Tests of the quark-line rule for the η-η' complex*

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The quark-line rule involving η and η' mesons has been tested for reactions $\pi^+p\to\eta\Delta^{++}$, $\pi^-p\to\eta n$, $p\bar{p}\to\pi^+\pi^-\eta$, etc. The results appear to be consistent with a general particle-mixing model for the η-η' complex, and suggest that the validity of the rule is fairly reasonable. Other experimental tests of the rule are also discussed.

The discovery[1] of ψ and ψ' has renewed interest in the study of the nonet ansatz and the related quark-line rule[2] (hereafter referred to as QLR), as well as of the probable mechanism[3-7] violating the rule. For some time, it has been well known[8] that the QLR with ideal nonet structure for ϕ and f mesons can explain well various decay rates of these mesons. The rule also predicts

$$\sigma(p\bar{p}\to\phi\pi^+\pi^-)/\sigma(p\bar{p}\to\omega\pi^+\pi^-)\ll 1, \tag{1}$$

$$\sigma(\pi^-p\to\phi n)/\sigma(\pi^-p\to\omega n)\ll 1, \tag{2}$$

$$\sigma(\pi^+p\to\phi\Delta^{++})/\sigma(\pi^+p\to\omega\Delta^{++})\ll 1. \tag{3}$$

These relations are experimentally well satisfied,[9-11] although it may be necessary[12] to take into account some specific mechanism breaking the rule. Moreover, if we utilize the standard SU(3) Regge analysis, then the QLR leads to[13,14]

$$\sigma(K^-p\to\omega Y)=\sigma(K^-p\to\rho^0 Y)\ , \tag{4a}$$

$$\sigma(K^-p\to\phi Y)=\sigma(\pi^-p\to K^{0*}Y), \tag{4b}$$

which are experimentally well satisfied.[14] Also, the QLR in the exact-SU(3) limit predicts[15]

$$\begin{aligned}\sigma(e\bar{e}\to\rho^0K^0\bar{K}^0) &=\sigma(e\bar{e}\to\omega K^0\bar{K}^0)\\ &=\sigma(e\bar{e}\to\omega\pi^+\pi^-)\\ &=\sigma(e\bar{e}\to K^{0*}\pi^0\bar{K}^0)\\ &=\tfrac{1}{2}\sigma(e\bar{e}\to\phi K^0\bar{K}^0),\end{aligned} \tag{5a}$$

$$\sigma(e\bar{e}\to\rho^0K^+K^-)=\sigma(e\bar{e}\to\omega K^+K^-), \tag{5b}$$

$$\sigma(e\bar{e}\to\phi\pi^+\pi^-)=0, \tag{5c}$$

$$\sigma(e\bar{e}\to\phi K^+K^-)=\sigma(e\bar{e}\to K^{0*}\pi^+K^-), \tag{5d}$$

which may be tested in the future. At any rate, the rule appears in general to be well satisfied[9-12] within at most 10% for processes involving ϕ and f mesons.

However, the situation is less clear with respect to the pseudoscalar nonet. First of all, the 0^- nonet does not satisfy the typical mass formula[2] of the 1^- and 2^+ nonets, although this fact may have something to do with the so-called η puzzle[16] as has been suggested by many authors. Second, we have two possible candidates,[17] η' (958 MeV) and E (1420 MeV) for the nonet partner of η (549 MeV), although a recent experimental study[18] strongly favors 1^+ spin-parity assignment for the latter, thus precluding the possibility of the nonet partner being the E meson. In this note, we assume that the η' (958 MeV) is the ninth member of the 0^- nonet, partly because of the above experiment[18] and partly because of the paucity of experimental data involving the E meson. Ordinarily, it is customary to assume the mass-mixing scheme for the η-η' complex.

$$\begin{aligned}|\eta\rangle &=\cos\theta\,|\eta_8\rangle-\sin\theta\,|\eta_0\rangle\ ,\\ |\eta'\rangle &=\sin\theta\,|\eta_8\rangle+\cos\theta\,|\eta_0\rangle\ ,\end{aligned} \tag{6}$$

where η_8 and η_0 are defined in terms of the SU(3) quarks as

$$\begin{aligned}\eta_0 &=\frac{1}{\sqrt{3}}(q_1\bar{q}_1+q_2\bar{q}_2+q_3\bar{q}_3),\\ \eta_8 &=\frac{1}{\sqrt{6}}(q_1\bar{q}_1+q_2\bar{q}_2-2q_3\bar{q}_3).\end{aligned} \tag{7}$$

