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Renormalization of a distorted gauge-invariant theory*

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We consider a new type of renormalizable theory involving massive Yang-Mills fields whose mass is generated by an intrinsic breakdown of the usual local gauge symmetry. However, the Lagrangian has a distorted gauge symmetry which leads to the Ward-Takahashi (WT) identities. Also, the theory is independent of the gauge parameter ξ. We completely carry out an explicit renormalization at the one-loop level by exhibiting counterterms, defining the physical parameters, and computing all renormalization constants to check the WT identities. Our results indicate that the physical scalar can be removed from the physical spectrum and that perhaps the theory could become asymptotically free.

I. INTRODUCTION

In a previous paper[1] we have discussed a renormalizable theory (by power counting) involving massive Yang-Mills fields in which the vector-boson masses are generated by an intrinsic breakdown of the usual local gauge symmetry. The vector-boson mass M cannot be obtained from spontaneous symmetry breakdown owing to the absence of the quartic potential of scalar fields.[2] Although the Lagrangian is no longer invariant under the usual local gauge transformation, it is still invariant under a generalized or "distorted" local gauge transformation involving M.[3,4] In the limit $M \to 0$, the distorted gauge transformation reduces to the usual one. The unitarity and gauge independence of the theory have been verified by calculations up to and including the two-loop level. Furthermore, based on the distorted gauge symmetry, one can give a general formal proof of unitarity and ξ independence of the theory.[3]

In this paper, we derive the Ward-Takahashi (WT) identities,[5,6] which lead to constraints among the renormalization constants (Z's). The renormalization of the theory[1] is carried out in a manifest way to supplement the general formal treatments.[3] All Z's are computed to confirm the WT identities and to ensure the consistency of renormalization. We show some of the interesting features of those theories with intrinsic symmetry breakdown that are not revealed in previous formal treatments and provide a theoretical framework for discussing the cancellations of divergences.

We choose a linear gauge condition and derive the fictitious Lagrangian (f Lagrangian) based on distorted-gauge-symmetry considerations.[7,3] We obtain an f Lagrangian which is apparently different from that obtained in the Lagrange-multiplier formalism.[8] Yet, in fact, they are equivalent when the class of linear gauge condition is chosen; this question has been discussed before.[9,10] When one chooses a bilinear gauge condition which is Abelian gauge invariant, the usual gauge formalism for non-Abelian theories leads to unnecessary compensating terms for the electromagnetic gauge. On the other hand, the Lagrange-multiplier formalism does not have these unnecessary compensating terms.[11]

The Lagrangian in this theory is renormalizable to all orders by power counting; it contains massive charged vector fields and a physical scalar $U(x)$ with a zero mass in the physical spectrum. We study this Lagrangian thoroughly because it offers a possiblity of constructing a new renormalizable and asymptotically free theory involving no residual physical scalar (see Sec. V).

II. DISTORTED GAUGE SYMMETRY

Let us consider the Lagrangian involving massive Yang-Mills fields $\vec{f}_\mu$ and scalar fields $\vec{\phi}$ and U:[1]

$$\begin{aligned} L_1 = &-\tfrac{1}{4}\vec{f}_{\mu\nu}\cdot\vec{f}^{\mu\nu} + \tfrac{1}{2}M^2\vec{f}_\mu\cdot\vec{f}^\mu + \tfrac{1}{2}\partial_\mu U\partial^\mu U \\ &+ \tfrac{1}{2}\partial_\mu\vec{\phi}\cdot(\partial^\mu\vec{\phi} + g\vec{f}^\mu\times\vec{\phi}) - \tfrac{1}{2}g\vec{f}_\mu\cdot(U\partial^\mu\vec{\phi} - \vec{\phi}\partial^\mu U) \\ &+ \tfrac{1}{8}g^2\vec{f}_\mu\cdot\vec{f}^\mu(\vec{\phi}^2 + U^2) + \tfrac{1}{2}gM\vec{f}_\mu\cdot\vec{f}^\mu U + M\vec{\phi}\cdot\partial_\mu\vec{f}^\mu, \end{aligned} \tag{1}$$

$$\vec{f}_{\mu\nu} = \partial_\nu\vec{f}_\mu - \partial_\mu\vec{f}_\nu - g\vec{f}_\mu\times\vec{f}_\nu. \tag{2}$$

