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Meson scattering in quantum chromodynamics in two dimensions

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We report on some properties of meson collisions in quantum chromodynamics in two space-time dimensions to leading order in $1/N$, where N is the number of colors. Following a review of "Regge-like" power behavior of two-body, nondiffractive scattering amplitudes (quark exchange), we turn to diffractive scattering. We have calculated the high-energy behavior of the "twisted loop" or "cylinder" graph and have shown that there is no "bare" Pomeron to leading order in $1/N$.

Quantum chromodynamics (QCD), the theory of colored quarks interacting via colored gauge fields, may be the fundamential theory of strong interactions.[1] Its properties at short distances can be calculated from perturbation theory, as if the virtual quanta are almost noninteracting. Thus, it possesses the approximate scaling behaviors characteristic of certain inelastic processes, such as deep-inelastic lepton scattering and electron-position annihilation. Attempts are being made[2] to show that the theory confines quarks and gluons to form the color-singlet bound states observed[3] as hadrons in the world. If indeed it does describe hadron physics, one should be able to derive certain general features of experimental data and to relate them to concepts previously employed to explain them. One striking characteristic of high-energy collisions is the transverse-momentum damping: When hadrons collide along an axis, nearly all the particles produced lie near the axis. This observation motivated Feynman to assume that in a hadron moving with very large momentum, the transverse-momentum distribution of its virtual constituents is also sharply damped.[4] Two other prominent phenomenological properties of scattering should also be noted: (1) Two-body, nondiffractive amplitudes manifest power-law behavior at high energies, the power depending on the quantum numbers exchanged and simply related to Regge trajectories of particles which can be exchanged. (2) Elastic and diffractive amplitudes grow approximately linearly with energy, corresponding to approximately constant total cross sections.

Some time ago, 't Hooft proposed[5] a classification of the Feynman diagrams of QCD based on an expansion in the inverse of the number N of colors. He showed that, in each order, the diagrams could be put into one-to-one correspondence with the dual perturbation theory.[6] Thus, to leading order, meson-meson scattering amplitudes consist of planar graphs with no internal quark loops. Assuming this approximation is sufficient to confine quarks and gluons, then, to this order, all channels should be pole-dominated, just as in the Veneziano model.[7] If these amplitudes manifest Regge asymptotic behavior, all the usual phenomenology based on duality diagrams can be anticipated, such as exoticity criteria and exchange degeneracy.[8] Unfortunately, no progress has been made toward solving QCD in four dimensions, even in this extreme approximation.

However, in two space-time dimensions, 't Hooft showed[9] that the $1/N$ expansion may be implemented and the properties of the solution demonstrated. The two-dimensional theory is prototypical of a nontrivial field theory which is both asymptotically free and confining.[10] A number of interesting theoretical and phenomenological questions may be analyzed in this model.[11,12] Experimentally, transverse momenta are observed to be strongly damped in high-energy hadron collisions, so it may even be that certain results obtained in two dimensions may be abstracted and usefully applied to the real world. Be that as it may, the two-dimensional model poses a well-defined problem wherein the properties of high-energy scattering of bound-state mesons may be evaluated. Obviously, questions related specifically to crossing symmetry, to spin, or to large-transverse-momentum behavior cannot be approached in two dimensions.

In this paper, we take up the question of the existence of a "bare Pomeron" in meson-meson scattering to leading order in $1/N$. Our terminology reflects the dual model[7] in which a particu-

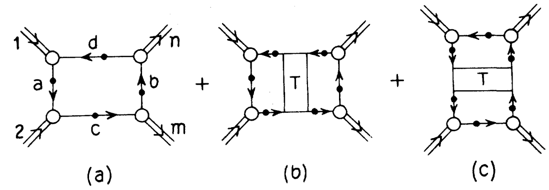

FIG. 1. The (*st*) amplitude to leading order in $1/N$ (⇛ = hadron; →→ = dressed quark).

lar topology (see Fig. 2 below) leads to the dominant contribution to elastic scattering, whose asymptotic energy dependence is not determined by the "Regge-pole" intercepts. To establish notation and terminology, we begin with a review of the Regge-like behavior resulting from quark exchange. We then turn to the discussion of the prototypical Pomeron contribution.

