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Parity-violating vector-meson-exchange internucleon potential in a modified factorization approach*

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The vector-meson-exchange potential relevant for parity-violating nuclear processes is calculated in a modified factorization approach. Two general parameterizations of the weak Hamiltonian are given: one useful for the free-field Hamiltonian, and one needed when strong-interaction corrections are included. A correspondence between the present method and the usual factorization approach is given, and is illustrated by several examples of parity-violating processes.

I. INTRODUCTION

Studies of parity-violating nuclear processes[1,2] are motivated by the desire to observe and understand the $\Delta S=0$ weak interaction. As has been previously noted, these processes are potentially useful in differentiating among various theories of the weak interaction, although neither theory nor experiment is yet adequate for such a task. The parity-violating internucleon potential will contain pieces due to one-pion exchange, which is purely $\Delta I=1$, and vector-meson exchange, which in general contains terms with $\Delta I=0,1,2$. Since some processes probe only $\Delta I\neq 1$, the vector-meson potential is important even if it is expected to be intrinsically weaker, because of its shorter range, than that due to pion exchange. This paper presents an evaluation of the vector-meson potential for several models of the weak interaction, using a recently proposed modified factorization approach.[3]

The original factorization approach was given by Michel[4] and consists of

$$\langle p\rho^-|H_w^{pv}(0)|n\rangle=\frac{G}{\sqrt{2}}\cos^2\theta_C\langle\rho^-|V_\mu^{(1-i2)}(0)|0\rangle \times\langle p|A^{(1+i2)\mu}(0)|n\rangle. \qquad (1)$$

This was later shown to be a consequence of the current-field identity and field algebra.[5] Questions have been raised over a possible cancellation of the factorization result by seagull terms,[1,6] and a possibility that the vector-meson amplitudes are divergent.[7] These questions have not been satisfactorily resolved, but the factorization method still appears to be the standard procedure for calculating the parity-violating NNV vertices. However, it is not compatible with the most common model of the weak currents, as we show below.

Present theoretical prejudice writes the weak currents as bilinears in fundamental quark fields. These quarks come in three colors and obey anticommutation relations. The currents are color singlets. To show that the usual factorization approximation is not consistent with this framework, we look at an example,

$$\langle p\rho^-|V_\mu^3A^{3\mu}|n\rangle, \qquad (2)$$

where

$$V_\mu^3(0)=\bar q_i(0)\gamma_\mu\tfrac{1}{2}\lambda^3q_i(0) =\tfrac{1}{2}[\bar{\mathcal{P}}_i(0)\gamma_\mu\mathcal{P}_i(0)-\bar{\mathcal{N}}_i(0)\gamma_\mu\mathcal{N}_i(0)]. \qquad (3)$$

Here i is a color index and is summed over the three colors. This matrix element vanishes in the usual factorization approach. However, if we rearrange terms in the current product using a Fierz identity, we will obtain a nonzero result using the same method. The identity is

$$\bar{\mathcal{P}}_i\gamma_\mu\mathcal{P}_i\bar{\mathcal{N}}_j\gamma_\mu\gamma_5\mathcal{N}_j+\bar{\mathcal{P}}_i\gamma_\mu\gamma_5\mathcal{P}_i\bar{\mathcal{N}}_j\gamma_\mu\mathcal{N}_j \equiv\bar{\mathcal{P}}_i\gamma_\mu\mathcal{N}_j\bar{\mathcal{N}}_j\gamma_\mu\gamma_5\mathcal{P}_i+\bar{\mathcal{P}}_i\gamma_\mu\gamma_5\mathcal{N}_j\bar{\mathcal{N}}_j\gamma_\mu\mathcal{P}_i. \qquad (4)$$

Since the currents and states are color singlets

$$\begin{aligned}\langle\rho^-|\bar{\mathcal{N}}_j\gamma_\mu\mathcal{P}_i|0\rangle&=\tfrac{1}{3}\delta_{ij}\langle\rho^-|\bar{\mathcal{N}}_k\gamma_\mu\mathcal{P}_k|0\rangle\\&=\tfrac{1}{3}\delta_{ij}\langle\rho^-|V_\mu^{1-i2}|0\rangle,\\ \langle p|\mathcal{P}_i\gamma_\mu\gamma_5\mathcal{N}_j|n\rangle&=\tfrac{1}{3}\delta_{ij}\langle p|A_\mu^{1+i2}|n\rangle.\end{aligned} \qquad (5)$$

This implies that, after Fierz reordering,

$$\langle p\rho^-|V_\mu^3A^{3\mu}|n\rangle=-\tfrac{1}{12}\langle\rho^-|V_\mu^{(1-i2)}|0\rangle \times\langle p|A^{\mu(1+i2)}|n\rangle, \qquad (6)$$

which is clearly nonzero. It is the possibility of Fierz reordering in the quark fields in the current product that is not compatible with the usual factorization method. The approach used in this paper is similar in spirit to the usual method but does not suffer from the above difficulty.

Recently, another approach has been proposed[8] that goes beyond the usual factorization approach. In it the vector-meson amplitudes are calculated in a quark model using Bethe-Salpeter wave functions. While comparison between this approach and the method of the present paper is difficult, both agree on the importance of terms neglected in the usual procedure.

In Sec. II, we present a derivation of our method. This has been given before,[3] but is included here to make this paper self-contained. The general parameterization of the weak Hamiltonian and our results for the vector-meson potential are given in Sec. III. Two parameterizations are given: one that is most useful for a general product of color singlet currents, and one that is useful for theories where strong-interaction corrections to the free-field Hamiltonian are given. In the last section we present a method for transforming calculations done for the usual factorization approach into our methods, and give several examples that allow us to compare our results with previous ones.

