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Rescattering in 100-GeV/c proton-deuteron and positive-pion–deuteron interactions

J. E. A. Lys, C. T. Murphy, and M. Binkley
Fermi National Accelerator Laboratory, Batavia, Illinois 60510[*]

S. Dado, A. Engler, G. Keyes, R. W. Kraemer, and G. Yekutieli[†]
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213[‡]

A. Brody and J. Hanlon
State University of New York at Stony Brook, Stony Brook, New York 11794[§]

A. A. Seidl, W. S. Toothacker, and J. C. VanderVelde
University of Michigan, Ann Arbor, Michigan 48104[‡]
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We have studied rescattering in 100-GeV/c proton and positive-pion interactions with deuterons in the Fermilab 30-in. bubble chamber. We study specifically events in which the deuteron breaks up and meson production occurs. After attempting to take account of elastic rescattering, we determine the fraction of events in which rescattering occurs, and the multiplicity distribution of the rescatter events. The rescatter fraction found for 100-GeV/c π^+d events is consistent with the values found in 21-GeV/c and 205-GeV/c π^-d experiments while the fraction found for 100-GeV/c pd events is slightly larger. The multiplicity distributions in the rescatter events are in reasonable agreement with 205-GeV/c pp or π^-p multiplicity distributions. We compare our results to predictions of energy-flux-cascade and coherent-tube models.

I. INTRODUCTION

We present results on a study of rescattering in proton-deuteron and positive-pion–deuteron interactions at 100-GeV/c incident-hadron momentum in the Fermilab 30-in. bubble chamber.

We use the word rescattering to describe interactions in which both nucleons in the deuteron take active part, as opposed to interactions in which one nucleon, the spectator nucleon, emerges with a momentum distribution essentially determined by the deuteron wave function. Rescattering may be thought of in terms of a model in which some product of a beam-particle–nucleon interaction subsequently interacts with the second nucleon in the deuteron. In such a model there is explicit rescattering. Alternatively, a rescatter event may be thought of as an interaction of a beam particle with a single target that has a baryon number of two. Evidence for rescattering is provided by a deficiency in the observed fraction of events with spectatorlike protons.

From our data we calculate the fraction of rescatter events, after correcting for deuteron-final-state events and for elastic rescattering, so that our results refer to inelastic rescattering in deuteron-breakup events. The calculation involves reasonable assumptions about the properties of spectator nucleons. We then calculate the multiplicity distribution for rescatter events.

Interactions with nuclear targets have received considerable attention recently because they present the possibility of studying the space-time evolution of multiparticle production. Experimental results (reviewed in Refs. 1 and 2) clearly rule out naive cascade models; it is concluded that in hadron-hadron interactions the individual particles found in the asymptotic final state are not active instantaneously. Various models[3-7] have been suggested to explain the available data.

Because of the simplicity of the deuteron, hadron-deuteron interactions can contribute uniquely to the understanding of hadron-nucleus interactions. For example, in heavier nuclei it is necessary to make some sort of average over the number of nucleons that interact, while with the deuteron we can hope to study interactions in which a specific number of nucleons, namely two, take part in the interaction.

The existence of rescattering, its approximate magnitude, and some of its manifestations in high-energy ($\gtrsim 20$ GeV/c) experiments have been pointed out recently by a number of authors.[8-13] The present paper gives one of the first detailed studies of rescattering in the deuteron at high energies, with an emphasis on multiplicity. For a review of rescattering in the deuteron at lower energies, see Ref. 14.

Notation

In this paper we consistently use the symbol M to denote number of events, and N to denote charged-particle multiplicity. We use P_N to denote the probability that an inelastic interaction (of a specified type) will have charge multiplicity N, with normalization such that the sum over all possible N values is unity. We take $P_N(hd)$ to refer to hadron-deuteron events in which the deuteron breaks up and meson production occurs, thus excluding deuteron-final-state and quasi-elastic events, and always add one to the prong count if odd to take account of the presumedly invisible proton. For rescatter events, N is the multiplicity of the whole event, not the presumed multiplicity of the interaction with the "first" nucleon in the deuteron; we will use N_1 at one point to denote the latter multiplicity.

II. EXPERIMENTAL DETAILS

The data were obtained from an exposure of the Fermilab 30-in. deuterium bubble chamber to a beam of 100-GeV/c positive particles. The beam composition was approximately 57% proton, 39%, π^+, 2%μ^+, and 2% K^+, and a tagging system[15] allowed a determination of the mass and location of each beam track in the bubble chamber. Results on multiplicity distributions have been published.[10]

The present paper uses a 23 000-picture sample of the film in which all tracks with momentum ≤ 1.5 GeV/c were measured in events with $N \geq 3$. Positive tracks with momentum below 1.4 GeV/c (as determined by the geometry-reconstruction program) were assigned a mass, assumed either proton or pion, in a special ionization scan.

Because of data-processing losses, a multiplicity-dependent and institution-dependent weight was assigned to each event. At least two independent prong counts were made on each event and discrepancies were edited by an experienced scanner or physicist. About 2.5% of the events found could not be assigned a unique prong count; such events are excluded from the data sample and a multiplicity-dependent weight corrects for this loss.

Short proton (or deuteron) tracks with lengths as small as 1–2 mm in the bubble chamber were included in the prong count and were measured. However, for this paper all tracks of length less than 5 mm have been classified as invisible and removed from the data, and prong counts amended accordingly (for a proton, a range <5 mm corresponds to a momentum <120 MeV/c). This was done to ensure that our results are not affected by any short-track scanning bias.