The mixing angle θ can then be determined from the SU(3) mass formula to be

$$\theta=-23.8^\circ, \tag{8a}$$

$$\theta=-10.6^\circ, \tag{8b}$$

respectively, depending upon whether we use a linear or a quadratic mass formula. Actually, the absolute sign of θ cannot be decided in this way. However, the choice of the negative rather than the positive sign in Eqs. (8) is dictated[19] from various studies of $\eta\to2\gamma$ and $\eta\to\pi^+\pi^-\gamma$ decays. As we shall see shortly, the negative (but *not* positive) sign for θ is also consistent with results of various reactions involving η and η'.

Now Lipkin[14] has derived a relation

$$\sigma(K^-p\to\eta\Lambda)+\sigma(K^-p\to\eta'\Lambda)=\sigma(K^-p\to\pi^0\Lambda)+\sigma(\pi^-p\to K^0\Lambda) \tag{9}$$

on the basis of the QLR together with the SU(3) and Regge pole analysis. He then finds that the right side of Eq. (9) at 3.9 GeV/c is experimentally[20] larger than the left side by a factor of 1.6, and concludes that the QLR is badly violated for the η-η' complex. However, in principle it could be possible[21,22] that a large SU(3) violation may be involved in this case or that the energy is not high enough to apply the Regge theory. Indeed, as we shall see below, there are some other relations which are consistent with the QLR without assuming the validity of the SU(3) symmetry and of the Regge theory. Secondly, it may be that the standard mass-mixing scheme, Eq. (6), on which the derivation of Eq. (9) depends may not be the correct one. Actually, on the basis of the dual unitary picture,[3] several authors[4,7,23] suggested a more complicated mixing scheme for the η-η' complex. Especially, Imani *et al.*[23] noted that this approach would lead to a general particle-mixing scheme[24]

$$|\eta\rangle = S_1(\cos\theta_1\,|\eta_8\rangle - \sin\theta_1\,|\eta_0\rangle)$$
$$|\eta'\rangle = S_2(\sin\theta_2\,|\eta_8\rangle + \cos\theta_2\,|\eta_0\rangle). \tag{10}$$

The simple mass-mixing scheme Eq. (6) is obviously a special case of Eq. (10) with

$$S_1 = S_2 = 1, \quad \theta_1 = \theta_2 \equiv \theta. \tag{11}$$

Although formula (10) contains four unknown real parameters, S_1, S_2, θ_1, and θ_2, two angles θ_1 and θ_2 can be determined from the masses of the 0^- nonet to be[7,23]

$$\theta_1 \simeq -6^\circ, \quad \theta_2 \simeq -20^\circ. \tag{12}$$

From the standard SU(3)-Regge analysis on reactions,

$$K^-p \to \eta\Lambda,\ \eta'\Lambda,\ \bar{K}^0 n,\ \pi^0\Lambda,$$
$$\pi^-p \to \eta n,\ \eta' n,\ K^0\Lambda,\ \pi^0 n, \tag{13}$$
$$K^+n \to K^0 p,$$

they also computed[23] values of S_1 and S_2 as

$$(S_2/S_1)^2 = 0.69 \pm 0.19, \tag{14a}$$

$$(S_1)^2 = 0.64 \pm 0.11\,. \tag{14b}$$

Note that relation (9) must be now replaced by a more complicated formula[23] which we will not write here. For the sake of comparison, we may remark that Rosenzweig[4] assigns a value of

$$S_1 = S_2 = 1, \quad \theta_1 \simeq -10^\circ, \quad \theta_2 \simeq -20^\circ. \tag{15}$$

The purpose of this note is to investigate further consequences of the quark-line rule involving η and η' mesons on the basis of either the mass-mixing scheme (6) or more general scheme (10).

