we have the isovector currents

$$O^1_{m\mu} = -\hat{O}^1_{m\mu} . \tag{4.6}$$

If σ is pseudoscalar ($\eta_{P\sigma}=\eta_{T\sigma}=-1$), then from (3.10) the first-class current $J^+_{m\mu}$ vanishes, and

$$J^-_{m\mu} = 2O^1_{m\mu} \tag{4.7}$$

is a second-class vector current. The charge-raising ($m=1$) current is

$$\begin{aligned} J^-_{1\mu} &= i\pi^{+\dagger}\overleftrightarrow{\partial}_\mu \sigma + i\sigma \overleftrightarrow{\partial}_\mu \pi^- \\ &= 2i\pi^{+\dagger}\overleftrightarrow{\partial}_\mu \sigma . \end{aligned} \tag{4.8}$$

If σ is a scalar ($\eta_{P\sigma}=\eta_{T\sigma}=+1$) then from (3.11) $J^-_{m\mu}=0$ and the first-class axial-vector current is

$$J^+_{m\mu} = 2iO^1_{m\mu} . \tag{4.9}$$

As a second example, let $\phi^{a\dagger}$ and $\phi^{b\dagger}$ be non-self-conjugate isodoublets, with components

$$\phi^{a\dagger} = \begin{pmatrix} \phi^{a\dagger}_p \\ \phi^{a\dagger}_n \end{pmatrix} , \tag{4.10}$$

and similarly for $\phi^{b\dagger}$. For $\eta_{Pa}=\eta_{Pb}=\eta_{Ta}=\eta_{Tb}=\pm 1$, the charge-raising isovector currents

$$J^\pm_\mu = -(i\phi^{a\dagger}_p \overleftrightarrow{\partial}_\mu \phi^b_n \pm i\phi^{b\dagger}_p \overleftrightarrow{\partial}_\mu \phi^a_n) \tag{4.11}$$

are first- and second-class vector currents. If $\eta_{Pa}=\eta_{Ta}=-\eta_{Pb}=-\eta_{Tb}=1$, then the first- and second-class axial-vector currents are

$$J^\pm_\mu = \phi^{a\dagger}_p \overleftrightarrow{\partial}_\mu \phi^b_n \mp \phi^{b\dagger}_p \overleftrightarrow{\partial}_\mu \phi^a_n . \tag{4.12}$$

Currents can also be constructed from a single isomultiplet of strongly coupled spin-0 fields. Define

$$O^{I_a}_{m\mu} = i\partial_\mu \phi^{a\dagger}_m \tag{4.13}$$

and

$$\hat{O}^{I_a}_{m\mu} = -i(-1)^m \partial_\mu \phi^a_{-m} , \tag{4.14}$$

which carry isospin I_a. In the special case in which ϕ^a is self-conjugate

$$O^{I_a}_{m\mu} = -\hat{O}^{I_a}_{m\mu} . \tag{4.15}$$

The P, C, and T properties and the construction of first- and second-class currents from O and $\hat{O}$ is the same as for the fermion case. Equations (3.6) through (3.9) hold for these single-meson currents if one replaces η_{Pb}, η_{Cb}, η_{Tb}, $\eta_{P\Gamma}$, and $\eta_{C\Gamma}$ by one.

The various first- and second-class currents constructed from one and two meson fields can emerge from weak-interaction gauge theories by mixing fields in the weak multiplets in exactly the same way as for fermion currents, as discussed in Sec. III. (Single meson currents appear when one of the members of the multiplet has a nonzero vacuum expectation value.) Axial-vector currents require that scalar and pseudoscalar fields be placed in the same weak multiplet (with appropriate factors of i).

There is little difficulty in obtaining nonzero matrix elements for these meson currents. Self-conjugate fields (which carry no conserved quantum numbers other than isospin) can couple directly to ordinary quark or nucleon fields. Alternately, ϕ^a and ϕ^b may carry nontrivial but identical quantum numbers. The necessary mixing may be supplied by either meson or Yukawa couplings.

