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Quantum-electrodynamic corrections to the gravitational interaction of the photon*

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The order-α quantum-electrodynamic modifications to the photon stress tensor are calculated using the finite, causal methods of source theory. The results are in agreement with those of Berends and Gastmans, who used dimensional regularization. Although the corrected stress tensor is conformally invariant, a "trace anomaly" does appear as a consequence of gravitational and electromagnetic gauge invariance.

I. INTRODUCTION

There is, as yet, no complete quantum theory of gravitation. Instead, we have a theory of massless helicity-two gravitons, which, when gravitational gauge invariance is extended to general coordinate invariance, becomes, in the classical limit, the strong-field, Einstein theory.[1,2] All of the observed features of gravitation follow from this weak-field graviton theory (they are the consequences of iterated single-graviton exchange between classical energy distributions), yet, theoretically, we require of a complete theory that quantum corrections (irreducible multiparticle-exchange processes) be calculable. Difficulties emerge, apparently, when modifications to gravity involving the exchange of other particles (electrons and photons, for example) are incorporated,[3-5] although, for the isolated graviton system, the theory seems to be renormalizable in the first nontrivial order, at least.[5]

Our concern here is with a related, but simpler, question, the purely quantum-electrodynamic corrections to the interaction of electrons and photons with gravity. These corrections may be regarded as modifications to the energy-momentum, or stress, tensors of the corresponding particles. That this question is nontrivial, going beyond the confines of electrodynamics, is made evident in the (physically unrealisitic) situation of spin-0 electrodynamics.[6] There, an unambiguous quantum correction can be computed only if the initial description of the interaction of the spin-0 "electron" with gravity is though the stress tensor that respects conformal or scale invariance in the limit of zero mass (the "conformal" stress tensor).[7] The realistic electrodynamic case automatically provides a completely calculable quantum correction because the canonical stress tensors for spin-$\frac{1}{2}$ and spin-1 are just the conformal ones. In conventional language, we are saying that this combined electrogravidynamic system is renormalizable (in this simple application) only for a particular class of interactions. This observation also suggests that partial conformal invariance may be important in quantizing gravitation. In any event, understanding how such calculations may be performed should be helpful in constructing a more satisfactory theory of gravitational phenomena, or in appreciating what the present theory is, in fact, saying. But it should be borne in mind that such calculations have no foreseeable practical importance, since the phenomena are quite small even in neutron-star environments.

In my earlier researches[6,8,9] I had calculated the order-α quantum corrections to the electron stress tensor, with primary emphasis upon the consistency of the theory. For some reason I failed to consider the corrections to the photon stress tensor. Now, stimulated by a recent computation by Berends and Gastmans,[10] I have returned to this subject in order to complete my treatment of this problem. My verification of their result is significant partly because we employ quite different methods. Berends and Gastmans use conventional field-theory methods together with dimensional regularization. I, on the other hand, use what I feel are the more physically perspicuous methods of source theory,[1,11] in which only physical coupling constants and masses occur, and renormalization does not arise. In the approach I adopt, the photon-graviton form factors are generated automatically in a spectral form reflecting the internal two-particle-exchange process.

Especially of interest in this problem are the questions of (1) gauge invariance with respect to both photon and graviton, and the corresponding choice of an appropriate tensor basis, and (2) conformal and scale invariance, and their possible breaking by quantum corrections. For (1), the basis given by Berends and Gastmans[10] will be seen not to be the best for physical interpretation, although not incorrect. In regard to (2), Berends and Gastmans assert that there occurs a scale violation, which they attribute to the use of dimensional regularization. While I agree with the value they obtain for this "trace anomaly," I show it to be an expected consequence of the dimension (4) of the contributing gauge-invariant basis tensors. It also does not reflect a violation of conformal invariance of the electromagnetic stress tensor, for the dilation Ward identity[8,12] is satisfied here.

II. PRIMITIVE INTERACTION AND BASIS TENSORS

The canonical stress tensor of the photon is

$$t^{\mu\nu}=F^{\mu\lambda}F^{\nu}{}_{\lambda}-\tfrac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\,, \tag{2.1}$$

where $F^{\mu\nu}$ is the field strength tensor. This stress tensor is conformally invariant, as expressed in part by

$$t^{\mu}{}_{\mu}=0\,. \tag{2.2}$$

The primitive interaction of photons with gravity is thus taken to be the action term

$$W=\int(dx)t^{\mu\nu}(x)h_{\mu\nu}(x)\,, \tag{2.3}$$

where $h_{\mu\nu}$ is the gravitational field.