We emphasize that (1) does not have a quartic potential of the scalar fields, an essential for spontaneous breaking of gauge symmetry. In this sense, the mass M of the vector field $\vec{f}_\mu$ in (1) has little to do with spontaneous symmetry breaking. It could be regarded as generated by an intrinsic symmetry breaking because if $M=0$ the Lagrangian L_1 in (1) will be invariant under the usual SU(2) local gauge transformation. One can verify that the Lagrangian L_1 is invariant under the distorted SU(2) gauge transformation

$$\begin{aligned} &f_\mu^a \to f_\mu'^a = f_\mu^a - g\epsilon^{abc}\omega^b f_\mu^c + \partial_\mu\omega^a, \\ &\phi^a \to \phi'^a = \phi^a - \tfrac{1}{2}g\epsilon^{abc}\omega^b\phi^c + \tfrac{1}{2}g\omega^a(U + 2M/g), \\ &U \to U' = U - \tfrac{1}{2}g\omega^a\phi^a, \end{aligned} \tag{3}$$

where $\vec{\omega} = \vec{\omega}(x)$ is an infinitesimal gauge function. When $M=0$, ϕ_a and U transform like components of a complex isodoublet field.

We choose as the gauge condition

$$\partial_\mu f_a^\mu + M\phi_a/\xi = b_a(x)\,, \tag{4}$$

where $b_a(x)$ is a suitable function independent of the fields and the gauge function $\omega_a(x)$. The vacuum-to-vacuum amplitude of the theory is[7]

$$W(b_a) = \int d[F]\, \exp\left(i\int d^4x\, L_1\right) \times \det Q \prod_a \delta(\partial_\mu f_a^\mu + M\phi_a/\xi - b_a)\,, \tag{5}$$

where F denotes the set of fields in L_1, $F \equiv \{\vec{f}_\mu, \vec{\phi}, U\}$. The functional determinant $\det Q$ is defined by[12]

$$1/\det Q = \int d[\vec{\omega}] \prod_a \delta(\partial_\mu f_a'^\mu + M\phi_a'/\xi - b_a)\,. \tag{6}$$

It follows from (6) that

$$Q^{ab} = \delta^{ab}(\Box + M^2/\xi) - g\epsilon^{abd}\overline{\partial}_\mu f_d^\mu - \frac{1}{2}\frac{Mg}{\xi}\epsilon^{abd}\phi_d + \tfrac{1}{2}\delta^{ab}\frac{Mg}{\xi}U\,, \tag{7}$$

$$\overline{\partial}_\mu f_d^\mu \equiv (\partial_\mu f_d^\mu) + f_d^\mu \partial_\mu\,, \quad \Box \equiv \partial_\mu\partial^\mu\,. \tag{8}$$

It can be shown that $W(b_a)$ is invariant under an infinitesimal change of $b_a(x)$ for all $b_a(x)$.[2] Thus, we may write $W(b_a)$ in (5) as

$$\begin{aligned} W &= \int W(b_a)\exp\left[-i\int d^4x\, \xi b_a^2(x)/2\right] d[\vec{b}(x)] \\ &= \int d[F]\, \det Q\, \exp\left\{i\int d^4x[L_1 - \tfrac{1}{2}\xi(\partial_\mu \vec{f}^\mu + M\vec{\phi}/\xi)^2]\right\}, \end{aligned} \tag{9}$$

to within unimportant multiplicative factors. The amplitude (9) corresponds to starting from the effective Lagrangian[1]

$$L_{\rm eff} = L_1 - \tfrac{1}{2}\xi(\partial_\mu \vec{f}^\mu + M\vec{\phi}/\xi)^2 - D_a' Q_{ab} D_b\,, \tag{10}$$

which is renormalizable by power counting. The Lagrangian (10) completely specifies the theory involving the physical Yang-Mills fields and scalar field U, together with the unphysical scalar fields $\partial^\mu f_\mu^a$, ϕ^a and the fictitious scalar-fermion fields D_a' and D_a.

The complete Feynman rules derived from (10) are given in a previous paper.[1] In the limit $\xi \to 0$, we have a formally unitary theory, in which the masses of all unphysical fields (i.e., $\partial^\mu \vec{f}_\mu$, $\vec{\phi}$, $\vec{D}'$, and $\vec{D}$) become infinite. For the tree diagrams, the unitarity of the S matrix in the limit $\xi \to 0$ is obvious. However, the effects of unphysical scalars remain because the fictitious loops degenerate to quartically divergent contact terms when $\xi = 0$:

$$\det Q \propto \exp\{i\delta^4(0)\,\mathrm{Tr}\ln[\delta^{ab} + g\epsilon^{abd}\phi_d/(2M) + \delta^{ab}gU/(2M)]\}\,.$$

In general, the limit $\xi \to 0$ is singular and could interfere with loop-momentum integrations.[13] Therefore, it must be examined carefully in the framework of renormalization and regularization.