II. THE MODIFIED FACTORIZATION APPROACH

To find the parity-violating (pv) vector-meson-exchange potential we need to know the weak pv $BB'V$ amplitudes. For an example we will look at

$$M_{bc}^{ad} = \langle N'\rho^a | : V_\mu^b(0) A^{c\mu}(0) : | N^d \rangle . \qquad (7)$$

This can be expressed in general as

$$\begin{aligned} M_{bc}^{ad} &= \epsilon_\lambda^* M^\lambda \\ &= \epsilon_\lambda^* \bar{u}(p')[\gamma^\lambda \gamma_5 h(q^2) + \sigma^{\lambda\sigma}\gamma_5 q_\sigma k(q^2) \\ &\qquad + iq^\lambda \gamma_5 l(q^2)] u(p) . \end{aligned} \qquad (8)$$

When $q_\mu \to 0$, the form factors $k(q^2)$ and $l(q^2)$ can be neglected. We use the Lehmann-Symanzik-Zimmerman reduction technique to write this as

$$M_{bc}^{ad} = i\epsilon_\lambda^* (q^2 + m_\rho^2) \int d^4x\, e^{-iq\cdot x} \langle N' | T(\rho^{a\lambda}(x) : V_\mu^b(0) A^{c\mu}(0) :) | N \rangle \qquad (9)$$

plus possible seagull terms. Then the current-field identity[9] (CFI)

$$\rho_\lambda^a(x) = \frac{f_\rho}{m_\rho^2} V_\lambda^a(x) \qquad (10)$$

can be used to replace the ρ field by the vector current. When the currents have the general form

$$V_\lambda^a(x) = \bar{q}(x)\gamma_\lambda \tfrac{1}{2}\lambda^a q(x) , \qquad (11)$$

with the λ^a being the SU(3) matrices, we can evaluate the time-ordered product of three currents using Wick's theorem to obtain

$$\begin{aligned} T(V_\lambda^a(x) : V_\mu^b(0) A^{c\mu}(0) :) = {} & : V_\lambda^a(x) V_\mu^b(0) A^{c\mu}(0) : \\ & + : \bar{q}(x)\gamma_\lambda \tfrac{1}{2}\lambda^a \langle 0 | T(q(x)\bar{q}(0)) | 0 \rangle \gamma_\mu \tfrac{1}{2}\lambda^b q(0)\bar{q}(0)\gamma^\mu\gamma^5 \tfrac{1}{2}\lambda^c q(0) : + \text{perm} \\ & + \langle 0 | T(V_\lambda^a(x) V_\mu(0)) | 0 \rangle A^{c\mu}(0) + : \bar{q}(0)\gamma_\mu \tfrac{1}{2}\lambda^b \langle 0 | T(q(0) V_\lambda^a(x) \bar{q}(0)) | 0 \rangle \gamma^\mu\gamma^5 \tfrac{1}{2}\lambda^c q(0) : \\ & + : \bar{q}(0)\gamma^\mu\gamma^5 \tfrac{1}{2}\lambda^c \langle 0 | T(q(0) V_\lambda^a(x)\bar{q}(0)) | 0 \rangle \gamma_\mu \tfrac{1}{2}\lambda^b q(0) : . \end{aligned} \qquad (12)$$

In the second line, the baryon interacts with the weak Hamiltonian first, propagates as a three-quark state, and then interacts via the vector current. This can be seen to be analogous to a baryon pole, and we interpret it as such. However, these contributions vanish, since the diagonal matrix element $\langle B(p) | H_w^{pv} | B(p) \rangle$ vanishes. The remaining contributions can be evaluated by reversing the CFI and reduction procedure to obtain

$$\begin{aligned} M_{bc}^{ad} = {} & \langle \rho^a | V^{b\mu}(0) | 0 \rangle \langle N' | A_\mu^c(0) | N^d \rangle \\ & - (\gamma_\mu)^{\alpha\beta} (\gamma^\mu\gamma_5)^{\gamma\delta} \tfrac{1}{2}\lambda_{pq}^b \tfrac{1}{2}\lambda_{rs}^c [\langle \rho^a | : \bar{q}_{\alpha i p}(0) q_{\delta j s}(0) : | 0 \rangle \langle N' | : \bar{q}_{\gamma j r}(0) q_{\beta i q}(0) : | N^d \rangle \\ & \qquad + \langle \rho^a | : \bar{q}_{\gamma j r}(0) q_{\beta i q}(0) : | 0 \rangle \langle N' | : \bar{q}_{\alpha i p}(0) q_{\delta j s}(0) : | N^d \rangle] + \epsilon_\lambda^* R^\lambda , \end{aligned} \qquad (13)$$

with

$$R_\lambda = i(q^2 + m_\rho^2) \frac{f_\rho}{m_\rho^2} \int d^4x\, e^{-iq\cdot x} \langle N' | : V_\lambda^a(x) V_\mu^b(0) A^{c\mu}(0) : | N^d \rangle . \qquad (14)$$

Here $\alpha, \beta, \gamma, \delta$ are Dirac indices, p, q, r, s are SU(3) indices, and i, j refer to color. The first term above corresponds to that obtained in the usual factorization approach. The second terms are those mentioned in the Introduction, which are obtained via a Fierz rearrangement of the fields. Finally $\epsilon_\lambda^* R^\lambda$ will not contribute to $h(q^2)$ at $q^2 = 0$ since $q_\lambda R^\lambda = 0$. Equation (13) then defines our approach.