However, in our analysis, we do *not* assume the validity of the SU(3) symmetry *nor* of the Regge theory. For simplicity, let us call any hadron nonstrange if its constituent quarks include only nonstrange quarks. Thus, pions and nucleons are nonstrange hadrons, but kaon, η, and η' are not. Let us set

$$q_3\bar{q}_3 = \frac{1}{\sqrt{3}}\,(\eta_0 - \sqrt{2}\,\eta_8)\,, \tag{16}$$

which corresponds to a fictitious 0^- bound state of strange quarks q_3 and $\bar{q}_3$. Consider a given exclusive reaction

$$A + B \to (q_3\bar{q}_3) + C_1 + C_2 + \cdots + C_n\,. \tag{17}$$

Suppose that all hadrons A, B, C_1, $C_2 \ldots, C_n$ in Eq. (17) are nonstrange hadrons, i.e., they do not contain any q_3 and $\bar{q}_3$ quarks. Then the QLR (or equivalently the nonet rule) demands that the matrix element for the reaction (17) should vanish. Then expressing $q_3\bar{q}_3$ in terms of physical η and η' by means of Eqs. (10) and (16), this leads to

$$\frac{\bar{\sigma}(A + B \to \eta' + C_1 + C_2 + \cdots + C_n)}{\bar{\sigma}(A + B \to \eta + C_1 + C_2 + \cdots + C_n)} = K, \tag{18}$$

$$K = \left(\frac{S_2}{S_1}\right)^2 \left(\frac{\sin\theta_2 + \sqrt{2}\cos\theta_2}{\cos\theta_1 - \sqrt{2}\sin\theta_1}\right)^2$$
$$= \left[\frac{S_2\sin(\theta_0 + \theta_2)}{S_1\cos(\theta_0 + \theta_1)}\right]^2, \tag{19a}$$

$$\theta_0 = \tan^{-1}\sqrt{2} = 54^\circ 44', \tag{19b}$$

where $\bar{\sigma}$ represents the production cross section divided by the phase volume. The derivation of Eq. (18) is obviously independent of the validity of the SU(3) symmetry as well as the Regge analysis, so that it directly tests the QLR in contrast to Eq. (9). Many tests of (18) for the mass-mixing case have been given already by Lipkin.[13,14] Equation (18) should be valid for the differential cross sections as well as for the total ones, so that the η and η' should have exactly the same angular and energy distributions in the reaction, provided that the mass difference between η and η' does not cause any serious kinematical problem. We remark that relation (18) is also valid even when some of C_1, $C_2, \ldots, C_n$ may coincide with η and/or η'.

The value of K can be computed from Eq. (19) and depends only upon specific choice of values assumed for θ_1, θ_2, S_1, and S_2. We shall discuss several cases of practical interest:

(a) Linear mass-mixing scheme with $S_1 = S_2 = 1$ and $\theta_1 = \theta_2 = -24^\circ$,

$$K = 0.35. \tag{20a}$$

(b) Quadratic mass-mixing scheme with $S_1 = S_2 = 1$

and $\theta_1 = \theta_2 = -10°$,

$$K = 0.96. \tag{20b}$$

(c) Rosenzweig model with Eq. (15),

$$K = 0.63. \tag{20c}$$

(d) General mixing scheme with Eqs. (12) and (14),

$$K = 0.50 \pm 0.14. \tag{20d}$$

(e) Linear mass-mixing scheme with positive sign for θ, i.e., $S_1 = S_2 = 1$ and $\theta_1 = \theta_2 = +24°$,

$$K = 23.9. \tag{20e}$$

(f) Quadratic mass-mising scheme with positive sign for θ, i.e., $S_1 = S_2 = 1$ and $\theta_1 = \theta_2 = +10°$,

$$K = 4.37. \tag{20f}$$

(g) No mixing, i.e., $S_1 = S_2 = 1$ and $\theta_1 = \theta_2 = 0$,

$$K = 2. \tag{20g}$$

After these preparations, we would like to test our prediction. First, let us consider

$$\frac{\bar{\sigma}(\pi^+ p \to \eta' \Delta^{++})}{\bar{\sigma}(\pi^+ p \to \eta \Delta^{++})} = K . \tag{21a}$$

Originally, this relation for the case of the mass-mixing model has been derived on the basis of the quark model,[13] and subsequently has been studied experimentally. So far, experimental values of K determined from (21a) are rather conflicting. One experiment[25] at 8 GeV/c gives

$$K = 0.90 \pm 0.40 \text{ or } K = 0.70 \pm 0.40, \tag{21b}$$

depending upon how the data are analyzed, while another experiment at 5.45 GeV/c reports[26]

$$K = 0.24 \pm 0.11. \tag{21c}$$

These values could be consistent with any of (20a)–(20d) but *not* with the positive-θ cases of (20e) and (20f) in conformity with the analysis[19] of $\eta \to 2\gamma$ and $\eta \to \pi^+\pi^-\gamma$ decays. Also, the non-resonant analog of (21a), i.e.,

$$\frac{\bar{\sigma}(\pi^\pm p \to \eta' \pi^\pm p)}{\bar{\sigma}(\pi^\pm p \to \eta \pi^\pm p)} = K, \tag{22}$$

could be experimentally tested in the future, although the presently available data[27] appear to imply the value of K is somewhat smaller than unity.