III. WT IDENTITIES

Let us define the generating functional $W(J)$ in the gauge specified by (4) as

$$W(J) = \int d[F, \vec{D}, \vec{D}']\exp\left[i\int d^4x(L_{\rm eff} + \vec{J}_\mu\cdot\vec{f}^\mu + \vec{J}_\phi\cdot\vec{\phi} + J_U U)\right]. \tag{11}$$

We consider the transformation (3) with the gauge function $\omega_a(x)$ restricted by

$$\omega_a(x) = [Q^{-1}(F)]_{ab}\rho_b\,, \tag{12}$$

where ρ_b is an arbitrary infinitesimal number independent of fields. After performing the transformation (3) with $\vec{\omega}(x)$ restricted by (12) on the variables of the path integral (11), one obtains the WT identity[3,6]

$$\begin{aligned} \Bigg\{ &-\xi\left(\partial_\mu \frac{\delta}{i\delta J_\mu^a} + \frac{M}{\xi}\frac{\delta}{i\delta J_\phi^a}\right)_y + \int\Bigg[J_\mu^c\left(-\epsilon^{cbd}g\frac{\delta}{i\delta J_\mu^d} + \delta^{cb}\partial^\mu\right) \\ &+ J_\phi^c\left(-\tfrac{1}{2}\epsilon^{cbd}g\frac{\delta}{i\delta J_\phi^d} + \tfrac{1}{2}g\delta^{cb}\frac{\delta}{i\delta J_U} + \delta^{cb}M\right) + J_U\left(-\tfrac{1}{2}\delta^{cb}g\frac{\delta}{i\delta J_\phi^c}\right)\Bigg]_z \left[Q_{zy}^{-1}\left(\frac{\delta}{i\delta J}\right)\right]^{ba} d^4z\Bigg\}\, W(J) = 0 \end{aligned} \tag{13}$$

for $W(J)$, where

$$(J_\mu^c\,\delta/\delta J_\mu^d)_z = J_\mu^c(z)\,\delta/\delta J_\mu^d(z)\,,$$

etc. The identity (13) implies relations between different renormalization constants. One may, for example, differentiate (13) with respect to $\vec{J}_\mu$ two and three times, and then let all external sources vanish to obtain the relations[3]

$$
\begin{aligned}
&Z_1/Z_3 = Y_1/Y_3\,,\\
&Z_4 = Z_1{}^2/Z_3\,,
\end{aligned}
\tag{14}
$$

where Z_3 (Y_3) is the wave-function renormalization for f^a_μ (D^a), Z_1 (Y_1) is the vertex renormalization for $f^a_\mu f^b_\nu f^c_\lambda$ ($f^a_\mu D'^b D^c$), and Z_4 is for $f^a_\mu f^b_\nu f^c_\lambda f^d_\sigma$. The WT identities (14) are the same as those occurring in the massless Yang-Mills theory,[5] though the actual distorted gauge transformations (3) are different from the usual gauge transformations. The nontrivial identities (14) will be checked in Sec. IV because the scalar particles, which have different interactions from those in previous theories,[5] contribute to Z's.

IV. RENORMALIZATION

The effective bare Lagrangian of the theory is given by (10), i.e.,

$$
\begin{aligned}
L_{\text{eff}} &= L_1 + L_\xi + L_{ff}(D'_a, D_a)\,,\\
L_\xi &= -\tfrac{1}{2}\xi(\partial_\mu f^\mu_a + M\phi_a/\xi)^2\,,\\
L_{ff} &= \partial_\mu D'_a \partial^\mu D_a - (M^2/\xi)D'_a D_a\\
&\quad - g\epsilon^{abd}D'_a\partial_\mu(f^\mu_b D_d)\\
&\quad - \tfrac{1}{2}(Mg/\xi)(\epsilon^{abd}D'_a\phi_d D_b + D'_a D_a U)\,,
\end{aligned}
\tag{15}
$$

where L_1 is given by (1). The renormalization program is formulated on the basis of L_1. We rescale fields and parameters in L_1 according to

$$
\begin{aligned}
&f^a_\mu \to Z_3{}^{1/2} f^a_\mu\,,\quad \phi^a \to Z_\phi{}^{1/2}\phi^a\,,\\
&U \to Z_U{}^{1/2} U\,,\quad g \to gZ_1/(Z_3)^{3/2}\,,\\
&M^2 \to Z_m M^2/Z_3\,.
\end{aligned}
\tag{16}
$$

