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Comments and Addenda

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Symmetry schemes, mixing angles, and meson decays*

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The $V \rightarrow P\gamma$, $P \rightarrow PP\gamma$, $P \rightarrow \gamma\gamma$, and $V \rightarrow PPP$ decays are investigated in two schemes that have had a measured degree of success in the understanding of $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ decays. These are (i) a nonet-breaking but SU(3)-preserving scheme with noncanonical angles θ_P and θ_V and (ii) an SU(3)-breaking scheme with canonical θ_P and θ_V. In a fit to all available data the problem of understanding the experimental $\rho \rightarrow \pi\gamma$ rate is much worse than the case where a fit is made to only the $V \rightarrow P\gamma$ data.

The recent measurements[1] of vector-meson radiative-decay widths have generated much interest in the symmetry and symmetry-breaking structure of the $VP\gamma$ vertex. O'Donnell[2] has investigated the feasibility of describing these rates in a nonet scheme; we[3,4] have suggested a variety of SU(3)-breaking structures and Boal, Graham, and Moffat[5] have tried a scheme which breaks the nonet symmetry while preserving SU(3) symmetry. The approaches of Refs. 3–5 have had a measured degree of success in understanding the $V \rightarrow P\gamma$ decays. Using mixing angles similar to the canonical ones of the quadratic mass formula, we were able to account for all radiative rates except $\Gamma(\rho \rightarrow \pi\gamma)$ in our *ABCD* model.[3,4] Boal, Graham, and Moffat[5] in their nonet-breaking scheme obtained good predictions with a vector-mixing angle of 24°, which is somewhat lower than that suggested by the inverse-square mass formula. The fits of Refs. 3–5 were confined to the $V \rightarrow P\gamma$ decays.

Vector-meson dominance (VMD) allows us to relate the amplitudes for such processes as $P \rightarrow 2\gamma$, $V \rightarrow PPP$, and $P \rightarrow PP\gamma$ to the $VP\gamma$ amplitude. In this paper we have calculated the rates for $\pi \rightarrow 2\gamma$, $\eta \rightarrow 2\gamma$, $X^0 \rightarrow 2\gamma$, $\omega \rightarrow 3\pi$, $\phi \rightarrow 3\pi$, $\eta \rightarrow \pi\pi\gamma$, and $X^0 \rightarrow \pi\pi\gamma$ for the *ABCD* model[3,4] and the nonet-symmetry-breaking model.[5] These models have been successful in the understanding of the radiative-decay rates with the exception of the $\rho \rightarrow \pi\gamma$ rate. We are aware of a few other calculations[6,7] which discuss the radiative decays in conjunction with other meson decays — some of these works were done before the recent experimental rates were available.

Suppressing the Lorentz structure, the $V^m \rightarrow P^i\gamma$ amplitudes for the two schemes are the following:

(i) SU(3) symmetry with broken nonet symmetry[5]:

$$g_{V^m P^i \gamma} = g d_{\min}\left(\delta_{n3} + \frac{1}{\sqrt{3}}\,\delta_{n8}\right) \quad (m, i = 1, \ldots, 8) \tag{1}$$

[when i (or m) is a singlet, g is replaced by f (or f')] and

(ii) the *ABCD* model[3,4]:

$$g_{V^m P^i \gamma} = \{A d_{\min} + \tfrac{1}{2}B(d_{8ik}d_{kmn} - d_{8mk}d_{kin} - d_{8nk}d_{kim}) + (C + \tfrac{1}{6}B)(\delta_{8m}\delta_{in} + \delta_{8n}\delta_{im}) + (D + \tfrac{1}{6}B)\delta_{8i}\delta_{mn}\}[\delta_{n3} + (1/\sqrt{3})\delta_{n8}] \quad (m, i = 0, \ldots, 8; k = 1, \ldots, 8). \tag{2}$$

The calculation of rates closely follows that of Ref. 7 with current[8] particle masses and widths.