Note that because, outside photon sources, the stress tensor is conserved,

$$\partial_{\mu}t^{\mu\nu}=0\,, \tag{2.4}$$

the action (2.3) is invariant under the substitution[13]

$$h_{\mu\nu}\rightarrow h_{\mu\nu}+(\partial_{\mu}\xi_{\nu}+\partial_{\nu}\xi_{\mu})\,. \tag{2.5}$$

This is a gravitational gauge transformation, which must represent an exact invariance of the theory.

When we consider the quantum corrections to the photon stress tensor, additional tensor structures arise. It is simplest to express these in momentum space. We write the general coupling between two photons and a graviton in the form

$$W=\frac{1}{2}\int\frac{(dq)}{(2\pi)^4}\frac{(dq')}{(2\pi)^4}A_{\mu}(-q')A_{\nu}(-q)\times I^{\mu\nu\kappa\lambda}(q,q')h_{\kappa\nu}(Q)\,, \tag{2.6}$$

where $Q=q+q'$ is the momentum carried by the graviton. In determining the form of the tensor $I^{\mu\nu\kappa\lambda}$ we impose the following four constraints:

(1) It should be symmetric in κ,λ:

$$I^{\mu\nu\kappa\lambda}=I^{\mu\nu\lambda\kappa}\,. \tag{2.7}$$

(2) It should be invariant under interchange of the photon variables:

$$I^{\mu\nu\kappa\lambda}(q,q')=I^{\nu\mu\kappa\lambda}(q',q)\,. \tag{2.8}$$

(3) It should be gauge invariant with respect to the graviton field:

$$Q_{\kappa}I^{\mu\nu\kappa\lambda}=0\,. \tag{2.9}$$

(4) It should be gauge invariant with respect to the photon field:

$$q'_{\mu}I^{\mu\nu\kappa\lambda}=0\,. \tag{2.10}$$

For real photons, obeying

$$q^2=q'^2=0\,, \tag{2.11}$$

there are six tensors satisfying these requirements. A convenient way of writing these is the following:

$$I^{\mu\nu\kappa\lambda}=\sum_{i=1}^{6}B_iT_i^{\mu\nu\kappa\lambda}\,, \tag{2.12}$$

where the B_i are scalar functions and the tensors are

$$\begin{aligned}T_1^{\mu\nu\kappa\lambda}=&(2q^{\mu}q'^{\nu}+2q'^{\mu}q^{\nu}-Q^2g^{\mu\nu})(2q^{\kappa}q'^{\lambda}+2q'^{\kappa}q^{\lambda}-Q^2g^{\kappa\lambda})\\&-(2q^{\nu}q'^{\lambda}+2q'^{\nu}q^{\lambda}-Q^2g^{\nu\lambda})(2q^{\mu}q'^{\kappa}+2q'^{\mu}q^{\kappa}-Q^2g^{\mu\kappa})\\&-(2q^{\nu}q'^{\kappa}+2q'^{\nu}q^{\kappa}-Q^2g^{\nu\kappa})(2q^{\mu}q'^{\lambda}+2q'^{\mu}q^{\lambda}-Q^2g^{\mu\lambda})\,,\end{aligned} \tag{2.13a}$$

$$T_2^{\mu\nu\kappa\lambda}=(2q^{\mu}q'^{\nu}+2q'^{\mu}q^{\nu}-Q^2g^{\mu\nu})(Q^{\kappa}Q^{\lambda}-Q^2g^{\kappa\lambda})\,, \tag{2.13b}$$

$$T_3^{\mu\nu\kappa\lambda}=(2q^{\mu}q'^{\nu}-2q'^{\mu}q^{\nu}-Q^2g^{\mu\nu})(Q^{\kappa}Q^{\lambda}-Q^2g^{\kappa\lambda})\,, \tag{2.13c}$$