This gives an invariant renormalized Lagrangian, denoted by L_{inv},

$$
\begin{aligned}
L_{\text{inv}} = &-\tfrac{1}{4}Z_3(\partial_\mu f^a_\nu - \partial_\nu f^a_\mu)^2 + \tfrac{1}{2}Z_m M^2 f^a_\mu f^\mu_a - Z_1 g\epsilon^{abc}(\partial_\mu f^a_\nu)f^b_\mu f^c_\nu - \tfrac{1}{4}(g^2 Z_1{}^2/Z_3)\epsilon_{abc}\epsilon_{ade} f^b_\mu f^c_\nu f^\mu_d f^\nu_e\\
&+\tfrac{1}{2}Z_U\partial_\mu U\partial^\mu U + \tfrac{1}{2}Z_\phi\partial_\mu\phi_a\partial^\mu\phi_a + \tfrac{1}{2}(gZ_1 Z_\phi/Z_3)\epsilon_{abc}(\partial^\mu\phi_a)f^b_\mu\phi^c\\
&-\tfrac{1}{2}[gZ_1(Z_U Z_\phi)^{1/2}/Z_3]f^a_\mu(U\partial^\mu\phi_a - \phi_a\partial^\mu U) + (Z_m Z_\phi)^{1/2}M\phi_a\partial^\mu f^a_\mu\\
&-\tfrac{1}{8}(g^2 Z_1{}^2/Z_3{}^2)(f^a_\mu)^2(Z_\phi\phi_b\phi_b + Z_U U^2) + \tfrac{1}{2}gZ_1(Z_m Z_U Z_3{}^{-2})^{1/2}MUf^a_\mu f^\mu_a\,.
\end{aligned}
\tag{17}
$$

The gauge-fixing term L_ξ in (15) is chosen for simple f^a_μ and ϕ^a propagators and gives no f^a_μ-ϕ^a transition propagator in the bare theory. But there will be an f^a_μ-ϕ^a transition propagator of order $L \sim g^2/(4-n)\big|_{n\to 4}$ and complicated f^a_μ and ϕ^a propagators in the renormalized theory. In order to have a convenient gauge for the renormalized theory, we choose

$$
L^R_\xi = -\xi(\partial_\mu f^\mu_a + M\phi_a/\xi)^2/2 \tag{18}
$$

to quantize the theory. We note that, in (17) and (18), g and M are now renormalized parameters and the fields are renormalized.

The renormalized L_{inv} in (17) is now invariant under the following renormalized transformation:

$$
\begin{aligned}
f'_{a\mu} &= f_{a\mu} - g(Z_1 Z_\omega/Z_3)\epsilon^{abc}\omega^b_r f^c_\mu + Z_\omega\partial_\mu\omega^a_r\,,\\
\phi'_a &= \phi_a - \tfrac{1}{2}g(Z_1/Z_3)\epsilon^{abc}\omega^b_r\phi^c Z_\omega\\
&\quad + \tfrac{1}{2}g(Z_1/Z_3)Z_\omega\omega^a_r[Z_U{}^{1/2}U/Z_\phi{}^{1/2}\\
&\qquad + Z_m{}^{1/2}Z_3 2M/(gZ_1 Z_\phi{}^{1/2})]\,,\\
U' &= U - \tfrac{1}{2}g[Z_1 Z_\omega Z_\phi{}^{1/2}/(Z_3 Z_U{}^{1/2})]\omega^a_r\phi^a\,,
\end{aligned}
\tag{19}
$$

where we have rescaled ω^a according to $\omega^a \to Z_\omega\omega^a_r/Z_3{}^{1/2}$ for convenience, i.e., Z_ω will be related to the wave-function renormalization constant Y_3 of the fictitious field D_a in a simple way [see Eq. (32) below]. We note that the renormalized Lagrangian (17) is invariant under (19), which is different from (3), and L^R_ξ in (18) is expressed in terms of renormalized quantities. Thus, we must now derive the renormalized f Lagrangian on the basis of (19) and (18). In analogy with the derivation of Q^{ab} in (8) and (10), we obtain the following renormalized f Lagrangian:

$$
\begin{aligned}
L^R_{ff} = &Z_\omega\partial_\mu D'_a\partial^\mu D_a - Z_\omega M^2 Z_m{}^{1/2}D'_a D_a/(Z_\phi{}^{1/2}\xi)\\
&+g(Z_1 Z_\omega/Z_3)D'_a\partial_\mu(f^\mu_d D_b)\epsilon^{abd}\\
&+\tfrac{1}{2}(gM/\xi)(Z_1 Z_\omega/Z_3)\epsilon^{abd}D'_a\phi_d D_b\\
&-\tfrac{1}{2}(gM/\xi)(Z_1 Z_\omega Z_U{}^{1/2}/Z_3 Z_\phi{}^{1/2})D'_a U D_a\,.
\end{aligned}
\tag{20}
$$