We present the results in Table I. The model parameters are obtained by fitting to the five available decays of the kind $V \rightarrow P\gamma$ and five other experimental rates of the kind $V \rightarrow PPP$, $V \rightarrow PP\gamma$, and $P \rightarrow 2\gamma$. These rates are indicated in brackets in Table I. The value $g_\rho^2/4\pi = g_{\rho\pi\pi}^2/4\pi = 2.93$ (Ref. 9) is

TABLE I. Meson-decay rates. Predictions based on all available rates. See text for explanation.

Decay mode	Nonet breaking (a)	Nonet breaking (b)	Nonet breaking (c)	*ABCD* (a)	Exp.
$\phi\rightarrow\pi\gamma$ (keV)	(5.9)	(4.6)	(6.1)	(5.9)	5.9 ± 2.1 (Ref. 1)
$\rho^-\rightarrow\pi^-\gamma$ (keV)	(81)	(49)	(87)	(86)	35 ± 10 (Ref. 1)
$K^{0*}\rightarrow K^0\gamma$ (keV)	(180)	(110)	(190)	(130)	75 ± 35 (Ref. 1)
$\omega\rightarrow\pi\gamma$ (keV)	(910)	(1200)	(890)	(970)	870 ± 80 (Ref. 8)
$\phi\rightarrow\eta\gamma$	(160)	(98)	(140)	(65)	70 ± 16 (Refs. 1 and 8)
$\rho\rightarrow\eta\gamma$ (keV)	53	42	50	45	<160[a]
$K^{+*}\rightarrow K^+\gamma$ (keV)	44	27	48	150[b]	<80 (Ref. 8)
$\omega\rightarrow\eta\gamma$ (keV)	10	32	28	3.4	<50 (Ref. 8)
$\phi\rightarrow X^0\gamma$ (keV)	1.0	1.7	0.54	0.17	...
$X^0\rightarrow\rho\gamma$ (keV)	160	270	4.5	150	<270 (Ref. 8)
$X^0\rightarrow\omega\gamma$ (keV)	14	7.0	0.48	11	<80 (Ref. 8)
$\omega\rightarrow 3\pi$ (MeV)	(8.3)	(10)	(8.0)	(8.8)	9.0 ± 0.4 (Ref. 8)
$\phi\rightarrow 3\pi$ (keV)	(670)	(520)	(680)	(660)	660 ± 90 (Ref. 8)
$\eta\rightarrow\pi\pi\gamma$ (keV)	(0.13)	(0.10)	(0.12)	(0.11)	0.13 ± 0.03 (Ref. 8)
$X^0\rightarrow\pi\pi\gamma$ (keV)	150	260	4.2	140	...
$\pi\rightarrow 2\gamma$ (eV)	(7.0)	(4.3)	(7.6)	(7.4)	7.92 ± 0.42[c]
$\eta\rightarrow 2\gamma$ (keV)	(0.41)	(0.37)	(0.39)	(0.33)	0.324 ± 0.046[d]
$X^0\rightarrow 2\gamma$ (keV)	9.0	15	0.74	6.1	<19 (Ref. 8)
$\Gamma(X^0\rightarrow 2\gamma)/\Gamma(X^0\rightarrow\rho\gamma)$	0.056	0.054	0.17	0.042	0.0693 ± 0.0120 (Ref. 8)

[a] M. E. Nordberg *et al.*, Phys. Lett. 51B, 106 (1974).
[b] See Ref. 4 for a discussion of this rate.
[c] A. Browman *et al.*, Phys. Rev. Lett. 33, 1400 (1974).
[d] A. Browman *et al.*, Phys. Rev. Lett. 32, 1067 (1974).

used throughout. The solutions labeled (a) use mixing angles $\theta_P=-10°$ and $\theta_V=35°$. Boal, Graham, and Moffat[5] used $\theta_P=-10°$ and then adjusted θ_V to obtain a good fit to $V\rightarrow P\gamma$ decays. The choice (b) of $\theta_P=-10°$ and $\theta_V=24°$ was the one used in Ref. 5. When fitting to all available rates, we found that if $\theta_P=-10°$, the best solutions occurred for θ_V in the range 32°–35°. The label (c) refers to $\theta_P=-24°$ and $\theta_V=37°$, the linear-mass-formula predictions. This combination of angles was also tried in Ref. 5.