$$T_4^{\mu\nu\kappa\lambda}=(2q^{\mu}q'^{\nu}+2q'^{\mu}q^{\nu}-Q^2g^{\mu\nu})(q-q')^{\kappa}(q-q')^{\lambda}\,, \tag{2.13d}$$

$$T_5^{\mu\nu\kappa\lambda}=(2q^{\mu}q'^{\nu}-2q'^{\mu}q^{\nu}-Q^2g^{\mu\nu})(q-q')^{\kappa}(q-q')^{\lambda}\,, \tag{2.13e}$$

$$\begin{aligned}T_6^{\mu\nu\kappa\lambda}=&(2q^{\nu}q^{\lambda}+2q'^{\nu}q^{\lambda}-Q^2g^{\nu\lambda})(2q'^{\mu}q'^{\kappa}+2q^{\mu}q'^{\kappa}-Q^2g^{\mu\kappa})\\&+(2q'^{\mu}q'^{\lambda}+2q^{\mu}q'^{\lambda}-Q^2g^{\mu\lambda})\\&\times(2q^{\nu}q^{\kappa}+2q'^{\nu}q^{\kappa}-Q^2g^{\nu\kappa})\,.\end{aligned} \tag{2.13f}$$

Note that $T_1^{\mu\nu\kappa\lambda}$ has been so chosen that it is orthogonal to q or q' in each of its indices (it is symmetric under $q\longleftrightarrow q'$):

$$\begin{aligned}&q_{\mu}T_1^{\mu\nu\kappa\lambda}=0\,,\\&q_{\nu}T_1^{\mu\nu\kappa\lambda}=0\,,\\&q_{\kappa}T_1^{\mu\nu\kappa\lambda}=0\,,\\&q_{\lambda}T_1^{\mu\nu\kappa\lambda}=0\,,\end{aligned} \tag{2.14}$$

and it is traceless with respect to the graviton indices

$$T^{\mu\nu\kappa}{}_{\kappa}=0\,. \tag{2.15}$$

Not surprisingly, it corresponds to the free photon interaction through (2.1).

To see this, we now recognize that the real photon fields in (2.6) are proportional to polarization vectors,

$$A_\mu(-q') = \epsilon'_\mu A(-q'),$$
$$A_\nu(-q) = \epsilon_\nu A(-q), \tag{2.16}$$

satisfying

$$\epsilon' q' = \epsilon q = 0. \tag{2.17}$$

Employing (2.17) we obtain three independent reduced tensors:

$$\begin{aligned}\theta_1^{\kappa\lambda} &= -\frac{1}{2Q^2}\,\epsilon'_\mu\epsilon_\nu\,T_1^{\mu\nu\kappa\lambda}\\ &= -\frac{1}{2Q^2}\{[2(\epsilon' q)(\epsilon q') - Q^2\epsilon\epsilon'](2q^\kappa q'^\lambda + 2q^\lambda q'^\kappa - Q^2 g^{\kappa\lambda})\\ &\quad - (2\epsilon q' q^\lambda - Q^2\epsilon^\lambda)(2\epsilon' q q'^\kappa - Q^2\epsilon'^\kappa)\\ &\quad - (2\epsilon q' q^\kappa - Q^2\epsilon^\kappa)(2\epsilon' q q'^\lambda - Q^2\epsilon'^\lambda)\},\end{aligned} \tag{2.18a}$$

$$\begin{aligned}\theta_2^{\kappa\lambda} &= \epsilon'_\mu\epsilon_\nu T_{2,3}^{\mu\nu\kappa\lambda}\\ &= [2(\epsilon' q)(\epsilon q') - Q^2\epsilon\epsilon'](Q^\kappa Q^\lambda - Q^2 g^{\kappa\lambda}),\end{aligned} \tag{2.18b}$$

$$\begin{aligned}\theta_3^{\kappa\lambda} &= \epsilon'_\mu\epsilon_\nu T_{4,5}^{\mu\nu\kappa\lambda}\\ &= [2(\epsilon' q)(\epsilon q') - Q^2\epsilon\epsilon'](q-q')^\kappa(q-q')^\lambda.\end{aligned} \tag{2.18c}$$

