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Amplitude reconstruction in NN scattering at 6 GeV/c: Where do we stand and what measurements should be done?*

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We study the problem of amplitude reconstruction at a fixed t ($-t \sim 0.3$ GeV/c^2). The five amplitudes are reconstructed, up to an overall phase, by 9 observables σ, P_0, C_{nn}, K_{nn}, D_{nn}, C_{ss}, C_{sl}, C_{ll}, and $I(sn0s)$. This set includes all possible single-scattering measurements. The status of the amplitude reconstruction is discussed. To study both discrete ambiguities and the effects of the measurement errors, we employ a Monte Carlo method. We find that D_{ls} and $I(ns0s)$ remove the prominent ambiguities. In addition, given the current precision, R and $I(ls0n)$ will significantly improve the amplitude determination.

The polarized proton beam of the Argonne zero-gradient synchrotron (ZGS), when used in conjunction with the polarized-nucleon-target facilities of the laboratory, provides an opportunity to study the nucleon-nucleon scattering amplitudes in detail at intermediate energies. These facilities allow one to make accurate measurements of certain NN spin-correlation parameters, as well as to make other measurements that were prohibitively difficult. From such measurements, one expects to be able to unravel the various nucleon-nucleon amplitudes. The initial experiments[1] have concentrated upon proton-proton scattering at $P_{1ab} = 6$ GeV/c. We shall be concerned exclusively with the extent to which one may determine pp amplitudes from data at that energy; furthermore, we have concentrated on a particular momentum transfer, $t = -0.3$ GeV/c^2, in this report.

There are five independent (complex) proton-proton amplitudes, and all two-body measurements may be expressed in terms of bilinear functions of these amplitudes. Consequently, one cannot determine an overall s- and t-dependent common phase, so there are nine independent real amplitudes to be obtained from data. We would expect that nine precisely measured independent experimental quantities would either determine these amplitudes uniquely, or more probably, would yield only a discrete set of ambiguities in their determination. Furthermore, these discrete ambiguities could presumably be eliminated by a few additional measurements.[2]

This rather simple situation is complicated considerably by the uncertainties on the measured quantities. Experimentally, single-scattering spin-correlation measurements may be done with great precision, whereas double-scattering measurements involve detecting the final spin direction of the recoil proton, and may be obtained only with limited precision. Theoretically, the five nucleon-nucleon scattering amplitudes are expected to be substantially different in magnitude, and therefore will not be determined with the same uncertainties by a given set of measurements; in fact, the exchange amplitudes of most interest in conventional Regge models are probably rather small, and thus will be harder to extract from the data.[3] It could also happen, and in fact *does* happen, that certain measurements provide no new information not already given by measurements completed. This latter point of *numerical* independence is really distinct from that of *algebraic* independence of measurements, since the feasible level of experimental precision is a crucial factor in this matter.[4]

We shall write the five independent nucleon-nucleon scattering amplitudes in terms of t-channel helicity amplitudes: N_0, N_1, and N_2 are "natural-parity exchange" amplitudes with 0, 1, and 2 units of helicity; whereas U_0 and U_2 are "unnatural-parity exchange" amplitudes with 0 and 2 units of helicity. These independent amplitudes need *not* be of comparable magnitude; in fact we expect that at 6 GeV/c and our small momentum transfer, the amplitude N_0 should be the dominant one. This theoretical expectation, which is readily accessible to experimental test, suggests that all terms quadratic in the "small amplitudes" (N_1, N_2, U_0, and U_2) will be rather small, and therefore difficult to determine in direct measurements. Furthermore, one expects that measurements with terms involving $|N_0|^2$ will allow us to test whether

N_0 is dominant, but give essentially no other information. Finally, one expects to obtain the most useful information by observables involving "interference terms," which are dominated by a term linear in N_0. Eight suitable interference terms are $\mathrm{Re}N_0N_1^*$, $\mathrm{Im}N_0N_1^*$, $\mathrm{Re}N_0N_2^*$, $\mathrm{Im}N_0N_2^*$, $\mathrm{Re}N_0U_0^*$, $\mathrm{Im}N_0U_0^*$, $\mathrm{Re}N_0U_2^*$, and $\mathrm{Im}N_0U_2^*$.

Upon the basis of these considerations, one may adopt the following simple strategy for determining amplitudes from data:

(1) Measure a sufficient number of the observables that involve $|N_0|^2$, to establish that N_0 is *in fact* the dominant amplitude. These measurements constitute a somewhat arbitrary choice, and they may be relatively crude, since they are not meant for a precise determination of $|N_0|^2$.

(2) Measure rather precisely the eight observables that involve the eight above interference terms, or independent linear combinations. While the choice of these observables is arbitrary to an extent, it is nevertheless a rather stringent requirement that one be able to obtain the eight interference terms from them.

