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Alternative space-time view of vector-meson dominance for virtual-photon–nucleus scattering*

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We clarify the meaning of vector-meson dominance for virtual photons via a coupled-channel formalism, in which the photon can interact only by converting itself into a vector meson, the conversion occurring anywhere in space. We calculate the relative contributions of the different conversion regions, discuss their physical interpretation, and establish the equivalence of this approach to the usual treatment.

I. INTRODUCTION

In this paper the meaning of vector-meson dominance (VMD) for virtual photons is clarified via an alternative, but equivalent, treatment to that of Refs. 1 and 2. In the usual treatment[1] the photon can interact either directly with the nucleus or by first converting itself into a vector meson; all interactions are described by potentials confined within the nucleus and are proportional to one another. In this approach maximum shadowing, comparable to hadronic shadowing, occurs when the energy is much larger than $(-q^2+m_V^2)/2M$ (M = nucleon mass m_V = vector-meson mass), owing to a cancellation of direct and indirect photon potentials. It is in this sense that the term VMD is valid, despite the presence of the direct photon potential. However, it is more customary to think of VMD as implying that the photon can interact only by converting itself into a vector meson, and that the conversion can occur anywhere in space. As we will show, this approach is equivalent to the usual one; this is to be expected, because of the canonical equivalence of these two approaches in Lagrangian field theory.[3] Nevertheless, to the best of our knowledge, this equivalence has never been demonstrated for the nuclear problem. Since the photon–vector-meson conversion can occur even in the absence of the target, a renormalization of the relevant amplitudes is needed before the equivalence can be shown.

In Sec. II we describe our approach, discuss the renormalization problem, and establish the equivalence to the usual approach. In Sec. III we discuss the case of a target of constant density and the meaning of VMD for $-q^2 \neq 0$. We restrict ourselves to the case of constant rather than more realistic density, because the problem can then be solved analytically, thus facilitating the physical interpretation of this alternative approach. For example, this physical picture requires that the photon–vector-meson conversion occur both outside as well as inside the nucleus. It would then be interesting to calculate the relative contributions of the different conversion regions. Although the total contribution from a nucleus of total constant density is well known,[1,2] this question of the relative contributions of the different conversion regions could not have been formulated in the usual approach.

II. ALTERNATIVE APPROACH TO VMD

We start, as in Ref. 1, with a set of coupled equations for the photon and vector-meson wave functions ψ_γ and ψ_V,

$$(\nabla^2+k^2)\psi_\gamma=\sum_V \tilde{U}_{\gamma V}\psi_V\,,$$
$$(\nabla^2+k_V^2)\psi_V=\tilde{U}_{V\gamma}\psi_\gamma+\tilde{U}_V\psi_V\,, \tag{1}$$

where k (k_V) is the virtual-photon (vector-meson) momentum, $\tilde{U}_{\gamma V}$ is the "potential" responsible for the conversion of the photon to the vector meson V, $\tilde{U}_{\gamma V}$ is that for the reverse conversion, and U_V is the potential describing the strong interaction of the vector meson V with the target. Unlike Ref. 1, there is no U_γ term (direct photon interaction) and the $\tilde{U}_{V\gamma}=\tilde{U}_{\gamma V}$ potentials are constants throughout space (not confined within the target area). This is because, as explained in the Introduction, the photon interaction in our approach can only take place through conversion, and conversion can occur anywhere in space. Thus if for convenience we let

$$\lambda_V' \equiv \frac{i\tilde{U}_{\gamma V}}{(2k)^{1/2}(2k_V)^{1/2}} \tag{2a}$$

and

$$\lambda_V \equiv \frac{i\tilde{U}_V}{2k_V}\,, \tag{2b}$$

then this approach requires λ_V' to be constant in space and λ_V to be zero outside the target. In order to preserve well-defined, asymptotic photon states, we limit the range of λ_V' to a distance $2z_0$ centered around the target, with z_0 ultimately going to infinity.

As in Ref. 1 we solve Eqs. (1) in the eikonal approximation. In terms of the eikonal wave functions

$$\chi_\gamma = e^{-ikz}\psi_\gamma, \quad \chi_V = e^{-ik_V z}\psi_V, \tag{3}$$

Eqs. (1) become approximately

$$\frac{d}{dz}\chi_V + \lambda_V\chi_V = -\lambda_V' e^{iz\delta_V}, \quad \frac{d}{dz}\chi_\gamma = -\sum_V e^{-iz\delta_V}\lambda_V'\chi_V, \tag{4}$$

where

$$\delta_V \equiv k - k_V \simeq \frac{-q^2 + m_V^2}{2\nu}, \quad k \simeq k_V \simeq \nu \tag{5}$$

