PHYSICAL REVIEW D VOLUME 15, NUMBER 1 1 JANUARY 1977

Multiparticle production in high-energy hadron-nucleus collisions

Meng Ta-chung

Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794
and Institut für theoretische Physik der Freien Universität Berlin, Berlin, Germany*

(Received 26 February 1976; revised manuscript received 14 September 1976)

An attempt is made to understand the gross features of multiparticle production in high-energy hadron-nucleus collisions (including the nuclear-size dependence of inclusive cross sections at large transverse momenta and the energy and atomic-number dependence of average multiplicity) in terms of a simple physical picture. The main characteristics of this picture are: (a) The time needed for the formation of multibody final states in hadron-hadron collisions at high energies is so long that in a high-energy hadron-nucleus multiparticle-production process the nucleons along the path of the incident hadron inside the target nucleus can be viewed as acting collectively, and in the first-order approximation can be considered as a single object—an "effective target." (b) This hadron–effective-target collision process can be described by the same physical picture as that used to describe the collision between two hadrons. In particular, such a collision is either gentle (fragmentation) or violent (violent collision). (c) The mass of the effective target is proportional to its "average thickness." For the sake of simplicity and definiteness, the ansatz $M = A^{1/3}M_1$ (here A is the atomic mass number of the nucleus, M and M_1 are the mass of the effective target and that of the proton, respectively) is used to carry out the illustrative examples. Arguments for this picture are presented. Further experiments are suggested.

I. MAIN FEATURES OF EXPERIMENTAL DATA

Striking results have been obtained in recent high-energy hadron-nucleus multiparticle-production experiments. Taken together with the earlier experimental findings, the main features of the existing data can be briefly summarized as follows[1]:

(*i*) *Nuclear-size dependence of inclusive distribution at large transverse momenta.* Experiments on the production of hadrons[2] and leptons[3] with large transverse momenta ($p_\perp \gtrsim 2$ GeV/c, say) at high incident energy ($E_{\text{lab}} = 300$ GeV) and large angles ($\theta \approx 90°$ in the c.m. system of the incident proton and a single nucleon at rest) show that the effective number of nucleons A_{eff} given by[2,3]

$$A_{\text{eff}} = \frac{d\sigma/d^3p(p_\perp, \theta_{\text{c.m.}} \approx 90°; E_{\text{lab}} = 300\ \text{GeV}, A)}{d\sigma/d^3p(p_\perp, \theta_{\text{c.m.}} \approx 90°; E_{\text{lab}} = 300\ \text{GeV}, 1)} = A^{\alpha(p_\perp)}, \tag{1}$$

where A is the atomic mass number of the nucleus and $d\sigma/d^3p$ is the single-particle inclusive cross section, has the following properties: The exponent $\alpha(p_\perp)$ increases with $p_\perp$ and it is greater than 1 for $p_\perp \gtrsim 2.5$ GeV/c. Furthermore, there are indications that the increase of α with $p_\perp \gtrsim 2.5$ GeV/c. Furthermore, there are indications that the increase of α with $p_\perp$ is faster in cases where the mass of the produced particle is larger.

(*ii*) *Energy and nuclear-size dependence of average multiplicity.* It has been observed at Fermilab[4] and at cosmic-ray energies[5] that the ratio

$$R_A = \langle n \rangle_A / \langle n \rangle_1 \tag{2}$$

is (a) roughly independent of the incident energy, and (b) slowly increasing with increasing A. Here $\langle n \rangle_A$ is the average number of charged relativistic particles ($\beta \geq 0.7$) produced in an inelastic collision between a hadron and a nucleus[6] with atomic mass number A, and $\langle n \rangle_1$ is the average number of charged relativistic particles produced in an inelastic collision of the same hadron with a proton. The A dependence in (b) has been parameterized in the form $R_A = A^\alpha$ where the following experimental results for α are given in the literature[7]. $\alpha = 0.15 \pm 0.06$: Feinberg.[1] $\alpha = 0.131 \pm 0.005$: Gurtu *et al.*[5] $\alpha \approx 0.12$: AALMT collaboration.[4] $\alpha \approx 0.13$: Jain *et al.*[4] $\alpha = 0.129 \pm 0.004$: Vishwanath.[5]

(*iii*) *Frequency distribution of N_h and correlation between N_h and $\langle n_s \rangle$ in emulsion experiments.* It is found in emulsion experiments[8] (a) the distribution with respect to N_h, the number of "heavy particles" with $\beta < 0.7$ (i.e., the sum of numbers of "black" and the "grey" tracks), and their mean values $\langle N_h \rangle$ are roughly independent of the incident energy; (b) For $N_h = 0, 1.2$, the value of the average number of "shower particles" $\langle n_s \rangle$ is independent of the nuclear size; (c) For large N_h ($N_h \geq 7$, say), $\langle n_s \rangle$ is increasing with increasing N_h.

(*iv*) *Angular dependence of the secondary charged relativistic particles.* It has been observed in both counter[9] and emulsion[10] experiments that the increase of multiplicity of relativistic particles (cf. ii and iii) with increasing nuclear size (A dependence) occurs entirely at large angles. Furthermore, it is also seen that the A dependence in different angular regions depends on n_s. While no difference in angular distributions in proton-nucleus and proton-proton reactions is seen for $n_s \leq 8$ and for $9 \leq n_s \leq 16$, the A dependence for $n_s \geq 17$ is very strong at large angles.

(*v*) *Production of energetic protons*. In bubble-chamber π^--Ne collision experiments[11] at 10.5 and at 200 GeV/c it is seen that an unexpected large number of energetic protons ($p_{\text{lab}} > 1$ GeV/c) are produced. The number of these protons seems to increase linearly with the number of "shower tracks" (lightly ionized or identified pions). Furthermore, the ratio of the number of fast protons produced in pion-Neon to the average number of fast protons produced in a pion-proton inelastic collision is found[12] to be approximately equal to $\bar{\nu} = A\sigma_{\text{in}}(\pi p)/\sigma_{\text{in}}(\pi\text{Ne})$, where $\sigma_{\text{in}}(\pi p)$ and $\sigma_{\text{in}}(\pi\text{Ne})$ are the total inelastic πp and πNe cross section, respectively.