The final numbers of weighted odd- and even-prong events are given in Table I for proton and pion beam particles. The average weight per event was 1.20. Also given in Table I are the numbers of even-prong events with a backward proton; such events will henceforth be termed backward-proton events (here and elsewhere in this paper, a backward proton means a backward-hemisphere proton and is defined as an outgoing proton with $\cos\theta < 0.0$, where θ is the laboratory angle between the incident-beam direction and the outgoing proton). Weighted event numbers may be converted to cross sections using the sensitivities for the present data sample of 8.11 ± 0.30 μb/event for pd events and 10.9 ± 0.5 μb/event for π^+d events.

TABLE I. Weighted events ($N \geq 3$).

	pd	π^+d
Odd prongs	2263 ± 51	1103 ± 36
All even prongs	4394 ± 74	1913 ± 49
Even prongs with a backward proton	237 ± 17	122 ± 12
Odd prongs, deuteron-final-state events removed	2160 ± 55	1041 ± 38
Even prongs, deuteron-final-state events removed	4361 ± 74	1893 ± 49
Microbarns per event	8.11 ± 0.30	10.9 ± 0.5

The multiplicity distributions of the weighted events in Table I are presented in Table II for pd and in Table III for π^+d events. The distributions in Tables II and III are close to, but not identical with, our best estimates of the odd- and even-prong multiplicity distributions as determined in this experiment,[10] since the event samples used are slightly different. It is worth noting that, although we will assume that both odd-prong events and backward-proton events arise primarily from pn or π^+n collisions, we expect their multiplicity distributions (i.e., columns 2 and 4 of Table II or Table III) to be slightly different. Differences are in fact observed and are attributable to three causes: the contribution to the odd-prong events of deuteron-final-state events, the small but dif-

TABLE II. Multiplicity distributions of weighted pd events; deuteron-final-state events not removed. (For neutron-target multiplicity distributions, see Ref. 10.)

N	Odd prongs [a]	Even prongs	Even prongs with a backward proton
4	529 ± 24	1030 ± 35	59 ± 8
6	584 ± 26	1143 ± 37	79 ± 10
8	504 ± 24	941 ± 34	43 ± 7
10	337 ± 20	636 ± 28	33 ± 6
12	180 ± 15	341 ± 21	12 ± 4
14	77 ± 10	175 ± 16	9 ± 4
16	37 ± 7	96 ± 13	3 ± 2
18	10 ± 4	25 ± 7	0
20	4 ± 3	6 ± 6	0
22	2 ± 2	0	0

[a] For this table, the presumed invisible proton or deuteron has been added to the prong count.

TABLE III. Multiplicity distributions of weighted $\pi^+ d$ events; deuteron-final-state events not removed. (For neutron-target multiplicity distributions, see Ref. 10.)

N	Odd prongs [a]	Even prongs	Even prongs with a backward proton
4	245 ± 16	419 ± 22	29 ± 6
6	264 ± 17	480 ± 24	32 ± 6
8	242 ± 16	439 ± 23	26 ± 6
10	158 ± 14	288 ± 19	18 ± 5
12	107 ± 12	165 ± 15	11 ± 4
14	60 ± 9	76 ± 11	4 ± 3
16	23 ± 5	29 ± 6	1 ± 1
18	2 ± 2	11 ± 5	0
20	0	4 ± 3	0
22	0	1 ± 1	0
24	0	1 ± 1	0

[a] For this table, the presumed invisible proton or deuteron has been added to the prong count.

ferent contributions of pp or $\pi^+ p$ collisions to both samples, and the slightly different average center-of-mass energies. Therefore neither column 2 nor column 4 on Tables II and III are to be construed as multiplicity distributions of pn and πn interactions. These latter distributions are given in another paper.[10]

Estimates of one- and two-prong multiplicities and deuteron-final-state cross sections

In what follows we need values for the quantities $P_1(pn)$, $P_1(\pi^+ n)$, $P_2(pd)$, and $P_2(\pi^+ d)$, and the cross sections at each multiplicity N for deuteron-final-state events, i.e., for the reactions $hd \to dX$. It is not possible to measure these quantities reliably in this experiment.

As before,[10] we use $\pi^- p$ data[16] and pp data[17–20] to estimate $P_1(hn)$. We take $P_1(pn) = (0.6 \pm 0.1)P_2(pp)$ and $P_1(\pi^+ n) = (0.6 \pm 0.1)P_2(\pi^- p)$, resulting in $P_1(pn) = 0.088 \pm 0.015$ and $P_1(\pi^+ n) = 0.056 \pm 0.012$. In both cases 0.6 ± 0.1 equals our estimate of the probability that a struck proton in a two-prong pp or $\pi^- p$ inelastic interaction will retain its charge state.[10]

We estimate $P_2(hd)$ assuming the relation

$$P_2(hd) = 0.5[P_1(hn) + P_2(hp)]F_2 \ . \quad (1)$$

Such a relation is suggested by simple impulse-approximation considerations, and similar relations for higher multiplicities, with $F = 1$, were found to be in reasonable agreement with the data.[10] The term F_2 is a correction factor which takes into account both deuteron-final-state events with $N = 1$ or 2 [since $P_d(hd)$ refers to deuteron-breakup events] and symmetry requirements of the two-nucleon wave function.[10] We estimate F_2 to be 0.90 ± 0.05. The resulting values are $P_2(pd) = 0.105 \pm 0.020$ and $P_2(\pi^+ d) = 0.081 \pm 0.020$ (we have arbitrarily doubled the errors to take account of uncertainties in the assumed relation).