We shall now consider one-loop corrections to the theory in the Feynman gauge, $\xi = 1$ in (18). We employ dimensional regularization,[14] which preserves distorted gauge symmetry, to define the divergent quantities. The divergences due to one-loop corrections have the form of simple poles at $n = 4$. Renormalization amounts to subtraction of the poles with their appropriate residues to render the theory finite. In general, in higher-order corrections, there are divergent quantities which must be canceled by counterterms. A theory which is renormalizable by power counting can be made finite by the addition of a finite number of counterterms.[15] The forms of some counterterms, e.g. the tadpole and the U-mass counterterms, do not appear in the bare Lagrangian (15) and must be included in the renormalized Lagrangian in order to renormalize the theory. The distorted gauge symmetry of the theory severely restricts the forms of these "new" counterterms. The situation is similar to the well-known γ_5 meson-nucleon interaction theory, where one must add a "new" counterterm of quartic meson coupling for renormalization.

The effective renormalized Lagrangian is

$$L^R_{\text{eff}} = L_{\text{inv}} + G(U,\phi_a) + L^R_\xi + L^R_{ff} \equiv L_{\text{inv}}(Z'\text{s}\to 1) + L^R_\xi + L^R_{ff}(Z'\text{s}\to 1) + L_{\text{ct}}, \quad (21)$$

where $Z'\text{s}\to 1$ denotes that all Z's are set equal to unity and the "new" counterterm $G(U,\phi_a)$ takes the form

$$G(U,\phi_a) = \delta((U+2M/g)^2+\phi_a{}^2)^2 + (\delta' - 16\delta)UM^3/g, \quad (22)$$

which is suggested by calculations. The parameters δ and δ' are to be determined later. The counterterm Lagrangian L_{ct} is

$$\begin{aligned}
L_{\text{ct}} = {}& -\tfrac{1}{4}(Z_3-1)(\partial_\mu f^a_\nu - \partial_\nu f^a_\mu)^2 + \tfrac{1}{2}(Z_m-1)M^2 f^a_\mu f^\mu_a \\
& - g(Z_1-1)\epsilon^{abc}(\partial_\mu f^a_\nu) f^b_\mu f^c_\nu - \tfrac{1}{4}g^2(Z_1{}^2/Z_3-1)\epsilon_{abc}\epsilon_{ade} f^b_\mu f^c_\nu f^\mu_d f^\nu_e \\
& + \tfrac{1}{2}(Z_U-1)\partial_\mu U\partial^\mu U + \tfrac{1}{2}(Z_\phi-1)\partial_\mu\phi_a\partial^\mu\phi^a + \tfrac{1}{2}g(Z_1Z_\phi/Z_3-1)\epsilon_{abc}(\partial^\mu\phi_a) f^b_\mu\phi^c \\
& - \tfrac{1}{2}g[Z_1(Z_UZ_\phi)^{1/2}/Z_3-1] f^a_\mu(U\partial^\mu\phi_a - \phi_a\partial^\mu U) - \tfrac{1}{8}g^2(Z_\phi Z_1{}^2/Z_3{}^2-1)\phi_a\phi_a - \tfrac{1}{8}g^2(Z_UZ_1{}^2/Z_3{}^2-1)U^2 \\
& + [(Z_mZ_\phi)^{1/2}-1]M\phi_a\partial^\mu f^a_\mu + \tfrac{1}{2}g[Z_1(Z_mZ_UZ_3{}^{-2})^{1/2}-1]MUf^a_\mu f^\mu_a \\
& + G(U,\phi_a) + (Z_\omega-1)\partial_\mu D'_a\partial^\mu D_a - (Z_\omega Z_m{}^{1/2}Z_\phi{}^{-1/2}-1)M^2 D'_a D_a/\xi \\
& + (Z_1Z_\omega Z_3{}^{-1}-1)g\epsilon^{abd}D'_a\partial_\mu(f^\mu_d D_b) + (Z_1Z_\omega Z_3{}^{-1}-1)(gM/2\xi)\epsilon^{abc}D'_a D_b\phi_c \\
& - (gM/2\xi)(Z_1Z_\omega Z_U{}^{1/2}Z_3{}^{-1}Z_\phi{}^{-1/2}-1)D'_a D_a U\,.
\end{aligned} \quad (23)$$

We shall ignore the tadpole contributions for the moment. Let us consider first the one-particle-irreducible diagrams for the vector-boson self-energy and the relevant counterterm, which are given in Fig. 1. The finite parts will be neglected in our discussions. The sum of contributions due to Fig. 1 is

$$\begin{aligned}
\pi^{ab}_{\mu\nu}(p) &= iM^2Lg_{\mu\nu}\delta_{ab} + i\tfrac{19}{6}L(g_{\mu\nu}p^2 - p_\mu p_\nu)\delta_{ab} + i(Z_m-1)M^2g_{\mu\nu}\delta_{ab} - i(Z_3-1)(g_{\mu\nu}p^2-p_\mu p_\nu)\delta_{ab} \\
&\equiv \pi_1(p^2)g_{\mu\nu}\delta_{ab} + \pi_2(p^2)p_\mu p_\nu\delta_{ab}\,, \quad (24)\\
L &\equiv (g/4\pi)^2(2-n/2)^{-1}.
\end{aligned}$$