(*vi*) *Linear dependence of dispersion on average multiplicity*. It is observed[13] that not only in hadron-hadron but also in hadron-nucleus processes the average multiplicity $\langle n\rangle$ and the dispersion $D = (\langle n^2\rangle - \langle n\rangle^2)^{1/2}$ obey a linear law: $D = a\langle n\rangle + b$, where a and b are constants.

A number of ideas and models have already been discussed[14] in connection with these experimental findings.

II. A SIMPLE PHYSICAL PICTURE

In the present paper an attempt is made to understand the empirical facts summarized in Sec. I in terms of a simple physical picture. The main characteristics of this picture for high-energy hadron-nucleus multiparticle-production processes are the following: (a) The time needed for the formation of multibody final states in hadron-hadron collisions at high energies is so long that, in a high-energy hadron-nucleus multiparticle-production process, the nucleons in the path of the incident hadron inside the target nucleus can be viewed as acting collectively and in the first-order approximation can be considered as a single object—an "effective target." (b) This hadron–effective-target process can be described by the same physical picture as that used to describe the collision between two hadrons. In particular, such a collision is either gentle (fragmentation) or violent (violent collision). (c) The mass of the effecttive target is proportional to its "average thickness." For the sake of simplicity and definiteness, the ansatz $M = A^{1/3}M_1$ (here A is the atomic mass number of the nucleus, M and M_1 are the mass of the effective target and that of the proton, respectively) is used to carry out the illustrative examples in this paper.

The idea that in high-energy hadron-nucleus collision the nucleons along the path of the incident hadron inside the nucleons may act collectively [see (a) of this section] is already very old.[15a] Also the ansatz for the mass of the effective target given in (c) has already been discussed several times in the literature.[15b] The purpose of the present paper is to show the following: The main features of the existing data, in particular those in connection with the large-$p_\perp$ phenomena, can be readily understood in terms of the assumptions (a) and (c) *provided that the simple physical picture*[16,17] *which has been used to describe hadron-hadron collision is applied to the hadron–effective-target process*.

Before applying the above-mentioned picture[16,17] to hadron–effective-target processes [see (b)], it seems useful to recall the following.

Experiments[18] show that high-energy hadron-hadron collision processes can be understood in terms of the hypothesis of limiting fragmentation,[19] provided that these collisions are *gentle* (in the sense that only an infinitesimal energy transfer, an infinitesimal longitudinal momentum transfer, and a finite transverse momentum transfer take place). The physical picture for such a process is that the projectile and the target are two extended objects going through each other, which in general become excited, and the excitation may cause them to break up. The most remarkable fact[18,19] is that these two objects, the target and the projectile, fragment separately. The fragments from the target approach a limiting probability distribution when viewed in the laboratory as the incident energy increases. Similarly, the fragments from the projectile approach a limiting probability distribution in the projectile system where the projectile was initially at rest.

It is observed experimentally[20,21] that in high-energy hadron-hadron collisions there are also processes which do not have the characteristic features of gentle collisions mentioned above. In particular, the inclusive single-particle $p_\perp$ distributions in such processes do not show limiting behavior at high incident energies. It is found that (a) such processes are associated with the production of large-$p_\perp$ particles and/or with high (relative to that of gentle processes) multiplicity, and (b) the products of these processes dominate the central region [i.e., near $\theta = 90°$ in the c.m. system of the colliding particles]. Collision processes of this type are known as *violent collisions*.[22,17] Such processes can be described by a statistical model[16,17] in which the production mechanism is evisaged to take place as follows: The colliding hadrons "hit each other so hard" that, instead of going through each other, they arrest each other and form a conglomerate which first expands and then decays when a critical volume (which in its rest system is assumed to be independent of the total energy) is reached. As a consequence, the temperature (T) of the conglomerate at the moment of decay increases with increasing incident energy ($E_{1\mathrm{ab}}$):

$$T \sim s^{1/8}, \tag{3}$$

where

$$s \approx 2ME_{1\mathrm{ab}} \quad \text{for} E_{1\mathrm{ab}} \gg M\,. \tag{4}$$

M is the mass of the target. In first-order approximation, the inclusive cross section for the production of a single large-$p_\perp$ particle at the production angle $\theta_{\mathrm{c.m.}} = 90°$ is

$$d\sigma/d^3p(p_\perp, 90°; E_{1\mathrm{ab}}) \propto \sigma_{\mathrm{in}} \exp(-bp_\perp s^{-1/8}), \tag{5}$$

where σ_{in} is the total inelastic cross section, and b is a real positive number which is independent of $p_\perp$, θ, and s but in general is a function of all other quantum numbers of the total system. Furthermore, the average multiplicity $\langle n \rangle$ of violent collision is given by[16]

$$\langle n \rangle \propto s^{3/8}. \tag{6}$$

Further experimental results, especially those obtained from the most recent studies,[21,23] strongly suggest that the gross features of high-energy hadron-hadron multiparticle-production processes can be understood in terms of the following simple picture[17]: Such a process is *either* gentle (fragmentation) *or* violent (violent collision).