Even though we have used the current-field identity in motivating our method, we need not use it in the evaluation of the matrix elements, since there is experimental information available. We will write

$$\langle V^{(a)} | J_\mu^{\text{em}} | 0 \rangle = C_a m_\rho^2 \epsilon_\mu^* . \qquad (15)$$

The constants C_a can be obtained from the decays[10] $V^{(a)} \to e^+e^-$, which yield $C_\rho = 0.19 \pm 0.02$, $C_\omega = 0.07$

± 0.01, $C_\phi = -0.14 \pm 0.01$. However, because the electromagnetic current is purely octet, we cannot obtain information about the SU(3)-singlet current. To do this, we rely on the quark model to tell us that

$$\langle \omega | J^0_\mu | 0 \rangle = \sqrt{6}\, \langle \omega | J^{\rm em}_\mu | 0 \rangle . \tag{16}$$

Likewise we need to know the matrix elements $\langle N' | A^\alpha_\mu | N \rangle$ for $\alpha = 0, 1, \ldots, 8$. For $\alpha = 1, \ldots, 8$ these can be obtained from an SU(3) parameterization of experimentally accessible matrix elements; however, we cannot extract $\alpha = 0$ without some additional assumption. Again we use the quark model to obtain[11]

$$\langle N | A^0_\mu | N \rangle = \sqrt{2}\, \langle N | A^8_\mu | N \rangle . \tag{17}$$

III. THE POTENTIAL

In the interest of generality and ease of application we will present two parameterizations of the weak Hamiltonian. The first is the form that follows from H_w being simply the product of color-singlet currents,

$$H_w = \frac{G}{\sqrt{2}} [A(V^{1+i2}_\mu A^{1-i2\mu} + V^{1-i2}_\mu A^{1+i2\mu}) + BV^3_\mu A^{3\mu} + CV^3_\mu A^{8\mu} + C'V^8_\mu A^{3\mu} + DV^8_\mu A^{8\mu} + EV^0_\mu A^{0\mu} + FV^3_\mu A^{0\mu} + F'V^0_\mu A^{3\mu}$$
$$+ GV^8_\mu A^{0\mu} + G'V^0_\mu A^{8\mu} + H(V^{6+i7}_\mu A^{6-i7\mu} + V^{6-i7}_\mu A^{6+i7\mu}) + I(V^{4+i5}_\mu A^{4-i5\mu} + V^{4-i5}_\mu A^{4+i5\mu})] , \tag{18}$$

where V^α_μ $(\alpha = 0, 1, \ldots, 8)$ is given by Eq. (11), and $\lambda^0 = (\frac{2}{3})^{1/2} I$. The parameters of several models of the weak interaction[12-18] are listed in Table I. We have omitted several recent models that have many unknown parameters.

Recently the strong-interaction corrections to the weak Hamiltonian have been calculated for asymptotically free theories.[8,19,20] In addition to providing enhancement or suppression of various terms in H_w, the strong interactions mix into H_w a new set of 4-quark operators, having the form

$$\bar{q}\gamma^\mu\gamma^5 t^A q \bar{q}\gamma^\mu t^A q ,$$

where t^A are the SU(3) color matrices. These theories generally cannot be accommodated by the parameterization given above. We will give another general form of H_w that is useful for these theories.

A general 4-quark parity-violating operator can be written as

$$O(M, N) \equiv \bar{q}\gamma^\mu\gamma^5 M q \bar{q}\gamma^\mu N q , \tag{19}$$

TABLE I. Weak Hamiltonian parameters. (We define $s \equiv \sin^2\theta_W$.)

Model	A	B	C	C'	D	E
Cabibbo, Ref. 12	$\cos^2\theta_C$	0	0	0	0	0
d'Espagnat, Ref. 13	1	2	$-2/\sqrt{3}$	$-2/\sqrt{3}$	$\frac{2}{3}$	0
Segrè, γ_5-invariant, Ref. 14	1	0	0	0	$\frac{4}{3}$	0
Segrè, γ_5-noninvariant, Ref. 14	1	0	$-2/\sqrt{3}$	0	$\frac{4}{3}$	0
Lee, Ref. 15	1	1	0	$-\sqrt{3}$	0	0
Oakes, Ref. 16	$\cos^2\theta_C$	2	$2\sqrt{3}$	$2\sqrt{3}$	6	0
Tomozawa, octet, Ref. 17	1	2	0	0	2	$\sin^2\theta_C$
Tomozawa, nonet, Ref. 17	1	2	$(2/\sqrt{3})\sin^2\theta_C$	$(2/\sqrt{3})\sin^2\theta_C$	2	1
Weinberg-Salam, Ref. 18	$\cos^2\theta_C$	$2(1-2s)$	$(2/\sqrt{3})(1-2s)$	$(2/\sqrt{3})(1-2s)$	$\frac{2}{3}(1-2s)$	$\frac{1}{3}$