To obtain deuteron-final-state cross sections, we first extrapolate published $pd \to dX$ data[21,22] to include the whole kinematic region, and assume a smooth variation of the cross section with beam momentum. The result is $\sigma(pd \to dX) = 2.0 \pm 0.3$ mb at 100 GeV/c. Then, since approximately 90% of the cross section occurs with recoil mass squared $M_x^2 < 20$ GeV^2, we assume the same multiplicity distribution as[23] for $pp \to pX$ events with $M_x^2 < 20$ GeV^2, and arrive at partial cross sections of 0.9 mb ($N = 2$), 0.9 mb ($N = 4$), 0.2 mb ($N = 6$), and <0.05 mb ($N \geq 8$). The t distribution of these events[21,22] is such that 76% will have an invisible (range <5 mm) deuteron and hence be classed as odd prongs.

No measurements of $\pi^+ d \to dX$ are available near our energy, so we make use of $\pi^- p$ diffraction-dissociation data,[24] plus charge symmetry. We assume

$$\sigma(\pi^- d \to d\pi^{-*})/\sigma(pd \to dp^*) = \sigma(\pi^- p \to p\pi^{-*})/\sigma(pp \to pp^*) \ , \quad (2)$$

where an asterisk indicates a diffractively produced excited state, and we assume that all deuteron-final-state events are diffractively produced. The result is $\sigma(\pi^+ d \to dX) = 1.3 \pm 0.3$ mb, with individual multiplicity contributions, from π^{-*} multiplicity distributions, of 0.40 mb ($N = 2$), 0.63 mb ($N = 4$), 0.22 mb ($N = 6$), 0.05 mb ($N = 8$), and <0.05 mb ($N \geq 10$).

The final numbers of odd- and even-prong deuteron-breakup events, obtained after subtracting the deuteron-final-state contributions, are given in Table I. In the remainder of this paper we use only deuteron-breakup events. The largest of the deuteron-final-state partial cross sections, for $N = 4$, is only 8% of the deuteron-breakup cross section for $N = 4$, so the corrections are small.

III. RESCATTER FRACTIONS

Rescatter fraction, *pd*

We make the simplifying assumption that deuteron-breakup events can be divided into three classes: spectator-proton events, spectator-neutron events, and rescatter events. It is assumed that the ratio of event numbers in the first two classes equals the ratio of the inelastic free-neutron and free-proton cross sections, which is approximately unity. The assumed properties of spectator protons allow us to determine the number of spectator-proton events, and hence the number of spectator-neutron events. The remaining events are assigned to the rescatter class.

To determine the number of spectator-proton events in our sample, we assume that all odd-prong and backward-proton events, apart from a small fraction which is calculated below, are spectator-proton events. We can then use either the number of odd-prong events or the number of backward-proton events to estimate the number of forward spectator protons. Use of backward-proton event numbers may be preferable, since the result is less dependent on knowledge of the deuteron wave function.

Before making a determination of the rescatter fraction, two effects must be considered. These are spectator-neutron events with an invisible or backward proton, and elastic rescattering.

The contributions of spectator-neutron events to odd-prong and backward-proton events are caused mainly or wholly by the Fermi motion of the struck proton. We have used 100-GeV/c pp data,[17] and smeared the target-proton momentum in accordance with the Hulthên-wave-function prediction (see, e.g., Fridman[25]; we use the same parameter values as Ref. 26) to determine these contributions. The deuteron form factor was included appropriately to take account of those events where the deuteron does not break up. The resulting fractions of spectator-neutron events with $N \geq 4$ that are expected to give odd-prong and backward-proton events are each 0.002 ± 0.001, in both cases 90% with $N=4$, 10% with $N=6$. Appropriate corrections are made below in determining the rescatter fraction.

We now consider elastic rescattering in detail. In a model with explicit rescattering, one special type of rescatter is an elastic scatter on what we will call the pseudospectator nucleon. The scattered particle may be a beam particle (prescattering) or a product of an inelastic interaction on the other nucleon; we refer to both as elastic rescattering. The forward peaking in elastic-scattering experiments means that the resulting pseudospectator momentum distribution will peak at a low (~300 MeV/c) value, and since pseudospectator and true spectator nucleons are indistinguishable there can be interference. Dean[27] has suggested how to take account of these elastic rescatters within Glauber theory, with neglect of any spin or isospin effects. Dean assumes that when the scattering follows an inelastic interaction the scattered particle has beamlike properties, that is, it has the same elastic-scattering amplitude as the beam particle. We have evaluated Dean's formula for the vector-momentum distribution of the spectator and pseudospectator nucleon [Eq. (9) of Dean's paper]. In our evaluation we used an imaginary elastic scattering amplitude, exponential in momentum transfer, and we included the longitudinal momentum transfer in elastic scattering. In the elastic-rescattering term we used the Hulthên deuteron wave function, while in the no-rescatter term we added a 6.5% D-wave component (explicitly, a multi-Gaussian fit to the Reid soft-core wave function[28]). Appropriate Moller flux-factor[29] terms were also included.

