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Hadron electromagnetic mass differences

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Mass differences between members of isotopic-spin multiplets of hadrons are calculated in the MIT bag model. All low-mass SU(4) multiplets of spin 0, 1/2, 1, and 3/2 particles are considered. Both Coulomb and magnetic contributions to mass differences are evaluated without the assumption of SU(6)-degenerate intermediate states. The mass difference between the "up" and the "down" quarks inside the bag is not (necessarily) taken to be a constant; we propose a parameterization for it that works well for its contribution to known hadron electromagnetic mass differences. Based on the parametrization we present a class of sum rules for charmed-particle electromagnetic mass differences.

I. INTRODUCTION

Calculating mass differences among hadrons in the same isospin multiplet has intrigued particle physicists for many years.[1] Since such differences should be due to just the electromagnetic interaction (admittedly in the presence of strong interactions) they are, in principle, calculable in terms of what is known. However, attempts to perform these calculations, putting in the strong interactions through form factors or using soft-meson techniques with the Weinberg sum rules, have met with very limited success.[2] It seems possible to calculate the pion mass difference and even, perhaps, all $\Delta I=2$ mass differences but $\Delta I=1$ mass differences, such as that between the proton and neutron, come out completely wrong or involve subtractions in dispersion relations with unknown subtraction constants.

The problems that the strong interactions create can perhaps be avoided by calculating in a quark model. The mass shift due to the electromagnetic interaction has two parts, one from the electric and magnetic interactions between different quarks, and a second from the self-energies of the quarks themselves. This second part leads to a mass difference between the "up" and the "down" quarks. One can then try to calculate the interactions between quarks while fixing the value of the quark mass difference by fitting to the experimental value of one hadronic mass difference. This is a first step which leaves the quark mass difference to be calculated in a later, more perfect, theory. Even such a modest attempt, however, requires a fairly complete quark theory if we are to avoid simply taking the electric interaction to be α/R for some R and if we are to calculate the magnetic interaction at all.

Fortunately, such a quark theory exists. Over the past few years a number of workers have studied the structure and consequences of a conceptually simple and well-defined realization of the idea of an underlying quark structure for hadrons—the MIT bag model.[3] The quarks in this model carry color and interact through zero-mass colored neutral vector gluons. The essence of the model is the introduction of a new term, $-g_{\mu\nu}\theta_s B$, into the energy-momentum tensor; B is a universal constant and θ_s is unity inside the bag in which the quarks are confined and zero outside. This new term makes color confinement explicit: It corresponds to the finite part left over after subtraction of the space-dependent zero-point field-theory vacuum energy.

The basic attribute of the bag model is calculability; a wide range of phenomena can be calculated in a well-defined way. This follows from the fact that the model has definite quark wave functions. So far, these wave functions have been found in the "static-bag" approximation in which the bag is treated classically, without its coordinates being quantized. This approximation has yielded sensible results for Regge trajectories,[4] for the spectroscopy of low-lying states,[5] and for high-energy scattering.[6]

For low-lying hadronic states of both baryons and mesons, calculations of masses and magnetic moments have been made.[5] These agree well with experiment. Predictions have also been made[5] for the masses of charmed[7] particles. As the reader has perhaps surmised, the present paper is an attempt to extend these calculations to the calculation of electromagnetic mass differences. Our work tests both the bag model itself and the static cavity approximation to it.

As discussed above, electromagnetic mass differences in a quark model are given by

$$\Delta M = (\Delta M)_{\text{el}} + (\Delta M)_{\text{mag}} + (\Delta M)_{\text{quark}} . \tag{1.1}$$

Since the bag model commits itself to definite wave functions, the first two terms can be calculated explicitly with no free parameters. The up and down quarks in the bag model are approximately massless. This feature accounts for the successful magnetic-moment predictions; here it makes the second (magnetic) term in (1.1) potentially important. We will find that the magnetic mass shift is usually much less than the electric mass shift in the static bag. The magnetic energy is, however, especially sensitive to the bag radius and there is reason to believe that it could play a more important role in nonstatic approximations. The third term in (1.1) is the mass difference between the up and the down quarks. This is, in principle, calculable in the bag theory; it requires knowing (the poles of) the quark propagator in the bag, but we will not attempt to calculate it.[8]

The need for a method of calculating electromagnetic mass shifts has become more acute with the observation of some charmed mesons. The D and the D^*,[9] which are the pseudoscalar and vector combinations of one charmed and one ordinary quark, are expected to have very close to the same mass. Thus, what combinations of D and D^* are produced, and how they decay, depends on the electromagnetic mass shift.[10] Some decays of $D^* \to D\pi$ may have negative Q values, for example. We will calculate the electromagnetic mass differences for all low-lying hadrons, i.e., for all the multiplets of charmed and noncharmed pseudoscalar, vector, spin $\frac{1}{2}$, and spin $\frac{3}{2}$ hadrons that in the bag model are formed from s-wave quarks without radial excitations or excitations of the bag's surface.

The paper is organized as follows: In Sec. II we review the parts of the bag model that we will need, in particular the wave functions and the results of Ref. 5 for strong masses. In Sec. III we derive the equations for the electric and magnetic mass shifts, and in Sec. IV we present and discuss the numerical results. Included in the discussion of Sec. III are tables of coefficients necessary in any quark model for determining the magnetic quark-quark interaction contribution to the self-mass when the approximations of SU(6)-symmetric intermediate states is *not* made.