The scale for these amplitudes is determined by measuring the differential cross section

$$\begin{aligned}\sigma &= I(0,0,0,0)\\ &= |N_0|^2+2|N_1|^2+|N_2|^2+|U_0|^2+|U_2|^2. \end{aligned} \tag{1}$$

One could establish that N_0 is the dominant amplitude by measuring the depolarizations D_{nn} and D_{zz}, which are expressed in terms of amplitudes as (Ref. 4)

$$\begin{aligned}\sigma D_{nn} &= |N_0|^2+2|N_1|^2+|N_2|^2-|U_0|^2-|U_2|^2,\\ \sigma D_{zz} &= I(0z0z) = -I(0l0l)\cos\theta_R + I(0l0s)\sin\theta_R\\ &= |N_0|^2-|N_2|^2+|U_0|^2-|U_2|^2. \end{aligned} \tag{2}$$

The quantity θ_R, the laboratory recoil angle of the target proton, is close to $90°$ for small-angle scattering (1.237 radians at 6 GeV/c for $t=-0.3$ GeV/c^2). Since $D_{ll}^2+D_{ls}^2 \leq 1$, it is just as useful to measure the more accessible quantity $\sigma D_{ls} = I(0l0s)$. The amplitude N_0 is dominant if, and only if, the quantities D_{nn} and $D_{zz}(D_{ls})$ are close to $+1$. The depolarization parameter D_{nn} has been measured and is close to $+1$, but neither D_{ls} nor any other suitable observable has been measured, so that one may *not* yet conclude that N_0 is dominant. However, we shall proceed under the assumption that D_{ls} has been measured and is close to $+1$.

Now we present a feasible choice of eight observables which are dominated by the right sorts of interference terms:

$$\begin{aligned}
&\sigma P_0 = I(0n00) \simeq -2\,\mathrm{Im}\,N_0N_1^*,\\
&\sigma R = I(0s0s) \simeq -\sigma\cos\theta_R - 2\,\mathrm{Re}N_0N_1^*,\\
&I(ns0s) \simeq 2\,\mathrm{Im}\,N_0N_2^*,\\
&\sigma C_{nn} = I(nn00) \simeq -2\,\mathrm{Re}N_0N_2^*,\\
&I(ls0n) \simeq -2\,\mathrm{Im}\,N_0U_0^*,\\
&\sigma C_{ll} = I(ll00) \simeq -2\,\mathrm{Re}N_0U_0^*,\\
&I(sn0s) \simeq 2\,\mathrm{Im}\,N_0U_2^*,\\
&\sigma C_{ss} = I(ss00) \simeq 2\,\mathrm{Re}N_0U_2^*.
\end{aligned} \tag{3}$$

These observables are approximately linear in N_0, as well as linear in the interfering amplitudes. Thus, there are no severe problems with correlated errors when all eight of these observables are determined.

This sample scheme may be quite sufficient for reducing the amplitude ambiguity. However, it is not known whether, and to what extent, N_0 is the dominant amplitude. Also, one must be able to assess the effect of measurements which have already been completed, but which are not on the above list. We shall describe a Monte Carlo strategy for generating amplitudes from data which is more general than the above.

As an aid in carrying out this strategy, we choose a "preferred set" of nine algebraically independent measurements. These measurements alone will not allow us to determine unique amplitudes however, since (i) there is an eightfold discrete ambiguity in determining amplitudes from these data and (ii) the uncertainties in these measurements lead to corresponding uncertainties in the amplitudes. Some of the uncertainties will be resolved by other measurements. It is useful to determine the level of amplitude uncertainty on the basis of this preferred set, to assess the effectiveness of additional measurements in further reducing the uncertainty.

Our preferred set is chosen to include all the high-precision measurements which are possible using a polarized beam and polarized target: σ, P_0, C_{nn}, C_{ss}, C_{sl}, and C_{ll}, which are (Ref. 5)

$$\begin{aligned}
&\sigma P_0 = I(0n00) = -2\,\mathrm{Im}(N_0-N_2)N_1^*,\\
&\sigma C_{nn} = I(nn00) = 2\,\mathrm{Re}(U_0U_2^* - N_0N_2^*) + 2|N_1|^2,\\
&\sigma C_{ss} = I(ss00) = 2\,\mathrm{Re}(N_0U_2^* - N_2U_0^*),\\
&\sigma C_{sl} = I(sl00) = 2\,\mathrm{Re}(U_0+U_2)N_1^*,\\
&\sigma C_{ll} = I(ll00) = -2\,\mathrm{Re}(N_0U_0^* - N_2U_2^*).
\end{aligned} \tag{4}$$

