Scaling of multiplicity distributions from p-nucleus collisions in emulsion at energies between 6.2 and 300 GeV

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The Koba-Nielsen-Olesen semi-inclusive scaling hypothesis has been tested for the two nearly homogeneous groups of target nuclei in nuclear emulsion. It is shown that the value of the parameter α in the scaling function used by Buras *et al.* is close to zero for the emulsion data, regardless of the size of the target nucleus. The experimental shower-particle multiplicity distributions from proton-emulsion experiments are found to be consistent with the scaling hypothesis from 300 GeV down to 6.2 GeV.

The Koba-Nielsen-Olesen (KNO) semi-inclusive scaling hypothesis[1] for proton-proton interactions has been verified by Slattery[2] for hydrogen bubble-chamber data in the momentum range 50–300 GeV/c. Subsequently attempts have been made to test the validity of the scaling hypothesis in the case of proton-nucleus interactions. Martin *et al.*[3] found that for nuclear emulsion the parameters in the Slattery scaling function have to be modified in order to get a reasonable fit to the experimental data in the energy range 30–200 GeV. Hébert *et al.*[4] pointed out that since emulsion is a composite target, it is not valid to reduce the multiplicity distribution to the form applicable to proton-proton collisions. One should test the scaling hypothesis for a homogeneous target or, alternatively, use the proton-proton scaling curve to deduce a multiplicity distribution for each component group of target nuclei in the emulsion and then combine the weighted distributions to obtain the distribution to be expected for the emulsion as a whole.

This technique was originally applied to p-emulsion data at 200 and 300 GeV,[4] using the Slattery scaling function. This function has the form

$$\psi(z) = (Az + Bz^3 + Cz^5 + Dz^7)e^{Ez} ,$$

where $z = n_{ch}/\langle n_{ch}\rangle$. The quantity n_{ch} is the number of charged particles in the final state of a proton-proton interaction. When applying the function to the case of proton-nucleus interactions in emulsion, n_{ch} was replaced by n_s, where n_s is the shower-particle multiplicity. Strictly speaking, n_s does not correspond to n_{ch}, since only relativistic particles ($\beta \geq 0.7$) are classified as shower particles. Slow, secondary protons and some of the created pions are, therefore, not included. In the energy range 200–300 GeV, this approximation (of replacing n_{ch} by n_s in the scaling function) was found to be satisfactory, but it could not be used at lower energies.

For this reason, we decided to test the KNO scaling hypothesis over a wider range of energies by using the scaling function of Buras *et al.*,[5] which was shown to be applicable to proton-proton interactions in the momentum range 5.5–300 GeV/c. For the scaling hypothesis to be valid, the moments of the multiplicity distribution, $C_N = \langle n^N\rangle/\langle n\rangle^N$, where $N = 2, 3, \ldots$, should be independent of energy. In the case of p-p interactions, Buras *et al.* noted that if the moments are rewritten in the form $C'_N = \langle (n-\alpha)^N\rangle/(\langle n\rangle - \alpha)^N$, the resulting modified moments are nearly energy independent when the value of the parameter α is close to

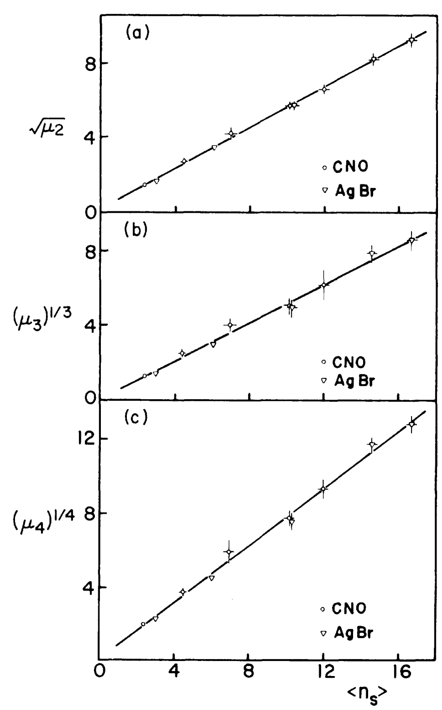

FIG. 1. Plots of $(\mu_N)^{1/N}$ (where μ_N is the Nth central moment) for $N = 2$, 3, and 4 as functions of $\langle n_s\rangle$ for p-nucleus collisions from 6.2 GeV up to 300 GeV.

TABLE I. Average shower-particle multiplicities and moments of the distributions.