Let us consider the Feynman diagrams corresponding to meson-meson scattering. To leading order in $1/N$, the scattering amplitude consists of planar graphs with no internal quark loops. In any gauge in which there is no self-coupling of the gluon field, the scattering amplitude may be depicted as in Fig. 1, where we have drawn one amplitude[13] for $1+2\rightarrow n+m$. In the figure, T denotes the quark-antiquark scattering amplitude, which has been explicitly calculated in the light-cone gauge, $A_-=0$.[11,12] To be general, we have supposed all mesons have different flavors and have labeled the quark lines by their flavors a,b,c,d. For example, meson 1 is composed of quark a and antiquark $\bar{d}$. Its wave function is $\phi_1^{a\bar{d}}(x)$, where x is the momentum fraction carried by quark a. The kinematics of this process has been described by Feynman[4] and will not be repeated here. However, in this model, we may go further. Recalling that[6] $\phi_1^{a\bar{d}}(x)\rightarrow x^{\beta a}$ as $x\rightarrow 0$, we can say that the probability amplitude to find quark a in meson 1 with "wee" momentum goes as $P_1^{-\beta a}$, where P_1 is the total momentum of the meson. Similarly, the amplitude to find antiquark $\bar{a}$ in meson 2 with wee momentum goes as $P_2^{-\beta a}$. Hence, the amplitude for exchanging a wee quark of flavor a between meson 1 and meson 2 goes as $s^{-\beta a}$. An analogous discussion applies to the exchange of quark b, so the asymptotic behavior of this amplitude is $s^{\alpha_{ab}}$, where $\alpha_{ab}=-\beta_a-\beta_b$. (It is noteworthy that the power is *additive* in the quarks. This result will be returned to later.) This intuitive result can be verified by direct calculation, and one can show that the asymptotic behavior of *all three* diagrams in Fig. 1 is the same.[14] Recall that β_i lies between 0 and 1, being determined by the equation[9]

$$\pi\beta_i \cot\pi\beta_i = -\pi^2\alpha'\tilde{m}_i^2\,, \qquad (1)$$

where the (asymptotic) level spacing α'^{-1} is related to the coupling constant by $\alpha'^{-1}=\pi g^2 N$ and $\tilde{m}_i$ is the renormalized quark mass, related to the bare quark mass m_i according to $\tilde{m}_i^2=m_i^2-g^2N/\pi$. ($m_i^2$ can be negative.)

The unitarity structure of the model is also interesting. Similar to the discussion of virtual Compton scattering,[12] it can be shown[14] that the disconnected diagram, Fig. 1(a), is canceled by a piece of Fig. 1(b) and that there are no quark discontinuities coming from the remaining contributions to Fig. 1(b) or from Fig. 1(c). Of course, this is to be expected of a field theory with confinement. The only discontinuities in s (for positive s) coming from Fig. 1 are meson poles, so this much of duality is retained by the two-dimensional model: *The sum of s-channel poles leads to a Regge-like asymptotic power behavior* $(-s)^\alpha$. (Similarly, the sum of u-channel poles leads to s^α.) The fact that we apparently must sum over s- and t-channel exchanges, Fig. 1(b) and 1(c), is gauge-dependent. In any gauge in which the self-coupling of gluons does *not* vanish, this decomposition of the planar diagram does not occur.[15] Unitarity is quite simple: To this leading order in $1/N$, the elastic scattering of mesons may be described by a phenomenological Lagrangian involving only three-point couplings of mesons.[16]

To next order, $O(N^{-2})$, many diagrams contribute corresponding to "Regge-Regge cuts", renormalizations of the "intercept" α_{ab}, etc. In this paper, we concentrate on the imaginary part of the class of planar diagrams having two quark boundaries and no handles, which is usually identified with the "bare" Pomeron[17] [Fig. 2(a)]. [To this graph must be added all planar gluon exchanges, just as Fig. 1(b) and 1(c) were added to Fig. 1(a).] Topologically, this graph with all its gluonic corrections may also be depicted as a cylinder, Fig. 2(b), or, alternatively, as the twisted-loop graph, Fig. 2(c). In four dimensions, one would describe this diagram in terms of gluon exchange between the quarks in each meson, as may be easily seen from Fig. 2(b). Indeed, one expects these gluons

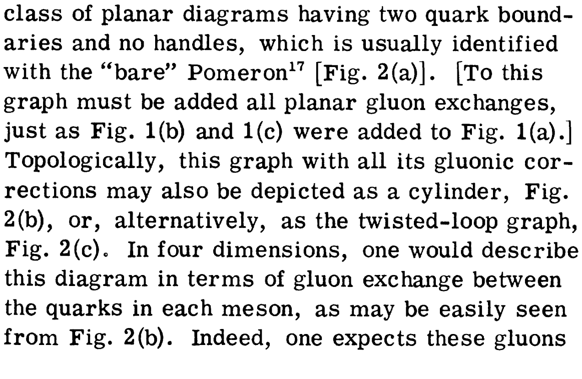

FIG. 2. Equivalent representations of the bare Pomeron graph: (a) planar graph with two quark boundaries, (b) cylinder or tube showing gluonic exchanges in the t channel, (c) twisted loop displaying mesonic intermediate states.