All these quantities have been measured accurately except C_{sl} and C_{ll}, and these will soon be determined at Argonne. The other three measurements are taken to be D_{nn}, K_{nn}, and $I(sn0s)$, which are expressed as

$$\begin{aligned}\sigma D_{nn} &= |N_0|^2 + 2|N_1|^2 + |N_2|^2 - |U_0|^2 - |U_2|^2 ,\\ \sigma K_{nn} &= -2\,\mathrm{Re}(U_0 U_2^* + N_0 N_2^*) + 2|N_1|^2 ,\\ I(sn0s) &= 2\,\mathrm{Im}(N_0 U_2^* + N_2 U_0^*)\sin\theta_R\\ &\quad + 2\,\mathrm{Im}(U_2 - U_0)N_1^* \cos\theta_R .\end{aligned} \tag{5}$$

These three measurements, which involve detection of recoil spin components, have been completed.

From precise measurements of the nine quantities given in (4) and (5), one may determine the nucleon-nucleon amplitudes (to within an overall phase) up to an eightfold discrete ambiguity. Before giving the details of the algebraic solution, we shall make a convenient change in notation. First, we shall divide each nucleon-nucleon amplitude by $\sigma^{1/2}$; this rescaling is equivalent to replacing σ by 1 in expressions (4) and (5). The calculation is easier in the transversity basis.[6] These standard transversity amplitudes are related to the helicity amplitudes by

$$\begin{aligned} T_1 &= T_{++++} = N_0 - N_2 - 2iN_1 ,\\ T_2 &= T_{----} = N_0 - N_2 + 2iN_1 ,\\ T_3 &= T_{+-+-} = N_0 + N_2 ,\\ T_4 &= T_{++--} = -U_0 - U_2 ,\\ T_5 &= T_{+--+} = U_0 - U_2 .\end{aligned} \tag{6}$$

The measurements of $\sigma(=1)$, P_0, C_{nn}, D_{nn}, and K_{nn} determine the moduli of these transversity amplitudes:

$$\begin{aligned} |T_1|^2 &= \tfrac{1}{2}(1 + D_{nn} + C_{nn} + K_{nn}) + 2P_0 ,\\ |T_2|^2 &= \tfrac{1}{2}(1 + D_{nn} + C_{nn} + K_{nn}) - 2P_0 ,\\ |T_3|^2 &= \tfrac{1}{2}(1 + D_{nn} - C_{nn} - K_{nn}) ,\\ |T_4|^2 &= \tfrac{1}{2}(1 - D_{nn} + C_{nn} - K_{nn}) ,\\ |T_5|^2 &= \tfrac{1}{2}(1 - D_{nn} - C_{nn} + K_{nn}) .\end{aligned} \tag{7}$$

The positivity of the five right-hand terms gives *five inequality constraints* which the data must satisfy.

Let us write the transversity amplitudes in polar form

$$T_i = |T_i|\, e^{i\eta_i} , \tag{8}$$

and fix an overall phase by setting $\eta_4 = 0$. We are to determine the phases $\eta_1, \eta_2, \eta_3, \eta_5$ from precisely known values of C_{ss}, C_{sl}, C_{ll}, and $I(sn0s)$. We note that C_{ss}, C_{sl}, and C_{ll} directly determine three of the relative phases. These observables are expressed in terms of transversity amplitudes as

$$\begin{aligned} C_{ss} &= -\tfrac{1}{2}\,\mathrm{Re}[(T_1 + T_2)T_4^* + 2T_3 T_5^*] ,\\ C_{sl} &= \tfrac{1}{2}\,\mathrm{Im}(T_1 - T_2)T_4^* ,\\ C_{ll} &= -\tfrac{1}{2}\,\mathrm{Re}[-(T_1 + T_2)T_4^* + 2T_3 T_5^*] .\end{aligned} \tag{9}$$

We add the expressions for C_{ss} and C_{ll} to obtain

$$\cos(\eta_5 - \eta_3) = -\frac{C_{ss} + C_{ll}}{2|T_5||T_3|} ; \tag{10}$$

from this expression the phase $\eta_5 - \eta_3$ may be determined to within a twofold ambiguity. The data are constrained such that the right-hand side is less than or equal to 1 in magnitude. We may obtain, in addition, two independent relations from (9):

$$\begin{aligned} a &= -\frac{C_{ss} - C_{ll}}{|T_4|} = |T_1|\cos\eta_1 + |T_2|\cos\eta_2 ,\\ b &= \frac{2C_{sl}}{|T_4|} = |T_1|\sin\eta_1 - |T_2|\sin\eta_2 .\end{aligned} \tag{11}$$