Second, the analogous relation

$$\frac{\bar{\sigma}(\pi^- p \to \eta' \Delta^0)}{\sigma(\pi^- p \to \eta \Delta^0)} = K \tag{23a}$$

has been measured[28] at $p_L = 7.1$ GeV/c to give

$$K = 0.25 \pm 0.025, \tag{23b}$$

which is consistent with (21c) in accordance with the charge independence, if the value of K is independent of the energy.

Next let us discuss[13]

$$\frac{\bar{\sigma}(\pi^- p \to \eta' n)}{\bar{\sigma}(\pi^- p \to \eta n)} = K , \tag{24a}$$

which has been investigated by many authors.[29] If we use experimental data[30] on the wide momentum range from $p_L = 3.8$ GeV/c to 48.0 GeV/c, then this reproduces (20d), i.e.,

$$K = 0.50 \pm 0.14, \tag{24b}$$

for all these momentum ranges since these data have been used by Imani *et al.*[23] to evaluate $(S_2/S_1)^2$ as in Eq. (14). Recently, the same reaction with the higher momentum range of $p_L = 20$–200 GeV/c has been reported.[31] A preliminary analysis shows that the value of K is essentially constant for all energies and for all values of t, which is the square of the momentum transfer. More importantly, it reproduces essentially the same value (24b) again for practically all momentum values under investigation. Also, the same reaction has been studied recently[32] at 8.4 GeV/c by Edwards *et al.* They discover the value of K to be dependent upon t, and we shall come back to its discussion later; however, we note here the fact that the average value of K is consistent with (24b).

The charge-symmetric analog of (24a)

$$\frac{\bar{\sigma}(\pi^+ n \to \eta' p)}{\bar{\sigma}(\pi^+ n \to \eta p)} = K \tag{25a}$$

as well as

$$\frac{\bar{\sigma}(\pi^+ d \to \eta' pp)}{\bar{\sigma}(\pi^+ d \to \eta pp)} = K \tag{26}$$

could be experimentally tested. So far, experimental data[33] on $\pi^+ d \to pp\eta(\eta')$ at 1.7-GeV/c pion momentum yield an upper bound for (25b),

$$K \lesssim 0.67 \pm 0.18. \tag{25b}$$

Comparing this with (24b), we see that both are consistent as is required by charge independence.

Next, consider a proton-antiproton reaction[34]

$$\frac{\bar{\sigma}(p\bar{p} \to \eta' \pi^+ \pi^-)}{\bar{\sigma}(p\bar{p} \to \eta \pi^+ \pi^-)} = K \tag{27a}$$

An experiment[35] at rest enables us to calculate

$$K = 0.73 \pm 0.15 . \tag{27b}$$

Also, the Dalitz plots of the η and η' for this process were experimentally found to be essentially identical to each other in accordance with our prediction. [See the discussion following Eq. (19).]

Similarly, for resonant ρ^0 production reaction[35]

$$\frac{\bar{\sigma}(p\bar{p}\to\eta'\rho^0)}{\bar{\sigma}(p\bar{p}\to\eta\rho^0)}=K \tag{28a}$$

we estimate[36]

$$K \geq 0.40\pm 0.35. \tag{28b}$$

Also for decays of the A_2 meson, we sould find that

$$\frac{\bar{\Gamma}(A_2\to\eta'\pi)}{\bar{\Gamma}(A_2\to\eta\pi)}=K, \tag{29b}$$

where $\bar{\Gamma}$ represents the decay width divided by the d-wave phase volume. Using the experimentally known decay rates[37] for these modes, this leads to

$$K<1.60, \tag{29b}$$

which excludes again cases (20e), (20f), and (20g).

