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$SU(6)_W$ analysis of sums of baryonic partial widths*

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An $SU(6)_W$ analysis is given of some decay amplitudes of the baryon resonances that correspond to the first excited state in the quark model. The squares of the amplitudes of resonances of the same spin, hypercharge, and isotopic spin are added, so that the results are independent of mixing angles. This technique is useful because the quark-model level is almost full. The D-wave and S-wave amplitudes are analyzed separately. In general, the fit is good in the D-wave case, even though decays with very different phase-space factors are compared.

I. INTRODUCTION

The probable existence of charm suggests that a fundamental symmetry of strong interactions is broken SU(4) symmetry. It is likely that the manner in which SU(4) is broken by the very heavy mass associated with charm is related to the manner in which SU(3) symmetry is broken by the moderately heavy mass associated with strangeness. The main purpose of this paper is to study the effects of mass differences, such as the K-π mass difference, on the interaction symmetries of decay amplitudes of odd-parity baryon resonances.

I will consider only resonances coupled directly to either πN or $\bar{K}N$ states and only decay modes of the type PB, where P is a pseudoscalar meson and B is a member of either the nucleon octet (multiplet B_8) or the Δ decouplet (B_{10}). Several authors have used "l-broken $SU(6)_W$" symmetry to analyze the decays of odd-parity baryon resonances.[1-3] This symmetry is the same as $SU(6)_W$ except that interactions of different decay orbital angular momenta are not compared.[4] The situation is complicated by mixing between quark-model states of the same "class" (spin, parity, hyperchange, and isotopic spin). In fact, there are 14 unknown mixing angles involved in the Δ, N, Λ, and Σ states associated with the first excited level in the quark model.

In this paper I sum the coupling constant squared over the resonances in the same class. Some experimental information is lost, but the analysis is simplified because all mixing angles drop out. There are only two theoretical parameters, the S- and D-wave coupling constants. This method of analysis is not new. However, it is more useful now than several years ago because almost all of the strangeness-zero and minus-one resonances predicted at the first quark-model level have been found, so not much is left out of the sums.

We will pay special attention to interaction ratios for decays into πN and $\bar{K}N$, and for decays into πN, πY, and $\pi\Delta$ (where Y is a Λ or Σ), to see if the meson and baryon mass differences lead to significant deviations from symmetry. Decay states including η mesons are more complicated because of the presence of an SU(3)-singlet component in the η. Therefore, η final states are deferred to the following paper.[5]

II. GENERAL PROCEDURE AND RESULTS

In the quark model of baryons the first excited states correspond to the SU(6) representation $\underline{70}$ and to quark internal orbital angular momentum one. The quark-spin and SU(3) multiplicities of the states in the $\underline{70}$ are $^4 8$, $^2 8$, $^2 1$, and $^2 10$. The orbital angular momentum is to be added in all possible ways with the quark spin. This leads to 21 predicted isotopic-spin multiplets of strangeness 0 and -1.

In order to be as objective as possible concerning which resonances are established experimentally, I have considered every odd-parity resonance given a definite spin and parity and a rating of two stars or better in the recent compilation of the Particle Data Group.[6] There are 19 such states with masses less than 1950 MeV; they all correspond to states of the first excited quark-model level, so there are only two $S=0$ or -1 slots unfilled. This is shown in Fig. 1; two stars are listed for the two-star (questionable) resonances, while no stars are listed for the well-established resonances (rated three or four stars in Ref. 6). There is no significance to the ordering of states of the same class.

The success of this classification suggests the use of l-broken $SU(6)_W$ to analyze the S-wave decays and the D-wave decays. The partial width for a particular decay mode is proportional to a barrier penetration factor. It has been found in previous SU(3) and l-broken $SU(6)_W$ analyses of various hadronic decays that the validity of the symmetry is not improved by using a nonzero range in the barrier factor.[1,3,7] Therefore, I assume a zero range. The partial width for the decay of a resonance r into a two-particle state i with definite orbital angular momentum l is related to a coupling constant g_{ri} by

$$\Gamma_{ri} = g_{ri}^2 p_i^{2l+1}/M_r^2 , \tag{1}$$

where M_r is the mass of the resonance r and p_i is the decay momentum.[8] The ratios of the g_{ri} for D-wave decays and the ratios for S-wave decays are to be compared with the predictions of l-broken SU(6)$_W$.