These two trigonometric relations from η_1 and η_2 may be rearranged to give

$$\cos(\eta_1 + \eta_2) = \frac{a^2 + b^2 - |T_1|^2 - |T_2|^2}{2|T_1||T_2|} , \tag{12}$$

so that the phase $\eta_1 + \eta_2$ may be determined to within a twofold sign ambiguity. Note again that the magnitude of the right-hand side of (12) being less than or equal 1 is an *inequality constraint* on the data. If we insert the value of $\eta_1 + \eta_2 \equiv \delta$ into the relations (11), we obtain coupled linear equations for $\cos\eta_2$ and $\sin\eta_2$ which may be solved to give

$$\begin{aligned} \cos\eta_2 &= \frac{a(|T_1|\cos\delta + |T_2|) + b(|T_1|\sin\delta)}{a^2 + b^2} ,\\ \sin\eta_2 &= \frac{a(|T_1|\sin\delta) - b(|T_1|\cos\delta + |T_2|)}{a^2 + b^2} .\end{aligned} \tag{13}$$

As a consequence, *both* η_1 and η_2 may be determined from (11), and there are two and *only* two solutions of these coupled equations.

Finally, we shall determine the phase η_5 from $I(sn0s)$, using the fact that η_1, η_2, and $\eta_5 - \eta_3 \equiv \epsilon$ are known. This quantity is written in terms of the transversity amplitudes as

$$\begin{aligned} I(sn0s) &= -\mathrm{Im}\, T_3 T_4^* \sin\theta_R\\ &\quad + \tfrac{1}{2}\,\mathrm{Re}(e^{i\theta_R} T_1 T_5^* - e^{-i\theta_R} T_2 T_5^*) .\end{aligned} \tag{14}$$

We may cast this equation into the form

$$I(sn0s) = \alpha\cos\eta_5 + \beta\sin\eta_5 , \tag{15}$$

where the coefficients α and β can be written in terms of known quantities as

$$\alpha = |T_3|\,|T_4|\sin\theta_R \sin\epsilon + \tfrac{1}{2}|T_5|[\,|T_1|\cos(\eta_1+\theta_R) - |T_2|\cos(\eta_2-\theta_R)]\,,$$
$$\beta = -|T_3|\,|T_4|\sin\theta_R \cos\epsilon + \tfrac{1}{2}|T_5|[\,|T_1|\sin(\eta_1+\theta_R) - |T_2|\sin(\eta_2-\theta_R)]\,. \tag{16}$$

Relation (15) may be written as

$$\frac{I(sn0s)}{[\alpha^2+\beta^2]^{1/2}} = \cos(\eta_5-\phi)\,, \tag{17}$$

with ϕ uniquely determined (modulo 2π) by the relations

$$\cos\phi = \frac{\alpha}{[\alpha^2+\beta^2]^{1/2}}\,,\quad \sin\phi = \frac{\beta}{[\alpha^2+\beta^2]^{1/2}}\,. \tag{18}$$

Note that η_5 is determined to within a twofold ambiguity by (17), and the data are *constrained* so that the magnitude of the left side of (17) is less than or equal to 1.

Let us label the sign of $\eta_5-\eta_3$ in (10) by I_1, the sign of $\eta_1+\eta_2$ in (12) by I_2, and that of $\eta_5-\phi$ in (17) by I_3. The eight solutions are represented by (I_1,I_2,I_3); $I_i=\pm 1$. For a given set of (exact) measurements, some or all of these eight solutions may be missing, when one or more of the above equations has no real solution.

We use the above *algebraic* solution to construct an efficient Monte Carlo program, which is used to study the relation of errors in experimental quantities to the uncertainties of the nucleon-nucleon amplitudes. Namely, we generate each of the experimental quantities randomly and independently about its measured value within its determined error. We compute sets of scattering amplitudes for each algebraic solution (I_1,I_2,I_3). From these sets, which are consistent with the nine measurements, we may compute other observables and decide what additional measurements should be done to determine the amplitudes more accurately.

Given the above formalism, we can answer the question of where we stand on the determination of the scattering amplitudes at 6 GeV/c. To use the above analysis, we note that these correlations are constrained[4] by the other measurements to lie roughly in the range -0.10 to 0.10. We therefore use the values 0.0 ± 0.10 for each of these parameters.

At 6 GeV/c, there are two measurements which have been completed which were not in the favored set[1]:

$$\begin{aligned}\sigma R &= I(0s0s)\\ &= -\cos\theta_R(|N_0|^2-|N_2|^2+|U_2|^2-|U_0|^2)\\ &\quad -\sin\theta_R[2\,\mathrm{Re}(N_0+N_2)N_1^*]\,,\\ \sigma K_{ss} &= I(s00s)\\ &= -\cos\theta_R[2\,\mathrm{Re}(N_0U_2^*+N_2U_0^*)]\\ &\quad -\sin\theta_R[2\,\mathrm{Re}(U_2-U_0)N_1^*]\,.\end{aligned} \tag{19}$$

These measurements are not necessary for the Monte Carlo algorithm, but could provide further information about the scattering amplitude at $P_{1ab}=6$ GeV/c; $t=-0.3$ GeV/c^2. Therefore, we incorporate them as additional constraints in the program.