Model	F	F'	G	G'	H	I
Cabbibo	0	0	0	0	0	$\sin^2\theta_C$
d'Espagnat	0	0	0	0	1	$\sin^2\theta_C$
Segrè, γ_5-invariant	0	0	0	0	1	1
Segrè, γ_5-noninvariant	0	0	0	0	1	1
Lee	0	0	0	0	$\sin^2\theta_C$	$\sin^2\theta_C$
Oakes	0	0	0	0	0	$\sin^2\theta_C$
Tomozawa, octet	$(\frac{3}{2})^{1/2}\sin^2\theta_C$	$(\frac{3}{2})^{1/2}\sin^2\theta_C$	$(1/\sqrt{2})\sin^2\theta_C$	$(1/\sqrt{2})\sin^2\theta_C$	1	1
Tomozawa, nonet	$(\frac{2}{3})^{1/2}\sin^2\theta_C$	$(\frac{2}{3})^{1/2}\sin^2\theta_C$	$(\sqrt{2}/3)\sin^2\theta_C$	$(\sqrt{2}/3)\sin^2\theta_C$	1	1
Weinberg-Salam	$-(2/\sqrt{6})(1-2s)$	$-2/\sqrt{6}$	$-(\sqrt{2}/3)(1-2s)$	$-\sqrt{2}/3$	0	$\sin^2\theta_C$

where M and N are arbitrary matrices in flavor and color spaces. For our purposes it is sufficient to define six matrices in flavor space. If we take as our basis $q=(p,n,\lambda)$ we have

$$A_1=\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix},\quad A_2=\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix},\quad A_3=\begin{pmatrix}0&0&0\\0&0&1\\0&0&0\end{pmatrix},\tag{20}$$

$$B_1=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\quad B_2=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix},\quad B_3=\begin{pmatrix}1&0&0\\0&1&0\\0&0&-1\end{pmatrix}.\tag{21}$$

Then H_w must have the form

$$H_w=\frac{G}{\sqrt{2}}\sum_i\left[\alpha_{ii}O(A_i^\dagger,A_1)+\beta_{ii}O(A_i^\dagger t^A,A_i t^A)+\text{H.c.}\right]+\frac{G}{\sqrt{2}}\sum_{i,j}\left[\gamma_{ij}O(B_i,B_j)+\delta_{ij}O(B_i t^A,B_j t^A)\right].\tag{22}$$

The coefficients $\alpha_{ii},\beta_{ii},\gamma_{ij},\delta_{ij}$ are listed in Table II for models[3,8,18-24] for which the strong-interaction effects have been calculated. The enhancement or suppression of various pieces of H_w depend on

$$a\equiv\left[1+\frac{g}{24\pi^2}(33-2n)\ln\left(\frac{M_w}{\mu}\right)\right]\tag{23}$$

raised to some exponent. Here g is the effective quark-gluon coupling constant at renormalization point $p^2=-\mu^2$, and n is the number of quark types (flavors) in the theory. This quantity, Eq. (23), can be variously estimated to be in the range 2 to 10. For the Weinberg-Salam model we quote $a=4,10$, while for other theories we follow Ref. 8 and use $a=10$. When $a=1$ we obtain the free-field limit where no effects of the strong interactions are included.

The vector-meson potential follows using standard techniques combined with the modified factorization approach. For $B=\rho,\omega,\phi$,

$$V_B=-\frac{g_{BNN}Gg_A m_\rho^2 C_B}{16\pi M_N}\Big(\vec{\sigma}^{(2)}\cdot\{\vec{p}_{12},\exp(-m_B r)/r\}I^B_{12}-\vec{\sigma}^{(1)}\cdot\{\vec{p}_{12},\exp(-m_B r)/r\}I^B_{21}$$
$$-i\vec{\sigma}^{(1)}\times\vec{\sigma}^{(2)}\cdot[\vec{p}_2,\exp(-m_B r)/r]\tilde{I}^B_{12}-i\vec{\sigma}^{(2)}\times\vec{\sigma}^{(1)}\cdot[\vec{p}_1,\exp(-m_B r)/r]\tilde{I}^B_{21}\Big).\tag{24}$$

Here the I^B_{ij} are isospin functions. For the first parameterization

$$I^\rho_{ij}=\sqrt{2}\left(A-\frac{B}{12}+\frac{D}{36}+\frac{E}{18}+\frac{G}{18\sqrt{2}}+\frac{G'}{18\sqrt{2}}\right)(\tau_+^{(i)}\tau_-^{(j)}+\tau_-^{(i)}\tau_+^{(j)})$$
$$+\frac{1}{\sqrt{2}}\left(-\frac{A}{3}+\frac{7}{12}B+\frac{D}{36}+\frac{E}{18}+\frac{G}{18\sqrt{2}}+\frac{G'}{18\sqrt{2}}\right)\tau_3^{(i)}\tau_3^{(j)}+\frac{1}{20\sqrt{6}}[7C+C'+\sqrt{2}\,(7F+F')]1^{(i)}\tau_3^j,\tag{25}$$

$$I^\omega_{ij}=\frac{3}{5\sqrt{2}}\left(2A+\frac{B}{2}+\frac{7}{6}D+\frac{7}{3}E+\frac{7G}{3\sqrt{2}}+\frac{7G'}{3\sqrt{2}}\right)1^{(i)}1^j+\frac{1}{2\sqrt{6}}[C+7C'+\sqrt{2}\,(F+7F')]\tau_3^{(i)}1^{(j)},\tag{26}$$

$$I^\phi_{ij}=-\frac{3}{5\sqrt{2}}(-D+2E-2G+G'+H+I)1^{(i)}1^{(j)}-\frac{1}{\sqrt{2}}(I-H+\sqrt{3}\,F'-\sqrt{3}\,C')\tau_3^i 1^j.\tag{27}$$