We perform conventional renormalizations of mass and wave function so that Z_m and Z_3 eliminate the divergent quantities in the expansion of $\pi_1(p^2)$ about $p^2 = M_{\text{phys}}{}^2 = M^2$. We obtain

$$Z_3 = 1 + 19L/6\,, \quad Z_m = 1 - L\,. \quad (25)$$

The one-loop correction to the Yang-Mills

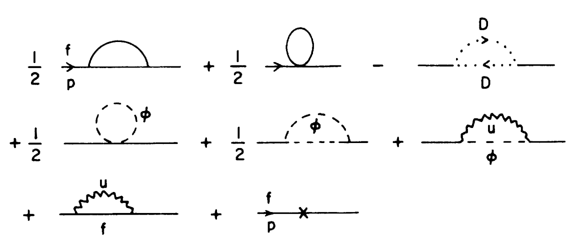

FIG. 1. The one-loop self-energy of f^a_μ. We use a solid line for f, a wavy line for U, a dashed line for ϕ, and a dotted line for D.

three-point function (Fig. 2) is

$$f^{abc}_{\mu\nu\lambda}[-7L/6 + (Z_1-1)], \quad (26)$$

where

$$f^{abc}_{\mu\nu\lambda} \equiv -g\epsilon_{abc}[g_{\mu\nu}(p-q)_\lambda + g_{\nu\lambda}(q-k)_\mu + g_{\lambda\mu}(k-p)_\nu]\,.$$

The constant Z_1 is determined by requiring the only contribution to the physical $f^a_\mu f^b_\nu f^c_\lambda$ coupling constant to be the tree diagram:

$$Z_1 = 1 + 7L/6\,. \quad (27)$$

The one-loop correction to the Yang-Mills four-point function (Fig. 3) is

$$f^{abcd}_{\mu\nu\lambda\sigma}[(L/6) + (2L/3) + (Z_1{}^2/Z_3-1)]\,, \quad (28)$$

where

$$\begin{aligned}
f^{abcd}_{\mu\nu\lambda\sigma} = -ig^2[&\epsilon_{fab}\epsilon_{fcd}(g_{\mu\lambda}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\lambda}) \\
&+ \epsilon_{fac}\epsilon_{fdb}(g_{\mu\sigma}g_{\nu\lambda} - g_{\mu\nu}g_{\lambda\sigma}) \\
&+ \epsilon_{fad}\epsilon_{fbc}(g_{\mu\nu}g_{\lambda\sigma} - g_{\mu\lambda}g_{\nu\sigma})]\,. \quad (29)
\end{aligned}$$

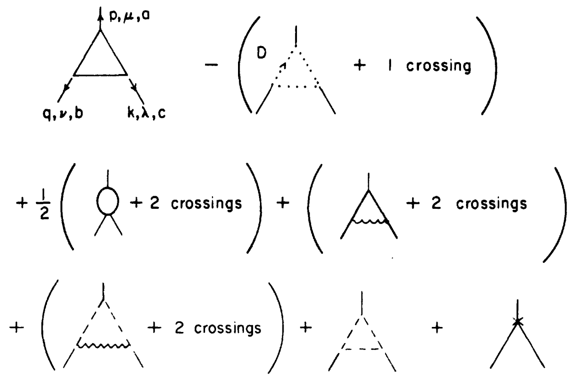

FIG. 2. Corrections to the Yang-Mills three-point function.

With the help of (25) and (27) we see that (28) does vanish as required. This implies that

$$Z_4 = Z_1^2/Z_3\,, \tag{30}$$

where Z_4 is the renormalization constant for $f_\mu^a f_\nu^b f_\lambda^c f_\sigma^d$ and, therefore, the second identity in (14) is confirmed. From the self-energy diagrams for ϕ and U, we get

$$Z_U = Z_\phi = 1 + 3L/2\,,\qquad \delta = -9Lg^2/64\,. \tag{31}$$

In the f Lagrangian (20), we may regard the scalar-fermion D_a and the $f_\mu D'D$ coupling constant as being rescaled according to

$$D^a \to Y_3^{1/2} D^a\,,\quad Y_3 = Z_\omega\,,\qquad g \to g Y_1/(Y_3 Z_3^{1/2})\,, \tag{32}$$

together with (16). From the diagrams in Fig. 4, we obtain the sum of the D_a self-energy and the