We shall neglect any uncertainty in the elastic differential cross section σ, and use the scale $\sigma=1.00$. The experimental error in σ amounts to a small error in the scale factor, which we ignore here. Our input measurements are as follows:

$$\begin{aligned}&\sigma=1.00\,,\\ &P_0=0.12\pm 0.01\,,\\ &C_{nn}=0.10\pm 0.02\,,\\ &D_{nn}=0.95\pm 0.08\,,\\ &K_{nn}=0.14\pm 0.08\,,\\ &C_{ss}=-0.15\pm 0.10\,,\\ &C_{sl}=0.0\pm 0.1\,,\\ &C_{ll}=0.0\pm 0.1\,,\\ &I(sn0s)/\sigma=0.20\pm 0.08\,,\\ &R=-0.40\pm 0.20\,,\\ &K_{ss}=0.03\pm 0.08\,.\end{aligned} \tag{20}$$

From these data we have obtained 200 consistent sets of amplitudes for each of the eight algebraic solutions (I_1,I_2,I_3). The amplitudes N_0, N_1, N_2, U_0, and U_2 are displayed in Fig. 1. We have specified the unmeasured phase by requiring N_0 to be purely imaginary with a positive-imaginary part. For each of these Monte Carlo amplitudes we compute the values of the additional measureable quantities, in order to determine those measurements that would be the most useful in reducing the rather substantial ambiguities displayed in Fig. 1. We find the quantities (D_{ls}) and $I(ns0s)$, as given below, to be the most useful.

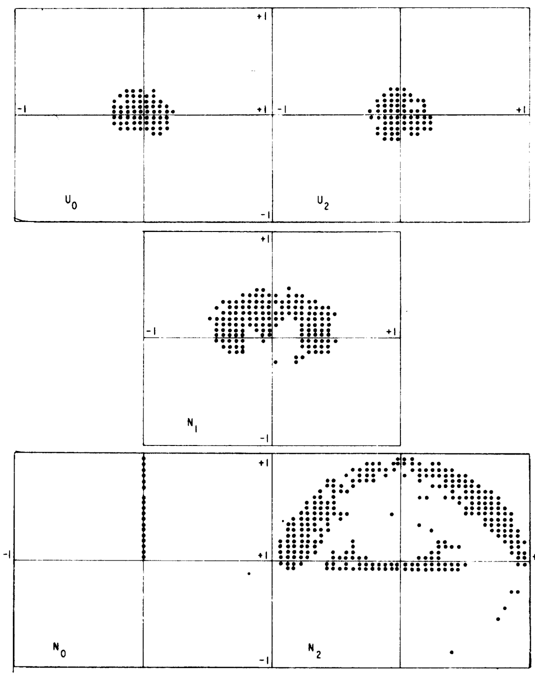

FIG. 1. Present uncertainties in the five independent pp elastic amplitudes based on the spin-correlation measurements [Eq. (20)] for a typical t value -0.3 (GeV/c)^2.

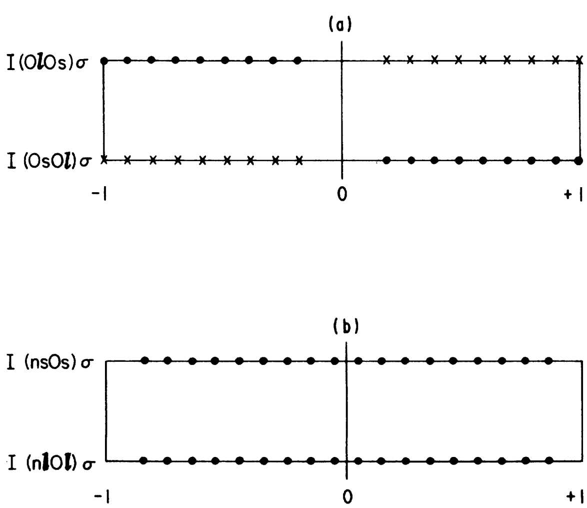

FIG. 2. Present spreads in the unmeasured quantities (a) D_{ls} and $I(0s0l)/\sigma$ and (b) $I(ns0s)/\sigma$ and $I(nl0l)/\sigma$ based on the measurements [Eq. (20)]. In (a), the "x's" illustrate that solutions $D_{ls} \geq 0$ correspond to $I(0s0l)/\sigma \leq 0$. In (b), we assume D_{ls} is positive.