In the second parameterization

$$I^\rho_{ij}=\sqrt{2}\,[\alpha_{11}+\tfrac{1}{3}(\gamma_{11}+\gamma_{13}-\gamma_{22}+\gamma_{31}+\gamma_{33})+\tfrac{16}{9}(\delta_{11}+\delta_{13}-\delta_{22}+\delta_{31}+\delta_{33})]T^+_{ij}$$
$$+\frac{1}{3\sqrt{2}}[-\alpha_{11}-\tfrac{16}{3}\beta_{11}+\gamma_{11}+\gamma_{13}+7\gamma_{22}+\gamma_{31}+\gamma_{33}+\tfrac{16}{3}(\delta_{11}+\delta_{13}+\delta_{22}+\delta_{31}+\delta_{33})]\tau_3^{(i)}\tau_3^{(j)}$$
$$+\frac{1}{5\sqrt{2}}[7\gamma_{12}+7\gamma_{32}+\gamma_{21}+\gamma_{23}+\tfrac{10}{3}(\delta_{12}+\delta_{32})+\tfrac{22}{3}(\delta_{21}+\delta_{23})]1^{(i)}\tau_3^{(j)},\tag{28}$$

$$I^\omega_{ij}=\frac{6}{5\sqrt{2}}[\alpha_{11}+\tfrac{16}{3}\beta_{11}+7(\gamma_{11}+\gamma_{13}+\gamma_{31}+\gamma_{33})+\gamma_{22}+\tfrac{16}{3}(\delta_{11}+\delta_{13}+\delta_{22}+\delta_{31}+\delta_{33})]1^{(i)}1^{(j)}$$
$$+\sqrt{2}\,[\gamma_{12}+\gamma_{32}+7\gamma_{21}+7\gamma_{23}+\tfrac{22}{3}(\delta_{12}+\delta_{32})+\tfrac{10}{3}(\delta_{21}+\delta_{32})]\tau_3^{(i)}1^{(j)},\tag{29}$$

$$I^\phi_{ij}=-\frac{3}{5\sqrt{2}}[\alpha_{22}+\alpha_{33}+\tfrac{16}{3}(\beta_{22}+\beta_{23})+6(\gamma_{11}-\gamma_{12}-\gamma_{13}+\gamma_{31}-\gamma_{32}-\gamma_{33})+2(\delta_{12}+2\delta_{13}-\delta_{21}-\delta_{23}-2\delta_{31}+\delta_{32})]1^{(i)}1^{(j)}$$
$$-\frac{1}{\sqrt{2}}[\alpha_{22}-\alpha_{33}+\tfrac{16}{3}(\beta_{22}-\beta_{33})+6(\gamma_{21}-\gamma_{22}-\gamma_{23})+2(\delta_{12}-\delta_{21}+\delta_{23}-\delta_{32})]\tau_3^{(i)}1^{(j)}.\tag{30}$$

TABLE II. Weak Hamiltonian parameters. (We define $s \equiv \sin^2\theta_W$.)

Model	α_{11}	α_{22}	α_{33}	β_{11}	β_{22}	β_{33}
Cabibbo	$\cos^2\theta_C$	$\sin^2\theta_C$	0	0	0	0
Weinberg-Salam, Ref. 18	$\cos^2\theta_C$	$\sin^2\theta_C$	0	0	0	0
Weinberg-Salam (a = 4), Refs. 3, 21	$1.14\cos^2\theta_C$	$1.14\sin^2\theta_C$	0	$-0.32\cos^2\theta_C$	$-0.32\sin^2\theta_C$	0
Weinberg-Salam (a = 10), Refs. 3, 20	$1.39\cos^2\theta_C$	$1.39\sin^2\theta_C$	0	$-0.61\cos^2\theta_C$	$-0.61\sin^2\theta_C$	0
Vectorlike, Refs. 21, 22	$\cos^2\theta_C$	$\sin^2\theta_C$	0	0	0	0
Vectorlike (a = 10), Ref. 8	$1.59\cos^2\theta_C$	$1.59\sin^2\theta_C$	0	$-0.80\cos^2\theta_C$	$-0.80\sin^2\theta_C$	0
4-quark DGG, Ref. 21	$\cos^2\theta_C$	$\sin^2\theta_C$	0	0	0	0
4-quark DGG (a = 10), Ref. 8	$1.39\cos^2\theta_C$	$1.39\sin^2\theta_C$	0	$-0.61\cos^2\theta_C$	$-0.80\sin^2\theta_C$	0
4-quark FGM, Refs. 8, 22, 23	$\cos^2\theta_C$	$\sin^2\theta_C$	0	0	0	0
4-quark FGM (a = 10), Ref. 23	$1.39\cos^2\theta_C$	$1.39\sin^2\theta_C$	0	$-0.61\cos^2\theta_C$	$-0.61\sin^2\theta_C$	0
LPZ, Ref. 24	$\cos^2\theta_C$	$\sin^2\theta_C$	0	0	0	0
LPZ (a = 10), Ref. 8	$1.59\cos^2\theta_C$	$1.59\sin^2\theta_C$	0	$-0.80\cos^2\theta_C$	$-0.61\sin^2\theta_C$	0