in the high-energy limit. In Eqs. (5) ν and q^2 are respectively the laboratory energy and the squared mass of the virtual photon, and m_V is the mass of the vector meson V. In order to study the cross section for the photon to scatter off the target, we need the solution of Eqs. (4) satisfying the boundary conditions $\chi_\gamma(z=-z_0)=1$ and $\chi_V(z=-z_0)=0$; this is

$$\chi_V(z) = -\lambda_V'\int_{-z_0}^{z} dz'\, e^{iz'\delta_V}\exp\left[-\int_{z'}^{z} dt\,\lambda_V(t)\right], \tag{6}$$

$$\chi_\gamma(z) - 1 = \sum_V (\lambda_V')^2\int_{-z_0}^{z} dz'\, e^{-iz'\delta_V} \times\int_{-z_0}^{z'} dz''\, e^{iz''\delta_V}\exp\left[-\int_{z''}^{z'} dt\,\lambda_V(t)\right]. \tag{7}$$

The scattering from the target is described by the difference of the eikonal wave functions in the presence and in the absence of the target potential λ_V evaluated at $z=z_0$. This difference can be calculated from Eqs. (6) and (7); we obtain

$$\chi_V(z_0) - \chi_V^{\lambda_V=0}(z_0) = -\lambda_V'\int_{-z_0}^{z_0} dz\, e^{iz\delta_V}\left\{\exp\left[-\int_{z}^{z_0} dt\,\lambda_V(t)\right] - 1\right\}, \tag{8}$$

$$\chi_\gamma(z_0) - \chi_\gamma^{\lambda_V=0}(z_0) = \sum_V(\lambda_V')^2\int_{-z_0}^{z_0} dz\, e^{-iz\delta_V}\left(\int_{-z_0}^{z} dz'\, e^{iz'\delta_V}\left\{\exp\left[-\int_{z'}^{z} dt\,\lambda_V(t)\right] - 1\right\}\right). \tag{9}$$

This subtraction is the renormalization alluded to in the Introduction. Performing an integration by parts in the z' integral (by differentiating the quantity in the curly brackets) Eq. (9) becomes

$$\chi_\gamma(z_0) - \chi_\gamma^{\lambda_V=0}(z_0) = \sum_V(\lambda_V')^2\int_{-z_0}^{z_0} dz\, e^{-iz\delta_V}\left(\frac{i}{\delta_V}e^{-iz_0\delta_V}\left\{\exp\left[-\int_{-z_0}^{z} dt\,\lambda_V(t)\right] - 1\right\} - \frac{1}{i\delta_V}\int_{-z_0}^{z} dz'\, e^{iz'\delta_V}\lambda_V(z')\exp\left[-\int_{z'}^{z} dt\,\lambda_V(t)\right]\right). \tag{10}$$

Since we are ultimately interested in the limit $z_0\to\infty$, the factor $e^{-iz_0\delta_V}$ will average to zero, so we drop it. Performing one more integration by parts in the z integral (by differentiating the quantity in the parentheses) in the limit $z_0\to\infty$ Eq. (10) finally becomes

$$\chi_\gamma(\infty) - \chi_\gamma^{\lambda_V=0}(\infty) = \sum\left(\frac{\lambda_V'}{\delta_V}\right)^2\int_{-\infty}^{\infty} dz\,\lambda_V(z) - \sum_V\left\{\left(\frac{\lambda_V'}{\delta_V}\right)^2\int_{-\infty}^{\infty} dz\, e^{-iz\delta_V}\lambda_V(z)\int_{-\infty}^{z} dz'\, e^{iz'\delta_V}\lambda_V(z')\exp\left[-\int_{z'}^{z} dt\,\lambda_V(t)\right]\right\}. \tag{11}$$

Similarly, performing an integration by parts, Eq. (8) becomes

$$\chi_V(\infty) - \chi_V^{\lambda_V=0}(\infty) = -i\frac{\lambda_V'}{\delta_V}\int_{-\infty}^{\infty} dz\, e^{iz\delta_V}\lambda_V(z) \times\exp\left[-\int_{z}^{\infty} dt\,\lambda_V(t)\right]. \tag{12}$$

Now, in the limit $\delta_V R\to 0$ (e.g., $\nu\to\infty$ for fixed q^2, R is the radius of the target) Eqs. (12) and (11) become respectively

$$\chi_V(\infty) - \chi_V^{\lambda_V=0}(\infty) = -i\frac{\lambda_V'}{\delta_V}\left\{\exp\left[-\int_{-\infty}^{\infty} dt\,\lambda_V(t)\right] - 1\right\}, \tag{13}$$

$$\chi_\gamma(\infty) - \chi_\gamma^{\lambda_V=0}(\infty) = -\sum_V\left(\frac{\lambda_V'}{\delta_V}\right)^2\left\{\exp\left[-\int_{-\infty}^{\infty} dt\,\lambda_V(t)\right] - 1\right\}. \tag{14}$$