(Note that

$$\epsilon'_\mu\epsilon_\nu T_6^{\mu\nu\kappa\lambda} = 2Q^2\theta_1^{\kappa\lambda} + \theta_2^{\kappa\lambda} - \theta_3^{\kappa\lambda}.)$$

Here, gravitational gauge invariance appears as

$$q_\kappa\theta_1^{\kappa\lambda} = q'_\kappa\theta_1^{\kappa\lambda} = 0, \quad Q_\kappa\theta_2^{\kappa\lambda} = Q_\kappa\theta_3^{\kappa\lambda} = 0. \tag{2.19}$$

It is permissible and necessary to divide by Q^2 in defining $\theta_1^{\kappa\lambda}$, since the numerator there vanishes at $Q^2 = 0$. The basis (2.18) is thus free from kinematic singularities and zeros.[14] The photon-graviton coupling of (2.6) can now be written in terms of the stress tensor coupling in (2.3), where

$$\begin{aligned}t^{\kappa\lambda}(Q) &= \frac{1}{2}\int\frac{(dq)}{(2\pi)^4}\frac{(dq')}{(2\pi)^4}(2\pi)^4\delta(q+q'-Q)\\ &\quad\times A(-q')A(-q)\sum_{i=1}^{3}\theta_i^{\kappa\lambda}(q,q')F_i(Q).\end{aligned} \tag{2.20}$$

In particular, the free photon coupling given by (2.1) corresponds just to $\theta_1^{\kappa\lambda}$:

$$\left.\begin{aligned}F_1 &= 1\\ F_{2,3} &= 0\end{aligned}\right\}\ \text{free photon}. \tag{2.21}$$

This observation provides the reason for defining the tensor $\theta_1^{\kappa\lambda}$ in the way we did, for it then alone describes the primitive interaction. We now ask how (2.21) is modified when the lowest-order electrodynamic corrections are included.

III. CAUSAL TWO-ELECTRON EXCHANGE

There is exactly one causal process which gives rise to the order-α modification of the above primitive interaction between graviton and photons. This is shown in Fig. 1, which represents the production of a real electron pair by a virtual graviton, and its subsequent annihilation into two real photons. The action term representing the $ee\to\gamma\gamma$ process is

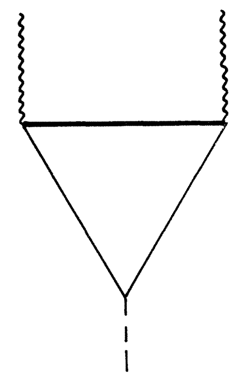

FIG. 1. Causal diagram representing the production of an electron pair by a virtual graviton, and its subsequent annihilation into two photons. Upon space-time extrapolation, this process gives rise to an electrodynamic modification of the photon stress tensor.

$$\begin{aligned}W = \tfrac{1}{2}\int (dx)(dx')\psi_2(x)\gamma^0 e\hat{q}\gamma A_1(x)\\ \times G_+(x-x')e\hat{q}\gamma A_1(x')\psi_2(x'),\end{aligned} \tag{3.1}$$

where the causal labels 2, 1 represent the fields of incoming and outgoing particles, respectively, and $\hat{q}$ is the charge matrix. (We are using real fields.) The relation of the electron fields to their (earlier acting) sources,

$$\begin{aligned}\psi(x) &= i\int d\omega_p e^{ipx}(m-\gamma p)\eta(p),\\ \psi(x)\gamma^0 &= i\int d\omega_{p'} e^{ip'x}\eta(p')\gamma^0(m+\gamma p'),\end{aligned} \tag{3.2}$$

where the momentum-space measure is

$$\begin{aligned}d\omega_p &= \frac{(d\vec{p})}{(2\pi)^3}\frac{1}{2p^0},\\ (p^0)^2 &= \vec{p}^{\,2} + m^2,\end{aligned} \tag{3.3}$$

allows us to rewrite (3.1) in terms of the two-electron effective source[6]

$$\begin{aligned}i\eta(p)\eta(p')\gamma^0\big|_{\text{eff}}\\ = \tfrac{1}{4}h_{\mu\nu}(p+p')[\gamma^\mu(p-p')^\nu + \gamma^\nu(p-p')^\mu],\end{aligned} \tag{3.4}$$

which involves the canonical stress tensor of the electron. Summing over charge space, we find the result for the causal exchange process may be written in the form of (2.6) with

$$I^{\mu\nu\kappa\lambda}=-\tfrac{1}{2}e^2 i\int d\omega_p d\omega_{p'}(2\pi)^4\delta(p+p'-Q)$$
$$\times\,\mathrm{Tr}\left((m-\gamma p)[\gamma^\kappa(p-p')^\lambda+\gamma^\lambda(p-p')^\kappa](m+\gamma p')\gamma^\mu\frac{m+\gamma(p'-q')}{(p'-q')^2+m^2}\gamma^\nu\right). \tag{3.5}$$