Model	γ_{11}	γ_{22}	γ_{33}	γ_{12}	γ_{21}	γ_{13}	γ_{31}	γ_{23}	γ_{32}
Cabibbo	0	0	0	0	0	0	0	0	0
Weinberg-Salam	0	$\frac{1}{2}(1-2s)$	0	0	$-\frac{1}{3}s$	0	0	0	0
Weinberg-Salam (a = 4)	$0.02(3-2s)$	$0.57(1-2s)$	0	0	$-0.39s$	0	0	0	0
Weinberg-Salam (a = 10)	$0.04(3-2s)$	$0.70(1-2s)$	0	0	$-0.52s$	0	0	0	0
Vectorlike	0	0	0	0	0	0	0	0	0
Vectorlike (a = 10)	0	0	0	0	0	0	0	0	0
4-quark DGG	0	$\frac{1}{4}(\frac{3}{2}-2s)$	$-\frac{1}{8}$	0	$-\frac{1}{6}$	0	$-\frac{1}{6}$	$-\frac{1}{8}$	$\frac{1}{4}(\frac{3}{2}-2s)$
4-quark DGG (a = 10)	$0.04(3-2s)$	$0.35(\frac{3}{2}-2s)$	-0.17	$0.09-0.1s$	$-0.12-0.17s$	$-0.09+0.02s$	$0.12-0.33s$	$0.07-0.28s$	$0.77-1.12s$
4-quark FGM	$-\frac{1}{12}(\frac{3}{2}+2s)$	$\frac{1}{2}(1-2s)$	$-\frac{1}{8}$	$\frac{1}{4}(1-2s)$	$-\frac{1}{6}(\frac{3}{2}+2s)$	$\frac{1}{8}$	$\frac{1}{12}(\frac{3}{2}+2s)$	$\frac{1}{4}$	$-\frac{1}{4}(1-2s)$
4-quark FGM (a = 10)	$-0.11-0.29s$	$0.70(1-2s)$	-0.17	$0.42-0.77s$	$-0.42(1+s)$	$0.25-0.05s$	$0.15+0.27s$	$0.42-0.07s$	$0.42+0.77s$
LPZ	$\frac{1}{4}(1-2s)$	$\frac{1}{4}(1-2s)$	0	$\frac{1}{4}(1-2s)$	$\frac{1}{4}(1-2s)$	0	0	0	0
LPZ	$0.56-0.8s$	$0.40(1-2s)$	0	$0.40-1.0s$	$0.40-1.1s$	0	0	0	0

	δ_{11}	δ_{22}	δ_{33}	δ_{12}	δ_{21}	δ_{13}	δ_{31}	δ_{23}	δ_{32}
Cabibbo	0	0	0	0	0	0	0	0	0
Weinberg-Salam	0	0	0	0	0	0	0	0	0
Weinberg-Salam (a = 4)	$-0.03(3-2s)$	$-0.16(1-2s)$	0	$0.13s$	$-0.01s$	0	0	0	0
Weinberg-Salam (a = 10)	$-0.06(3-2s)$	$-0.30(1-2s)$	0	$0.31s$	$-0.03s$	0	0	0	0
Vectorlike	0	0	0	0	0	0	0	0	0
Vectorlike (a = 10)	0	0	0	0	0	0	0	0	0
4-quark DGG	0	0	0	0	0	0	0	0	0
4-quark DGG (a = 10)	$-0.06(3-2s)$	$-0.23(1-2s)$	0.11	0.06	$-0.14+0.13s$	$-0.06+0.08s$	$0.14-0.05s$	$-0.65+0.45s$	$0.05-0.25s$
4-quark FGM	0	0	0	0	0	0	0	0	0
4-quark FGM (a = 10)	$0.23+0.03s$	$-0.31(1-2s)$	0.08	$0.35-0.11s$	$-0.35+0.47s$	$0.07-0.19s$	$0.27-0.16s$	$0.35-0.50s$	$-0.35+0.20s$
LPZ	0	0	0	0	0	0	0	0	0
LPZ (a = 10)	$0.32+0.02s$	$-0.20(1-2s)$	0	$-0.20+0.28s$	$-0.20+0.62s$	0	0	0	0

For both forms

$$\tilde{I}^{\rho}_{ij}=(1+\mu_p-\mu_n)I^{\rho}_{ij}\,,\quad \tilde{I}^{\omega}_{ij}=I^{\omega}_{ij}\,,\quad \tilde{I}^{\phi}_{ij}=I^{\phi}_{ij}\,. \tag{31}$$

The nucleon-nucleon vector-meson coupling g_{BNN} arises in the effective strong-interaction Lagrangian

$$\mathcal{L}_s=g_{\rho NN}\vec{\rho}_\mu\bar{N}\left[\gamma^\mu+\frac{i(\mu_p-\mu_n)}{2M_N}\sigma^{\mu\nu}\partial_\nu\right]\tfrac{1}{2}\vec{\tau}N+g_{\omega NN}\omega_\mu\bar{N}\gamma^\mu N+g_{\phi NN}\phi_\mu\bar{N}\gamma^\mu N\,. \tag{32}$$

There are many ways[25] to determine $g_{\rho NN}$, generally yielding $g_{\rho NN}{}^2/4\pi\simeq 2.5$. The situation is less clear for $g_{\omega NN}$ and $g_{\phi NN}$, and is complicated by ω-ϕ mixing.[26] The reader is of course free to supply these constants himself. However, using SU(3) and known hadron dynamics we can obtain plausible values for $g_{\omega NN}$ and $g_{\phi NN}$ in terms of $g_{\rho NN}$. To do this we note that in the language of SU(3), the strong BBV coupling can be written

$$\mathcal{L}_s=-g_8\mathrm{Tr}(V_\mu[\bar{B},\gamma^\mu B])-g_0V^0\mathrm{Tr}(\bar{B}B)\,, \tag{33}$$

where $V_\mu(B)$ is the octet of vector mesons (baryons) and V^0 is the SU(3) singlet vector meson. With ω-ϕ mixing the ω and ϕ coupling constants will be linear combinations of g_8 and g_0. However, we use hadron dynamics in the form of the Zweig-Iizuka rule,[27] which tells us that if ω and ϕ are ideally mixed (e.g. $\phi=\bar{\lambda}\lambda$), then $g_{\phi NN}=0$. This implies that $g_0=\sqrt{3}\,g_8$ and we have

$$\mathcal{L}_s=\sqrt{2}\,g_8\left\{\vec{\rho}_\mu\bar{N}\gamma^\mu\tau N+\left[(\tfrac{3}{2})^{1/2}\cos\theta+\frac{\sqrt{3}\,\sin\theta}{2}\right]\omega_\mu\bar{N}\gamma^\mu N+\left[(\tfrac{3}{2})^{1/2}\sin\theta-\frac{\sqrt{3}\,\cos\theta}{2}\right]\phi_\mu\bar{N}\gamma^\mu N\right\}. \tag{34}$$