The contribution of single meson currents to β decay is actually very small (proportional to the electron mass) because from (4.13) and (4.14) it is apparent that the matrix elements of these currents are always proportional to the hadronic momentum transfer q_μ.

In conclusion, one can construct second-class currents from strongly coupled spin-0 fields without introducing extra quarks. The price one pays, however, is large: One probably loses asymptotic freedom[5] and introduces parity violation[6] to order α.

V. DISCUSSION AND ALTERNATE MECHANISMS

The experimental results of Calaprice[1] *et al.* and Sugimoto[2] *et al.* strongly suggest the presence of large second-class axial-vector current effects which are invariant under time reversal. However, the experimental situation is still confused and additional experiments are urgently needed, both in β decay and in charged-current elementary particle reactions. (Second-class neutral currents are extremely unlikely, as is discussed in Sec. II.)

We now comment on the plausible explanations of the observed second-class effects, assuming of course that they are verified by later experiments.

(1) The observed effects cannot be due entirely to nuclear physics complications, because the analyses of the experiments are independent of the impulse approximation.

(2) The next possibility is that fundamental second-class currents are present in the weak Lagrangian. It is easy to introduce second-class currents into nonrenormalizable theories. As we have seen in this article, however, second-class currents which contribute to β decay cannot be introduced into renormalizable theories without violating popular theoretical notions. Perhaps the simplest way is to introduce strongly coupled spin-0 fields into the theory and to construct the second-class currents from them. Alternatively, the second-class currents can be constructed from fermion fields. However, there are still difficulties in this case. In order to have nonzero β-decay matrix elements one must either introduce strongly coupled spin-0 fields (and, usually, extra quarks) into the theory or else consider gauge theories in which the weak and strong gauge groups do not commute. In the latter case the physical hadrons cannot be singlets under the strong gauge group unless it is dynamically broken. Other difficulties are discussed in Secs. III and IV.

(3) The third possibility is that the observed effects are due to isospin-violating corrections to the matrix elements of first-class currents. The corrections that enter at the quark or nucleon level would be expected to be of order α, while those which enter at the nuclear physics level should be of order $Z\alpha$. The reported effects,[1,2] however, are large (of order unity after kinematic factors are removed).

As a simple example, consider the first-class current

$$J_\mu = \bar{\psi}_p \Gamma_\mu \psi_n \,, \tag{5.1}$$

where ψ_p and ψ_n are either quarks or nucleons. Isospin violation (either electromagnetic or due to a small term in the strong interactions) can be introduced by letting m_p and m_n be different. The lowest-order strong vertex correction to the matrix element of J_μ between p and n states is shown in Fig. 3. The exchanged boson B can be a scalar, pseudoscalar, vector, or axial vector. One finds by explicit calculation that the diagram leads to "second-class" form factors $f_3(0)$ and $g_2(0)$ which are of order $g^2(m_p - m_n)/(m_p + m_n)$ with respect to the first-class form factors, where g is the coupling between the B field and the fermions. This result suggests that the isospin-violating corrections will be small.

It is of course possible that the $\mathcal{P}$ and $\mathcal{N}$ quarks are very light and that $|m_\mathcal{P} - m_\mathcal{N}|/(m_\mathcal{P} + m_\mathcal{N})$ is not small. (A recent estimate of the quark masses by Gasser and Leutwyler[41] has yielded this situation.) In fact, Halprin, Lee, and Sorba[42] have recently pointed out that if this is the case a naive calculation could lead to large values for $g_2(0)$ in the nucleon matrix element [enhanced by $2M/(m_\mathcal{P} + m_\mathcal{N})$ over the quark form factor, where M is the nucleon mass]. However, they argued that when one folds in the nucleon bound-state wave functions, the induced second-class form factors in the nucleon matrix element will be small. This is certainly reasonable; if this suppression did not occur one would expect large isospin violations in all physical processes. It therefore seems unlikely that isospin violation can account for the observed effects. However, more detailed estimates, especially at the nuclear physics level, are desirable.