Predictions from the evaluation of Dean's formula are compared to our data in Fig. 1. Also shown are the predictions of a pure spectator model, that is, with no pseudospectator term but still including D-wave and flux-factor terms. In Fig. 1(a), which shows the momentum distribution of backward protons from even-prong events, the predicted values are normalized to the expected number of odd-prong events that have an invisible backward proton. Figure 1(b) shows the angular distribution

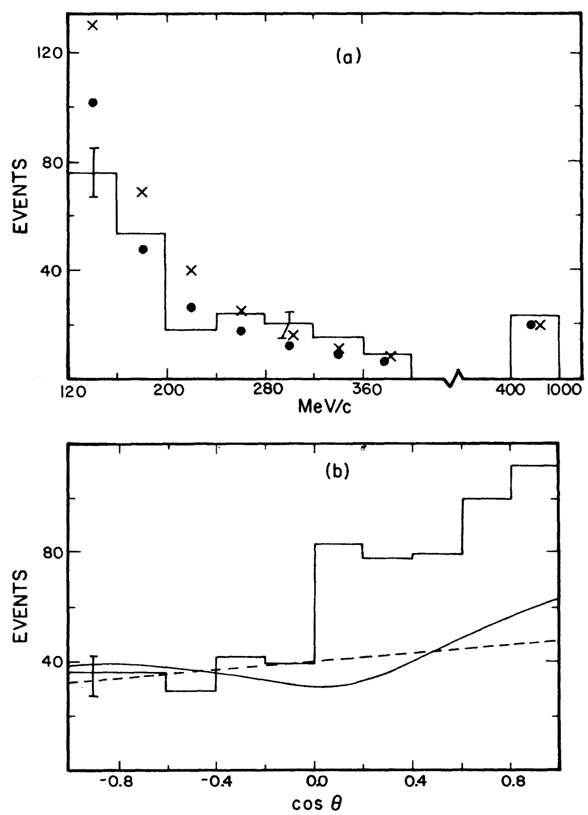

FIG. 1. Proton distributions from (weighted) pd events. (a) The momentum distribution of backward protons in even-prong events. The solid circles and crosses are, respectively, the predictions with and without elastic rescattering (see text), normalized using the number of odd-prong events. (b) The angular distribution of seen spectator protons (see text) with momentum between 120 and 300 MeV/c. The curves are the predictions with (solid curve) and without (broken curve) elastic rescattering, both normalized to the $\cos\theta < 0.0$ region.

of seen spectator protons that have a momentum in the range 120–300 MeV/c. Here a seen spectator is defined to be a proton from an even-prong event, and if an event has two protons we select the backward proton if there is one, or the slower proton if neither is backwards. The predicted curves in Fig. 1(b) are normalized to the $\cos\theta < 0.0$ region; in the $\cos\theta > 0.0$ region we expect a considerable extra contribution from proton recoils from spectator-neutron events. The reasonable agreement between the backward-proton data and the predictions from Dean's formula suggests that the latter does not give a gross misestimate of the elastic-rescattering effect.

In order to use the backward-proton events to estimate the number of forward-spectator-proton events, we need a theoretical value for the ratio r of forward to backward visible spectator (including pseudospectator) protons. Our evaluation of Dean's formula gives a value of 1.45, to be compared with values of 1.29 if elastic rescattering is omitted, and 1.23 if both the D-wave component and elastic rescattering are omitted. The latter two values reflect simply the Møller flux factor (all values refer to spectator momenta above 120 MeV/c). Much of the contribution of the larger r value when elastic rescattering is included comes from proton momenta above 300 GeV/c, and is not apparent in Fig. 1(b). Simply weighting our backward-proton events with appropriate Møller flux factors leads to $r = 1.27$. We then take $r = 1.45 \pm 0.16$. The error is such that the no-elastic rescatter value is only one standard deviation away.

We expect that the rescatter fraction calculated with this value of r will refer only to inelastic rescatters, that is, events in which new particle production occurs on both nucleons. We note that we may not have taken into account elastic rescatters by low-energy particles; however, the small change in the value for r above (1.29 to 1.45) suggests that our result will not be substantially altered by low-energy elastic rescatters.

We now proceed to calculate F_{rs}, the fraction of rescatter events, where we now explicitly refer to inelastic rescatters. Denoting the numbers of spectator-proton, spectator-neutron, and rescatter events with $N \geq 3$ by $M(p_s), M(n_s), M(\text{rs})$, respectively, we write

$$M(p_s) = M(\text{odd}) + (1 + r)M(\text{back } p) , \tag{3}$$

$$M(n_s) = M(p_s)\sigma(pp,\ N \geq 4)/\sigma(pn,\ N \geq 3) , \tag{4}$$

$$M(\text{rs}) = M(\text{tot}) - M(p_s) - M(n_s) , \tag{5}$$

where $M(\text{odd})$, $M(\text{back}p)$, and $M(\text{tot})$ are, respectively, the number of odd-prong events, backward-proton even-prong events, and total events in our data sample.