II. THE BAG MODEL

The quarks in a spherical bag of radius R describing a low-lying hadron state satisfy the free Dirac equation[11,12]

$$(-i\vec{\gamma}\cdot\vec{\nabla} + \gamma^0\omega + m_q)q = 0 , \tag{2.1}$$

where color[13] and flavor indices have been suppressed, and m_q is the mass of the light ($m_q \simeq 0$), strange, or charmed quark as appropriate. ω is the frequency of the mode. Equation (2.1) is supplemented by two conditions: (i) a linear condition that ensures the vanishing of all (vector) currents carrying quantum numbers at the surface of the bag, and (ii) a nonlinear condition that ensures that no energy or momentum is carried across the surface. The linear condition (i) is

$$-i\vec{\gamma}\cdot\hat{r}q(\vec{r}) = q(\vec{r}) \quad \text{at} \quad |\vec{r}| = R . \tag{2.2}$$

A similar condition is satisfied by the colored gluon fields. The nonlinear condition (ii) is

$$B = -\frac{1}{2}\frac{\partial}{\partial r}\left(\sum_\alpha \bar{q}_\alpha q_\alpha\right) + \text{gluon pressure terms,} \quad \text{at } |\vec{r}| = R . \tag{2.3}$$

The sum in (2.3) is (effectively) over all quarks in the particular bag under consideration. B is a universal constant and thus a free parameter to be adjusted according to the strong masses in the theory. Two other free parameters are buried in the terms not written out in (2.3): the gluon-gluon coupling constant and the energy associated with zero-point fluctuations of the quantum modes contained in the finite but particle-dependent bag volume. These two constants are similarly adjusted to strong masses. The result of solving (2.1) subject to (2.2) is the ground-state cavity-approximation wave function for a quark with flavor α:

$$q_\alpha(\vec{r},t) = \frac{N_\alpha}{(4\pi)^{1/2}} \begin{bmatrix} \left(\frac{\omega_\alpha - m_\alpha}{\omega_\alpha}\right)^{1/2} i j_0\left(\frac{x_\alpha r}{R}\right) U_\alpha \\ -\left(\frac{\omega_\alpha - m_\alpha}{\omega_\alpha}\right)^{1/2} j_1\left(\frac{x_\alpha r}{R}\right) \vec{\sigma}\cdot\hat{r}\, U_\alpha \end{bmatrix} \times e^{-i\omega_\alpha t} , \tag{2.4}$$

where $r = |\vec{r}|$. j_l are spherical Bessel functions and the U_α are two-component Pauli spinors. $q_\alpha^\dagger q_\alpha$, integrated over the bag, is normalized to unity for each α. Thus,

$$\frac{1}{N_\alpha^{\,2}} = R^3 j_0^{\,2}(x_\alpha)\,\frac{2\omega_\alpha(\omega_\alpha - 1/R) + m_\alpha/R}{\omega_\alpha(\omega_\alpha - m_\alpha)}\,, \qquad (2.5)$$

with the frequency of the lowest mode given by

$$\omega_\alpha = \frac{1}{R}\left[x_\alpha^{\,2} + (m_\alpha R)^2\right]^{1/2}. \qquad (2.6)$$

Equation (2.2) implies an eigenvalue equation for the x_α

$$\tan x_\alpha = \frac{x_\alpha}{1 - m_\alpha R - [x_\alpha^{\,2} + (m_\alpha R)^2]^{1/2}}\,. \qquad (2.7)$$

Application of the nonlinear condition (2.3) in Ref. 5 then yielded the bag radius, R, for all low-lying hadrons; (2.7) then determines x_α for each quark in each hadron and (2.4) gives the explicit wave function for each quark in each hadron. In Table I we list R for all low-lying hadrons and x_α for each of their quarks.

It should be emphasized that all quarks are treated *ab initio*, by Eqs. (2.1)–(2.3), relativistically. On the other hand, the condition (2.2) forces the "small" component of the wave function to be smaller than the large component throughout the interior of the bag. [Indeed (2.7) is just the condition that the large and small components be equal in magnitude.] This suppression of the small components makes plausible the sometimes surprising accuracy of nonrelativistic quark-model predictions. The bag model is, at the same time, successful in finding hadronic magnetic moments[5]; for this the small components are essential.

Finally, it should be noted that the masses and magnetic momenta calculated in Refs. 5 probe hadronic structures, basically in the "independent-quark" approximation; quark-quark wave-function correlations are not tested there directly. Electromagnetic mass differences are a much stronger test of the model since they are sensitive to these correlations.

TABLE I. The radius R and the quark eigenvalues x_α, $\alpha = u, d, s, c$ ($x_u = x_d$) for each of the hadrons. The masses of the quarks are $m_u = m_d = 0$, $m_s = 0.279$ GeV, $m_c = 1.551$ GeV. R is given in units of GeV^{-1}. The experimental mass and the mass as predicted by the bag are both given (in units of GeV). The charmed states are also listed by their quark content.

Particle	R	x_u	x_s	x_c	$M_{\rm exp}$	$M_{\rm bag}$
N	5.00	2.040			0.938	0.938
Σ	4.95	2.040	2.485		1.189	1.144
Ξ	4.91	2.040	2.484		1.321	1.289
N^*	5.48	2.040			1.236	1.233
Σ^*	5.43	2.040	2.515		1.385	1.382
Ξ^*	5.39	2.040	2.505		1.533	1.529
C_1 (cuu)	4.79	2.040		2.945		2.357
S^+ $[c(su)_{\rm sym}]$	4.75	2.040	2.475	2.935		2.507
A^+ $[c(su)_{\rm anti}]$	4.58	2.040	2.465	2.935		2.396
X_u (ccu)	4.27	2.040		2.915		3.538
C_1^* (cuu)	5.12	2.040		2.955		2.461
S^* $[c(su)_{\rm sym}]$	5.07	2.040	2.495	2.955		2.603
X^* (ccu)	4.69	2.040		2.935		3.661
π	3.34	2.040			0.139	0.280
K	3.26	2.040	2.375		0.495	0.497
D	2.80	2.040		2.825	1.865	1.726
ρ	4.71	2.040			0.77	0.783
K^*	4.65	2.040	2.465		0.892	0.928
D^*	4.18	2.040		2.915	2.0(?)	1.969

III. ELECTROMAGNETIC MASS DIFFERENCES

The electromagnetic self-energy of a particle with momentum p is given, in field theory, to lowest order in e^2, by

$$\frac{e^2}{2}\int d^4x\, D^{\mu\nu}(x)\langle p|(J_\mu(x)J_\nu(0))_+|p\rangle\,, \qquad (3.1)$$

where $J_\mu(x)$ is the electromagnetic current.[14] This expression is not appropriate to the bag, which is not translationally invariant.[15] The states in (3.1) are normalized covariantly. Introducing states normalized to unity by

$$|p\rangle = [(2\pi)^3 2E\delta_p^3(0)]^{1/2}|T\rangle \qquad (3.2)$$

allows the $\delta_p^3(0)$ to be absorbed into a spatial coordinate for the second current. In the rest frame, where the E in (3.2) is M, the correction to the mass squared, for the bag model, is

$$\frac{e^2}{2}\,2M\int dx^0\int_{\rm bag} d^3x\,d^3y\,D^{\mu\nu}(\vec{x}-\vec{y},x^0) \times\langle T|(J_\mu(x)J_\nu(\vec{y},0))_+|T\rangle\,. \qquad (3.3)$$

The difference of the mass squared is therefore a difference of (3.3) for two different states T. Dividing by $2M$ we have a linear mass difference; this is the expression we will use.