Energy	Light group				Heavy group			
(GeV)	$\langle n_s \rangle$	C_2	C_3	C_4	$\langle n_s \rangle$	C_2	C_3	C_4
6.2	2.4 ± 0.1	1.35 ± 0.15	2.2 ± 0.4	4.1 ± 0.9	3.0 ± 0.1	1.31 ± 0.08	2.0 ± 0.2	3.5 ± 0.5
22.5	4.5 ± 0.2	1.37 ± 0.21	2.3 ± 0.5	4.3 ± 1.4	6.1 ± 0.1	1.31 ± 0.11	2.0 ± 0.3	3.6 ± 0.6
67	7.0 ± 0.4	1.35 ± 0.32	2.3 ± 0.8	4.4 ± 2.2	10.3 ± 0.3	1.30 ± 0.16	2.0 ± 0.4	3.5 ± 0.9
200	10.2 ± 0.3	1.31 ± 0.15	2.1 ± 0.4	3.7 ± 0.9	14.6 ± 0.3	1.32 ± 0.10	2.1 ± 0.2	3.9 ± 0.6
300	12.0 ± 0.4	1.30 ± 0.17	2.0 ± 0.4	3.7 ± 1.0	16.7 ± 0.3	1.31 ± 0.09	2.1 ± 0.2	3.7 ± 0.6

unity. Consequently, the Nth roots of the central moments $[\mu_N^{1/N} = \langle (n - \langle n \rangle)^N \rangle^{1/N}]$ of the multiplicity distributions when plotted against $\langle n \rangle$ should yield a set of straight lines intercepting the $\langle n \rangle$ axis at α. This observation led Buras *et al.* to use a scaling function in the variable, $z' = (n - \alpha)/(\langle n \rangle - \alpha)$. The function has the form

$$\psi(z') = A(z' + B)\exp(Cz' + Dz'^2) \ .$$

In order to apply this expression to p-nucleus interactions in emulsion, one must first determine the values of α for the light group (C, N, O) and the heavy group (Ag, Br) of target nuclei. The separation of the events into these two groups is carried out on the basis of the number of charged particles resulting from the evaporation or fragmentation of the target nucleus. All events having more than six such particles are attributed to the heavy group. The remaining events (small stars) are either interactions in the light group (hydrogen included) or peripheral interactions in silver or bromine. In the case of our own data at 200 and 300 GeV, the separation was carried out on the basis of the criterion (proposed by Lohrman and Teucher[6]) that the presence of a highly ionizing particle (black track) having a range less than 65 μ indicates that the struck nucleus belonged to the (C, N, O) group. On the other hand, the presence of a very short, black track (1–2 μ long) indicates a recoiling heavy nucleus.[7] The multiplicity distributions obtained from the separation of the small stars were found to be essentially the same for the light and the heavy groups of nuclei. Hence, the multiplicity distribution for the (C, N, O) group can be assumed to be the same as the distribution for the small stars after the contribution from hydrogen has been extracted. In the case of the heavy nuclei, this distribution has to be weighted so that, when combined with the multiplicity distribution from the large stars, the total number of events is in accordance with geometrical cross section for inelastic interactions in silver and bromine.

This technique was applied to analyze the available data from proton-emulsion experiments in the energy range from 6.2 GeV to 300 GeV.[8-10] The central moments of the multiplicity distributions for each group of target nuclei, when plotted against the average number of shower particles, yield a family of straight lines passing almost through the origin, as shown in Fig. 1. This result implies that the value of α is approximately zero for both groups of target nuclei. This conclusion is confirmed by the fact that the values of the absolute moments, $C_N = \langle n_s^N \rangle / \langle n_s \rangle^N$, listed in Table I for $N = 2$, 3, and 4, are independent of energy within the limits of the experimental errors.