(4) The experimental results[1,2] could be accounted for without the presence of axial-vector second-class currents if the weak-magnetism terms are approximately twice as large as predicted by CVC. (We are referring to the strong form of CVC, which is more accurately called the isovector triplet hypothesis.) In fact, a recent

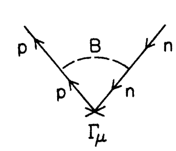

FIG. 3. The lowest-order strong vertex correction to the current in (5.1).

paper by Calaprice and Holstein[27] suggests on other grounds that CVC may be violated.

The problem is that CVC has been well tested for the first-class charge form factors in pion β decay and $0^+ \to 0^+$ nuclear β decay. Hence, violations of CVC would most likely be due to the existence of second-class vector currents,[43] which have all the same difficulties as second-class axial-vector currents.

It is of course possible that there exists a CVC-violating first-class vector current that contributes to weak magnetism, but not to charge form factors at zero momentum transfer. An example is

$$V_\mu = \partial^\nu(\bar{\psi}_p \sigma_{\mu\nu} \psi_n)\,. \qquad (5.2)$$

This current, however, cannot arise in a renormalizable theory. The first-class renormalizable currents, which are constructed in Secs. III and IV, will in general[44] contribute to charge form factors. (The contribution could vanish by a dynamical accident, but this would be unsatisfactory and unlikely.)

(5) We have so far assumed that β decay is described entirely by the exchange of a single vector boson between the leptons and the hadrons. There are, of course, other contributions. Higher-order weak processes, such as shown in Fig. 4(a), are too weak to be relevant. Another possibility involves the exchange of Higgs bosons (alone or with gauge bosons) as shown in Fig. 4(b). By Higgs bosons we simply mean scalar fields which couple both to leptons and hadrons. It is usually assumed that Higgs fields couple extremely weakly to fermions. However, one can devise models in which the Higgs couplings are sufficiently large that the diagrams in Fig. 4(b) are important. A careful investigation of these models is still in progress, but the preliminary result is that such models either fail to produce the apparent second-

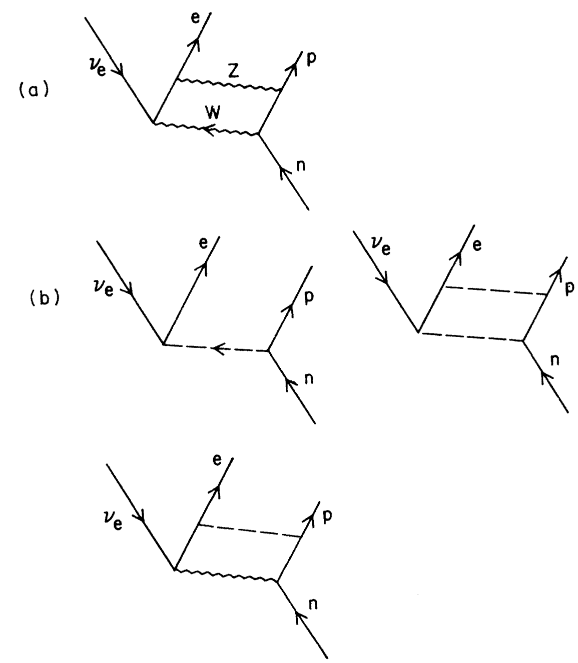

FIG. 4. (a) A higher-order weak diagram. (b) Some typical diagrams involving Higgs field exchange. Higgs fields are shown as dashed lines.

class effects or else lead to other results that conflict with experiment.

(6) One might abandon the framework of renormalizable field theory. Alternately, one might consider the possibility of nonperturbative solutions to a field theory, as was hinted at in Sec. III C.

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