$$\begin{aligned} \sigma D_{ls} &= I(0l0s) \\ &= \sin\theta_R(|N_0|^2 - |N_2|^2 + |U_0|^2 - |U_2|^2) \\ &\quad - 2\cos\theta_R \operatorname{Re}(N_0 + N_2)N_1^* , \\ I(ns0s) &= -2\sin\theta_R \operatorname{Im}(U_0 U_2^* - N_0 N_2^*) \\ &\quad - 2\cos\theta_R \operatorname{Im}(N_0 + N_2)N_1^* . \end{aligned} \tag{21}$$

The spreads in values of these quantities, as obtained from the Monte Carlo amplitudes, are displayed in Fig. 2. The Monte Carlo solutions have two obvious discrete ambiguities. One ambiguity is the sign of $|N_0|^2 - |N_2|^2$ and the other is the sign of $\operatorname{Re} N_2$. Neither of the ambiguities is removed by present data, or improved measurements of the set in (17). In fact, the ambiguities are crudely, though not strictly, labeled by the phase ambiguities I_1 and I_3, respectively.

The first ambiguity is removed by a measurement of D_{ls}, which is very nearly equal to $|N_0|^2 - |N_2|^2$ since the unnatural-parity exchange amplitudes are small. Figure 2(a) shows D_{ls} and its spread for the two discrete regions. A relatively imprecise measurement will remove the ambiguity. We of course expect $D_{ls} \simeq +1$ since $|N_0|$ dominates $|N_2|$ at $-t=0$, and this dominance is expected to extend to $-t \simeq 0.3$ (GeV/c)^2. We also note that $I(0s0l)$ accomplishes the same task, but is harder to measure at the ZGS. A measurement of $I(0s0l)$, averaged over $0.2 \leq -t \leq 0.6$ (GeV/c)^2 has been made at Saclay.[7,8] They find a value of -0.4 ± 0.5 corresponding to $|N_0|^2 - |N_2|^2 > 0$.

The second sign ambiguity is resolved by the measurement of $I(ns0s)$ which is approximately $2|N_0| \operatorname{Re} N_2$, where by convention N_0 is positive-imaginary. If the calculation of Field and Stevens can be used as a guide, $I(ns0s)$ will be negative. Note that $I(nl0l)$ will also resolve the ambiguity, though it appears to be less practical. Figure 2(b) shows the spread in $I(ns0s)$ for the case that D_{ls} is positive.

To study the sensitivity of our results to the values of R and K_{ss}, we have looked at solutions with these parameters absent, but with the values of (20) otherwise unchanged. The conclusions about the need for D_{ls} and $I(ns0s)$ do not depend on the values of R and K_{ss}. Thus, our results apply at energies for which R and K_{ss} have *not* been measured.

To make the above points in a more dramatic way, we show the solutions generated by adding fictitious measurements for D_{ls} and $I(ns0s)$, obtained with an accuracy 8%, which may be possible at the ZGS.[9] In addition the single-scattering measurements C_{ss}, C_{sl}, and C_{ll} are assumed to be known to 2% accuracy. Specifically, we take the values:

$$\begin{aligned}
&\sigma = 1.00\,,\\
&P_0 = 0.12 \pm 0.01\,,\\
&C_{nn} = 0.10 \pm 0.02\,,\\
&D_{nn} = 0.95 \pm 0.08\,,\\
&K_{nn} = 0.14 \pm 0.08\,,\\
&C_{ss} = -0.15 \pm 0.02\,,\\
&C_{sl} = 0.0 \pm 0.02\,,\\
&C_{ll} = 0.0 \pm 0.02\,,\\
&I(sn0s)/\sigma = 0.20 \pm 0.08\,,\\
&D_{ls} = 0.90 \pm 0.10\,,\\
&I(ns0s)/\sigma = -0.16 \pm 0.10\,.
\end{aligned} \tag{22}$$

The solutions $(-,+,-)$ and $(-,-,-)$ are most consistent with these measurements. We find that the sign ambiguity due to I_2 results in no obvious distinctions in the amplitudes for these two solutions. To get a complete picture, one must of course study all eight discrete solutions. The resultant amplitudes are displayed in Fig. 3. They show that $\mathrm{Re}\,U_0$ and $\mathrm{Im}\,N_1$ could be measured more accurately.

One of the measurements left out of the set Eq. (22) is R, which can be measured to 8%, and is expected to determine $\mathrm{Im}\,N_1$. Another measurement already completed is K_{ss}. On the basis of the values in (22), these two measurements are constrained:

$$\begin{aligned}
&-0.5 \leq R \leq 0.4\,,\\
&-0.04 \leq K_{ss} \leq 0.08\,.
\end{aligned} \tag{23}$$

We see that an 8% measurement of K_{ss} gives no new information about the amplitudes, except indirectly as a check of consistency of the data. The R measurement, on the other hand, does reduce the errors on $\mathrm{Im}\,N_1$.