Finally, for an extremely-high-energy reaction, relation (18) will give

$$\frac{\sigma(A+B\to\eta'+\text{any nonstrange hadrons})}{\sigma_0(A+B\to\eta+\text{any nonstrange hadrons})}=K \tag{30}$$

for semi-inclusive reactions. However, some caution is necessary, since the cross section $\sigma(A+B\to\eta+\cdots)$ may contain an indirect contribution from $\sigma(A+B\to\eta'+\cdots)$ followed by the decay $\eta'\to\eta\pi\pi$. The modified cross section $\sigma_0(A+B\to\eta+\cdots)$ in Eq. (30) represents the cross-section subtracted for this indirect extra contribution. Even after this correction, relation (30) is still approximate for the following reason. Some fraction of both η and η' could be indirect decay products of resonances $A_2\to\pi\eta$ (or η') and $E\to\pi\pi\eta$ (or η') as well as $N^*\to N\eta$ (or η') production. Because of large phase-volume differences for n and η' modes, more η mesons than η' will be produced for such processes in comparison to that indicated by Eq. (30). Also, because of these decay processes, the left side of (30) will produce an apparent t-dependence instead of the constant behavior demanded by (30). Therefore, the best place to test Eq. (30) is perhaps limited to the forward direction near $t\simeq 0$, where the direct production mechanism for η and η' (rather than the indirect resonant decays) is expected to be more important.

A preliminary experiment by Lai *et al.*[38] measures cross sections for reactions

$$\begin{aligned}&\pi^-\text{Be}\to\eta'+\text{neutrals},\\&\pi^-\text{Be}\to\eta+\text{neutrals}\end{aligned} \tag{31}$$

at 100 GeV/c. Unfortunately, we cannot directly compare these data with (30), since the final neutral particles in (31) may contain strange-neutral hadrons K_S, K_L, Λ, Σ^0, and Ξ^0 which (except for K_L) decay quickly into neutral particles. Assuming that this contamination due to neutral strange particles is small and noting the known neutral-decay branching ratio of

$$\frac{\Gamma(\eta'\to\eta\pi^0\pi^0)}{\Gamma(\eta'\to all)}=0.23,$$

we may approximately rewrite (30) as

$$\frac{\sigma(\pi^-\text{Be}\to\eta'+\text{neutrals})}{\sigma(\pi^-\text{Be}\to\eta+\text{neutrals})}\simeq\frac{K}{1+0.23K}, \tag{32a}$$

we would hope, at least near $t\simeq 0$. The preliminary experimental data by Lai *et al.* appear to indicate the left side of (32a) to be roughly 0.45 at $0\leq|t|\leq 0.1(\text{GeV}/c)^2$, and 0.30 around $|t|\simeq 0.2\,(\text{GeV}/c)^2$ with smaller values for higher $|t|$. As we remarked already, we should test the validity of (32a) only near $t\simeq 0$. We then see that the value of K determined in this way is consistent with (24b), i.e.,

$$K\simeq 0.50. \tag{32b}$$

Summarizing what we have found in our analysis, we see that the experimental values of K are still confusing, but are mostly within a range of 0.25–0.75 with a good likelihood around $K\simeq 0.5$. The fact that this excludes the cases of positive mixing angle θ is reassuring in view of the same situation for $\eta\to 2\gamma$ and $\eta\to\pi^+\pi^-\gamma$ decays.[19] Also, these values are obtained from various experiments made at energies ranging from 1 GeV/c to 200 GeV/c. Some of the experimental values may not be mutually consistent for the following reason. All reactions $\pi^+p\to\eta\Delta^{++}$, $\pi^-p\to\eta\Delta^0$, and $\pi^-p\to\eta n$ as well as their corresponding η' modes are expected to proceed via the A_2-Regge-trajectory exchange at the high energy under consideration. Then the value of K for all these reactions should be the same, irrespective of the validity of the QLR. Hence, if the discrepancy between $K=0.5$ of (24b) and $K=0.25$ of (23b) is real, this would imply a large absorption correction, i.e., a large contribution from the Regge cut which would at the same time violate the QLR. In this connection the new experiment[32] on $\pi^-p\to\eta n$ and $\pi^-p\to\eta' n$ at 8.4 GeV/c is even more puzzling. At the momentum transfer of $t'\simeq 0$, the value of K is near 1.0 while it assumes $K\simeq 0.5$ at $|t'|\simeq 0.2$ (GeV/c)^2 and drops to $K\simeq 0.2$ at $|t'|\simeq 0.7$ (GeV/c)^2. However, the overall average of K appears to be $K\simeq 0.5$.