To evaluate the x^0 integral we write the photon propagator in momentum space, expand the time-ordered product, and insert a complete set of states between the currents. To be general we allow the intermediate state to have a different mass than the external state and write the expression in terms of ΔE, with ΔE defined as the mass of the intermediate state minus the mass of the external state. The two terms in the time-ordered product look different; but, after the energy integral of the photon propagator is calculated, they can be seen to be equal. This expression is then

$$\frac{e^2}{(2\pi)^2}\sum_n \int_{\text{bag}} d^3x\, d^3y \int_0^\infty \frac{dk}{k+\Delta E}\, \frac{\sin|\vec{x}-\vec{y}|k}{|\vec{x}-\vec{y}|}\, \langle T|J_\mu(x,0)|T_n\rangle\langle T_n|J^\mu(\vec{y},0)|T\rangle\,, \tag{3.4}$$

where the k integral is the remnant of the photon propagator integral. If ΔE is zero, the k integral is

$$\int_0^\infty \frac{dk}{k}\sin|\vec{x}-\vec{y}|k = \frac{\pi}{2}\,, \tag{3.5}$$

and the usual expression for the electric or magnetic energy, modified for the bag, is obtained:

$$\frac{e^2}{2}\int_{\text{bag}} d^3x\, d^3y\, \frac{1}{4\pi}\frac{1}{|\vec{x}-\vec{y}|}\langle T|J^\mu(\vec{x},0)J_\mu(\vec{y},0)|T\rangle\,. \tag{3.6}$$

We will continue to keep ΔE nonzero and work with (3.4).

In a quark model the current is

$$J_\mu(x) = \sum_\alpha \bar{q}_\alpha(x)\gamma_\mu q_\alpha(x) Q_\alpha\,, \tag{3.7}$$

where Q_α is the charge on the αth quark and q_α is its field operator. When (3.7) is substituted into (3.4) there are two types of terms:

(i) terms in which the virtual photon is emitted and absorbed by the same quark,

(ii) terms in which the virtual photon is exchanged between two different quarks.

The first class of terms yields the quark electromagnetic self-energies. These are difficult to calculate because the sum over intermediate states contains an infinite number of terms. An attempt to compute the quark self-energy has been made by Chodos and Thorn,[8] but further work is needed. Finding the correct sign for the up-down mass differences will be a strong test of the bag (or any other) quark model. Here we note its dependence on R. To the approximation that the light quarks have zero mass they would have no electromagnetic self-energy if free. When they are confined, however, one expects them to have an electromagnetic self-energy and there to be an up-down mass difference. Since the existence of this mass difference depends on the quarks being bound, we expect it to vary as $1/R$. This variation is not very important in the mass differences of the baryons because they all have roughly the same radius. It can be a large effect, however, for the mesons where, as seen from Table I, the radii have a large spread. This $1/R$ variation may be modified by the presence of strange and/or charmed quarks since the latter provide an alternative mass scale. In the following section we will find the known hadron mass differences are well described by an up-down mass difference of the form $A/R + B\times$ (number of strange quarks).

From the discussion of Sec. II the currents (3.7) are easily found. For the charge density we have

$$J^0(\vec{x},0) = \sum_\alpha \sum_m \frac{N_\alpha^{\,2}}{4\pi} b_\alpha^\dagger(m) Q_\alpha b_\alpha(m)\left\{\left(\frac{\omega_\alpha + m_\alpha}{\omega_\alpha}\right)\left[j_0\left(\frac{x_\alpha x}{R}\right)\right]^2 + \left(\frac{\omega_\alpha - m_\alpha}{\omega_\alpha}\right)\left[j_1\left(\frac{x_\alpha x}{R}\right)\right]^2\right\}, \tag{3.8}$$

where x is the magnitude of $\vec{x}$, α denotes the type of quark, R is the radius of the bag, and x_α and ω_α were defined in Sec. II.

$J^0(\vec{x},0)$ is a diagonal operator, so in (3.4) we must have T_n equal to only T, ΔE is thus zero, and (3.5) can be used. The shift in mass due to the electric energy between the quark pairs is a sum over the combinations of quarks

$$(\Delta M)_{\text{el}} = \sum_{\substack{\alpha,\beta\\ \alpha>\beta}} C_{\alpha\beta}[I_{\alpha\beta}(R) + I_{\beta\alpha}(R)]\,, \tag{3.9}$$

where $C_{\alpha\beta} = Q_\alpha Q_\beta$, the product of the quark charges. We must omit the quark self-energy part from the sum over α and β, but we do not restrict $\alpha > \beta$. The $\frac{1}{2}$ in (3.1) means we must take the sum of $I_{\alpha\beta}(R)$ and $I_{\beta\alpha}(R)$. This means the coefficient is symmetrized in α and β. The quark-quark interaction is

$$\begin{aligned} I_{\alpha\beta}(R) = \frac{e^2}{4\pi} N_\alpha^{\,2} N_\beta^{\,2}\int_0^R dy\, y &\left\{\left(\frac{\omega_\beta + m_\beta}{\omega_\beta}\right)\left[j_0\left(\frac{x_\beta y}{R}\right)\right]^2 + \left(\frac{\omega_\beta - m_\beta}{\omega_\beta}\right)\left[j_1\left(\frac{x_\beta y}{R}\right)\right]^2\right\} \\ &\times \frac{R^2}{4x_\alpha^{\,2}}\left\{\left(\frac{\omega_\alpha + m_\alpha}{\omega_\alpha}\right)\left(2y - \frac{R}{x_\alpha}\sin\frac{2x_\alpha y}{R}\right)\right. \\ &\left.\quad + \left(\frac{\omega_\alpha - m_\alpha}{\omega_\alpha}\right)\left[2y + \frac{R}{x_\alpha}\sin\left(\frac{2x_\alpha y}{R}\right) - 4\,\frac{R^2}{x_\alpha^{\,2}}\,\frac{\sin^2(x_\alpha y/R)}{y}\right]\right\}. \end{aligned} \tag{3.10}$$