The rescatter fraction, for $N \geq 3$, is defined by

$$F_{rs}(\geq 3) = M(\text{rs})/M(\text{tot}) , \tag{6}$$

or, using Eqs. (4) and (5),

$$F_{rs}(\geq 3) = 1 - \frac{M(p_s)}{M(\text{tot})}\left[1 + \frac{\sigma(pp,\ N \geq 4)}{\sigma(pn,\ N \geq 3)}\right] . \tag{7}$$

The rescatter fraction for all events (we mean all deuteron-breakup events in which meson production occurs) is given by

$$F_{rs} = (1 - P_2)F_{rs}(\geq 3) + P_2 F_{rs}(2) , \tag{8}$$

where $F_{rs}(2)$ is the rescatter fraction for $N = 2$ events. We assume $F_{rs}(2) = 0.10 \pm 0.05$, a value halfway between zero and the value of $F_{rs}(\geq 3)$, and consistent with any reasonable extrapolation of the number of rescatter events at each N determined below. A nonzero value for $F_{rs}(2)$ is indicated by the small value deduced later on for $\langle N_2 \rangle$, the mean multiplicity in the presumed incoherent interaction on the second nucleon in the deuteron.

Finally, the results from our data, utilizing Eqs. (3), (7), and (8), and assuming that the ratio of pn to pp inelastic cross sections is the same as for total cross sections,[30] are $F_{rs}(\geq 3) = 0.20 \pm 0.02$ and $F_{rs} = 0.19 \pm 0.02$. The errors include contributions from errors in $P_2(pd)$, $P_1(pn)$, $P_2(pp)$, r, $F_{rs}(2)$, and the pn and pp total cross sections, as well as the statistical errors in the event numbers. The last-named is the largest contributor. The error in r of ± 0.16 by itself gives an error in F_{rs} of ± 0.010, so that if $r = 1.29$ then $F_{rs} = 0.20 \pm 0.02$.

Rescatter fraction, $\pi^+ d$

We have determined the $\pi^+ d$ rescatter fraction in the same way as for pd. Evaluation of Dean's formula gives a value for the forward-backward visible-spectator ratio r of 1.34, smaller than in the pd case because of the smaller pion-nucleon elastic cross section. Hence we take $r = 1.34 \pm 0.05$. The cross sections in the analogies of Eqs. (4) and (7) are now $\pi^+ p$ and $\pi^+ n$ cross sections, and from charge symmetry we take the latter as equal to the $\pi^- p$ cross section. For $F_{rs}(2)$ in Eq. (8) we take now a value of 0.08 ± 0.04. Finally we obtain $F_{rs}(\geq 3) = 0.15 \pm 0.03$ and $F_{rs} = 0.14 \pm 0.03$. Again the statistical error is the largest contributor to the errors in F_{rs}. The error in r of ± 0.05 by itself gives an error in F_{rs} of ± 0.004.

The pd and $\pi^+ d$ rescatter fractions are summarized in Table IV.

Comparison with rescatter fractions from other experiments

Two $\pi^- d$ experiments[12,13] have reported rescatter fractions for $N \geq 3$, calculated with the as-

TABLE IV. Rescatter fractions, F_{rs}, from this experiment. The quantity r is the value assumed for the ratio of forward to backward visible spectators; the smaller r values neglect elastic-rescattering corrections (see text).

N range	$r(pd)$	$F_{rs}(pd)$	$r(\pi^+d)$	$F_{rs}(\pi^+d)$
All N	1.45 ± 0.16	0.187 ± 0.022	1.34 ± 0.05	0.142 ± 0.026
	1.29	0.197 ± 0.019	1.29	0.145 ± 0.026
$N \geq 3$	1.45 ± 0.16	0.197 ± 0.022	1.34 ± 0.05	0.148 ± 0.026
	1.29	0.208 ± 0.019	1.29	0.152 ± 0.026

sumption that $\sigma(\pi^-p,\ N \geq 4) = \sigma(\pi^-n,\ N \geq 3)$, and making use of odd-prong plus backward-proton events. We have recalculated the rescatter fractions from the published data without making the aforementioned assumption. The results are given in Table V (we note that the change from the authors' value of F_{rs} for 205-GeV/c π^-d is small). Our calculations used published cross sections[30-33] and used the same equations as for our data, with the same assumptions for $P_1(\pi n)$ and $P_2(\pi d)$, plus $F_{rs}(2) = 0.08 \pm 0.04$. Deuteron-final-state events were assumed to contribute equally to odd- and even-prong events. Elastic-rescattering corrections were not included. For comparison, Table V also gives rescatter-fraction values from the present experiment when elastic-rescattering corrections are omitted (we expect that the elastic-rescattering correction will be approximately independent of beam momentum above ~20 GeV/c).

The values in Table V suggest that the rescatter fraction for πd interactions is constant over the incident-beam momentum interval 21–205 GeV/c. In contrast, the πp mean charged-particle multiplicity changes from 4.6 to 8.0 over this interval. These points argue against a simple cascade model in which the rescatter probability is proportional to the hadron-nucleon mean multiplicity, but cannot rule out some multiplicity dependence.

The results of the present experiment suggest that the rescatter fraction is larger for pd than for πd interactions. This suggestion is reinforced if we take the average πd value in Table V. Thus it appears that the rescatter fraction, while independent of beam momentum above ~20 GeV/c, does depend on the nature of the beam particle. Clearly it is important for other deuterium experiments to confirm these interesting observations. The connection with models of hadron-nuclei interactions is examined in a later section of this paper.

IV. MULTIPLICITY DISTRIBUTIONS OF RESCATTER EVENTS

In this section we determine the numbers of rescatter events at each charge multiplicity N, after making a few reasonable assumptions.