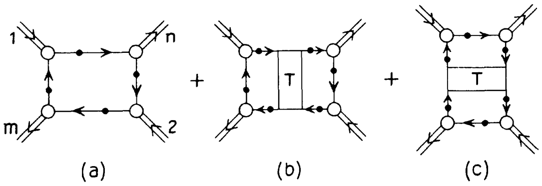

FIG. 3. The (ut) amplitude to leading order in $1/N$.

to form quarkless mesonic bound states which would be expected to lie on the Pomeron trajectory.[8] In two dimensions, the gluon field contains no dynamical degrees of freedom; there are no gluons, hence no quarkless mesonic bound states to support the bare Pomeron. Nevertheless, the quarks can scatter via the potential represented by the gluon field. Might this lead to an asymptotic behavior which dominates the Regge-like behavior discussed above and which might therefore be identified as the "bare" Pomeron in two dimensions?

To simplify the calculation, we proceed as follows: Because quarks are confined, we are assured that the only discontinuities come from mesonic intermediate states, so that the imaginary part is given by the square of the planar (ut) amplitude, Fig. 3,

$$\mathrm{Im}P = \sum_{m,n} |A_{12\to nm}(s)|^2 \rho_{nm}(s) \ . \tag{2}$$

Here, P denotes the amplitude corresponding to Fig. 2, and $A_{12\to nm}$ denotes the amplitude represented by Fig. 3. The phase space $\rho_{nm}(s)$ may easily be shown to be proportional to $\lambda^{-1/2}(s, \mu_m^2, \mu_n^2)$, where λ is the familiar triangular function, and μ_m and μ_n are the masses of mesons m and n, respectively. The sum extends over all mesons m,n allowed by momentum conservation ($\lambda > 0$). We wish to discuss the asymptotic behavior as $s\to\infty$ of Eq. (2), to determine whether a new power emerges which dominates the "Regge" behavior determined previously. One would anticipate that the dominant contribution would come from the region where both μ_m^2 and μ_n^2 are of order s. This may be easily understood intuitively, for example, in the center-of-mass frame with meson 1 (2) moving to the right (left). There is a finite, scaling amplitude $\phi_1(x_1)$ to find a right-moving quark in meson 1 carrying momentum fraction x_1 and, similarly, a scaling amplitude $\phi_2(x_2)$ to find a left-moving antiquark in meson 2 carrying momentum fraction $1 - x_2$. Interacting via the long-range Coulomb potential (in four dimensions, we would say "wee-gluon exchange"), this quark-antiquark pair bind to form meson n with $\mu_n^2 \cong x_1(1-x_2)s$. Similarly, the remaining quark-antiquark pair bind to form meson m with $\mu_m^2 \cong x_2(1-x_1)s$. Thus, one might *guess* that the production amplitude would scale as $s\to\infty$ for fixed x_1, x_2:

$$\lim_{s\to\infty} A_{12\to nm}(s) \overset{??}{\longrightarrow} A(x_1, x_2) \ . \tag{3}$$

The sum in Eq. (1) may then be represented as an integral over quark momentum fractions:

$$\sum \rho_{nm}(s) \approx \int d\mu_m^2\, d\mu_n^2\, \lambda^{-1/2}(s, \mu_n^2, \mu_m^2) \approx s \int\int dx_1 dx_2 \ . \tag{4}$$

Combining these two expressions, Eqs. (3) and (4), would then lead to $\mathrm{Im}P$ growing linearly with energy.

We have performed the calculation of $A_{12\to nm}(s)$ in the $A_- = 0$ gauge. The exact form of the (ut) amplitude, depicted in Fig. 3, can be written as[18]