The angular integrals were done by expanding $|\vec{x}-\vec{y}|^{-1}$ as a sum of a product of spherical harmonics in the usual way.[16]

The spatial part of the current is

$$J^i(\vec{x},0) = -\frac{1}{2\pi}\sum_\alpha \sum_{m,m'} N_\alpha{}^2 j_0\left(\frac{x_\alpha x}{R}\right) j_1\left(\frac{x_\alpha x}{R}\right) b_\alpha^\dagger(m) Q_\alpha b_\alpha(m') U_m^\dagger (\hat{x}\times\vec{\sigma})^i U_{m'}, \tag{3.11}$$

where the $b_\alpha(m)$ are destruction operators for quarks of type α with spin projection m. J^i, of course, can connect different states, and the full expression (3.4) must be used. The shift in mass due to the magnetic interactions between quarks is then a sum over intermediate states

$$(\Delta M)_{\rm mag} = \sum_Q (\Delta M)_{\rm mag}^{PQ}, \tag{3.12}$$

where the only states Q that contribute are those whose quarks are in the same spatial states as those of the external particle.

The contribution to the magnetic energy of particle P from an intermediate state Q is the sum over the quark-quark interactions

$$(\Delta M)_{\rm mag}^{PQ} = -\sum_{\substack{\alpha,\beta \\ \alpha>\beta}} C^{PQ}[J_{\alpha\beta}(\Delta E,R) + J_{\beta\alpha}(\Delta E,R)], \tag{3.13}$$

where the comments about the sum over α and β given after (3.9) also apply here. ΔE is $m_Q - m_P$. $J_{\alpha\beta}(\Delta E,R)$ is given by

$$J_{\alpha\beta}(\Delta E,R) = \frac{e^2}{4\pi}\frac{8}{3\pi} N_\alpha{}^2 N_\beta{}^2 \frac{R^3}{x_\alpha{}^3}\int_0^\infty dk\,\frac{k}{k+\Delta E}\int_0^R dy\, y^2 j_0\left(\frac{x_\beta y}{R}\right) j_1(ky) \times\left[\int_0^{x_\alpha y/R} dz\,\frac{\sin^2 z}{z}\,\sin\frac{kR_z}{x_\alpha} - j_1(ky)\sin^2\left(\frac{x_\alpha y}{R}\right)\right]. \tag{3.14}$$

The angular integrals were evaluated by using the expansion[16]

$$\frac{e^{ik|\vec{x}-\vec{y}|}}{|\vec{x}-\vec{y}|} = 4\pi i k \sum_{l=0}^\infty j_l(kx) h_l^{(1)}(ky) \sum_{m=-l} Y_{l,m}^*(\theta_x,\phi_x) Y_{l,m}(\theta_y,\phi_y),\ x<y \tag{3.15}$$

where $h_l^{(1)}$ is a spherical Hankel function.

The coefficients in (3.13) are given by

$$C_{\alpha\beta}^{PQ} = \sum_{m,m'}\sum_{k,k'} \langle P|b_\alpha^\dagger(m) Q_\alpha b_\alpha(m')|Q\rangle\langle Q|b_\beta^\dagger(k) Q_\beta b_\beta(k')|P\rangle U_m^\dagger \sigma^i U_{m'} U_k^\dagger \sigma^i U_{k'}. \tag{3.16}$$

These coefficients and the electric coefficients $C_{\alpha\beta} = Q_\alpha Q_\beta$ of (3.9) play the same role as the SU(6) coefficients given by Thirring.[17] In the electric case, since the wave functions for different quarks are different, it is important to keep separate the contribution from each pair of quarks and to perform the space integrals before summing the contributions. In the magnetic case, each intermediate state must be considered separately. For each intermediate state one must evaluate the interaction of each pair of quarks, being careful not to include self-energy effects. The explicit wave functions written for SU(6) in Ref. 17 are very useful for this purpose. We have used the generalization of these wave functions that includes the multiplets of SU(4). We limit ourselves to 0^- and 1^- intermediate states for mesons, and spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ intermediate states for baryons. Effects from neglected states should be quite small because ΔE for such states is larger, reducing their contribution to the magnetic energy, which is itself usually much less than the electric energy. The neglected states can contribute, even though they are in a different representation of SU(8), because of the restriction $\alpha\neq\beta$ on the sum in (3.13). To see this consider

$$\sum_\alpha\sum_\beta \langle P|Q_\alpha\vec{\sigma}_\alpha|n\rangle\cdot\langle n|Q_\beta\vec{\sigma}_\beta|P\rangle = \left\langle P\left|\sum_\alpha Q_\alpha\vec{\sigma}_\alpha\right|n\right\rangle\cdot\left\langle n\left|\sum_\beta Q_\beta\vec{\sigma}_\beta\right|P\right\rangle - \sum_\alpha |\langle P|Q_\alpha\vec{\sigma}_\alpha|n\rangle|^2. \tag{3.17}$$

In the first term on the right-hand side of (3.17) the intermediate states must be in the same representation as the external state since the operator is an SU(8) generator. In the second term, however, since $q_\alpha\sigma_\alpha$ for one quark is not an SU(8) generator, the intermediate state need not be in the same representation. We have included as intermediate states only states in the same representation. Thirring, on the other hand, takes a complete set of intermediate states and thus implicitly assumes a degenerate mass for all the states which can contribute to (3.17). Other states will contribute to (3.17) only for the cases where the external state involves two or more identical quarks. Thus, for mesons no other intermediate states come in, and the sum of our coefficients for a given particle equals Thirring's coefficient for that particle. The SU(8) representation in the decomposition of $8\times 8\times 8$ that includes the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons has the only wave functions that are totally symmetric in spin and flavor. When the external state contains two identical quarks the operator $q_\alpha \sigma_\alpha$ does not preserve this symmetry. Thus the sum of our coefficients for the interaction between identical quarks in a baryon will be greater than Thirring's coefficient since he effectively includes more states in the

TABLE II. The magnetic coefficients of the mesons.