At each value of N (N even) we consider those even-prong events that do not have a visible backward proton, of number $M_N(a)$. We assume that all rescatter events, $M_N(\text{rs})$, and almost all spectator-neutron events, $M_N(n_s)$, (the exceptions were discussed earlier) are included in $M_N(a)$. Some spectator-proton events are also included in $M_N(a)$, and we assume that their number may be obtained from the $(N-1)$-prong events by the equation

$$M_N^a(p_s) = Hf_N M_{N-1} \,. \tag{9}$$

Here H is the ratio of all visible forward-spectator-proton events to all odd-prong events, and our evaluation of Dean's formula leads to $H = 0.161$ for pd events and $H = 0.164$ for π^+d events. Thus H results from the deuteron wave function, the Møller flux factor, and the elastic-rescattering effect. The term f_N in Eq. (9) corrects for the fact that the average beam-neutron total center-of-mass energy for the slow neutrons that give odd-prong events is different from that for the faster, backward-moving neutrons that give visible forward spectator protons. We evaluated f_N assuming that the average multiplicity of hn events has the same s dependence as found for pp events,[34] and assum-

TABLE V. Rescatter fractions, F_{rs}. No elastic-rescattering corrections are included. The values in parentheses are those given in Refs. 13 and 12, respectively.

N range	21-GeV/c π^-d[a]	100-GeV/c π^+d	205-GeV/c π^-d[b]	100-GeV/c pd
All N	0.139 ± 0.019	0.145 ± 0.026	0.146 ± 0.018	0.197 ± 0.019
$N \geq 3$	0.148 ± 0.019	0.152 ± 0.026	0.150 ± 0.019	0.208 ± 0.019
	(0.11 ± 0.01)		(0.14 ± 0.01)	

[a] Data from Ref. 13.

[b] Data from Ref. 12.

TABLE VI. Multiplicity distributions of inelastic-rescatter events, with means $\langle N \rangle$, mean squares $\langle N^2 \rangle$, and dispersions D.

N	$M_N(\mathrm{rs})$, pd	$M_N(\mathrm{rs})$, π^+d
2	77 ± 42	21 ± 12
4	189 ± 41	68 ± 24
6	287 ± 44	70 ± 27
8	295 ± 39	115 ± 25
10	199 ± 32	70 ± 21
12	136 ± 23	47 ± 16
14	84 ± 16	28 ± 11
16	63 ± 13	12 ± 7
18	17 ± 7	7 ± 5
20	3 ± 6	3 ± 3
22	0	1 ± 1
$\langle N \rangle$	8.27 ± 0.29	8.36 ± 0.44
$\langle N^2 \rangle$	82.6 ± 4.9	83.7 ± 8.0
D	3.76 ± 0.19	3.71 ± 0.28

ing that hn multiplicity distributions follow the same "universal curve" that Slattery[35] fit to pp distributions. Values for f_N varied from $f_4 = 0.96$ to $f_{20} = 1.37$. Odd-prong rather than backward-spectator events were used to determine forward-spectator numbers because the f_N depart less from unity for the former.

We assume that the numbers of spectator-neutron events are given by

$$M_N(n_s) = \frac{F_p M(\mathrm{tot}) P_N(hp)(1 - F_{\mathrm{rs}})}{1 - P_2(hd)} , \tag{10}$$

$$F_p = \frac{\sigma(hp,\ \mathrm{inel.})}{\sigma(hn,\ \mathrm{inel.}) + \sigma(hp,\ \mathrm{inel.})} . \tag{11}$$

Again $M(\mathrm{tot})$ is the total number of events in our sample. The hp inelastic multiplicity probabilities $P_N(hp)$ in Eq. (10) are taken from pp and π^+p experiments.[17–20]

Finally, we have

$$M_N(\mathrm{rs}) = M_N(a) - M_N^a(p_s) - M_N(n_s) . \tag{12}$$

Equations (9)–(12) were used to determine $M_N(\mathrm{rs})$ for $N \geq 4$. Values for $M_2(\mathrm{rs})$ follow from the values for $F_{\mathrm{rs}}(2)$ assumed earlier. The results, together with the means for N and N^2 and the dispersion D, are given in Table VI. Errors include statistical errors, errors in F_p and $P_N(hp)$, and an assumed error of ± 0.03 in H; the statistical errors dominate. The multiplicity distributions (normalized to sum to unity) in rescatter events are plotted in Fig. 2, along with 205-GeV/c pp and π^-p distributions.[32,36]

Discussion of rescatter-event multiplicites

In Table VII some properties of the multiplicity distributions in rescatter events are

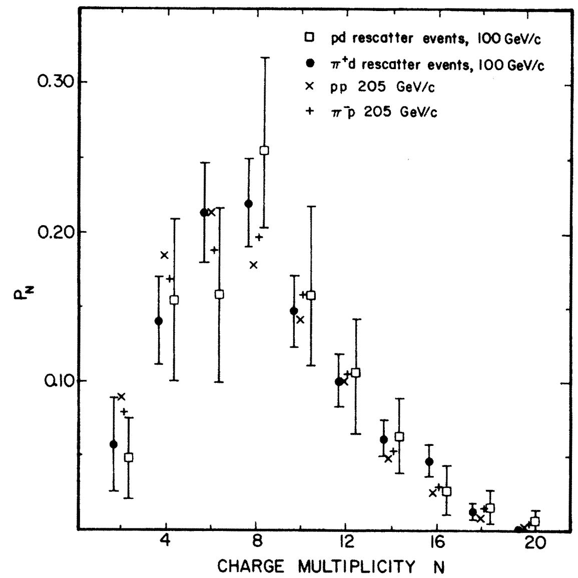

FIG. 2. The normalized charged-particle multiplicity distributions in 100-GeV/c pd and πd rescatter events. Also shown are 205-GeV/c pp and π^-p distributions from Refs. 36 and 32, respectively. The errors on the pp and π^-p points are less than 0.01.

