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Ward-Slavnov identities in the axial gauge

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A new derivation of the Ward-type identities for non-Abelian gauge theories in the axial gauge is given. The merit of this derivation is its extreme simplicity. It makes the basic feature of this gauge transparent and allows an extension, in a trivial manner, of the identities on the renormalization constants, written down by Kummer, to the case in which fermions are also included.

I. INTRODUCTION

In 1975 Kummer[1] presented a derivation of the Ward-Slavnov identities for a self-coupled non-Abelian gauge theory in the axial gauge. This gauge is characterized by the condition that

$$n^\mu A_\mu^a = 0, \tag{1}$$

where n_μ is any constant spacelike or timelike vector with $n^2 \neq 0$. His method follows the functional integral techniques of Slavnov[2] and Taylor.[3] The purpose of this note is to rederive some of his results with a very simple technique that takes advantage of a special feature of this gauge condition. We shall also extend the results fermion case relevant to quantum chromodynamics.

II. THE IDENTITIES

For simplicity we shall consider the Yang-Mills Lagrangian for SU(2). Generalization to other groups is trivial. The Lagrangian is

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}F_{\mu\nu}^{\,a}F^{\mu\nu a} - \bar\psi\gamma_\mu(\partial_\mu - ig\vec\tau\cdot\vec{A}^{\,\mu})\psi \\ &-m\bar\psi\psi - (n^\mu A_\mu^a)C^a.\end{aligned} \tag{2}$$

Here

$$F_{\mu\nu}^{\,a} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\epsilon_{abc}A_\mu^b A_\nu^c \tag{3}$$

and C^a is a Lagrange-multiplier field that enforces the gauge condition. For our purposes the essential feature of this gauge condition is that it is invariant under both global *and local* rotations in isospin space. Gauge conditions that involve *derivatives* of the gauge field are only invariant under isorotations with constant *global* phases.

If we subject $\mathcal{L}$ to a rotation in isospace with a local infinitesimal phase $\vec\Lambda(x)$,

$$\mathcal{L}'(x) = \mathcal{L}(x) - \vec{J}_\mu(x)\cdot\partial^\mu\vec\Lambda(x), \tag{4}$$

where J_μ is the *full* isospin current satisfying

$$\partial_\mu \vec{J}^{\,\mu}(x) = 0. \tag{5}$$

To find the Ward identities all we need do is subject the Green's functions in question to a local infinitesimal isospin rotation.[4] Consider, for example,

$$S_F(x-y) = \langle 0\,|(\psi(x)\bar\psi(y))_+|\,0\rangle \tag{6}$$

and subject the theory to the transformation

$$\begin{aligned}\psi'(x) &= [1 + i\vec\Lambda(x)\cdot\vec\tau]\,\psi(x), \\ A_\mu^{a\prime}(x) &= A_\mu^a(x) + \epsilon_{abc}\Lambda_b(x)A_\mu^c(x),\end{aligned} \tag{7}$$

so that $\mathcal{L}$ transforms as indicated in Eq. (4).

The reader will note that the transformations of Eq. (7) are *not* the local isotopic *gauge* transformations since the inhomogeneous gradient terms in the gauge-field transformations are missing. Under the gauge transformations $\mathcal{L}$ is invariant, whereas under these isotopic rotations $\mathcal{L}$ acquires an additional piece as indicated in Eq. (4):

$$\Delta S_F(x-y) = i\vec\Lambda(x)\cdot\vec\tau S_F(x-y) - iS_F(x-y)\vec\Lambda(y)\cdot\vec\tau. \tag{8}$$

We can also compute this change by regarding $\vec{J}_\mu(x)\cdot\partial^\mu\vec\Lambda(x)$ as a perturbation on $\mathcal{L}$ whose effect is to be taken in lowest order. Thus, in momentum space

$$\begin{aligned}\Delta S_F(P,P') = &-i(P_\mu - P'_\mu)S_F(P) \\ &\times\vec\Lambda(P-P')\cdot\vec{V}^{\,\mu}(P',P)S_F(P'),\end{aligned} \tag{9}$$

where the above is essentially the Fourier transform of

$$\langle 0\,|(\psi(x)\vec{J}_\mu(z)\bar\psi(y))_+|\,0\rangle. \tag{10}$$

Thus

$$i\vec\tau S_F(P) - iS_F(P')\vec\tau = -i(P-P')_\mu S_F(P)\vec{V}^{\mu}(P,P')S_F(P'), \tag{11}$$

which is the Ward identity for fermions in this gauge. Likewise we derive for the vector-meson Green's functions

$$\begin{aligned}&\epsilon_{abc}[W_{\mu\nu}(P) - W_{\mu\nu}(P')] \\ &\quad= -i(P-P')_\lambda W_{\mu\mu'}(P)V^{abc}_{\mu'\lambda\nu'}(P,P')W_{\nu'\nu}(P'),\end{aligned} \tag{12}$$

where we use the fact that the $\vec{A}_\mu$ propagators are diagonal in isospin indices.

The reader will have noticed that these identities involve the full conserved isospin current which can be written using the equations of motion

$$J^a_\mu + n_\mu C^a = \Box A^a_\mu - \partial_\mu(\partial_\lambda A^a_\lambda) + g(\vec{A}_\mu \times \partial^\nu \vec{A}_\nu)_a,$$
$$n^\mu \partial_\mu C^a = 0. \tag{13}$$

On the other hand, the Dyson equations are characterized by vertices which involve nonconserved currents—quantities generated by

$$\langle 0 | T(A^a_\mu(x) A^b_\nu(y) A^c_\lambda(z)) | 0 \rangle.$$

Such objects do not obey any simple Ward identity in any gauge but rather they obey the more complicated Ward-Slavnov identities as in Kummer's paper.[1] However, at the mass-shell pole,

$$P = P',$$

the values of these vertex functions are identical[5] to those computed with the conserved $\vec{J}_\mu$. Thus it follows from Eqs. (11) and (12) that in this gauge

$$Z^F_1 = Z^F_2 \tag{14}$$

and

$$Z^A_3 = Z^A_1. \tag{15}$$

The latter has been noted in Ref. 1.

With additional work the reader can apply this technique to the four-point vector-meson function and so on.

In summary, this technique does *not* lead to anything simple for the gauges in which the gauge conditions depend on derivatives of the gauge field since Eq. (4) is false and also ghosts must be included. Hence these simple relations among the Z's also fail, as is well known.

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[1] W. Kummer, Acta Phys. Austriaca 41, 315 (1975). See also, R. Delbourgo, Abdus Salam, and J. Strathdee, Nuovo Cimento 23A, 237 (1974), where identities were derived using functional techniques.

[2] A. Slavnov, Teor. Mat. Fiz. 10, 153 (1972) [Theor. Math. Phys. 10, 99 (1972)].

[3] J. C. Taylor, Nucl. Phys. B33, 436 (1971).

[4] This technique was used for weak interactions by J. Bernstein, M. Gell-Mann, and L. Michel [Nuovo Cimento 16, 560 (1960)] and is explained in detail by J. Bernstein, in *Elementary Particles and Their Currents* (Freeman, San Francisco, 1968).

[5] I am grateful to M. A. B. Bég for a simple argument using Eq. (13) that proves this and to H. Pagels for forcefully raising this point.