Particle P	Intermediate state Q	Coefficient $C^{PQ}_{\alpha\beta}$
π^+	ρ	$-\frac{2}{3}$
π^0	ρ	$\frac{5}{12}$
	ω	$\frac{5}{12}$
K^+	K^{*+}	$-\frac{2}{3}$
K^0	K^{*0}	$\frac{1}{3}$
D^+	D^{*+}	$-\frac{2}{3}$
D^0	D^{*0}	$\frac{4}{3}$
ρ^+	ρ^+	$\frac{4}{9}$
	π^+	$-\frac{2}{9}$
ρ^0	ρ^0	$-\frac{5}{18}$
	ω	$-\frac{5}{18}$
	π^0	$\frac{5}{36}$
	η	$\frac{5}{36}$
K^{*+}	K^{*+}	$\frac{4}{9}$
	K^+	$-\frac{2}{9}$
K^{*0}	K^{*0}	$-\frac{2}{9}$
	K^0	$\frac{1}{9}$
D^{*+}	D^{*+}	$\frac{4}{9}$
	D^+	$-\frac{2}{9}$
D^{*0}	D^{*0}	$-\frac{8}{9}$
	D^0	$\frac{4}{9}$

TABLE III. The magnetic coefficients of the spin-$\frac{1}{2}$ baryons.

Particle P $(q_1q_2q_3)$	Intermediate state Q	Coefficients $C^{PQ}_{\alpha\beta}$ q_1q_2	q_1q_3	q_2q_3
p (uud)	p	$\frac{16}{27}$	$\frac{4}{27}$	$\frac{4}{27}$
	N^{*+}	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{8}{27}$
n (udd)	n	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{4}{27}$
	N^{*0}	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{2}{27}$
Σ^+ (uus)	Σ^+	$\frac{16}{27}$	$\frac{4}{27}$	$\frac{4}{27}$
	Σ^{*+}	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{8}{27}$
Σ^- (dds)	Σ^-	$\frac{4}{27}$	$-\frac{2}{27}$	$-\frac{2}{27}$
	Σ^{*-}	$\frac{2}{27}$	$-\frac{4}{27}$	$-\frac{4}{27}$
Σ^0 (uds)	Σ^0	$-\frac{8}{27}$	$\frac{4}{27}$	$-\frac{2}{27}$
	Λ^0	$\frac{6}{27}$	0	0
	Σ^{*0}	$-\frac{4}{27}$	$\frac{8}{27}$	$-\frac{4}{27}$
Ξ^0 (uss)	Ξ^0	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{4}{27}$
	Ξ^{0*}	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{2}{27}$
Ξ^- (dss)	Ξ^-	$-\frac{2}{27}$	$-\frac{2}{27}$	$\frac{4}{27}$
	Ξ^{-*}	$-\frac{4}{27}$	$-\frac{4}{27}$	$\frac{2}{27}$
C_1^{++} (cuu)	C_1^{++}	$-\frac{8}{27}$	$-\frac{8}{27}$	$\frac{16}{27}$
	C_1^* (cuu)	$-\frac{16}{27}$	$-\frac{16}{27}$	$\frac{8}{27}$
C_1^+ (cud)	C_1^+	$-\frac{8}{27}$	$\frac{4}{27}$	$-\frac{8}{27}$
	C_0^+	0	0	$\frac{6}{27}$
	C_1^* (cud)	$-\frac{16}{27}$	$\frac{8}{27}$	$-\frac{4}{27}$
C_1^0 (cdd)	C_1^0	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{4}{27}$
	C_1^* (cdd)	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{2}{27}$
X_u^{++} (ccu)	X_u^{++}	$\frac{16}{27}$	$-\frac{8}{27}$	$-\frac{8}{27}$
	X_u^* (ccu)	$\frac{8}{27}$	$-\frac{16}{27}$	$-\frac{16}{27}$
X_u^+ (ccd)	X_u^+	$\frac{16}{27}$	$\frac{4}{27}$	$\frac{4}{27}$
	X_u^* (ccd)	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{8}{27}$
S^+ (cus)	S^+	$-\frac{8}{27}$	$\frac{4}{27}$	$-\frac{8}{27}$
	A^+	0	0	$\frac{6}{27}$
	S^* (cus)	$-\frac{16}{27}$	$\frac{8}{27}$	$-\frac{4}{27}$
S^0 (cds)	S^0	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{4}{27}$
	A^0	0	0	$-\frac{3}{27}$
	S^* (cds)	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{2}{27}$
A^+ (cus)	A^+	0	0	0
	S^+	0	0	$\frac{6}{27}$
	A^* (cus)	0	0	$\frac{12}{27}$
A^0 (cds)	A^0	0	0	0
	S^0 (cds)	0	0	$-\frac{3}{27}$
	A^* (cds)	0	0	$-\frac{6}{27}$

TABLE IV. The magnetic coefficients of the spin-$\frac{3}{2}$ resonances.