compared to hd and hp distributions[10,17–20,32,36] (the comparison is made for $N \geq 4$, since we do not experimentally measure $N = 2$). We see reasonable agreement between the rescatter-event moments at 100-GeV/c and the respective 205-GeV/c moments. Such agreement is interesting in that the center-of-mass energies of 100-GeV/c hd collisions and 205-GeV/c hp collisions are approximately equal. The comparison can also be seen in Fig. 2 (where $N = 2$ is now included).

It is of interest to determine a quantity $\langle \Delta N \rangle$, defined as the difference between the mean multiplicity of hadron-deuteron rescatter events and that of hadron-nucleon interactions at the same incident-beam energy. Our results give $\langle \Delta N \rangle = 1.95 \pm 0.30$ (pd) and 1.69 ± 0.45 (π^+d). If, in a model with explicit rescattering, the rescatter is an interaction between a materialized hadron from a beam-nucleon collision and the second nucleon in the deuteron, then the mean multiplicity of the materialized-hadron–second-nucleon interaction is given by $\langle N_2 \rangle \approx \langle \Delta N \rangle + 0.67$. The term 0.67 comes from assuming that in 67% of the rescatters the aforementioned materialized hadron is charged. We find $\langle N_2 \rangle = 2.62 \pm 0.30$ (pd) and 2.36 ± 0.45 (π^+d). These values are comparable with the mean multiplicity of ~2-GeV/c πp interactions.

In Eq. (10) above, it was implicitly assumed that the multiplicity distribution of spectator-neutron events was the same as that of hadron–free-proton interactions. This assumption might be called

TABLE VII. Comparison of multiplicity moments for $N \geq 4$.

	$\langle N \rangle$	$\langle N^2 \rangle$	D
100-GeV/c pd rescatters	8.65 ± 0.23	87.3 ± 4.5	3.53 ± 0.14
100-GeV/c pd [a]	7.65 ± 0.04	68.3 ± 0.8	3.13 ± 0.03
100-GeV/c pp [b]	7.14 ± 0.03	59.3 ± 0.6	2.87 ± 0.03
205-GeV/c pp [c]	8.23 ± 0.06	80.2 ± 1.2	3.54 ± 0.04
100-GeV/c π^+d rescatters	8.68 ± 0.42	87.7 ± 8.0	3.51 ± 0.25
100-GeV/c π^+d [a]	7.75 ± 0.07	70.2 ± 1.3	3.17 ± 0.06
100-GeV/c π^+p [b]	7.37 ± 0.05	62.8 ± 0.9	2.91 ± 0.04
205-GeV/c π^-p [d]	8.52 ± 0.06	85.4 ± 1.2	3.58 ± 0.05

[a] Ref. 10.
[b] Refs. 17–20.
[c] Ref. 36.
[d] Ref. 32.

a "no cascade" assumption. In a model with explicit rescattering, an equivalent assumption is that the rescatter probability is independent of the multiplicity of the first interaction. We could alternatively take a cascade-type model, in which the probability of a rescatter following a beam-nucleon interaction of charged plus neutral multiplicity N_1' increases with N_1', although evidence from experiments with heavier nuclei tends to argue against such models.[1,2,37] To indicate the effect that such a cascade-type model would have on our results, we have recalculated the rescatter-event multiplicities under the rather extreme assumption that the rescatter probability is equal to $\beta N_1'$, with $\beta = F_{rs}/\langle N_1' \rangle = 0.020$ (pd) and 0.014 (π^+d). The resulting rescatter-event mean multiplicities would be 8.60 ± 0.30 (pd) and 8.70 ± 0.45 (π^+d). We see that cascade-type models can increase the calculated mean multiplicities by up to ~0.3 units. However, after recalculating appropriately the mean multiplicities of the beam–first-nucleon interactions in rescatter events, we find that cascade-type models will reduce the calculated values of $\langle N_2 \rangle$ by ~0.8 units.

V. COMPARISON WITH THEORETICAL MODELS

In this section we compare our results with predictions of energy-flux-cascade (EFC) models and coherent-tube models.

In the EFC model,[1–3] the result of a hadron-nucleon interaction is an energy flux which, after traveling one mean free path in a nucleus, corresponds on the average to two components, a "head" and a "tail." The head has energy $\sim E_b$ (the beam energy), and undergoes $(\bar{\nu} - 1)$ further collisions, while the tail has energy $\sim E_b^{1/3}$ and is assumed not to undergo any multiplicity-increasing collisions. The parameter $\bar{\nu}$ is defined, and estimated to be 1.096 and 1.069, respectively, for pd and πd interactions, with some theoretical uncertainty, in the Appendix. Thus in deuterium the EFC model predicts a fraction $(\bar{\nu} - 1)$ of rescatters by the head component. Our data, however, yield a fraction which is twice as large. Explicitly, $F_{rs}/(\bar{\nu} - 1)$ is 2.0 ± 0.3 (pd) and 2.1 ± 0.4 (π^+d). There is no inconsistency here if the tail component rescatters independently of and approximately as often as the head. In this picture then, since $\bar{\nu}$ is almost independent of beam momentum above ~20 GeV/c, we expect F_{rs} to be similarly independent, and since $\bar{\nu}(pd) > \bar{\nu}(\pi d)$ we expect $F_{rs}(pd) > F_{rs}(\pi d)$. The data agree with both these expectations.