Evidently, this result is gravitationally and electromagnetically gauge invariant, in fact, it satisfies conditions (2.7)–(2.10), because of the causal restriction. Hence, it can be written in the form (2.12).

The evaluation of the scalar functions there is extremely straightforward. If one first works out the integration over momentum space, one has only to compute the trace of four Dirac matrices. Reducing the six resulting gauge-invariant tensors by use of the polarization vectors according to (2.18), we find ($\alpha=e^2/4\pi$)

$$\epsilon'_\mu\epsilon_\nu I^{\mu\nu\kappa\lambda}=\sum_{i=1}^{3} i\mathcal{B}_i(M^2)\theta_i^{\kappa\lambda}, \tag{3.6}$$

where

$$\mathcal{B}_1=\frac{\alpha}{12}\left(18\zeta-10\zeta^3-3(1-\zeta^2)(3-\zeta^2)\ln\frac{1+\zeta}{1-\zeta}\right), \tag{3.7a}$$

$$\mathcal{B}_2=-\frac{\alpha}{4M^2}(1-\zeta^2)\left(2\zeta-(1-\zeta^2)\ln\frac{1+\zeta}{1-\zeta}\right), \tag{3.7b}$$

$$\mathcal{B}_3=\frac{\alpha}{4M^2}(1-\zeta^2)\left(6\zeta-(3-\zeta^2)\ln\frac{1+\zeta}{1-\zeta}\right). \tag{3.7c}$$

Here we have introduced the mass M of the exchanged excitation,

$$M^2=-Q^2, \tag{3.8}$$

and the abbreviation

$$\zeta^2=1-\frac{4m^2}{M^2},\quad 1>\zeta\geq 0. \tag{3.9}$$

In the following, we will require the behavior of the amplitudes as $M^2\to\infty$, or $\zeta\to 1$:

$$\mathcal{B}_1\sim\tfrac{2}{3}\alpha, \tag{3.10a}$$

$$\mathcal{B}_{2,3}\sim\frac{1}{M^2}O\left(\frac{m^2}{M^2},\ \frac{m^2}{M^2}\ln\frac{M^2}{m^2}\right). \tag{3.10b}$$

IV. SPACE-TIME EXTRAPOLATION: FORM FACTORS

In the previous section we derived the action expression describing the causal exchange of an electron pair between its production by a virtual graviton field and its *subsequent* annihilation into a photon pair. In momentum space this corresponds to a timelike momentum of the exchanged state

$$-Q^2=M^2\geq 4m^2. \tag{4.1}$$

We remove this restriction by the process of space-time extrapolation: In essence, if x' represents the point where the exchanged pair is produced, and x the point where it is annihilated

$$\int\frac{(dQ)}{(2\pi)^4}e^{iQ(x-x')}=\int\frac{dM^2}{2\pi}d\omega_Q e^{iQ(x-x')}$$
$$=\int\frac{dM^2}{2\pi i}\Delta_+(x-x';\ M^2),\quad (x-x')^0>0$$
$$\to\int\frac{(dQ)}{(2\pi)^4}\int\frac{dM^2}{2\pi i}\frac{e^{iQ(x-x')}}{Q^2+M^2-i\epsilon},$$
$$(x-x')^0\ \text{unrestricted}. \tag{4.2}$$

Appearing in the last form here is the general propagation function of an excitation having mass M. After the causal restriction is removed in this manner, Q is, of course, no longer confined to timelike values.