In practice, for the performance of data-analyses it is useful to keep in mind the following differences between these two kinds of inelastic processes: (a) A violent collision event can be recognized through the observation of a large-transverse-momentum particle among the products and/or through the fact that the products of this event predominantly populate the central region and that the multiplicity is relatively high (compared with a gentle process between the same colliding particles at the same energy). Products of violent collisions can be found at all angles. [If the effects of noncentral collisions could be neglected, the distribution of particles produced in violent collisions would be isotropic in the rest frame of the conglomerate. Conservation of angular momentum in noncentral violent collisions between the two incoming objects (hadron-hadron, hadron–effective-target, "effective-projectile"–effective-target in hadron-hadron, hadron-nucleus and nucleus-nucleus processes, respectively) leads to plane and peaklike structures in angular distribution of the produced particles. The preferred emission angles in the production plane are not necessarily the forward and backward c.m.s. angles. This is because the expansion time (that is, the time interval between the formation and the decay) of the conglomerate is long, and hence the originally rather flat conglomerate (viewed in its rest system) may rotate a finite angle before it decays; the rotation angle depends on the incident energy and the masses of the colliding objects. Details of this calculation will be given elsewhere.[24]] This is to be compared with the fragmentation events, in which the multiplicity is in general lower, and the products are found mainly in the forward and backward angles (in the c.m. frame of the incoming objects) (b) The increase with energy of the average multiplicity in violent collisions ($\sim s^{3/8}$) is much faster than that of fragmentation processes (~const or ln$s$). Hence, at sufficiently large s, the dominating term in the expression for average multiplicity of all processes is that of violent collisions alone. That is to say, Eq. (6) is a reasonable approximation for the average of all inelastic processes (gentle and violent) at sufficiently high energies.[25]

III. ARGUMENTS FOR THE PICTURE

Arguments for the proposed physical picture for high-energy multiparticle-production processes are:

1. Noninstantaneous formation of multiparticle final states in high-energy hadron-hadron collisions.
Based on the empirical fact mentioned in (ii b) of Sec. I, it has already been suggested by a number of authors[26] that multiparticle final states in high-energy hadron-hadron collisions are *not* produced instantaneously. This point is of fundamental importance for the present picture, as we shall see in the discussion below (see, in particular, subsections 3–7). It should be emphasized that since the production processes, both in the case of fragmentation and in the case of violent collision, go through several stages, the conjecture that in a high-energy hadron-nucleus multiproduction process either fragmentation or violent collision takes place is clearly in accordance with the noninstantaneous nature of the production mechanism mentioned above.[27]

2. Existence of two kinds of inelastic hadron-nucleus processes. The experimental findings cited in (iii) and (iv) of Sec. I, as well as that given in (vi) taken together with the observation made by Van Hove,[28] suggest the following:

(α) There are two different kinds of inelastic hadron-nucleus processes.

(β) One of them is associated with low, and the other with high multiplicities.

(γ) The observed A dependence of $\langle n_s \rangle$ is predominantly due to processes associated with high multiplicities. As we shall see in the discussion below, these two kinds of processes can be identified as the fragementation and the violent collision, respectively.[29]

3. *Collective behavior of the nucleons along the path of the incident hadron.* The noninstantaneous nature of multiparticle-production processes in hadron-hadron collisions mentioned above strongly suggests[26,15] that in high-energy hadron-nucleus reactions multiparticle states are not produced while the incident hadron is still inside the nucleus, and that the nucleons along the path of the incident hadron inside the nucleus may act collectively. We shall see in the following discussion (subsections 4, 5, 6, and 7) that the main features of the high-energy hadron-nucleus multiparticle-production data can be understood by considering these nucleons as an effective target that behaves like a single hadron, the mass of which is proportional to the "average thickness" $\bar{\nu}$ of the target nucleus. (To be more precise, $\bar{\nu}$ is the average number of nucleus in the target nucleus along the path of the incident hadron.) That is,

$$M_A = \bar{\nu} M_1 , \tag{7}$$

where M_A and M_1 are the mass of the effective target and that of the nucleon, respectively. The quantity $\bar{\nu}$ can be obtained from experiments by measuring the total inelastic hadron-nucleon and hadron-nucleus cross sections or it can be calculated from a nuclear model.

4. *Fragmentation of the effective target.* The experimental result mentioned in (v) of Sec. I supports the idea that the nucleons along the path of the incident hadron inside the nucleus act collectively. In this picture, the energetic protons[30] as well as the "shower" particles are the fragments of the effective target which is "pushed out" of the nucleus by the incident pion.[31] The heavy tracks are caused by the parts of the "leftovers" of the target. They are not fragments of the effective target. Furthermore, we note that the observed energy independence of $\langle N_h \rangle$ and that of the distribution with respect to N_h [see (iii) of Sec. I] is a natural consequence of this picture.[31]

5. *Fragmentation of the projectile.* Viewed from the rest system of the projectile, the target nucleus is a thin slab, the thickness of which is proportional to $\bar{\nu}(M/E_{\mathrm{lab}})$. Here $\bar{\nu}$ is the (dimensionless) average thickness of the nucleus, E_{lab} is the energy of the projectile in the target rest frame, and M is the projectile mass. The first factor is due to the nuclear size, while the last factor is due to Lorentz contraction. As we have learned from hadron-hadron reactions, in gentle collisions[18,19] the process of momentum and quantum-number transfer between the "stuff" in the projectile and the "stuff" in the target does not appreciably change when the thickness of the target varies (because of the Lorentz contraction), provided that the incident energy is so high that the limiting distribution is already reached. Hence it is conceivable that, at sufficiently high energies, the replacement of the target hadron by a nucleus will not influence the fragmentation of the projectile. This is because the nuclear size contributes a factor $\bar{\nu}$ to the thickness, the effect of which is the same[32] as that due to a decrease in incident energy by the factor $1/\bar{\nu}$. Earlier experiments on reactions of the type[33] $\pi A \to (\pi\pi\pi)A$ (where A is a nucleus), as well as the most recent inclusive experiments mentioned in (iii b) and (iv) of Sec. I, indeed show that also in high-energy hadron-nucleus processes the projectile behaves according to the hypothesis of limiting fragmentation.[19]

6. *Production of large-transverse-momentum particles as a consequence of violent hadron–effective-target collisions.* As we have seen in Sec. II, large-$p_\perp$ particles are produced in violent hadron-hadron collisions.[22,16,17] The fact that large-$p_\perp$ particles are observed in high-energy hadron-nucleus reactions,[2,3] taken together with the arguments presented at the beginning of this section (subsections 1 and 3), lead us to the following conjecture: Such particles are produced in events where violent collisions between the incident hadron and the effective target take place.