Let us now present the following argument. Since we expect that the QLR will be better satisfied for higher energies, we should choose the ideal value of K to be $K=0.5$ determined as in (24b). This also a rough average of all values of K so far studied. Then we may interpret any deviation of K from this value 0.5 to be an indication of the violation of the QLR. Since most of the experimental values of K studied in this note range between 0.25 and 0.75, we assume this fact to imply that the violations of the QLR for most of the various reactions are at most 50% or so in magnitude for cross sections, and hence at most 22% in the scattering amplitude. As we noted in the beginning, the violation of the QLR involving the ϕ meson may amount to[9-12] as much at 10% in amplitude.[39] Comparing both cases, we could say that the quark-line rule is still reasonably well satisfied even for the η-η' complex. It appears to be certainly better than ordinarily believed to be possible for the 0^- nonet. Of course, more experimental tests are required to verify this point. We could test the rule also in various other reactions[40] such as

$$\bar{\sigma}(\bar{p}d\to\eta'\pi^-p)/\bar{\sigma}(\bar{p}d\to\eta\pi^-p)=K, \tag{33}$$

$$\bar{\sigma}(\gamma p\to\eta' p)/\bar{\sigma}(\gamma p\to\eta p)=K, \tag{34}$$

$$\bar{\sigma}(ep\to e\eta' p/\bar{\sigma}(ep\to e\eta p)=K, \tag{35}$$

$$\bar{\sigma}(e\bar{e}\to\eta'\pi^+\pi^-)/\bar{\sigma}(e\bar{e}\to\eta\pi^+\pi^-)=K, \tag{36}$$

where the symbol d in (33) represents the deuteron target. Moreover, if the decays of ψ and/or ψ' are dominantly mediated by exchanges of three SU(3)-color gluons, then we expect to have [as in Eq. (18)]

$$\frac{\bar{\Gamma}(\psi\to\eta'+C_1+C_2+\cdots+C_n)}{\bar{\Gamma}(\psi\to\eta+C_1+C_2+\cdots+C_n)}=K. \tag{37}$$

However, a relatively large QLR-breaking decay rate for $\psi\to\pi^+\pi^-\phi$ may indicate[41] a more complicated mechanism for the ψ decay, thus possibly invalidating relation (37).

If we apply the QLR to decays $\eta'\to\rho^0\gamma$ and $\rho^0\to\eta\gamma$, we find that

$$\frac{\Gamma(\eta'\to\rho^0\gamma)}{\Gamma(\rho^0\to\eta\gamma)}=3\left(\frac{q'}{q}\right)^3 K, \tag{38}$$

where q and q' are energies of the final photons in the rest frames of $\eta'\to\rho^0\gamma$ and $\rho^0\to\eta\gamma$, respectively. Assuming the value $K\simeq 0.5$, this predicts

$$\Gamma(\eta'\to\rho^0\gamma)\simeq\Gamma(\rho^0\to\eta\gamma). \tag{39}$$

Similarly, we predict

$$\Gamma(\eta'\to\omega\gamma)\simeq\Gamma(\omega\to\eta\gamma). \tag{40}$$

A recent experimental measurement[42] of $\Gamma(\rho^0\to\eta\gamma)$ and $\Gamma(\omega\to\eta\gamma)$ gives the following two possible values of

$$\Gamma(\rho^0\to\eta\gamma)=50\pm13\ \text{keV},\quad \Gamma(\omega\to\eta\gamma)=3.0^{+2.5}_{-1.8}\ \text{keV}, \tag{41a}$$

or

$$\Gamma(\rho^0\to\eta\gamma)=76\pm15\ \text{keV},\quad \Gamma(\omega\to\eta\gamma)=29\pm7\ \text{keV}. \tag{41b}$$

As we shall see shortly, the second choice (41b) conflicts with (40) so that we accept the solution (41a). Then, together with the known branching ratio[37] of

$$\Gamma(\eta'\to\rho^0\gamma)/\Gamma(\eta'\to\text{all})=0.274,$$

we predict

$$\Gamma(\eta'\to\text{all})\simeq 180\ \text{KeV}. \tag{42}$$

Also, Eqs. (39), (40), and (41a) lead to

$$\frac{\Gamma(\eta'\to\omega\gamma)}{\Gamma(\eta'\to\rho^0\gamma)}\simeq\frac{\Gamma(\omega\to\eta\gamma)}{\Gamma(\rho^0\to\eta\gamma)}\simeq 0.06, \tag{43a}$$

which is consistent with the present experimental upper bound[37] for $\Gamma(\eta'\to\omega\gamma)$. On the other hand, the solution (41b) gives a very large value of

$$\frac{\Gamma(\eta'\to\omega\gamma)}{\Gamma(\eta'\to\rho^0\gamma)}\simeq 0.4, \tag{43b}$$

which contradicts the present experimental values for this ratio.