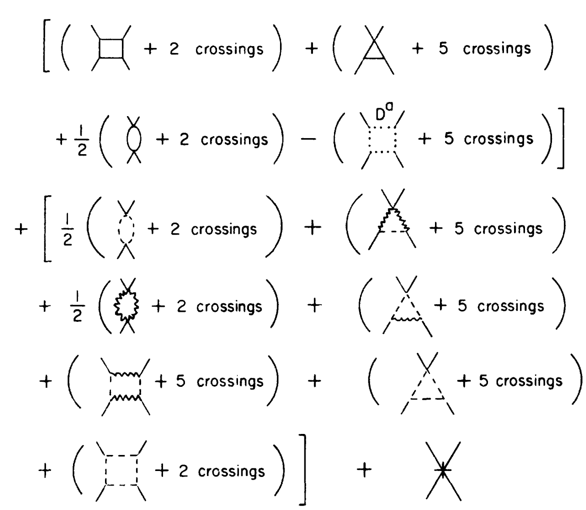

FIG. 3. Corrections to the Yang-Mills four-point function.

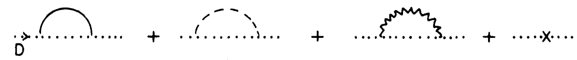

FIG. 4. The self-energy of D_a.

counterterm:

$$\pi_D^{ab}(p^2) = -i(5L/4)\delta^{ab} - iL(p^2 - M^2)\delta^{cb} - i(Y_3 Z_m^{1/2}/Z_\phi^{1/2} - 1)M^2\delta^{ab} + i(Y_3 - 1)p^2\,. \tag{33}$$

The constant Y_3 is determined by

$$\frac{\partial}{\partial p^2}\pi_D^{ab}(p^2)\bigg|_{p^2=M^2} = 0\,, \tag{34}$$

i.e.,

$$Y_3 = 1 + L\,. \tag{35}$$

Note that $\pi_D^{ab}(M^2)$ vanishes automatically,

$$\pi_D^{ab}(M^2) = 0\,,\quad \xi = 1 \tag{36}$$

as required for consistency. Also note that the D_a mass counterterm in (33) contains no new Z constants.

The corrections to the $f_\mu^a D^b D^c$ three-point function and the relevant counterterm in Fig. 5 give

$$-g\epsilon_{abc}k_\mu[L + (y_3 Z_1/Z_3 - 1)]\,, \tag{37}$$

which vanishes as can be seen with the help of (25), (27), and (35). From the bare f Lagrangian L_{ff} in (15), the scalings (32) and (16), and the result (37), we get

$$Y_1 = Y_3 Z_1/Z_3\,, \tag{38}$$

which confirms the first identity in (14). This and (30) ensure that g has been renormalized consistently. Other one-loop vertex corrections are given in the Appendix.

Let us now consider the tadpole diagrams. The contribution of tadpoles is divergent and must be canceled by a counterterm. The tadpole counterterm in (22) is not gauge invariant. This does not matter because the counterterms may not be gauge invariant in general and tadpoles are to be omitted in calculating with the renormalized Lagrangian.[16] The tadpole diagrams and the related counterterm in Fig. 6 give

$$i[+9LM^3/(2g^3) + \delta' M^3/g^3]\,, \tag{39}$$

which must vanish, and so we obtain

$$\delta' = -9L/2\,. \tag{40}$$

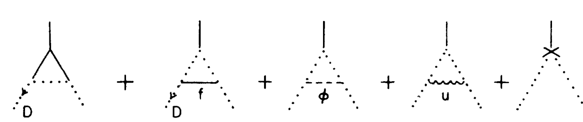

FIG. 5. Corrections to the $f_\mu^a D_b D_c$ vertex.

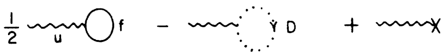

FIG. 6. Tadpole and its counterterm.

V. DISCUSSIONS AND CONCLUSION

The motivation and goal of this investigation have been to construct a renormalizable and asymptotically free theory without involving a physical scalar field eventually. For the purpose at hand we must study the renormalization thoroughly at the one-loop level and the Ward-Takahashi identities. The presence of the physical scalar field $U(x)$ prevents the theory from being asymptotically free because of the U-U scattering in higher orders. Thus, the whole crux of the matter is to properly remove the scalar U from the physical spectrum. This can be accomplished by adding a mass term $-m^2U^2/2$ to the Lagrangian (1) and taking the limit $m\to\infty$ eventually.[17] Of course, this limit must be examined very carefully because it may interfere with loop-momentum integrations. These will be discussed in a separate paper. We do not want to take things for granted because such a limiting procedure in the Lagrangian (1) is mathematically *different* from Lee and Yang's ξ-limiting procedure[13] and has not been explored before. The calculations in this paper are carried out to make sure that everything is all right and that they can be used for further study. Furthermore, our result here substantiates the formal proof of renormalizability and WT identities of distorted gauge-invariant theories.[3,6,18]