This conjecture can be readily checked experimentally. Here we note that the total c.m.s. energy $\sqrt{s_A}$ of the projectile–effective-target system is A-dependent. This is because the ansatz given in Eq. (7) implies

$$s_A \approx \bar{\nu} s_1 , \tag{8}$$

where $s_1 \approx 2M_1 E_{\mathrm{lab}}$ for $E_{\mathrm{lab}} \gg M_1$, M_1 is the nucleon mass, and E_{lab} is the incident energy of the projectile in the laboratory system.

In order to see the characteristic features of the present model, we first discuss the case[34] in which we use the simple ansatz

$$\bar{\nu} = A^{1/3} \tag{9}$$

for the average thickness, and the relation

$$\sigma_{\mathrm{in}}(pA) = A^{2/3} \sigma_{\mathrm{in}}(pp) \tag{10}$$

to take care of the "surface effect". Here $\sigma_{\mathrm{in}}(pA)$ and $\sigma_{\mathrm{in}}(pp)$ are the total inelastic cross section for proton-nucleus and proton-proton collisions, respectively. From Eqs. (5), (8), (9), and (10) we immediately obtain

$$\frac{d\sigma/d^3p(p_\perp, 90°; E_{\rm lab}; pA)}{d\sigma/d^3p(p_\perp, 90°; E_{\rm lab}; pp)} \approx A^{2/3}\exp[bp_\perp(2ME_{\rm lab})^{-1/8}(1-A^{-1/24})], \quad (11)$$

where b is the parameter that has already been determined[16,35] by the large-$p_\perp$ data for proton-proton processes.[20]

In order to compare the result of this model with the experimental data of Cronin *et al.*[2] we consider the function $\alpha_h(p_\perp, E_{\rm lab})$ defined by

$$A^{\alpha_h(p_\perp, E_{\rm lab})} = \frac{d\sigma/d^3p(p_\perp, 90°; E_{\rm lab}; hA)}{d\sigma/d^3p(p_\perp, 90°; E_{\rm lab}; hp)}, \quad (12)$$

where h stands for an arbitrary hadron. It follows from Eqs. (11) and (12)

$$\alpha_p(p_\perp, E_{\rm lab}) \approx \tfrac{2}{3} + bp_\perp(2ME_{\rm lab})^{-1/8}\frac{1-A^{-1/24}}{\ln A}. \quad (13)$$

Obviously, the function $\alpha_h(p_\perp, E_{\rm lab})$ is identical with $\alpha(p_\perp)$ in Eq. (1) for h = proton and $E_{\rm lab} = 300$ GeV. It is very interesting to see that the factor $(1-A^{-1/24})/\ln A$ is, as expected, in first approximation independent of A for $10 \lesssim A \lesssim 200$. The function $\alpha_p(p_\perp, E_{\rm lab} = 300$ GeV) can be readily calculated by using the known[16,35] value for the only parameter b. The result is given in Fig. 1. We see that the most striking features of $\alpha(p_\perp)$, namely, the fact that it increases with $p_\perp$ and exceeds unity for large $p_\perp$, can be readily understood in terms of the present picture. But the extremely simple version of this model is not in a position to reproduce the observed nonlinear $p_\perp$ dependence of $\alpha(p_\perp)$ for pions at large $p_\perp$ values.[36]

The following points should be mentioned in this connection: (a) No attempt has been made in this paper to fit the data with more sophisticated ansatz for the average thickness and for the A dependence of the total inelastic proton-nucleus cross sections. In a future, refined version of this model, these parameters have to be determined either directly from the experimental data or from a more realistic nuclear model. Furthermore, the effect of cluster production[36] as well as that of noncentral hadron–effective-target collision[24] should be taken into account. (b) General relations can be derived from the basic characteristics of this picture without making use of the detailed properties such as the ansatz given in Eqs. (9) and (10). For instance, we obtain from Eqs. (5) and (8)

$$\frac{d\sigma}{d^3p}(90°, p_\perp; E_{\rm lab}; hA) \approx \frac{d\sigma}{d^3p}(90°, p_{0\perp}; E_{\rm lab}; hA)\exp[-b(p_\perp - p_{0\perp})s_A{}^{-1/8}], \quad (14)$$

where $p_{0\perp}$ is a given value of the transverse momentum of the observed particle [for example, the lowest $p_\perp$ value of a set of $d\sigma/d^3p(90°, p_\perp, E_{\rm lab}; pA)$ data]. From this and Eq. (12) we obtain for the same kind of projectile (e.g., proton) at different incident energies $E_{\rm lab}$ and $E'_{\rm lab}$

$$\frac{\alpha_h(p_\perp, E_{\rm lab}) - \alpha_h(p_{0\perp}, E_{\rm lab})}{\alpha_h(p_\perp, E'_{\rm lab}) - \alpha_h(p_{0\perp}, E'_{\rm lab})} \approx \left(\frac{E'_{\rm lab}}{E_{\rm lab}}\right)^{1/8}. \quad (15)$$

7. *Existence of high-multiplicity events as a consequence of violent collision between the projectile hadron and the effective target.* It follows from the discussion in subsection 6 and Sec. II that, if this picture is correct, the pionization events with high multiplicity in hadron-nucleus collisions are caused by the same kind of production as those in which large-$p_\perp$ particles are observed, namely, by the violent collisions. Hence, we expect in particular that with respect to nuclear size, such events show the same characteristics as the large-$p_\perp$ events. The experimental result cited in (iv) of Sec. I shows that high-multiplicity events indeed have the expected strong A dependence at large angles as well as the expected shift in (pseudo)rapidity distributions. (I.e., a shift of $-\frac{1}{6}\ln A$ in Fig. 10 and Fig. 11 of Busza's review paper, Ref. 1. We note that the fragmentation products are not segregated in these