Each rescatter by the head component, in the EFC model, produces a mean multiplicity increase of η times the hadron-nucleon mean multiplicity, with the parameter η variously estimated[2–6,38] at 0.25–0.50. Heavy-nuclei experiments[2,39] suggest $\eta \approx 0.5$. If tail-nucleon rescatters produce negligible multiplicity increase, then the assumed fraction $(\bar{\nu} - 1)$ of head rescatters produces a mean multiplicity increase given by $\langle \Delta N \rangle F_{rs}/(\bar{\nu} - 1)$. Dividing by the hadron-nucleon mean multiplicity, our experiment[40] then yields values for η of 0.60 ± 0.12 (pd) and 0.52 ± 0.17 (π^+d). Again, if our picture of tail rescatters is correct there is agreement with the EFC model and with heavy-nuclei experiments.

In the coherent-tube model[7] the incident hadron is assumed to see a target composed of i nucleons, where i can be 1 to A (A = atomic number). The multiplicity distribution, for a given i value, is a function of the total energy, and averaging over i gives the overall multiplicity distribution. Hence our rescatter events, taken as $i = 2$ events, should have the same multiplicity distribution as 200-GeV/c hadron-nucleon interactions, in reasonable agreement with the data. The rescatter fraction in deuterium in this model is just $(\langle i \rangle - 1)$, but it is not clear[41] what the prediction is for $\langle i \rangle$.

We conclude that our results are in agreement with simple predictions of both the EFC and the coherent-tube models. Stronger conclusions must await either more experimental data on rescatter events, for example, rapidity distributions of outgoing particles, or more specific predictions from the models.

VI. CONCLUSIONS

Our main conclusions are as follows:

We have determined rescatter fractions for events in which the deuteron breaks up and meson production occurs. We have attempted to take account of elastic rescatters, so that these fractions refer to inelastic rescatters. We find $F_{rs}=0.19 \pm 0.02$ (pd) and 0.14 ± 0.03 ($\pi^+ d$).

We have determined the multiplicity distributions in inelastic rescatter events. The mean multiplicities are 8.3 ± 0.3 (pd) and 8.4 ± 0.4 ($\pi^+ d$).

Our 100-GeV/c $\pi^+ d$ rescatter fraction is in agreement with recalculated 21-GeV/c and 205-GeV/c $\pi^- d$ rescatter fractions.

The pd rescatter fraction is consistent with being larger than the πd rescatter fraction.

The data are consistent with the relation $F_{rs} = 2(\bar{\nu}-1)$.

The multiplicity distributions in the rescatter events appear to agree with predictions of energy-flux-cascade and coherent-tube models.

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APPENDIX

The parameter $\bar{\nu}$, for a hadron-nucleus collision, is the average number of inelastic collisions the hadron would make with nucleons in the nucleus if, following each collision, the hadron remained as a single hadron.[1,2] The value of $\bar{\nu}$ can be obtained from the formula $\bar{\nu}=A\sigma(hN,\text{ inel.})/\sigma(hA,\text{ inel.})$, where $\sigma(hN)$ and $\sigma(hA)$ are, respectively, the average hadron-nucleon cross section and the hadron-nucleus cross section. For deuterium we take $\sigma(hd,\text{ inel.})$ as the cross section for production of new particles, and since no measurements of this quantity are available we estimate it using the Glauber formula[42]:

$$\sigma(hd,\text{ inel.})=\sigma(hp\ \ \text{inel.})+\sigma(hn,\text{ inel.}) - 2\sigma(hp,\text{ inel.})\sigma(hn,\text{ inel.})\langle r^{-2}\rangle/4\pi. \tag{A1}$$

We then have

$$\frac{1}{\bar{\nu}}=1-\frac{2\sigma(hp,\text{ inel.})\sigma(hn,\text{ inel.})\langle r^{-2}\rangle}{4\pi[\sigma(hp,\text{ inel.})+\sigma(hn,\text{ inel.})]} \tag{A2}$$

or, approximately,

$$\bar{\nu}\approx 1+\frac{\sigma(hN,\text{ inel.})\langle r^{-2}\rangle}{4\pi}\ . \tag{A3}$$

Inserting appropriate inelastic cross sections[30,33] into Eq. (A2) we obtain, at 100 GeV/c, $\bar{\nu}(pd) = 1.096$ and $\bar{\nu}(\pi d)=1.069$, where we have taken values[30] for $\langle r^{-2}\rangle$ of 0.035 mb^{-1}(pd) and 0.040 mb^{-1}(πd). Equation (A3) shows that $\bar{\nu}$ is relatively independent of beam momentum in the range 20–200 GeV/c, where cross sections are relatively constant. Uncertainties in our $\bar{\nu}$ values arise from uncertainties in the values of $\langle r^{-2}\rangle$ and in the validity of Eq. (A1).

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