Thus, we see that in this limit [using Eqs. (2a) and (5)]

$$\chi_\gamma(\infty)-\chi_\gamma^{\lambda_V=0}(\infty)=-i\,\frac{\lambda_V'}{\delta_V}\{\chi_V(\infty)-\chi_V^{\lambda_V=0}(\infty)\}$$
$$\equiv\frac{\tilde{U}_{\gamma V}}{-q^2+m_V^2}\{\chi_V(\infty)-\chi_V^{\lambda_V=0}(\infty)\}\,. \tag{15}$$

Hence, if we define

$$f_V\equiv\frac{em_V^2}{\tilde{U}_{\gamma V}}\,, \tag{16}$$

then Eq. (15) is the usual VMD relation.

For finite values of $\delta_V R$ (e.g., low-energy real-photon scattering or in the Bjorken limit) a simple relation such as Eq. (15) is no longer valid. The physical reason for this is discussed in Sec. III.

Let us now compare Eqs. (11) and (12) with the results of Ref. 1; combining Eqs. (2) and (3) of Ref. 1 we obtain

$$\chi_\gamma(\infty)-\chi_\gamma^{\lambda_V=0}(\infty)$$
$$=\int_{-\infty}^{\infty}dz\,\frac{U_\gamma(z)}{2ik}+\sum_V\int_{-\infty}^{\infty}dz\,e^{-iz\delta_V}\,\frac{U_{\gamma V}(z)}{2ik}$$
$$\times\int_{-\infty}^{z}dz'\,e^{iz'\delta_V}\,\frac{U_{V\gamma}(z')}{2ik}$$
$$\times\exp\left[\int_{z'}^{z}dt\,\frac{U_V(t)}{2ik_V}\right]. \tag{17}$$

It is then clear that our Eq. (11) is consistent with Eq. (17), and Eq. (12) is consistent with Eq. (2) of Ref. 1, provided the following relations hold:

$$U_\gamma=\sum_V c_V^2U_V\equiv\sum\frac{e^2}{f_V^2}\,\frac{m_V^4}{(-q^2+m_V^2)^2}\,U_V\,,$$
$$U_{V\gamma}=U_{\gamma V}=c_VU_V\,, \tag{18}$$

and

$$\tilde{U}_V=U_V\,. \tag{19}$$

Equations (18) are the usual vector-dominance relations with the vector-meson propagator put in.

We have thus shown that the following two descriptions are equivalent. A photon interacts with the target by first converting to a vector meson anywhere in space ($\tilde{U}_{\gamma V}$). Or else, it either interacts directly (U_γ) or through conversion into a vector meson ($U_{V\gamma}$ and U_V), all interactions being confined within the target, and the potentials satisfying Eqs. (18). The equivalence of the results of these two descriptions reflects the fact that in Lagrangian field theory the Lagrangian that describes VMD by coupling the vector-meson and photon fields to each other and both to the hadronic current is equivalent, by a canonical transformation, to a Lagrangian, in which only the vector meson couples directly to this current.[3] The vector-meson propagator ($\sim 1/\delta_V$) appears automatically in Eq. (15) because this is a fully quantum-mechanical treatment with relativistic kinematics.

III. TARGET OF CONSTANT DENSITY

We proceed now to the evaluation of Eqs. (6) and (7) for the case of a target of constant density and radius R. Since the conversion can take place inside or outside the target, it is instructive to divide the z axis accordingly. Thus for every impact parameter b (see Fig. 1) we divide the z axis into three regions: A: $-z_0\le z\le -z_1\equiv-(R^2-b^2)^{1/2}$, B: $-z_1\le z\le z_1$, C: $z_1\le z\le z_0$ (with $z_0\to\infty$). Then

$$\lambda_V(z)=\lambda_V\,\theta(z_1-|z|)\,, \tag{20}$$

and Eq. (8) becomes

$$\chi_V(\infty)-\chi_V^{\lambda_V=0}(\infty)=-\lambda_V'(\Delta J_A+\Delta J_B+\Delta J_C)\,, \tag{21}$$

where

$$\Delta J_A=\frac{1}{i\delta_V}\,e^{-iz_1\delta_V}(e^{-2\lambda_Vz_1}-1)\equiv J_A-J_A^{\lambda_V=0}\,,$$
$$\Delta J_B=\frac{1}{\xi_V}\,e^{-\lambda_Vz_1}(e^{\xi_Vz_1}-e^{-\xi_Vz_1})$$
$$-\frac{1}{i\delta_V}(e^{i\delta_Vz_1}-e^{-i\delta_Vz_1})\,, \tag{22}$$
$$\Delta J_C=0\,.$$