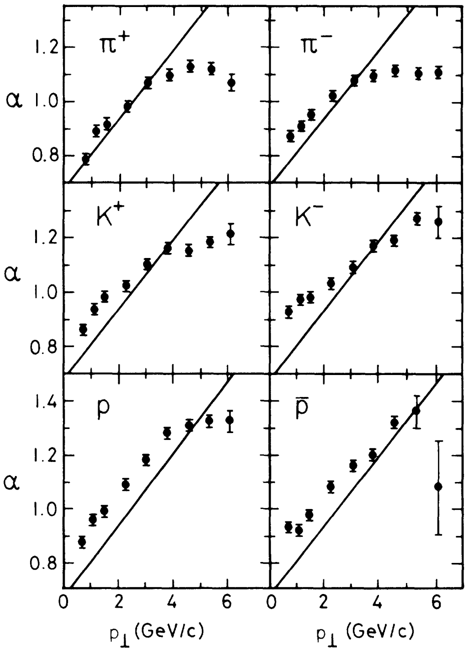

FIG. 1. Power α of the A dependence vs $p_\perp$ for the production of hadrons ($\pi^\pm$, $K^\pm$, p, and $\bar{p}$) by 300 GeV protons. Data are taken from Cronin *et al.* (Ref. 2). The curves are calculated from Eq. (13) where the parameter b is that determined in Ref. 16 and 35.

experiments. See also the discussions in Sec. II and Sec. IV.)

8. *Average multiplicity at high energies.* We now turn our attention to the experimental results given in (ii) of Sec. I. From Eqs. (2), (6), and (8) we immediately obtain, for violent hadron–effective-target collisions

$$R_A(E_{1ab}) = (\bar{\nu})^{3/8} , \tag{16}$$

where $\bar{\nu}$ is the (dimensionless) average thickness of the target nucleus, a quantity which has to be determined by experiments or to be calculated from a nuclear model. In particular, in the case[34] where the simple naive ansatz given in Eq. (9) is used as an illustrative example, we have

$$R_A(E_{1ab}) = A^{1/8} . \tag{17}$$

Furthermore, since the energy dependence of the averaged multiplicity in violent collisions is much stronger than that in gentle processes (see Sec. II and the references given therein), we expect that Eq. (16) and that, to a certain degree, also Eq. (17) are reasonable approximations of $R_A(E_{1ab})$ for all inelastic hadron-nucleus processes at sufficiently high energies.[37] In other words, it is the asymptotic expression for $R_A(E_{1ab})$ for inelastic hadron-nucleus collision processes.

It is interesting to see that the result given in Eq. (17) is energy-independent (as experimental data[4, 5] show), and that it is also in remarkably good agreement with the experimental results on A dependence given in (ii) of Sec. I. It should be pointed out, however, that since some of these results are obtained from experiments at $E_{lab} = 200$ GeV, where the ratio between fragmentation and violent-collision contributions is not negligible and thus finite-energy correction terms (see footnote 37) are still present, we should be cautious and not try to draw strong conclusions from this numerical agreement.

IV. CONCLUDING REMARKS AND SUGGESTED EXPERIMENTS

The physical picture proposed in this paper to account for the gross features of high-energy hadron-nucleus multiparticle-production processes is a simple and naive one. But, if it indeed provides a qualitative understanding of the existing experimental facts, its simplicity and its transparency should be considered as its virtues.

In addition to the experiments mentioned in Sec. I, the results of which have already been compared with this picture (see Sec. III), we would like to suggest the following experimental investigations which seem readily performable on the one hand, and of considerable interest on the other. We think they will provide further insight in hadron-nucleus multiparticle-production processes in general, and can be used as crucial tests for the proposed picture in particular.

(*a*) *Measurement of nuclear-size dependence of relative multiplicity in violent collisions for different angular regions.* If there are indeed different kinds of processes in high-energy hadron-nucleus multiparticle production as the existing data strongly suggest, then it is certainly of general interest to know what nuclear-size effect each kind of processes has. In particular, it is very desirable to have more experimental information on processes in which large-$p_\perp$ particles are produced. Suppose we can add to the experimental setup of Busza *et al.*[9] a detecting device which allows us to trigger on a large-$p_\perp$ pion, say. If the proposed picture is correct, what we shall see is that the relative multiplicity of relativistic charged particles produced in inelastic collision events *with large-$p_\perp$ trigger will be A-dependent also in the forward angles.* This is because one of the main features of this picture is that the A dependence of inclusive cross sections is due to the effective target through its fragmentation, or its violent collision with the incident hadron. Now, in the projectile–effective-target c.m. system, fragments from the effective target are concentrated in the backward hemisphere, but products of violent collisions do *not* only contribute to the backward angles. Hence, if this picture is correct, we expect to see the characteristic A dependence due to violent collisions also in the *forward* angles, *provided that* the overwhelming projectile fragments, which are A-independent, are segregated. Now, one of the basic characteristics of violent collisions is that such processes can produce particles with large transverse momentum ($p_\perp > 2$ GeV/c, say), and therefore a large-$p_\perp$ trigger will be sufficient to separate violent hadron–effective-target collision events from the rest.