$$\begin{aligned}
\frac{N}{4\pi} A_{12\to nm}(s) = {} & p_2 p_m \int\int du\, dv\, \frac{\phi_1\left(\frac{q+vp_2}{p_1}\right) - \phi_1\left(\frac{q+up_m}{p_1}\right)}{(vp_2 - up_m)^2}\, \phi_n\left(\frac{q+vp_2}{p_n}\right) \phi_2(v)\phi_m(u) \\
& + p_2 p_n \int\int du\, dv\, \frac{\phi_1\left(\frac{q+vp_2}{p_1}\right) - \phi_1\left(\frac{up_n}{p_1}\right)}{[p_2(1-v) - p_n(1-u)]^2}\, \phi_n(u)\phi_2(v)\phi_m\left(\frac{vp_2}{p_m}\right) \\
& + q^2 p_2 p_m \int\int\int\int dx\, dy\, du\, dv\, G(x,y;U)\phi_m(u)\phi_2(v) \frac{\phi_n\left(\frac{yq}{p_n}\right) - \phi_n\left(\frac{q+p_2 v}{p_n}\right)}{[p_2 v + q(1-y)]^2}\, \frac{\phi_1\left(\frac{xq}{p_1}\right) - \phi_1\left(\frac{q+up_m}{p_1}\right)}{[p_m u + q(1-x)]^2} \\
& + \Delta^2 p_2 p_n \int\int\int\int dx\, dy\, du\, dv\, G(x,y;T)\phi_n(u)\phi_2(v) \frac{\phi_m\left(\frac{p_2+y\Delta}{p_m}\right) - \phi_m\left(\frac{vp_2}{p_m}\right)}{[\Delta y + p_2(1-v)]^2}\, \frac{\phi_1\left(\frac{p_n+x\Delta}{p_1}\right) - \phi_1\left(\frac{up_n}{p_1}\right)}{[\Delta x + p_n(1-u)]^2} \ .
\end{aligned} \tag{5}$$

All momenta appearing in the formula refer to their minus components, i.e., $q/p_1 \equiv q_-/p_{1-}$; all integrals run over (0, 1). Here, $\phi_i(z)$ is the wave function of hadron i (=1, 2, n, m), $q^\mu \equiv p_1^\mu - p_m^\mu$, $\Delta^\mu \equiv p_1^\mu - p_n^\mu$, $U \equiv q_\mu{}^2$, $T \equiv \Delta_\mu{}^2$; G is the Green's function[12]

$$G(x, y; S) \equiv \sum_n \frac{\phi_n(x)\phi_n(y)}{\mu_n{}^2 - S} .$$

The first term comes from Fig. 3(a) and the Born term (single-gluon exchange) of Fig. 3(b); the third term comes from the rest of Fig. 3(b). The second term comes from Fig. 3(a) plus the Born term of Fig. 3(c); the fourth term comes from the rest of Fig. 3(c). Unfortunately, the intuitive discussion given above for the center-of-mass frame does not apply in the $A_- = 0$ gauge where parity invariance is not manifest. However, one can consider the limit $s \to \infty$ for fixed $\mu_n{}^2/s$, $\mu_m{}^2/s$. (In this limit, one also has U/s and T/s fixed.) Using scaling relations derived earlier,[12] one may easily determine the dominant contribution of each term. In each case, the dominant behavior comes from the infrared region for the gluon "propagator." Multiplying out the factors in Eq. (5) gives a sum of 12 terms, each of which *does* scale, and the argument of ϕ_1 becomes, in every case, $p_{n-}/p_{1-} \equiv x_1$, as was anticipated. However, recombining terms again, it is easy to see that this contribution cancels, leaving a contribution which falls by (at least) s^{-1}. We find, therefore, that the asymptotic behavior of $A_{12\to nm}(s)$ does *not* scale as described by Eq. (3); in fact, it vanishes at least as rapidly as s^{-1}, which, in turn, leads to a contribution to ImP which falls at least as rapidly as s^{-1}. In general, this vanishes more rapidly than the "Regge" term vanishes, and the contribution cannot be interpreted as a Pomeron. The precise reason for the cancellation remains obscure. Figure 3(a) and the Born contributions to Fig. 3(b) and 3(c) are separately gauge-dependent. Only when combined as in the first two integrals in Eq. (5) do we obtain gauge-invariant expressions. Thus, to some extent, the cancellation of scaling amplitudes may be regarded as an infrared or a gauge cancellation. This reflects the vanishing of the gluon "propagators" within the regions of integration. The same cannot be said of the remaining two integrals whose gluon propagators cannot vanish. Each term in the integrals appears to be separately gauge invariant so, even though the cancellation appears to be similar, we hesitate to call it an expression of gauge invariance. If we, nevertheless, conjecture that the absence of the Pomeron is related to the absence of gluons in two dimensions, it could be that when one calculates to higher order in $1/N$ and begins to build the quark-antiquark "sea", *their* exchange will lead to Pomeron-like behavior Such speculations we leave for future investigations.[19]

In four dimensions, where the transverse gluon field is an independent dynamical degree of freedom, the mesonic wave function must also describe the probability amplitude to find any number of gluons in addition to the valence quark-antiquark pair. It may well be that the analogous calculation involving the exchange of these wee gluons will produce a bare Pomeron already in order $1/N^2$.[20]

Note added. While preparing this manuscript for publication, we received a preliminary copy of a report[21] in which conclusions similar to those presented here have been reached. (We would like to thank R. Savit for informing us of this work and G. F. Chew for making this report available to us.)