In carrying out this extrapolation it is essential that the gauge-invariance properties of gravitation be maintained. Now $\theta_2^{\kappa\lambda}$ (2.18b) is identically conserved,

$$Q_\kappa\theta_2^{\kappa\lambda}=0, \tag{4.3}$$

while the corresponding property for $\theta_3^{\kappa\lambda}$ (2.18c) requires only the kinematic restriction

$$q^2=q'^2. \tag{4.4}$$

But the conservation of $\theta_1^{\kappa\lambda}$ (2.18a) necessitates the use of the photon field equations, embodied in (2.17). More generally, $\theta_1^{\kappa\lambda}$, representing the Maxwell stress tensor, (2.1), expresses the full gravitational gauge response of the photon field. The quantum corrections to this term must be invariant under the gauge transformation (2.5). This will be achieved if, before space-time extrapolation, we make the gauge transformation

$$h_{\mu\nu}(Q)\to\frac{1}{M^2}(-Q^2h_{\mu\nu}+Q_\nu Q^\lambda h_{\mu\lambda}+Q_\mu Q^\lambda h_{\nu\lambda}-Q_\mu Q_\nu h)$$
$$=-\frac{1}{M^2}R_{\mu\nu}(Q) \tag{4.5}$$

in which

$$R_{\mu\nu}(Q)=(Q^2g_{\alpha\mu}g_{\beta\nu}-Q_\nu Q_\alpha g_{\beta\mu}$$
$$-Q_\mu Q_\alpha g_{\beta\nu}+Q_\mu Q_\nu g_{\alpha\beta})h^{\alpha\beta}(Q) \tag{4.6}$$

is the gauge-invariant field strength, the (linearized) Riemann tensor.[15] [Note that in a gravitational Lorentz gauge, where

$$Q^\lambda h_{\mu\lambda} - \tfrac{1}{2} Q_\mu h = 0, \tag{4.7}$$

the Riemann tensor becomes

$$R_{\mu\nu}(Q) = Q^2 h_{\mu\nu}(Q).\,] \tag{4.8}$$

Ensuring gauge invariance by making this replacement, we are free to make a space-time extrapolation, as sketched in (4.2). After extrapolating, we can simplify $R_{\mu\nu}$ by use of (2.18a) and write the result in the form (2.20), with the form factors now appearing as (with the primitive interaction now incorporated)

$$F_1(Q) = 1 - Q^2 \frac{1}{2\pi} \int_{4m^2}^{\infty} \frac{dM^2}{M^2} \frac{\mathcal{B}_1(M^2)}{Q^2 + M^2 - i\epsilon}, \tag{4.9a}$$

$$F_{2,3}(Q) = \frac{1}{2\pi} \int_{4m^2}^{\infty} dM^2 \frac{\mathcal{B}_{2,3}(M^2)}{Q^2 + M^2 - i\epsilon}. \tag{4.9b}$$

An alternative way of appreciating the factor $-Q^2/M^2$ in (4.9a) is to regard it as enforcing the normalization of the "gravitational charge" of the photon, expressed through the stress tensor (2.1).

Note that the extrapolation embodied in letting the upper limit of the integrals in (4.9) range up to infinity is valid, since the spectral functions have significant values only for low-mass states; that is, (3.10) indicates that the spectral integrals are convergent.

As emphasized in Ref. 9, the spectral representation (4.9) is quite sufficient and convenient for most purposes. (For example, see below.) However, in order to make contact with the results of Ref. 10, we note that we can very easily evaluate the integrals occurring in (4.9), when $Q^2 > -4m^2$. Only elementary functions appear, unlike in the electron case[9,10]: in particular, we encounter

$$\int_0^1 \zeta d\zeta \frac{1}{v^2 - \zeta^2} \ln \frac{1+\zeta}{1-\zeta} = \ln^2 x, \tag{4.10}$$

where we have introduced

$$v^2 = 1 + \frac{4m^2}{Q^2}, \quad x^2 = \frac{v-1}{v+1}. \tag{4.11}$$

The integral (4.10) may be, for example, simply evaluated by contour methods. The form factors (4.9) are thus explicitly found to be

$$F_1 = 1 - \frac{2\alpha}{\pi} \left[\frac{1}{72} \frac{(-35 + 226x^2 - 35x^4)}{(1-x^2)^2} - \frac{1}{3} \frac{1+x^2}{(1-x^2)^3} (1 - 7x^2 + x^4) \ln x + \frac{x^2}{(1-x^2)^4} (1 - 4x^2 + x^4) \ln^2 x \right], \tag{4.12a}$$