Particle P^* $(q_1q_2q_3)$	Intermediate state Q	Coefficients $C_{\alpha\beta}^{PQ}$ q_1q_2	q_1q_3	q_2q_3
N^{*++} (uuu)	N^{*++}	$\frac{20}{27}$	$\frac{20}{27}$	$\frac{20}{27}$
N^{*+} (uud)	N^{*+}	$\frac{20}{27}$	$-\frac{10}{27}$	$-\frac{10}{27}$
	p	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{4}{27}$
N^{*0} (udd)	N^{*0}	$-\frac{10}{27}$	$-\frac{10}{27}$	$\frac{5}{27}$
	n	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{1}{27}$
N^{*-} (ddd)	N^{*-}	$\frac{5}{27}$	$\frac{5}{27}$	$\frac{5}{27}$
Σ^{*+} (uus)	Σ^{*+}	$\frac{20}{27}$	$-\frac{10}{27}$	$-\frac{10}{27}$
	Σ^{+}	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{4}{27}$
Σ^{*0} (uds)	Σ^{*0}	$-\frac{10}{27}$	$-\frac{10}{27}$	$\frac{5}{27}$
	Λ^{0}	$\frac{2}{9}$	0	0
	Σ^{0}	$\frac{4}{27}$	$-\frac{2}{27}$	$-\frac{2}{27}$
Σ^{*-} (dds)	Σ^{*-}	$\frac{5}{27}$	$\frac{5}{27}$	$\frac{5}{27}$
	Σ^{-}	$\frac{1}{27}$	$-\frac{2}{27}$	$-\frac{2}{27}$
Ξ^{*0} (uss)	Ξ^{*0}	$-\frac{10}{27}$	$-\frac{10}{27}$	$\frac{5}{27}$
	Ξ^{0}	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{1}{27}$
Ξ^{*-} (dss)	Ξ^{*-}	$\frac{5}{27}$	$\frac{5}{27}$	$\frac{5}{27}$
	Ξ^{-}	$-\frac{2}{27}$	$-\frac{2}{27}$	$\frac{1}{27}$
C_1^{*++} (cuu)	C_1^{*} (cuu)	$\frac{20}{27}$	$\frac{20}{27}$	$\frac{20}{27}$
	C_1^{++}	$-\frac{8}{27}$	$-\frac{8}{27}$	$\frac{4}{27}$
C_1^{*+} (cud)	C_1^{*+} (cud)	$\frac{20}{27}$	$-\frac{10}{27}$	$-\frac{10}{27}$
	C_1^{+}	$-\frac{8}{27}$	$\frac{4}{27}$	$-\frac{2}{27}$
	C_0^{+}	0	0	$\frac{2}{9}$
C_1^{*0} (cdd)	C_1^{*0} (cdd)	$-\frac{10}{27}$	$-\frac{10}{27}$	$\frac{5}{27}$
	C_1^{0}	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{1}{27}$
S^{*+} (cus)	S^{*+} (cus)	$\frac{20}{27}$	$-\frac{10}{27}$	$-\frac{10}{27}$
	S^{+}	$-\frac{8}{27}$	$\frac{4}{27}$	$-\frac{2}{27}$
	A^{+}	0	0	$\frac{2}{9}$
S^{*0} (cds)	S^{*0} (cds)	$-\frac{10}{27}$	$-\frac{10}{27}$	$\frac{5}{27}$
	S^{0}	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{1}{27}$
	A^{0}	0	0	$-\frac{1}{9}$
X_u^{*++} (ccu)	X_u^{*++} (cuu)	$\frac{20}{27}$	$\frac{20}{27}$	$\frac{20}{27}$
	X_u^{++}	$\frac{4}{27}$	$-\frac{8}{27}$	$-\frac{8}{27}$
X_u^{*+} (ccd)	X_u^{*+} (ccd)	$\frac{20}{27}$	$-\frac{10}{27}$	$-\frac{10}{27}$
	X_u^{+}	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{4}{27}$

negative-definite second term of (3.17). Our results for the magnetic coefficients C^{PQ} are summarized in Table II for mesons, Table III for spin-$\frac{1}{2}$ baryons, and Table IV for spin-$\frac{3}{2}$ resonances.

TABLE V. Electric-interaction integrals, $I_{\alpha\beta}(R) + I_{\beta\alpha}(R)$, for mesons. $I_{\alpha\beta}(R)$ is defined in Eq. (3.10) of the text.

Particles	$I_{\alpha\beta}(R)+I_{\beta\alpha}(R)$ (MeV)
π	2.78
K	2.95
D	3.68
ρ	1.97
K^*	2.07
D^*	2.47

IV. NUMERICAL RESULTS AND DISCUSSION

We have calculated numerically the integrals $I_{\alpha\beta}(R)$ [Eq. (3.10)] and $J_{\alpha\beta}(\Delta E, R)$ [Eq. 3.14)]. The results are given in Tables V and VI for $I_{\alpha\beta}$ for mesons and for baryons, respectively, and in Tables VII, VIII, and IX for $J_{\alpha\beta}$ for mesons, for spin-$\frac{1}{2}$ baryons, and for spin-$\frac{3}{2}$ baryons, respectively. The consequent values of (i) $(\Delta M)_{el}$ and (ii) $(\Delta M)_{mag}$ are given in Table X for those isotopic multiplets with measured mass differences. Also shown in Table X are (iii) R, (iv) the experimental value of each mass difference, and (v) the size of the up-down mass difference deduced from (i), (ii), and (iv) for those with the more reliable measurements, N, Σ, Ξ, and K, K^* but not Ξ^*.

The pion mass difference does not depend on the up-down difference. As previously discussed[18] the pion is not well described by the bag parameters used here, so that we do not expect a good answer for the pion electromagnetic mass difference and we do not get a good answer. By using values for

TABLE VI. Electric-interaction integrals, $I_{\alpha\beta}(R) + I_{\beta\alpha}(R)$, for each quark pair in each baryon. l stands for the light quark, either up or down.

Particle P $(q_1q_2q_3)$	Integrals $I_{\alpha\beta}(R)+I_{\beta\alpha}(R)$ (MeV) q_1q_2	q_1q_3	q_2q_3
N (lll)	1.86	1.86	1.86
Σ (lls)	1.87	1.96	1.96
Ξ (lss)	1.99	1.99	2.11
N^* (lll)	1.69	1.69	1.69
Σ^* (lls)	1.71	1.81	1.81
Ξ^* (lss)	1.78	1.78	1.86
C_1 (llc)	1.94	2.25	2.25
X_u (lcc)	2.32	2.32	2.62
S (lsc)	2.05	2.08	2.23
A (lsc)	2.12	2.30	2.47
C_1^* (llc)	1.81	2.07	2.07
X_u^* (lcc)	2.15	2.15	2.49
S^* (lsc)	1.93	2.13	2.30

TABLE VII. Magnetic-interaction integrals, $J_{\alpha\beta}(\Delta E,R)+J_{\beta\alpha}(\Delta E,R)$, for mesons. $J_{\alpha\beta}(\Delta E,R)$ is defined in Eq. (3.14) of the text. For the pseudoscalars the $J_{\alpha\beta}(0,R)$ are included for comparison purposes even though the $C_{\alpha\beta}^{PQ}$ in (3.13) are zero.