(*b*) *Precise measurement of single-particle distributions in multiple-production processes at different energies and on different nuclei.* The importance of such experiments, especially in connection with the possiblity of differentiating the existing models, has already been pointed out by many authors.[38] In this picture, as we have seen in Secs. II and III, the dependence of average multiplicity on nuclear size in violent hadron-nucleus collisions is a direct consequence of the s dependence of $d\sigma/d^3p$ [cf. Eq. (5); here s is the total c.m.s energy squared of the projectile–effective-target system] *and* the conjecture that the effective target acts as a single hadron, the mass of which depends on the atomic number of the nucleus [see Eqs. (4) and (7)]. Now, the s dependence of $d\sigma/d^3p$ for violent collisions is itself a characteristic feature of this picture. Taken together with the fundamental properties of fragmentation processes, the present picture asserts that *$d\sigma/d^3p$ in central region (near $\theta = 90°$ in the projectile–effective-target c.m. system) is dominated by the s-dependent contributions from violent-collisions events*. That is to say, if the picture proposed in this paper is correct, measurements of single-particle (pion, say) inclusive cross section in high-energy hadron-nucleus multiparticle processes will show that, similar to the striking results obtained by Bøggild *et al.*[21] for proton-proton collision at CERN ISR energies, *it increases with s much faster than the total inelastic cross section.* Furthermore, since the total energy $\sqrt{s}$ of the projectile–effective-target complex (in its c.m. system) depends not only on the incident energy of the projectile E_{lab} but also on the mass of the effective target M, which is given by Eq. (7),[39] it would also be very interesting to study the A dependence and the E_{lab} dependence of *$d\sigma/d^3p$ separately*. In particular, in the high-$p_\perp$ region we expect to see the validity of the simple relation for $\alpha_h(p_\perp, E_{\text{lab}})$ given in Eq. (15).

Added note. After this paper was submitted for publication, works by Fredriksson[40] and by Afek, Berlad, Eilam and Dar[41] were received in which large-$p_\perp$-phenomena in hadron-nucleus reactions are discussed in terms of a similar ansatz for the effective target. Since the present model is based on our picture[16,17,24] for hadron-hadron interactions which these authors did not use, their starting points as well as their results are different from ours.

It is extremely interesting to see that the most recent cosmic-ray data[42] indicates that the energy dependence of the average multiplicity is stronger than that obtained from extrapolation of data at lower energies. This strongly supports the idea that the contribution of violent collisions dominates at high energies (see Sec. II and Sec. III 8).

The very recent experimental findings of the BNL-CIT-LBL group[43] shows that exciting results will also be expected from large-$p_\perp$ production experiments on nuclei by using meson beams.[44]

ACKNOWLEDGMENT

The author wishes to thank Professor C. N. Yang, whose inspiring lectures on this and other related topics have initiated this investigation, and Professor A. S. Goldhaber, Professor S. Y. Lo, and Professor H. T. Nieh for valuable information, helpful discussions, and critical comments. He also wishes to thank Professor C. N. Yang and other members of the Institute for Theoretical Physics at Stony Brook for their hospitality.

---

*Permanent address.

[1] This summary consists only of an outline. For further information we refer to the following review articles, the references cited therein and those given in this paper: M. Miesowicz, in *Progress in Elementary Particles and Cosmic Ray Physics*, edited by J. G. Wilson and S. A. Wouthuysen (North-Holland, Amsterdam, 1971), Vol. X, p. 165; E. L. Feinberg, Phys. Rep. 6C, No. 5 (1972); K. Gottfried, in *High Energy Physics and Nuclear Structure*, proceedings of the Fifth International Conference, Uppsala, Sweden, 1973, edited by G. Tibell (North-Holland, Amsterdam/American Elsevier, New York, 1974), p. 79; W. Busza, in *High Energy Physics and Nuclear Structure—1975*, proceedings of the Sixth International Conference, Los Alamos and Santa Fe, 1975, edited by D. E. Nagle *et al.* (A.I.P., New York, 1975), p. 211; L. M. Lederman, in *ibid.* p. 303.

[2] J. W. Cronin *et al.*, Phys. Rev. D 11, 3105 (1975); L. M. Lederman, Ref. 1 and the papers cited therein.

[3] J. P. Boymond *et al.*, Phys. Rev. Lett. 33, 112 (1974); J. W. Cronin, in lectures given at the International School of Subnuclear Phys., Erice, 1975 (unpublished).

[4] I. Otterland *et al.*, in *High Energy Physics and Nuclear Structure*, edited by G. Tibell (see Ref. 1), p. 427; Alma-Ata-Lenengrad-Moscow-Touhkent collaboration, Yad. Fiz. 19, 1046 (1974) [Sov. J. Nucl. Phys. 19, 536 (1974)]; W. Busza *et al.*, Phys. Rev. Lett. 34, 836 (1975); P. L. Jain *et al.*, *ibid.* 34, 972 (1975). See also Ref. 1 and the papers cited therein.

[5] E. Lohrmann and M. W. Teucher, Nuovo Cimento 25, 957 (1962); A. Gurtu *et al.*, Phys. Lett. 50B, 391 (1974); P. R. Vishwanath *et al.*, Phys. Lett. 53B, 479 (1975). See also Ref. 1 and the papers cited therein.

[6] In emulsion experiments $\langle n\rangle_A$ denotes the average number of "shower particles" n_s, where A is the average atomic mass number.

[7] We note that (a) not *all* experimental results on R_A have been empirically parametrized in this way; (b) the definition for R_A is unfortunately *not* unique (see for example, Busza, Ref. 1 and the papers cited therein).

[8] See, e.g., Otterland *et al.* (Ref. 4); AALMT collaboration (Ref. 4); Gurtu *et al.* (Ref. 5).

[9] Busza *et al.* (Ref. 4); Vishwanath *et al.* (Ref. 5).

[10] See, e.g., AALMT collaboration (Ref. 4); J. Babecki *et al.*, Acta Phys. Pol. 35, 315 (1974); Jain *et al.* (Ref. 4).

[11] J. R. Elliott *et al.*, Phys. Rev. Lett. 34, 607 (1975).

[12] Busza (Ref. 1).

[13] See, e.g., Busza (Ref. 1); Gurtu *et al.* (Ref. 5) and the papers cited therein.