What we have done is to carry out an explicit and complete renormalization of a theory in which the vector-boson mass is generated by an intrinsic breakdown of the usual gauge symmetry. Yet the Lagrangian is nevertheless invariant under a distorted gauge transformation. This is the essential feature for the theory to be renormalizable. In spontaneously broken gauge theories, the Lagrangians are also strictly invariant under a distorted gauge transformation.[4,3] Conceptually, the distorted gauge transformation is a generalization of the usual gauge transformation, as one can see from (3). The distorted gauge symmetry is a general concept in the sense that it includes different symmetries as special cases, e.g. spontaneously broken gauge symmetry, intrinsically broken gauge symmetry, and the usual local gauge symmetry.

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APPENDIX

The renormalization constants in Sec. IV are summarized as follows:

$$
\begin{aligned}
&Z_3=1+19L/6\,,\quad Z_m=1-L\,,\\
&Z_1=1+7L/6\,,\quad Z_U=Z_\phi=1+3L/2\,,\\
&Y_3=1+L=Z_\omega\,,\quad Y_1=1-L\,,\\
&\delta=-9Lg^2/64\,,\quad \delta'=-9L/2\,,\\
&L\equiv(g/4\pi)^2(2-n/2)^{-1}
\end{aligned}
\tag{A1}
$$

for $\xi=1$. Because of the distorted gauge symmetry, the renormalizations of various couplings in (21) are related. There are also many types of "new" counterterms. The renormalized Lagrangian is

$$
\begin{aligned}
L^R_{\rm eff}=&-\tfrac{1}{4}(1+19L/6)(\partial_\mu f^a_\nu-\partial_\nu f^a_\mu)^2+\tfrac{1}{2}(1+L)M^2f^a_\mu f^\mu_a\\
&-(1+7L/6)g\epsilon^{abc}(\partial_\mu f^a_\nu)f^\mu_b f^\nu_c-\tfrac{1}{4}(1-5L/6)\epsilon^{abc}\epsilon^{ade}f^b_\mu f^c_\nu f^\mu_d f^\nu_e+\tfrac{1}{2}(1+3L/2)(\partial_\mu U\partial^\mu U+\partial_\mu\phi_a\partial^\mu\phi_a)\\
&-(27L/8)M^2U^2-(9L/8)M^2{\phi_a}^2+\tfrac{1}{2}(1-L/2)g\epsilon_{abc}(\partial^\mu\phi_a)f^b_\mu\phi^c\\
&-\tfrac{1}{2}(1-L/2)gf^a_\mu(U\partial^\mu\phi^a-\phi^a\partial^\mu U)+(1-5L/2)(g^2/8)f^a_\mu f^\mu_a(\phi_b\phi_b+U^2)\\
&+(1-7L/4)gMf^a_\mu f^\mu_a U+(1+L/4)M\phi^a\partial^\mu f^a_\mu-(9L/8)MgU({\phi_a}^2+U^2)-(9L/2)M^3U/g\\
&-(9L/64)(U^2+{\phi_a}^2)^2-(\partial_\mu f^\mu_a+M\phi_a)^2/2+(1+L)\partial^\mu D'_a\partial_\mu D_a-(1-L/4)M^2D'_aD_a\\
&+(1-L)[g\epsilon^{abc}D'_a\partial^\mu(f^c_\mu D^b)+gM\epsilon^{abc}D'_aD_b\phi_c/2-gMD'_aD_aU/2]\,.
\end{aligned}
\tag{A2}
$$

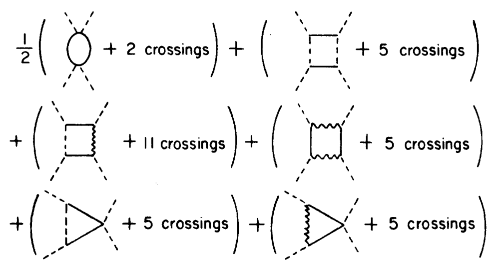

FIG. 7. Diagram for the new counterterm $(\phi_a \phi_a)^2$.

We have checked each term of (A2) to make sure that the theory is renormalized consistently in accordance with the distorted gauge symmetry. The coefficient of the new counterterm $(\phi_a \phi_a)^2$, for example, is obtained by calculating the diagrams in Fig. 7.

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