Particle	Intermediate state	$J_{\alpha\beta}(\Delta E,R)+J_{\beta\alpha}(\Delta E,R)$ (MeV)
π	π	0.295
π	ρ,ω	0.147
K	K	0.270
K	K^*	0.155
D	D	0.186
D	D^*	0.114
ρ	ρ,ω	0.209
ρ	π	0.333
ρ	η	0.450
ρ	η'	0.161
K^*	K^*	0.176
K^*	K	0.476
D^*	D^*	0.093
D^*	D	0.231

the quark masses which give a better value for the pion mass[5] a much better value for the mass difference is obtained. The $I=2$ part of the Σ mass difference is also independent of the up-down difference. From Table X we find

$$\Sigma^+ + \Sigma^- - 2\Sigma^0 = 1.61 \text{ MeV} \tag{4.1}$$

in very good agreement with the experimental value of 1.78 ± 0.20 MeV.

TABLE VIII. Magnetic-interaction integrals, $J_{\alpha\beta}(\Delta E,R)+J_{\beta\alpha}(\Delta E,R)$, for spin-$\frac{1}{2}$ baryons. l stands for the light quark, either up or down.

Particle P $(q_1q_2q_3)$	Intermediate state Q $(q_1q_2q_3)$	Integrals $J_{\alpha\beta}(\Delta E,R)+J_{\beta\alpha}(\Delta E,R)$ (MeV) q_1q_2	q_1q_3	q_2q_3
N (lll)	N	0.197	0.197	0.197
N (lll)	N^*	0.123	0.123	0.123
Σ (lls)	Σ	0.199	0.164	0.164
Σ (lls)	Σ^*	0.141	0.116	0.116
Ξ (lss)	Ξ	0.167	0.167	0.140
Ξ (lss)	Ξ^*	0.114	0.114	0.094
C_1 (llc)	C_1	0.206	0.076	0.076
C_1 (llc)	C_1^*	0.168	0.061	0.061
C_1 (llc)	C_0	0.221	0.082	0.082
X_u (lcc)	X	0.086	0.086	0.034
X_u (lcc)	X^*	0.068	0.068	0.026
S (lsc)	S	0.173	0.061	0.071
S (lsc)	S^*	0.143	0.060	0.057
S (lsc)	A	0.231	0.090	0.084
A (lsc)	A	0.181	0.081	0.070
A (lsc)	S	0.149	0.064	0.055
A (lsc)	S^*	0.128	0.056	0.048

TABLE IX. Magnetic-interaction integrals, $J_{\alpha\beta}(\Delta E,R)+J_{\beta\alpha}(\Delta E,R)$, for spin-$\frac{3}{2}$ baryons.

Particle P $(q_1q_2q_3)$	Intermediate state Q	Integrals $J_{\alpha\beta}(\Delta E,R)+J_{\beta\alpha}(\Delta E,R)$ (MeV) q_1q_2	q_1q_3	q_2q_3
N^* (lll)	N^*	0.180	0.180	0.180
N^* (lll)	N	0.381	0.381	0.381
Σ^* (lls)	Σ^*	0.182	0.148	0.148
Σ^* (lls)	Σ	0.329	0.268	0.268
Σ^* (lls)	Λ	0.379	0.309	0.309
Ξ^* (lss)	Ξ^*	0.145	0.145	0.116
Ξ^* (lss)	Ξ	0.277	0.277	0.225
C_1^* (llc)	C_1^*	0.193	0.067	0.067
C_1^* (llc)	C_1	0.262	0.093	0.093
C_1^* (llc)	C_0	0.379	0.133	0.133
S^* (lsc)	S^*	0.161	0.069	0.058
S^* (lsc)	S	0.212	0.089	0.075
S^* (lsc)	A	0.292	0.127	0.107
X_u^* (lcc)	X_u^*	0.074	0.074	0.028
X_u^* (lcc)	X	0.107	0.107	0.040

Following the discussion of Sec. III on the variation of $(\Delta M)_q$ with R and the number of heavy quarks, we parametrize $(\Delta M)_q$ by

$$(\Delta M)_q = A/R + Bn\,, \tag{4.2}$$

where n is the number of strange quarks. Consider first the baryons. Fixing A from the proton-neutron mass difference and B from the other three, we find

$$A = -8.95\times10^{-3}\,, \tag{4.3a}$$

$$B = -1.64\pm0.12 \text{ MeV}. \tag{4.3b}$$

The error quoted for B is 1 standard deviation; we consider it remarkably small.

TABLE X. Mass differences for which experimental values are known. Each of the columns is in units of MeV except for R which is measured in units of $(\text{GeV})^{-1}$ The values for $(\Delta M)_q$ are those deduced from the experimental values, using (1.1). $(\Delta M)_q$ is the mass of the up quark minus the mass of the down quark.

Mass difference	$(\Delta M)_{el}$	$(\Delta M)_{mag}$	R	$(\Delta M)_{expt}$	$(\Delta M)_q$
$p-n$	0.62	−0.12	5.00	−1.29	−1.79
$\Sigma^+-\Sigma^0$	0.60	−0.28	4.95	-3.10 ± 0.14	−3.42
$\Sigma^0-\Sigma^-$	−1.28	−0.02	4.95	-4.88 ± 0.06	−3.58
$\Xi^0-\Xi^-$	−1.32	−0.18	4.91	-6.4 ± 0.6	−4.90
$\Xi^{*0}-\Xi^{*-}$	−1.19	0.04	5.39	-3.3 ± 0.9	
$\pi^+-\pi^0$	1.39	0.22	3.34	4.60	
K^+-K^0	0.99	0.16	3.26	-3.99 ± 0.13	−5.14
$K^{*+}-K^{*0}$	0.69	0.04	4.65	-4.1 ± 0.6	−4.83

The parameterization (4.1) is an improvement over the $(\Delta M)_q$ = constant we used in Ref. 18 for mesons. There we have only two pieces of data, but we can use them to fix A and B for mesons. We find

$$A_M = -3.38\times 10^{-3}\,,\quad B_M = -4.10\,. \tag{4.4a}$$

In Table XI we give the predictions of the model for all noncharmed electromagnetic mass differences. Our only bad answer is the Ξ^* mass difference which is measured as -3.3 ± 0.9 MeV. The N^* mass differences are worth examining. It now seems reasonably well established[19,20] that $E_0(N^{*0})$ is greater than $E_0(N^{*++})$ where E_0 is the real resonance parameter in the Breit-Wigner formula, while $E_1(N^{*++})$ is greater than $E_1(N^{*0})$ where E_1 is the real part of the pole position. Consistency with the treatment of the quoted K^* mass difference in Ref. 19 and with the calculations of Ref. 2 would indicate comparison with the difference of the former while, at first glance, the latter difference would seem the comparable quantity. The question obviously deserves further study. It should also be noted that our results for the noncharmed baryons are close to those that Celmaster[2] has found from a gluon perturbed linear potential model.