[14] See, e.g., A. Dar and J. Vary, Phys. Rev. D 6, 2412 (1972); A. S. Goldhaber, in *High Energy Physics and Nuclear Structure*, edited by G. Tibell (see Ref. 1), p. 133; K. Gottfried, Phys. Rev. Lett. 32, 957 (1974);

P. M. Fishbane and J. S. Trefil, Phys. Lett. 51B, 139 (1974); G. Galucci, R. Jengo, and A. Pignotti, Phys. Rev. D 10, 1468; A. S. Goldhaber, Phys. Rev. Lett. 33, 47 (1974); A. Bialas and W. Czyz, Phys. Lett. 51B, 179 (1974); E. S. Lehman and G. A. Winbow, Phys. Rev. D 10, 2962 (1974); G. R. Farrar, Phys. Lett. 56B, 185 (1975); L. Bertocchi, in *High Energy Physics and Nuclear Structure—1975* (see Ref. 1), p. 238; P. F. Fishbane and J. S. Trefil, Phys. Rev. D 12, 2113 (1975); J. Koplik and A. H. Mueller, *ibid.* 12, 3638 (1975); A. Krzywicki, *ibid.* 14, 152 (1976); Y. Kazama and C. N. Yang (private communication).

[15] (a) See, e.g., M. G. Kaplon and D. M. Ritson, Phys. Rev. 88, 386 (1952); F. C. Roesler and C. B. A. McCusker, Nuovo Cimento 10, 127 (1953); W. Heitler and C. H. Terreaux, Proc. Phys. Soc. London A66, 929 (1953); Miesowicz (Ref. 1); L. D. Landau, *Collected Papers of L. D. Landau*, edited by D. Ter Haar (Pergamon, London, 1965), p. 665; S. Z. Belen'kij and L. D. Landau, Nuovo Cimento Suppl. 3, 15 (1956); Gottfried (Ref. 1); A. M. Baldin, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 1, p. 277; A. Z. Patashinskii Zh. Eksp. Teor. Fiz. Pis'ma Red. 19, 546 (1974) [JETP Lett. 19, 338 (1974)]; G. Berlad, A. Dar, and G. Eilam, Phys. Rev. D 13, 161 (1976) and the papers cited therein.
(b) Gottfried (Refs. 1 and 15a); Pataschinskii (Ref. 15a); Berlad, Dar, and Eilam (Ref. 15a).

[16] Meng Ta-chung, Phys. Rev. D 9, 3062 (1974). The spirit of this statistical model is similar to that of E. Fermi's [Prog. Theor. Phys. 5, 570 (1950); Phys. Rev. 81, 683 (1951)], although they are entirely different in the following essential points: Firstly, Fermi's model is proposed for *all* production processes while the present one is designed *only* for violent collisions. Secondly, according to Fermi, the conglomerate formed by the two colliding particles remains in a *frozen* state until it decays, but in the present model the conglomerate first *expands* and then decays when a critical volume is reached. This leads to the particular s dependence of the temperature given in Eq. (3). The possibility of expansion of the conglomerate before its decay has been suggested by I. Ya Pomeranchuk [Dokl. Akad. Nauk SSSR 78, 889 (1951)] in the framework of a statistical model and by Landau and coworkers (Ref. 15) in the framework of a hydrodynamical model. However, because of the fundamental differences in the assumptions (like Fermi's, their models are proposed to describe *all* production processes, but the expansion and decay mechanism are different from Fermi's as well as from ours), the results, in particular the s dependence of the temperature, are completely different from those given in the present paper (see the comparison made in Sec. II of the 1974 paper mentioned above). For details about other statistical and hydrodynamical models we refer to Feinberg (Ref. 1), Gottfried (Ref. 1), and the papers cited therein.

[17] Meng Ta-chung and E. Moeller, Phys. Rev. D 14, 1449 (1976).

[18] Direct experimental tests of limiting fragmentation have been performed at the ISR by G. Bellettini *et al.*, Phys. Lett. 45B, 69 (1973). Other experimental results in this connection can be found, e.g., in the review article by M. Jacob, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 3, p. 373.

[19] J. Benecke *et al.*, Phys. Rev. 188, 2159 (1969); C. N. Yang, in *High Energy Collisions*, proceedings of the Third International Conference, Stony Brook, 1969, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969), p. 509; T. T. Chou and C. N. Yang, Phys. Rev. D 7, 2005 (1971).

[20] F. W. Büsser *et al.*, Phys. Lett. 46B, 471 (1973). Other experimental results on large-$p_\perp$ phenomena can be found in the review article by Lederman (Ref. 1) and the paper given there.

[21] H. Bøggild *et al.*, contributed paper discussed by B. G. Duff, in *High Energy Physics*, proceedings of the European Physical Society International Conference, Palermo, 1975, edited by A. Zichichi (Editrice Compositori, Bologna, Italy, 1976), p. 976.

[22] A. W. Chao and C. N. Yang, Phys. Rev. D 9, 2505 (1974).

[23] M. Della Negra *et al.*, Phys. Lett. 59B, 401 (1975); K. Eggert *et al.*, Nucl. Phys. B98, 73 (1975).

[24] Meng Ta-chung and E. Moeller, FU Berlin report, 1976 (unpublished).

[25] As reasonable fits for all mean-charged-multiplicities data [see for example, the compilation by E. Albini, P. Capiluppi, G. Giacomelli, and A. M. Rossi, Nuovo Cimento 32A, 101 (1976) and the papers cited therein] for hadron-hadron collisions at $E_{\text{lab}} \gtrsim 100$ GeV, we can, for example, write

$$\langle n_{\text{ch}} \rangle = a + b s^{3/8}, \quad a = 2.8, \; b = 0.45$$

or

$$\langle n_{\text{ch}} \rangle = a \ln s + b s^{3/8}, \quad a = 0.5, \; b = 0.4,$$

where s is given in (GeV)^2, and a and b are given in corresponding units. A detailed discussion on this and other related points is given by E. Moeller [FU-Berlin Thesis, 1976 (unpublished)]. It should be emphasized that the $s^{3/8}$ term consists of contributions from *all violent-collision processes*, and not only from those in which large-$p_\perp$ particles are produced. Detailed discussions on, and experimental evidences for this mechanism can be found in Refs. 16 and 17.