We have used the values[1,8] $R = -0.30 \pm 0.10$ and the values for the observables in (22) to obtain a new set of Monte Carlo amplitudes. The results are shown in Fig. 4. The level of ambiguity of Fig. 4 is substantially better than that of Fig. 1, even though only a *few* well-chosen observables have been given reasonable, but fictitious, values. The most serious ambiguity that still remains is the determination of $\mathrm{Re}\,U_0$. One may reduce the uncertainty on $\mathrm{Re}\,U_0$ by measuring the quantity $I(ls0n)$, which involves three measured spins:

$$I(ls0n) = 2\,\mathrm{Im}(U_0 N_0^* - U_2 N_2^*) \simeq -2\,\mathrm{Re}\,U_0\,. \tag{24}$$

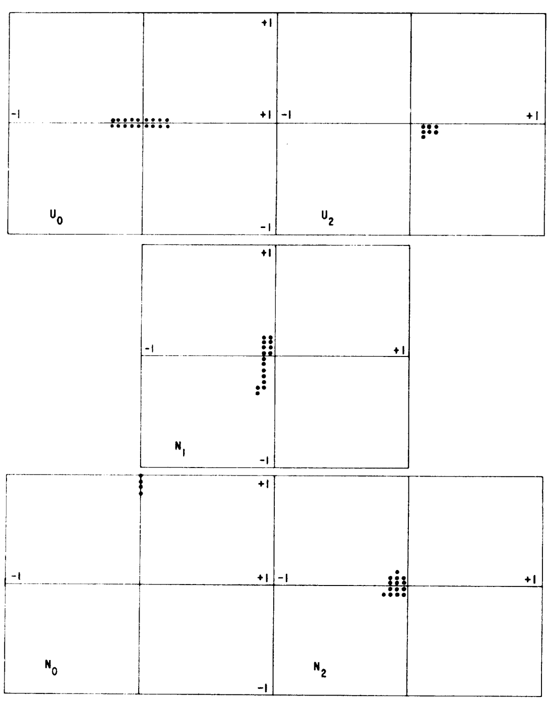

FIG. 3. Typical uncertainties in the amplitudes when D_{ls} and $I(ns0s)$ have been measured. The 11 quantities [Eq. (22)] are used.

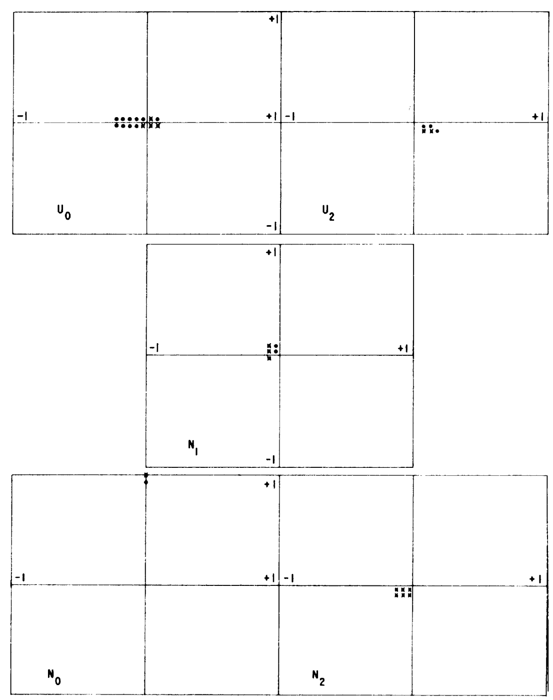

FIG. 4. The impact of the R measurement on the set [Eq. (22)]. We use $R = -0.30 \pm 0.10$. The "×'s" show how the uncertainty on the amplitudes is reduced by the additional measurement $I(ls0n)$; for illustration, $I(ls0n)/\sigma = -0.10 \pm .08$.

The additional effect of this measurement on $\mathrm{Re}\,U_0$ is also illustrated in Fig. 4.

We have mentioned earlier that the spin parameters must in general obey inequality and equality constraints. Two good examples involve the quantities D_{nn} and K_{nn}. The 8% errors on these quantities are difficult to reduce. However, for the solutions generated by (22) plus R and $I(ls0n)$, D_{nn} and K_{nn} lie in the restricted ranges

$$0.94 < D_{nn} < 0.98\,, \qquad 0.06 < K_{nn} < 0.12\,. \tag{25}$$

Thus the combined measurements imply a relatively accurate determination.