It is interesting to note that the bag model "explains" the validity of the Coleman-Glashow relation[21]

$$p - n + \Xi^0 - \Xi^- = \Sigma^+ - \Sigma^-\,.$$

This is satisfied by $(\Delta M)_{el}$, $(\Delta M)_{mag}$, and $(\Delta M)_q$ individually because the dynamics of the model give, in Ref. 5, degeneracy, to a good approximation for the N, Σ, and Ξ bag radii, R.

Finally,[22] we consider the predictions of the model for the electromagnetic mass differences of charmed particles. In Ref. 18 we predicted 7.82 MeV and 6.81 MeV for the D and D^* differences based on assuming a constant value for $(\Delta M)_q$ of -5.14 MeV determined from the K mass difference. Lane and Weinberg,[23] also keeping $(\Delta M)_q$ constant, but determining the electromagnetic energy from symmetry arguments give 6.7 MeV for the D mass difference. Here, however, we extend the simple, but more effective, parametrization (4.2) by writing

$$(\Delta M)_q = A/R + Bn + Cn_c\,, \tag{4.5}$$

where n_c is the number of charmed quarks. Once one charmed mass difference is measured reliably, thereby fixing C, the experience above with strange particles indicates that (4.5) should be successful in giving all the remaining differences. Two possible estimates for C are:

$$C = B\,, \tag{4.6}$$

$$C = (m_c/m_s)B\,. \tag{4.7}$$

The predictions based on these two possibilities, which would seem to be opposite extremes, are given in Table XII. It will, of course, be particularly interesting to verify the parametrization (4.5) in the case of baryons with two charmed quarks or one charmed and one strange quark.

TABLE XI. The predicted mass differences for all the noncharmed particles. Each M is in units of MeV. $(\Delta M)_{el}$ is calculated from (3.9), $(\Delta M)_{mag}$ from (3.13). $(\Delta M)_{total}$ then comes from (3.1) using the parameterization (4.2). The N, K, and K^* mass differences are input. The ρ mass difference does not include a contribution of ~1 MeV due to ρ-photon mixing.

Mass difference	$(\Delta M)_{el}$	$(\Delta M)_{mag}$	$(\Delta M)_{total}$
$p-n$	0.62	−0.12	−1.29
$\Sigma^+-\Sigma^0$	0.60	−0.28	−3.13
$\Sigma^0-\Sigma^-$	−1.28	−0.02	−4.75
$\Xi^0-\Xi^-$	−1.32	−0.18	−6.60
$N^{*++}-N^{*+}$	2.26	−0.23	0.40
$N^{*+}-N^{*0}$	0.56	−0.14	−1.21
$N^{*0}-N^{*-}$	−1.13	0.27	−2.69
$\Sigma^{*+}-\Sigma^{*0}$	0.54	−0.15	−2.90
$\Sigma^{*0}-\Sigma^{*-}$	−1.17	0.06	−4.40
$\Xi^{*0}-\Xi^{*-}$	−1.19	0.04	−6.09
$\pi^+-\pi^0$	1.39	0.22	1.61
K^+-K^0	0.99	0.16	−3.99
$\rho^+-\rho^0$	0.99	−0.05	0.94
$K^{*+}-K^{*0}$	0.69	0.04	−4.10

TABLE XII. Predicted mass differences for the charmed particles. Each ΔM has units of MeV. $(\Delta M)_{total}$ and $(\Delta M)'_{total}$ correspond to the two extreme values, (4.6) or (4.7), in the parameterization (4.5).

Mass difference	$(\Delta M)_{el}$	$(\Delta M)_{mag}$	$(\Delta M)_{total}$	$(\Delta M)'_{total}$
$C_1^{++}-C_1^+$	2.79	−0.12	−0.84	−8.32
$C_1^+-C_1^0$	0.85	0.17	−2.49	−9.97
$X_u^{++}-X_u^+$	3.10	0.13	−2.15	−17.10
S^+-S^0	0.70	0.12	−4.34	−11.82
A^+-A^0	0.83	−0.14	−4.54	−12.02
$C_1^{*++}-C_1^{*+}$	2.59	−0.22	−1.02	−8.50
$C_1^{*+}-C_1^{*0}$	0.77	+0.02	−2.60	−10.08
$X_u^{*++}-X_u^{*+}$	2.87	−0.10	−2.42	−17.37
$S^{*+}-S^{*0}$	0.77	−0.02	−4.30	−11.77
D^+-D^0	2.46	0.22	7.99	26.68
$D^{*+}-D^{*0}$	1.64	0.03	6.58	25.27

It should be remarked that the parametrization (4.5) is a purely phenomenological one unrelated to the bag model.

The predictions in Table XII for D and D^* corresponding to the choice (4.6) for C are very much the same as those of Ref. 18 which were based on the assumption of constant $(\Delta M)_q$. The possibility (4.7) leads, however, to dramatically larger charmed mass differences. In the absence of dynamical saturation considerations in the calculation of $(\Delta M)_q$, it is perhaps to be preferred on just dimensional grounds. On the other hand, one would be surprised if, when calculated, C could be very much larger than A/R.

In the absence of any knowledge about C one can eliminate C between two charmed-particle mass differences, thereby obtaining a sum rule relating the two. The most useful of these at present is that between the D and the D^*

$$m_{D^{*+}} - m_{D^{*0}} = m_{D^+} - m_{D^0} - 1.41 \text{ MeV}. \qquad (4.8)$$

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