[26] See, e.g., Miesowicz (Ref. 1); Gottfried (Ref. 1); Goldhaber (Ref. 14); W. Busza (Ref. 1) and the papers cited therein.

[27] The following remarks should be made: (a) It is, of course, in principle also possible to describe the observed weak A dependence by assuming that the production of multibody final states in high-energy hadron-hadron collisions takes place instantaneously, *provided that the details of the production mechanism collisions are such that intranuclear cascade can be avoided.*
(b) Besides the weak A dependence of R_A mentioned in the text, there are also other indications (see in this connection also the discussions in Ref. 26 and the papers cited therein) that the time for the production of multiparticle final states in high-energy hadron-hadron collisions in much longer than a few fermi/c

(in the lab frame). For example, in our model for violent collisions, the expansion time of the conglomerate can be estimated from the magnitude of the critical V. In its rest frame, it is of the order F/c. Hence, taken together with the dilatation factor $(E_{\text{lab}}/2M_1)^{1/2}$, where E_{lab} is the incident lab energy and M_1 is the proton mass, the time of expansion in the lab frame is much larger than few F/c at high energies ($E_{\text{lab}} \gtrsim 200$ GeV, say).

[28] L. Van Hove, Phys. Lett. 43B, 65 (1973).

[29] We recall that in hadron-hadron processes the average multiplicity of fragmentation processes are lower than that of violent collisions, and violent collisions are responsible for the following three classes of events: (a) high multiplicity *with* large-$p_\perp$ particle, (b) high multiplicity *without* large-$p_\perp$ particles and, (c) low multiplicity with large-$p_\perp$ particles. It is known empirically that (a) and (b) are the dominating parts of violent-collision processes.

[30] Evidence for energetic protons with laboratory momenta above 1 GeV/c is found (see Ref. 11) by examining the net charge of the shower tracks $n_+ - n_-$ for each event, where n_+ and n_- are the number of positive and negative shower tracks, respectively.

[31] We recall that the average binding energy per nucleon in nucleus is about 8 MeV. Hence it is extremely easy for a fast ($E_{\text{lab}} \geqslant 10$ GeV, say) projectile to break the bonds between the effective target and the rest of the nucleus.

[32] E.g., the projectile proton (p) sees effective targets of approximately the *same* thickness in the following two collision processes: p-Be at $E_{\text{lab}} = 200$ GeV and p-p at $E_{\text{lab}} = 100$ GeV.

[33] It has already been pointed out by Yang and collaborators (Ref. 19) several years ago that this type of process will supply information on the *fragmentation of the pion* into three pions, etc. For experimental material, see, e.g., the papers given in Benecke *et al.* (Ref. 19) and Gottfried (Ref. 1).

[34] In a future, refined version of this model, the average thickness as well as the A dependence of total inelastic hadron-hadron cross sections should either be determined from experiments or be calculated from a more realistic nuclear model.

[35] S. D. Ellis, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. V-23; P. V. Landhoff, *ibid.*, p. V-57.

[36] It is clear that the nonlinearity of $\alpha(p_\perp)$ in $p_\perp$ is closely related to the deviation of $d\sigma/d^3p$ from a simple exponential function of $p_\perp s^{-1/8}$. Hence in the framework of the present model this effect is due to cluster production and noncentral violent collisions.

[37] At energies where the ratio (r) between the contribution from fragmentation processes and that from violent-collision processes is not negligibly small compared to unity ($r \ll 1$), correction terms should be added to the asymptotic expression given in Eq. (17). To be more precise, in terms of the fits for $E_{\text{lab}} \gtrsim 100$ GeV such as those given in Ref. 25, $R_A(E_{\text{lab}})$ can be written as

$$R_A(E_{\text{lab}}) = \frac{A^{1/8} + f^{-1}}{1 + f^{-1}}$$
$$= A^{1/8} + (1 - A^{1/8}) f^{-1} + O(f^{-2}) ,$$

where f denotes $b s_1^{3/8}/a$ or $b s_1^{3/8}/(a \ln s_1)$, $s_1 \approx 2ME_{\text{lab}}$, and M is the nucleon mass. We note that according to the discussions given in Secs. II and III, only contributions from violent collisions are A dependent.

[38] See, e.g., Busza (Ref. 1) and Bertocchi (Ref. 14).

[39] The relation given in Eq. (7) has many extremely interesting consequences. Beside the property of $d\sigma/d^3p$ discussed in text, we would like to mention the following points: (a) In order to increase the total c.m.-system energy of the projectile–effective-target system, one can also use a heavier nucleus as target instead of increasing the incident laboratory energy of the projectile. (b) Taken together with the statistical model (Refs. 16, 17) for violent collisions, this relation leads us to speculate that high-density nuclear matter may be produced in violent high-energy hadron nucleus and nucleus-nucleus collisions.

[40] S. Fredriksson, Stockholm Royal Institute of Technology report, 1975 (unpublished).

[41] Y. Afek, G. Berlad, G. E. Eilam, and A. Dar, Phys. Rev. Lett. 37, 947 (1976); Phys. Rev. D (to be published).

[42] G. Yodh, in Proceedings of the Topical Meeting on Multiparticle Production on Nuclei at Very High Energies, Trieste, 1976 (unpublished).

[43] G. Donaldson *et al.*, Phys. Rev. Lett. 36, 1110 (1976); for theoretical intepretation in terms of the present picture, see Kwan-Wu Lai and Meng Ta-chung, *ibid.* 37, 241 (1976).

[44] See J. W. Cronin, in Proceedings of the Topical Meeting on Multiparticle Production at Very High Energies, Trieste, 1976 (unpublished); C. Halliwell *ibid.* for a theoretical discussion in the framework of the present picture, see Meng Ta-chung, *ibid.*