For ideal mixing $[\cos\theta=(\frac{2}{3})^{1/2}]$

$$g_{\omega NN}=\tfrac{3}{2}g_{\rho NN}\,,\quad g_{\phi NN}=0 \tag{35}$$

while for $\theta=40°$, as suggested by the mass formula and mixing angles,[28]

$$g_{\omega NN}=1.49\,g_{\rho NN}\,,\quad g_{\phi NN}=+0.12\,g_{\rho NN}\,. \tag{36}$$

IV. CONCLUSION

We have obtained the contribution to the parity-violating internucleon potential using a modified factorization approach. Our potential differs only in the isospin functions from that of the usual factorization method. By now there is a body of work which has relied on the previous potential. That work will apply to our potential if we make a correspondence of the isospin parameters between the two methods. This is done in the Appendix. As examples of parity-violating processes in our framework, we adapt the results of calculations in the literature to our methods.

Rustgi and Pirner[29] have published a parameterization for the circular polarization observed in the process $n+p\to d+\gamma$. Converting to our notation yields

$$P_\gamma=(3.84A-1.5B+0.10D+0.20E+0.14G+0.14G')\times10^{-8} \tag{37}$$

$$=[3.84\,\alpha_{11}-3.19\,\beta_{11}+1.22(\gamma_{11}+\gamma_{13}+\gamma_{31}+\gamma_{33})-6.0\,\gamma_{22}+2.09(\delta_{11}+\delta_{13}+\delta_{31}+\delta_{33})-8.46\,\delta_{22}]\times10^{-8}\,. \tag{38}$$

Following the suggestion by Desplanques and Craver *et al.*[30] we have corrected the sign of the results of Ref. 29. The polarization is greater in our method because the $\Delta I=2$ contribution to the potential, which P_γ is primarily sensitive to, is one-third stronger using our techniques. However, this slight effect is not significant enough to shed any light on the serious problems that exist in attempting to understand this process.[30,31]

From the review paper by Box *et al.*[2] we can extract the vector-meson contribution to the symmetry of the 110-keV γ-ray transition in ^{19}F:

$$A_\gamma=(8.42A+2.6B-2.4C-3.84C'+1.95D+3.89E-3.39F-5.4F'+2.75G+2.75G')\times10^{-5} \tag{39}$$

$$=[8.42\alpha_{11}+7.41\beta_{11}+23.4(\gamma_{11}+\gamma_{13}+\gamma_{31}+\gamma_{33})+10.4\gamma_{22}-16.6(\gamma_{12}+\gamma_{32})-26.6(\gamma_{21}+\gamma_{23})+35.1(\delta_{11}+\delta_{13}+\delta_{31}+\delta_{33})+10.0\delta_{22}-32.1(\delta_{12}+\delta_{32})-25.5(\delta_{21}+\delta_{32})]\times10^{-5}\,. \tag{40}$$

There is also a contribution due to pion exchange that we do not list since it involves methods and assumptions outside of the scope of this paper.

Finally we compare predictions for a transition in a heavy nucleus. Box *et al.*[2] calculate the circular polarization of the 396-keV γ transition in ^{175}Lu. We extract the vector-meson contribution as

$$P_\gamma=(1.04A+0.29B+0.11C-0.37C'+0.25D+0.50E+0.15F-0.56F'+0.35G+0.35G')\times10^{-5} \tag{41}$$

$$=[1.04\alpha_{11}+1.12\beta_{11}+3.0(\gamma_{11}+\gamma_{13}+\gamma_{31}+\gamma_{33})+1.19\gamma_{22}+0.68(\gamma_{12}+\gamma_{32})-2.60(\gamma_{21}+\gamma_{23})+4.1(\delta_{11}+\delta_{13}+\delta_{31}+\delta_{33})+1.1\delta_{22}-2.37(\delta_{12}+\delta_{32})-0.19(\delta_{21}+\delta_{23})]\times10^{-5}\,. \tag{42}$$

The values of these quantities for some of the models are listed in Table III, along with the value obtained in the usual method for the Cabibbo model, and the experimental result.[32-34] It is instructive to note that the effects of the strong-interaction enhancement are rather unpredictable. This is due to cancellation among the various contributions that can become more or less complete as we change the enhancement. These conditions make it difficult to extract detailed information from experiment about the structure of the $\Delta S=0$ weak Hamiltonian.