So far, we refrained deliberately from the use of the SU(3) symmetry. If we had utilized exact SU(3) together with the QLR as is usually done, we would then have obtained more predictions. However, this will be left as a future study in view of a possible complication due to the SU(3) violation. Coming back to the discussion of Eq. (9) in this connection, the discrepancy of the factor 1.6 (assuming now the mass-mixing scheme and the A_2 Regge-exchange mechanism) may still be due to a combined effect of roughly 20% QLR violation and of another 10% SU(3) breaking, since the combined 30% violation in amplitude is enough to account for the discrepancy.

Note added. After this paper was completed we came across a paper by N. H. Fuchs, [Phys. Rev. D 14, 1912 (1976)], in which the general mixing scheme, Eq. (10), has been derived and discussed from the viewpoint of color-gluon theory. Also in a recent paper, P. L. Woodworth *et al.*, [Phys. Lett. 65B, 89 (1976)], have reported a measurement of

$$\frac{\sigma(\pi^-p\to\phi\pi^+\pi^-\pi^-p)}{\sigma(\pi^-p\to\omega\pi^+\pi^-\pi^-p)}\simeq 0.005$$

at 19 GeV/c, which again supports the QLR for the vector mesons.

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[10]D. S. Ayres *et al.*, Phys. Rev. Lett. 32, 1463 (1974), give a value 0.0035 to the left side of Eq. (2) for $p_L = 5-6$ GeV/c.

[11]W. R. Butler *et al.*, Phys. Rev. D 7, 3177 (1973), report an upper bound of 0.0033 to the left side of Eq. (3) for $p_L = 3.7$ GeV/c. They also find a relatively large rate for nonresonant $\pi^+ p \to \pi^+ \phi\, p$ cross sections.

[12]See E. L. Berger and C. Sorensen, Phys. Lett. 62B, 303 (1976), for an analysis of $\pi^- p \to \phi n$. For $p\bar{p} \to V_9 P_8 P_8$ reaction at $p_L = 1.1$ GeV/c, an analysis of R. K. Logan, S. Konitz, and S. Tanaka, Univ. Toronto report, 1976 (unpublished), concludes that the QLR violation with ideal mixing angle for ω-ϕ complex may amount to 10% in amplitude.

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[14]H. J. Lipkin, Phys. Lett. 60B, 371 (1976).

[15]S. Okubo, Phys. Rev. D 14, 108 (1976).

[16]S. Weinberg, Phys. Rev. D 11, 3583 (1975).

[17]It is known that the quartic Schwinger mass formula is better satisfied by a choice $(\pi, K, \bar{K}, \eta, E)$ rather than $(\pi, K, \bar{K}, \eta, \eta')$ for the 0^- nonet. On the basis of this fact, M. D. Slaughter and S. Oneda (as well as some other authors) in Univ. of Maryland Tech. Report No. 76-116, 1976 (unpublished), favor the E meson for the nonet partner. On the other hand, Y. Ueda [Phys. Rev. D (to be published)] arrives at the opposite conclusion from a study of decay rates for $\eta' \to \eta\pi^+\pi^-$ and $E \to K\bar{K}\pi$.

[18]V. Vuillenin *et al.*, Lett. Nuovo Cimento 14, 165 (1975).

[19]E.g., see F. D. Gault, H. F. Jones, M. D. Scadron, and R. L. Thews, Nuovo Cimento 24A, 259 (1974). For some recent works on decays of η and η' mesons, see e.g. M. S. Chanowitz, Phys. Rev. Lett. 35, 977 (1975); G. Giocogna and F. Strocchi, Phys. Lett. 58B, 313 (1975).

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[21]We note that experimental decay rates for $\rho^+ \to \pi^+\gamma$, $K^{0*} \to K^0\gamma$, and $\omega \to \pi^0\gamma$ require either a large SU(3) violation or a large QLR violation or both. See B. J. Edwards and A. N. Kamal, Univ. of Alberta report, 1976 (unpublished); Phys. Rev. Lett. 36, 714 (1976); P. J. O'Donnell, Univ. of Toronto report, 1976 (unpublished); Phys. Rev. Lett. 36, 177 (1976); D. H. Boal, R. H. Graham, and J. W. Moffat, *ibid.* 36, 714 (1976); E. Takasugi and S. Oneda, in Proceedings of the International Symposium on Mathematical Physics, Mexico City, 1976, Report No. COO-1545-176 (unpublished).