$$F_2 = -\frac{\alpha}{\pi} \frac{1}{Q^2} \left[\frac{1}{12} \frac{1 + 34x^2 + x^4}{(1-x^2)^2} + \frac{2x^2(1+x^2)}{(1-x^2)^3} \ln x - \frac{4x^4}{(1-x^2)^4} \ln^2 x \right], \tag{4.12b}$$

$$F_3 = \frac{\alpha}{\pi} \frac{1}{Q^2} \left[\frac{1}{12} \frac{(-1 + 86x^2 - x^4)}{(1-x^2)^2} + \frac{6x^2(1+x^2)}{(1-x^2)^3} \ln x + \frac{2x^2}{(1-x^2)^4} (1 - 4x^2 + x^4) \ln^2 x \right]. \tag{4.12c}$$

Note that, in spite of their appearance, F_2 and F_3 are finite at $Q^2 = 0$. These explicit results agree with those of Berends and Gastmans,[10] when the linear combinations corresponding to their basis are formed. Particularly interesting is the limit

$$Q^2 \to \infty \text{ or } x \to 0;$$

then

$$F_1 \sim 1 + \frac{\alpha}{\pi} \left(\frac{35}{36} + \frac{1}{3} \ln \frac{m^2}{Q^2} \right), \tag{4.13a}$$

$$F_{2,3} \sim \frac{-\alpha}{\pi} \frac{1}{Q^2} \frac{1}{12}. \tag{4.13b}$$

The leading logarithm in (4.13a) follows immediately from the asymptotic behavior of $\mathcal{B}_1$ (3.10a), while the behavior in (4.13b) is most simply found directly by evaluating the elementary integrals

$$\frac{1}{8\pi} Q^2 \int_0^1 d\left(\frac{4m^2}{M^2}\right) f_{2,3}(\zeta), \tag{4.14}$$

where $f_{2,3}(\zeta)$ are given by the quantities in large parentheses in (3.7b), (3.7c). Note that one consequence of using the proper basis is that the leading logarithm is concentrated in a single term.

V. SCALE INVARIANCE

There are two limiting situations relating to scale invariance in this problem. On the one hand we ask how the conformal invariance of the electromagnetic field, as expressed initially by the vanishing trace (2.2), is maintained when quantum corrections are included. This question is answered by the dilation Ward identity,[8] which here for real photons requires

$$\sum_{i=1}^{3} F_i(Q) \theta^{\kappa}_{i\kappa}(q, q') \Big|_{Q=q+q'=0} = 0. \tag{5.1}$$

Since

$$\theta^{\kappa}_{1\kappa} = 0, \tag{5.2}$$

which just reexpresses (2.2), and

$$\theta^{\kappa}_{2,3\kappa} \propto Q^2, \tag{5.3}$$

the requirement (5.1) is trivially satisfied since $F_i(Q=0)$ is finite.

The situation in the opposite limit, $Q^2\to\infty$, is not so simple. We would naively anticipate that scale invariance would be achieved in the limit as the electron mass tends to zero,

$$m^2\to 0\ . \tag{5.4}$$

In fact, this is true before space-time extrapolation, since according to (3.10b)

$$\lim_{m\to 0}\mathfrak{B}_{2,3}=0\ . \tag{5.5}$$

But the trace of the stress tensor does not vanish after space-time extrapolation, since

$$\begin{aligned}\sum_{i=1}^{3}F_i(Q)\theta^{\kappa}_{i\kappa}(q,q')\\ &=-[2(\epsilon' q)(\epsilon q')-Q^2\epsilon\epsilon']Q^2(3F_2(Q)+F_3(Q))\\ &\to[2(\epsilon' q)(\epsilon q')-Q^2\epsilon\epsilon']\frac{\alpha}{3\pi}\end{aligned} \tag{5.6}$$

as $m\to 0$, according to (4.13b). This is the trace anomaly discussed by Berends and Gastmans.[10]

The origin of this phenomenon lies in an incompatibility between gauge invariance (gravitational and electromagnetic) and scale invariance. The former requires that the basis tensors $\theta_2^{\kappa\lambda}$ and $\theta_3^{\kappa\lambda}$, which are free from kinematic singularities and zeros, have (mass) dimension 4. Thus, since

$$\sum_{i=1}^{3}\theta_i^{\kappa\lambda}(q,q')F_i(Q)$$

has dimension 2, $F_{2,3}$ have dimension -2.