We have thus arrived at a complete set of measurements necessary to determine the pp amplitudes. Of course, we used assumed values for the yet unmeasured quantities. Actual values may change the set. Also, substitutions are possible. For example, we recall that $I(sn0s)$, which has been measured, is expected to give $\mathrm{Re}\,U_2$. An equally good way to obtain this quantity is through the observable

$$I(sl0n) = -2\,\mathrm{Im}(U_2 N_0^* - U_0 N_2^*) \simeq -2\,\mathrm{Re}\,U_2\,. \tag{26}$$

This may be useful if the final error on $I(sn0s)$ is greater than 8%. The unnatural-parity exchange amplitudes U_0 and U_2 are particularly interesting since they are less well understood than the natural-parity exchanges.

Our more detailed strategy is rather similar in its conclusions to the simple strategy outlined in the beginning, which did not take into consideration all of the available data. In particular, the detailed strategy would require measurement of the 11 quantities σ, D_{nn}, D_{ls}, P_0, R, $I(ns0s)$, C_{nn}, $I(ls0n)$, C_{ll}, $I(sn0s)$, and C_{ss} which should be obtained for the simple strategy. In addition, the detailed strategy brings in the measurements of K_{nn} and C_{sl}, whose value can be thought of as insurance against the unexpected. The measurement of K_{nn} has been completed. It turns out not to be useful in reducing ambiguity at low t since $K_{nn} \approx C_{nn}$, and K_{nn} has $\pm 8\%$ experimental uncertainty. At low t, one expects that C_{sl} will be small, since it is quadratic in the "small" amplitudes, cf. Eq. (4). However, since C_{sl} can readily be determined to $\pm 2\%$ accuracy, or better, it may be a good strategy to measure it, especially since at large t it is not forced to be zero.

In summary, we find at the level of accuracy currently obtainable, that the most useful set of measurements are the 11 in Eq. (22), and that measurements of R and $I(ls0n)$ reduce the errors further. *Some substitutions* can be made in our set, but there are no additional measurements which give significant new information.

The Monte Carlo strategy permits us to obtain the amplitude uncertainties from the data in a relatively efficient manner and it gives a direct indication of those measurements that are most effective in reducing ambiguities. The analysis can obviously be extended to other values of t (in particular, large t) and other values of s. However, it is only a preliminary step in determining amplitudes from data, since one does not really obtain the "most probable" values of the amplitudes by such an approach. We expect a standard χ^2 analysis will be most efficient for that purpose. Our main purpose here is to suggest additional measurements that would be most helpful in unambiguous determination of amplitudes; this work is a preliminary step to a t-dependent amplitude analysis. We plan to do a complete amplitude analysis as data become available.

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[1]At 6 GeV/c a number of spin measurements have been completed at the ZGS, some of which have been published, some of which exist in only preliminary form. In our analysis we have used values from the following sources: (a) R. C. Fernow *et al.*, Phys. Lett. 52B, 243 (1974); L. G. Ratner *et al.*, Phys. Rev. D 15, 604 (1977); and T. A. Mulera, Univ. of Michigan Report No. UM-HE-76-26 (unpublished). These references contain data at 6 GeV/c on C_{nn}, K_{nn}, D_{nn}, and $A = P_0$. Data at 12 GeV/c on C_{nn} also exist: K. Abe *et al.*, Phys. Lett. 63B, 239 (1976). (b) G. Hicks *et al.*, Phys. Rev. D 12, 2594 (1975); D. Miller *et al.*, Phys. Rev. Lett. 36, 763 (1976). These references give data on C_{nn} from 2–6 GeV/c. (c) Data from Argonne experiment No. E385 on $I(sn0s)$ and $I(s00s)$ are preliminary. The values we use in our analysis are a private communication from A. Beretvas *et al.*, and were taken as only an indication. See B. Sandler, in *High Energy Physics with Polarized Beams, and Targets*, edited by M. L. Marshak (A.I.P., New York, 1976), p. 77. (d) Data from Argonne experiment

No. E402 on C_{ss} are also preliminary. The values are based on private communication of I. P. Auer *et al.*, and were again taken as a guide only. Final values can be found in I. P. Auer, *et al.*, Phys. Rev. Lett. 37, 1727 (1976). (e) G. W. Abshire *et al.*, Phys. Rev. D 12, 3393 (1975). This experiment gives 6-GeV/c values for D_{nn}.

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[4]G. H. Thomas, see Ref. 3, p. 83.

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[7]Saclay has information on $I(0s0\ell)$ which they call $-\sigma R'$: J. Deregel *et al.*, Phys. Lett. 43B, 338 (1973). Similar data at 3.83 GeV/c and more discussion are given in J. Deregel *et al.*, Nucl. Phys. B103, 269 (1976). The 6-GeV/c data are also reanalyzed in this reference.

[8]Saclay also measures R (Ref. 7).

[9]A. Yokosawa (private communication).