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APPENDIX

We give here the correspondence between the usual method and our results. We do this for the potentials of Ref. 5 (labeled with FTT) and Ref. 2 (labeled BM). For the first parameterization the correspondence is

$$1_{\mathrm{FTT}}\cos^2\theta_C = H_{\mathrm{BM}} = C_\rho g_{\rho NN}\left(A-\frac{B}{12}+\frac{D}{36}+\frac{E}{18}+\frac{G}{18\sqrt{2}}+\frac{G'}{18\sqrt{2}}\right), \tag{A1}$$

$$B_{\mathrm{FTT}}\cos^2\theta_C = 2K_{\mathrm{BM}} = C_\rho g_{\rho NN}\left(-\frac{2A}{3}+\frac{7B}{6}+\frac{D}{18}+\frac{E}{9}+\frac{G}{9\sqrt{2}}+\frac{G'}{9\sqrt{2}}\right), \tag{A2}$$

$$\xi C_{\mathrm{FTT}}\cos^2\theta_C = \frac{I_{\mathrm{BM}}}{3\sqrt{3}} = C_\rho\frac{g_{\rho NN}}{20\sqrt{3}}\left[7C+C'+\sqrt{2}\,(7F+F')\right], \tag{A3}$$

$$C'_{\mathrm{FTT}}\cos^2\theta_C = 2I'_{\mathrm{BM}} = \frac{C_\omega g_{\omega NN}}{3}\left[7C'+C+\sqrt{2}(F+7F')\right], \tag{A4}$$

$$\xi D_{\mathrm{FTT}}\cos^2\theta_C = \frac{L_{\mathrm{BM}}}{3\sqrt{2}} = \frac{\sqrt{3}\,C_\omega g_{\omega NN}}{5}\left(2A+\frac{B}{2}+\frac{7D}{6}+\frac{7E}{3}+\frac{7G}{3\sqrt{2}}+\frac{7G'}{3\sqrt{2}}\right). \tag{A5}$$

For the second form we have

$$1_{\mathrm{FTT}}\cos^2\theta_C = H_{\mathrm{BM}} = C_\rho g_{\rho NN}\left[\alpha_{11}+\tfrac{1}{3}(\gamma_{11}+\gamma_{13}-\gamma_{22}+\gamma_{31}+\gamma_{33})+\tfrac{16}{9}(\delta_{11}+\delta_{13}-\delta_{22}+\delta_{31}+\delta_{33})\right], \tag{A6}$$

$$B_{\mathrm{FTT}}\cos^2\theta_C = 2K_{\mathrm{BM}} = \frac{2C_\rho g_{\rho NN}}{3}\left[-\alpha_{11}-\tfrac{16}{3}\beta_{11}+\gamma_{11}+\gamma_{13}+7\gamma_{22}+\gamma_{31}+\gamma_{33}+\tfrac{16}{3}(\delta_{11}+\delta_{13}+\delta_{22}+\delta_{31}+\delta_{33})\right], \tag{A7}$$

TABLE III. Parity-violating processes. (We use $\sin^2\theta_W=0.35$.)

Model	$n+p\to d+\gamma$ $10^8 P_\gamma$	^{19}F $10^5 A_\gamma$	^{175}Lu $10^5 P_\gamma$
Cabibbo	3.7	8.1	1.0
d'Espagnat	0.9	22.1	2.1
Segrê, γ_5-invariant	4.0	11.	1.4
Segrê, γ_5-noninvariant	4.0	13.7	1.5
Lee	2.3	17.7	2.0
Oakes	1.3	3.4	2.2
Tomozawa octet	1.0	17.6	2.1
Tomozawa nonet	1.2	21.5	2.6
Weinberg-Salam	2.8	12.8	1.5
Weinberg-Salam ($a=4$)	4.5	9.0	1.0
Weinberg-Salam ($a=10$)	6.3	6.5	1.2
Vectorlike	3.7	8.1	1.0
Vectorlike ($a=10$)	5.7	7.2	0.7
4-quark DGG	2.1	7.8	0.8
4-quark DGG ($a=10$)	5.9	21	1.5
4-quark FGM	2.8	12.8	1.5
4-quark FGM ($a=10$)	7.7	31	3.6
LPZ	3.3	7.3	1.2
LPZ ($a=10$)	8.5	7.2	1.0
Cabibbo usual factorization	2.8	6.8	0.8
Experiment	-130 ± 45 (Ref. 32)	18 ± 9 (Ref. 33)	4 ± 1 (Ref. 34)

$$\xi C_{\mathrm{FTT}} \cos^2\theta_C = \frac{I_{\mathrm{BM}}}{3\sqrt{3}}$$

$$= \frac{C_\rho g_{\rho NN}}{5} [7\gamma_{12} + 7\gamma_{32} + \gamma_{21} + \gamma_{23} + \tfrac{10}{3}(\delta_{12} + \delta_{32}) + \tfrac{22}{3}(\delta_{21} + \delta_{23})] , \tag{A8}$$

$$C'_{\mathrm{FTT}} \cos^2\theta_C = 2 I'_{\mathrm{BM}}$$

$$= \frac{4}{\sqrt{3}} C_\omega g_{\omega NN} [\gamma_{12} + \gamma_{32} + 7\gamma_{21} + 7\gamma_{23} + \tfrac{22}{3}(\delta_{12} + \delta_{32}) + \tfrac{10}{3}(\delta_{21} + \delta_{23})] , \tag{A9}$$

$$\xi D_{\mathrm{FTT}} \cos^2\theta_C = \frac{L_{\mathrm{BM}}}{3\sqrt{2}}$$

$$= \frac{2\sqrt{3}}{5} C_\omega g_{\omega NN} \times [\alpha_{11} + \tfrac{16}{3}\beta_{11} + 7(\gamma_{11} + \gamma_{13} + \gamma_{31} + \gamma_{33}) + \gamma_{22} + \tfrac{16}{3}(\delta_{11} + \delta_{13} + \delta_{22} + \delta_{31} + \delta_{33})] . \tag{A10}$$

In the above I have not included ϕ exchange because it contains a different spatial dependence, due to the ϕ mass, and because it is expected that ϕ will be weakly coupled to nucleons.

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