The table indicates that the solution obtained when a fit is made to all available data has larger $\rho\rightarrow\pi\gamma$ and $K^{0*}\rightarrow K^0\gamma$ widths and, in general, displays smaller symmetry breaking than the solution where fit is made[3-5] to $V\rightarrow P\gamma$ data only. This can be understood as follows. Independent of the symmetry-breaking mechanism and mixing angles, VMD alone fixes the ratios $\Gamma(\omega\rightarrow 3\pi)/\Gamma(\omega\rightarrow\pi\gamma)$, $\Gamma(\phi\rightarrow 3\pi)/\Gamma(\phi\rightarrow\pi\gamma)$, and $\Gamma(\rho\rightarrow\pi\gamma)/\Gamma(\pi\rightarrow 2\gamma)$.[10] The situation can be summarized as follows:

$$\frac{\Gamma(\omega\rightarrow 3\pi)}{\Gamma(\omega\rightarrow\pi\gamma)}:\ \text{expt. ratio}=10.3\pm1.0,$$

$$\text{VMD ratio}=9.1, \tag{3}$$

$$\frac{\Gamma(\phi\rightarrow 3\pi)}{\Gamma(\phi\rightarrow\pi\gamma)}:\ \text{expt. ratio}=112\pm42,$$

$$\text{VMD ratio}=112, \tag{4}$$

$$\frac{\Gamma(\rho\rightarrow\pi\gamma)}{\Gamma(\pi\rightarrow 2\gamma)}:\ \text{expt. ratio}=(4.4\pm1.3)\times10^3,$$

$$\text{VMD ratio}=11.5\times10^3. \tag{5}$$

As VMD is consistent with the experimental numbers for the first two ratios, inclusion of $\omega\rightarrow 3\pi$ and $\phi\rightarrow 3\pi$ simply produces more bias toward the measured $\omega\rightarrow\pi\gamma$ and $\phi\rightarrow\pi\gamma$ rates. Inclusion of $\pi\rightarrow 2\gamma$ on the other hand produces a bias away from the measured value of $\rho\rightarrow\pi\gamma$. Note also that $\Gamma(\eta\rightarrow 2\gamma)/\Gamma(\eta\rightarrow\pi\pi\gamma)$ has an experimental value of (2.49 ± 0.67). The theoretical value of this ratio is not determined by VMD alone. It depends on the symmetry scheme and also on the mixing angles. The table shows that the *ABCD* model gives a value of 3.0 for this ratio.

We conclude that the nonet-breaking scheme[5] fares well when fitted to the known $V\rightarrow P\gamma$ rates with mixing angle "b" but does not fare so well in predicting the $V\rightarrow PPP$, $P\rightarrow 2\gamma$, and $P\rightarrow PP\gamma$ rates. When all available rates are fitted with mixing angle "a" or "c," the $V\rightarrow PPP$, $P\rightarrow 2\gamma$, and $P\rightarrow PP\gamma$ rates do agree with experiment; however, the $\rho\rightarrow\pi\gamma$, $K^{0*}\rightarrow K^0\gamma$, and $\phi\rightarrow\eta\gamma$ rates are high. The *ABCD* model[3,4] when fitted to all rates produces $\rho\rightarrow\pi\gamma$ and $K^{0*}\rightarrow K^0\gamma$ rates which are larger than those obtained by fitting the model to the $V\rightarrow P\gamma$ rates only. There is no problem with other P

$\to PP\gamma$ and $P\to 2\gamma$ rates. A fit to all known data only aggravates the problem we had[3,4] in understanding the experimental $\rho\to\pi\gamma$ rate.

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