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[1]See the review by H. D. Politzer, Phys. Rep. 14C, 130 (1974).

[2]K. G. Wilson, in Proceedings of Erice Lectures, 1975, Cornell Report No. CLNS-321, 1975 (unpublished); in Proceedings of the 1976 Coral Gables Conference, Cornell Report No. CLNS-327, 1976 (unpublished); J. Kogut, D. K. Sinclair, and L. Susskind, Nucl. Phys. B114, 199 (1976); T. Banks *et al.*, Phys. Rev. D 15, 1111 (1977); W. A. Bardeen and R. B. Pearson, *ibid.* 14, 547 (1976); W. A. Bardeen, R. B. Pearson, and E. Rabinovici, in preparation.

[3]Particle Data Group, Rev. Mod. Phys. 48, S1 (1976).

[4]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972).

[5]G. 't Hooft, Nucl. Phys. B72, 461 (1974).

[6]K. Kikkawa, B. Sakita, and M. A. Virasoro, Phys. Rev. 184, 1701 (1969); K. Kikkawa, S. A. Klein, B. Sakita, and M. A. Virasoro, Phys. Rev. D 1, 3258 (1970).

[7] See the collection in ***Dual Theory,*** Physics Reports Reprint Book Series: Vol. 1 edited by M. Jacob (North-Holland, Amsterdam, 1974).

[8] An interesting discussion based on 't Hooft's work plus extensions to a "topological expansion" has been given by G. Veneziano, CERN Report No. Ref. TH.2200 CERN, 1976 (unpublished).

[9] G. 't Hooft, Nucl. Phys. B75, 461 (1974); and lectures given at Erice and Copenhagen Summer Schools, 1975, ITP, Utrecht report, 1975 (unpublished).

[10] These properties emerge almost trivially in two dimensions because (a) the theory is superrenormalizable and (b) the Coulomb potential is linear.

[11] C. G. Callan, N. Coote, and D. J. Gross, Phys. Rev. D 13, 1649 (1976).

[12] M. B. Einhorn, Phys. Rev. D 14, 3451 (1976).

[13] Figure 1 is often referred to as the (*st*) diagram in discussions of duality and/or the Veneziano model (see Ref. 7). To Fig. 1 must be added the two cyclically inequivalent permutations of the external mesons, commonly called (*su*) and (*ut*). The (*ut*) diagram (Fig. 3) will be discussed below in connection with the Pomeron.

[14] Full details and further discussion will be presented elsewhere: M. B. Einhorn, in preparation. It can also be shown that the coefficient manifests t-channel factorization. (A word of caution must be added, however, since we have not proved that the coefficient of $s^{\alpha_{ab}}$ is nonzero.) The (*ut*) diagram behaves as u^{α} which, when added to the (*st*) diagram, may be interpreted as giving "signature" to the exchange. Finally, the (*su*) diagram behaves as $s^{\alpha-2}$ for forward scattering.

[15] In four dimensions, one cannot gauge away the self-coupling of the gluon field, and we anticipate that full duality will be recovered: The planar amplitude will have meson poles in both the s and t channels and manifest true Regge asymptotic behavior.

[16] Assuming ordinary analyticity for meson amplitudes, there can be no "contact term," contrary to the claim of Ref. 11. See Ref. 14 for further discussion.

[17] In the context of the topological expansion, this correspondence has been made more precise recently, where it has been identified with the "bare" Pomeron which must be iterated (e.g., via Gribov's Reggeon calculus) to obtain a fully unitary S matrix. For a summary with references to the original literature, see Ref. 8. Its identification with the Pomeron in some general sense goes back as far as P. G. O. Freund, Lett. Nuovo Cimento 4, 147 (1970); P. G. O. Freund and R. J. Rivers, Phys. Lett. 29B, 510 (1969).

[18] We have assumed, without loss of generality, that $p_{1-} \geq p_{n-}$, $p_{m-} \geq p_{2-}$.

[19] Elsewhere, we will discuss vertex and intercept renormalizations coming from Eq. (5): M. B. Einhorn and E. Rabinovici, in preparation.

[20] For approaches in four dimensions, see F. E. Low, Phys. Rev. D 12, 163 (1975); S. Nussinov, Phys. Rev. Lett. 34, 1286 (1975); Phys. Rev. D 14, 246 (1976).

[21] R. C. Brower, J. Ellis, M. G. Schmidt, and J. H. Weis, Phys. Lett. 65B, 249 (1976).