When Eqs. (22) are substituted in Eq. (21), the

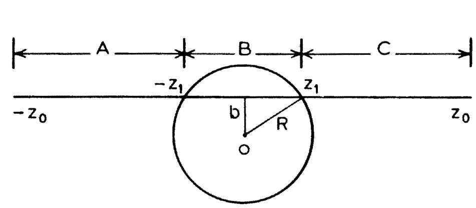

FIG. 1. The different regions introduced for the evaluation of Eqs. (6) and (7).

final result is

$$\chi_V(\infty)-\chi_V^{\lambda_V=0}(\infty)=-i\lambda_V\lambda_V'\frac{e^{iz_1\delta_V}}{\delta_V\xi_V}(1-e^{-2\xi_V z_1})\,. \tag{23}$$

Similarly we obtain from Eq. (9)

$$\chi_\gamma(z_0)-\chi_\gamma^{\lambda_V=0}(z_0)=\sum_V(\lambda_V')^2(\Delta I_{AA}+\Delta I_{AB}+\Delta I_{AC}+\Delta I_{BB}+\Delta I_{BC}+\Delta I_{CC})\,, \tag{24}$$

where ΔI_{ij} means that the region of integration is such that z is in the interval (i) and z' is in the interval (j). Explicitly

$$\Delta I_{AA}\equiv I_{AA}-I_{AA}^{\lambda_V=0}=0\,,$$
$$\Delta I_{CC}=0\,,$$
$$\Delta I_{BB}=-\frac{2\lambda_V z_1}{i\delta_V\xi_V}-\frac{\lambda_V(\lambda_V+2i\delta_V)}{\delta_V{}^2\xi_V{}^2}+\left(\frac{e^{-2\lambda_V z_1}}{\xi_V{}^2}+\frac{1}{\delta_V{}^2}\right)e^{-2i\delta_V z_1}\,, \tag{25}$$
$$\Delta I_{AC}=\frac{1}{\delta_V{}^2}e^{-2i\delta_V z_1}(1-e^{-2\lambda_V z_1})\,,$$
$$\Delta I_{AB}=\Delta I_{BC}=-\frac{i}{\delta_V}\left[\frac{-\lambda_V}{i\delta_V\xi_V}-e^{-2iz_1\delta_V}\left(\frac{e^{-2\lambda_V z_1}}{\xi_V}+\frac{i}{\delta_V}\right)\right]\,.$$

Collecting all terms we finally have

$$\chi_\gamma(\infty)-\chi_\gamma^{\lambda_V=0}(\infty)=\sum_V(\lambda_V')^2\left\{-\frac{\lambda_V}{i\delta_V\xi_V}\left[2z_1+\frac{\lambda_V}{i\delta_V\xi_V}(1-e^{-2\xi_V z_1})\right]\right\}\,. \tag{26}$$

We next discuss the physical content of Eqs. (22) and (25). First notice that

$$\Delta I_{iC}=\frac{e^{-iz_1\delta_V}}{i\delta_V}\Delta J_i\quad(i=A,B,C)\,. \tag{27}$$

This is the generalization of the VMD relation (15) to the case $\delta_V\neq 0$. The proportionality factor between ΔI_{iC} and ΔJ_i is nothing more than the matrix element for V-γ transition in region C and is therefore proportional to

$$\int dz\,e^{-ikz}e^{ik_V z}=\frac{e^{-iz_1\delta_V}}{i(\delta_V-i\epsilon)}\equiv G_V\,. \tag{28}$$

Similarly one can show that the γ-V transition in region A is also proportional to G_V.

Thus, in the spirit of vector dominance, one expects that the vector-meson scattering amplitude is proportional to $\Delta J_A/G_V=e^{-2\lambda_V z_1}-1$. Indeed, this is exactly what one gets if one solves Eqs. (1) in the eikonal approximation for the vector-meson elastic scattering amplitude. Of course, the full elastic photon amplitude contains in addition the terms ΔI_{iB} $(i=A,B)$, which have no counterpart in the vector-meson production amplitude. Thus, unless the ΔI_{iC} terms dominate, the above proportionality will not hold for the full amplitude. Since $\Delta I_{AB}=\Delta I_{BC}$, this dominance will occur only if ΔI_{AC} is large compared to all other terms. This is the case when $\delta_V R\to 0$, since then one can easily show from Eqs. (25) that

$$\Delta I_{AC}\simeq\frac{1}{\delta_V{}^2}(1-e^{-2\lambda_V z_1})\,,$$
$$\Delta I_{BB}\simeq\frac{2iz_1}{\delta_V}+\frac{1}{\lambda_V{}^2}e^{-2\lambda_V z_1}\,, \tag{29}$$
$$\Delta I_{AB}=\Delta I_{BC}\simeq\frac{i}{\delta_V\lambda_V}e^{-2\lambda_V z_1}\,.$$

After this paper was submitted for publication, it was brought to our attention that recently another discussion of space-time aspects of vector dominance has been given.[4]

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