If we did not have the causal scaling property (5.5), we would anticipate

$$\mathfrak{B}_{2,3}\sim\frac{1}{M^2},\quad M^2\to\infty\,, \tag{5.7}$$

which is sufficient to ensure that the extrapolated spectral integrals (4.9b) converge. But then we would have

$$F_{2,3}\sim\frac{1}{Q^2}\ln\frac{Q^2}{m^2}\,,\quad Q^2\to\infty\ , \tag{5.8}$$

leading to a logarithmically growing anomaly instead of (5.6). The actual situation involves improved high-M^2 behavior of the spectral functions $\mathfrak{B}_{2,3}$, (3.10b), which leads to the improved high-Q^2 behavior of the form factors $F_{2,3}$, and hence to the "softer" anomaly given by (5.6).

The situation is clarified by contrasting with the case of quantum corrections to the electron stress tensor.[6,9] There, the dimension of the new basis tensors which emerge from electrodynamic corrections is 2, so the form factors are dimensionless. (We here incorporate an extra factor of m from the spinors.) Scaling behavior of the spectral functions then implies scaling for the form factors: (We use the notation of Refs. 6 and 9, which should not be confused with that used above.)

$$m\Pi_{2,3}\sim m^2/M^2 \tag{5.9a}$$

implies

$$\begin{aligned}-2\pi mF_{2,3}&=\int dM^2\,\frac{m\Pi_{2,3}(M^2)}{M^2+Q^2}\\ &\sim\frac{m^2}{Q^2}\ln\frac{Q^2}{m^2}\to 0,\quad m^2\to 0\ .\end{aligned} \tag{5.9b}$$

Actually, a slightly more delicate argument is required, for the spectral integration corresponding to the two-photon intermediate state extends down to zero. We find, from the explicit results given in Ref. 9, that the precise asymptotic behavior is, as $m^2\to 0$ or $Q^2\to\infty$,

$$F_2\sim\frac{\alpha}{\pi m}\,2\,\frac{m^2}{Q^2}\,, \tag{5.10a}$$

$$F_3\sim\frac{\alpha}{\pi m}\,\frac{m^2}{Q^2}\left(\ln\frac{Q^2}{m^2}-4\right)\,, \tag{5.10b}$$

leading to a vanishing trace at $m=0$:

$$\begin{aligned}t^{\mu}{}_{\mu}(Q)&=\int\frac{(dp)}{(2\pi)^4}\frac{(dp')}{(2\pi)^4}(2\pi)^4\delta(p+p'-Q)\\ &\quad\times Q^2(F_2+3F_3)\tfrac{1}{8}\psi(p)\gamma^0\psi(p')\to 0.\end{aligned} \tag{5.11}$$

The photon situation that we have discussed in this paper leads to a "scaling anomaly" precisely because of the higher dimensionality required of the basis by gauge invariance. The asymptotic form of $\mathfrak{B}_{2,3}$, as $m^2\to 0$, leads to a threshold singularity in the spectral integration in the same limit, necessarily implying a constant limiting value for the trace of the stress tensor. But the breakdown of scale invariance so induced is the weakest possible.

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[4]However, an unambiguous source-theory calculation of the particle-exchange corrections to the graviton propagation function has been carried out by A. F. Radkowski [Ann. Phys. (N.Y.) 56, 319 (1970)]; and F. A. Berends and R. Gastmans [Phys. Lett. 55B, 311 (1975)] find a cancellation of divergences in the graviton correction to the electron magnetic moment.

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[13]To make this invariance true everywhere, we incorporate a model of the photon source. See the paper by Radkowski cited in Ref. 4.

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[15]This argument, first given in Ref. 6 (with the wrong sign of $\Gamma_{\mu\nu\lambda}$), is a generalization of that used for electromagnetic form factors. See, for example, Ref. 11.