The fact that $\alpha \approx 0$ for proton-nucleus interactions, whereas $\alpha \approx 1$ for proton-proton collisions, implies that $\langle n_s \rangle$ corresponds to $\langle n_{ch} \rangle - 1$. This observation suggests that the target protons do not always emerge as shower particles. In fact, in a p-p collision, according to Calucci *et al.*,[11] there are on the average 0.48 slow protons and 0.14 slow charged pions. Knowing the ratio of protons to neutrons in a given target nucleus and the average number of collisions the incoming particle undergoes inside that nucleus,[12] it is possible to estimate a correction factor for $\langle n_s \rangle$. Such calculations for the two constituent groups of target nuclei in emulsion show that the

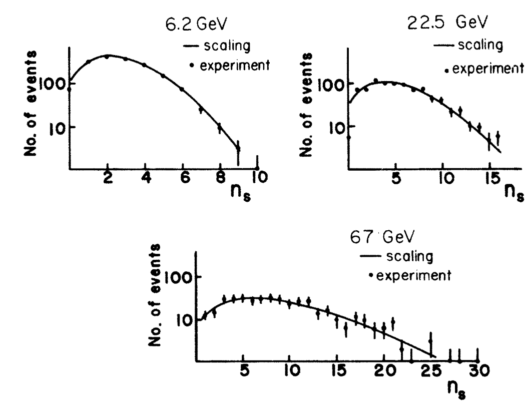

FIG. 2. Shower-particle distributions from p-emulsion experiments. The experimental points are from Refs. 8 and 9 and the curves are calculated from the scaling function.

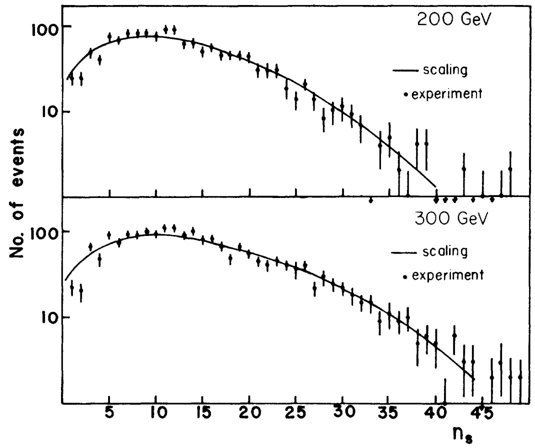

FIG. 3. Shower-particle distributions from p-emulsion experiments. The experimental points are from Ref. 10 and the curves are calculated from the scaling function.

contribution to $\langle n_s \rangle$ from this correction factor is 0.2, which is very close to the values of errors on $\langle n_s \rangle$.

It is now evident that the Slattery scaling function is valid only as long as $\langle n_{ch} \rangle$ is much greater than 1, whereas the Buras function should be applicable to proton-nucleus data down to much lower energies.[13] It still remains to be verified that no change in parameters is required. The slope of the straight line graph in Fig. 1(a) gives the ratio usually referred to in emulsion work as $D/\langle n_s \rangle$, where the dispersion D is equal to the square root of the second central moment. The value of $D/\langle n_s \rangle$ for a homogeneous target is found to be 0.55 ± 0.02, which is in good agreement with the Wroblewski fit[14] to the proton-proton data, expressed by the equation

$$D = (0.576 \pm 0.008)(\langle n_{ch} \rangle - 1) .$$

This indicates that the scaling function of Buras *et al.* using the variable $z = n_s/\langle n_s \rangle$, should be applicable to proton-nucleus interactions without any change in parameters (except for a factor of $\frac{1}{2}$ which accounts for the inclusion of odd values of n_s; all the values of n_{ch} are even).

TABLE II. χ^2 test for scaling fits. At every energy considered, events with $n_s = 0$ were excluded in the computation of χ^2 since the efficiency of detection of such events is low.

Energy (GeV)	χ^2	Degrees of freedom
6.2	8.2	8
22.5	19.7	17
67	22.9	26
200	57.0	42
300	71.7	49

This scaling function was used to compute the shower-particle multiplicity distributions for three groups of target nuclei in emulsion (H; C, N, O; Ag, Br) at incident proton energies between 6.2 GeV and 300 GeV. The average values of n_s used for the light and heavy groups are given in Table I. The values of $\langle n_s \rangle$ for hydrogen were taken from hydrogen bubble-chamber experiments at the same incident energies. The three distributions were weighted in accordance with the probability of inelastic interactions occurring in each group of target nuclei, (0.04, 0.25, and 0.71, respectively, for H; C, N, O; and Ag, Br), and the weighted distributions were combined to obtain the distribution to be expected for the whole emulsion. The predicted distributions are compared with the experimental shower-particle distributions in Fig. 2 and Fig. 3. The χ^2 values for the fits are given in Table II. The χ^2 tests indicate that there is good agreement between theory and experiment over the whole energy range. Hence, we can conclude that the emulsion data is consistent with the KNO semi-inclusive scaling hypothesis, provided that the scaling law is applied to a homogeneous target.

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