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[23]T. Imani, K. Kawarabayashi, and S. Kitakado, Univ. of Tokyo Report No. UT-Komaba 76-5, 1976 (unpublished). However, we changed their notations as $S_1 = S_{\eta'}$, $S_2 = S_{\eta'}$, $\theta_1 = \theta_\eta - \theta_0$, $\theta_2 = \theta'_\eta - \theta_0$, with $\theta_0 = \tan^{-1}\sqrt{2} = 54.°44 = 54°44'$.

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[25]M. Bardin-Otwinoska *et al.*, Phys. Rev. D 4, 2711 (1971).

[26]I. J. Bloodworth *et al.*, Nucl. Phys. B39, 525 (1972) and earlier references quoted therein.

[27]K. Boesebeck *et al.*, Nucl. Phys. B4, 501 (1968); G. Ascoli *et al.*, Phys. Rev. Lett. 20, 1321 (1968). We have to exclude the A_2 region for the test of (22) since $\Gamma(A_2 \to \pi\eta')$ is kinematically suppressed in comparison to $\Gamma(A_2 \to \pi\eta)$.

[28]K. W. Lai *et al.*, in a paper presented at the XVI International Conference on High Energy Physics, NAL, 1972 (quoted by F. D. Gault *et al.* in Ref. 19).

[29]A. D. Martin and C. Michael, Phys. Lett. 37B, 513 (1971); H. Nagai and A. Nakamura, Prog. Theor. Phys. 53, 523 (1975).

[30]W. D. Apel *et al.*, Phys. Lett. 46B, 459 (1973); V. N. Bolotov *et al.*, *ibid.* 48B, 280 (1974); W. D. Apel *et al.*,

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[31]The data for $\pi^- p \to \eta n$ can be found in O. I. Dahl *et al.*, Phys. Rev. Lett. 37, 80 (1976). The experimental values used for $\pi^- p \to \eta' n$ are due to a private communication from Professor A. Barnes of Carnegie Institute of Technology to Professor T. Ferbel. We would like to thank Professor Barnes and Professor Ferbel for giving the experimental data.

[32]K. W. Edwards *et al.*, a paper submitted for the XVIII International Conference on High Energy Physics, Tbilisi, 1976; University of Toronto Report No. COO-1545-193, 1976 (unpublished).

[33]T. C. Bacon *et al.*, Phys. Rev. 157, 1263 (1967). We have corrected for the phase-volume ratio of 1.9 and used the known decay rate $\Gamma(\eta' \to \eta\pi^+\pi^-)/\Gamma(\eta' \to \text{all}) \simeq 0.47$.

[34]This relation (27) as well as (36) has been given in Ref. 15 for the case of no mixing, i.e., $K=2$.

[35]M. Foster *et al.*, Nucl. Phys. B8, 174 (1968). The ratio of the phase volumes for the two reactions amounts to a factor of 3.

[36]The ratio of the S-wave phase volumes for $\bar{p}p \to \eta\rho^0$ and $\bar{p}p \to \eta'\rho^0$ at rest is 1.82. Since we do not know whether the final state has angular momentum $l=0$ or 1, the estimate (28b) gives a lower bound. For example, if the initial $p\bar{p}$ system is in the $^3S_1$ state (rather than the $^3P_1$ state), we have to multiply a factor of 3.3 to the right side of (28b).

[37]Particle Data Group, Phys. Lett. 50B, 1 (1974).

[38]K. W. Lai, private communication (1976).

[39]In this estimate, we assumed the ideal mixing for the ω-ϕ complex. If we had used a more realistic mixing angle, then the amount of the QLR violation would have become smaller.

[40]Although the photon could couple to the strange quarks, a straight-forward generalization of the QLR in one-photon-exchange approximation still leads to the validity of Eqs. (34), (35), and (36). The relation (36) with $K=2$ has been noted in Ref. 15.

[41]H. Harari, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California* (Ref. 1), p. 317. However, for a different viewpoint on $\psi \to \phi\pi^+\pi^-$ decay, see S. Okubo, Phys. Rev. D 13, 1994 (1976); 14, 108 (1976).

[42]C. Bemporad [in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California* (Ref. 1), p. 113] gives an upper bound of $\Gamma(\rho^0 \to \eta\gamma) < 160$ keV. A recent measurement by D. E. Andrews *et al.* [Univ. of Rochester report (1976) (unpublished)] finds two possible solutions, given in Eqs. (41a) and (41b).