The conventional assumption is made that the wave functions of the resonances ψ_r are related to those of definite quark spins and SU(3) representations ψ_q by an orthogonal transformation, i.e.,

$$\psi_r = \sum_q a_{rq}\psi_q . \tag{2}$$

For example, the three spin-$\frac{3}{2}$ Σ's of Fig. 1 are related by Eq. (2) to states of the quark-spin and SU(3) multiplicities ^{4}8, ^{2}8, and ^{2}10. The coupling constants for decays into any specific final state are also related by Eq. (2). It follows from the orthogonality of the a_{ij} matrix that the sum over a class $\sum_r g_{ri}^2$ is independent of the a_{rq} and so may be compared directly with the prediction of l-broken SU(6)$_W$. This is the procedure followed here, and is the point of departure from the previous analyses.[1-3]

The experimental sums $G_i^2 = \sum_r g_{ri}^2$ for many of the D-wave decays are compared with the predictions in Table I, while S-wave sums are compared in Table II. The manner of determining the

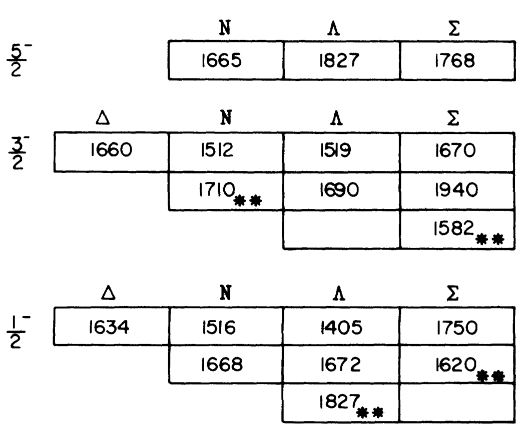

FIG. 1. Correspondence of odd-parity baryon resonances with masses less than 1950 MeV to states of the first excited quark-model level. The numbers in the boxes are masses in MeV.

TABLE I. Experimental and theoretical values of $G_i^2 = \sum_r g_{ri}^2$ for D-wave decays, in GeV^{-2}.

Decay	G_i^2 (exp)	Error in G_i^2 (exp)	G_i^2 (theory)
	Spin-$\frac{5}{2}$ resonances		
$N\to\pi N$	3.51	0.5	1.39
$\Lambda\to\bar{K}N$	0.37	0.21	0
$\Lambda\to\pi\Sigma$	4.6	2.1	4.2
$\Sigma\to\bar{K}N$	5.0	0.4	3.7
$\Sigma\to\pi\Lambda$	1.37	0.19	1.39
$\Sigma\to\pi\Sigma$	0.16	0.06	0.93
$N\to\pi\Delta$	38	14	20
$\Sigma\to\pi\Sigma_{1382}$	11.4	2.7	3.3
	Spin-$\frac{3}{2}$ resonances		
$\Delta\to\pi N$	1.60	0.48	1.17
$N\to\pi N$	8.7	1.5	9.5
$\Lambda\to\bar{K}N$	25.6	3.5	20.9
$\Lambda\to\pi\Sigma$	16.8	2.9	18.2
$\Sigma\to\bar{K}N$	0.8 to 3.7		1.4
$\Sigma\to\pi\Lambda$	0.4 to 4		1.4
$\Sigma\to\pi\Sigma$	7.5	4.3	10.2
$\Delta\to\pi\Delta$	<8		11.6
$N\to\pi\Delta$	68	44	24
	Spin-$\frac{1}{2}$ resonances		
$\Delta\to\pi\Delta$	47	28	23
$N\to\pi\Delta$	<17		23

values and errors of the experimental G_i^2 will be discussed in Sec. III, and the results will be interpreted in Sec. IV. Here I discuss the choice of decay modes and the manner of calculating the theoretical predictions.

The l-broken SU(6)$_W$ determines all the theoretical D-wave coupling ratios and all the S-wave coupling ratios.[9] The only two undertermined theoretical parameters are the overall strengths of the D and S wave couplings. These are chosen to minimize the quantities $\sum_i [G_i^2(\text{exp}) - G_i^2(\text{th})]^2$, where the sum is over the decays with the smaller experimental errors. In the D-wave case this

TABLE II. Experimental and theoretical values of G_i^2 for S-wave decays, in GeV^2.

Decay	G_i^2 (exp)	Error in G_i^2 (exp)	G_i^2(theory)
	Spin-$\frac{3}{2}$ resonances		
$\Delta\to\pi\Delta$	50	27	37
$N\to\pi\Delta$	<120		103
	Spin-$\frac{1}{2}$ resonances		
$\Delta\to\pi N$	24.5	5.7	3.7
$N\to\pi N$	56	16	37
$\Lambda\to\pi\Sigma$	68	12	78

includes all decays for which the error in $G_1{}^2$ is less than 2 GeV^{-2}, and in the S-wave case it includes the $\Delta \to \pi N$, $N \to \pi N$, and $\Lambda \to \pi\Sigma$ decays.

In the case of the spin-$\frac{3}{2}$ Λ's, there are only two observed states included in the experimental sum. It is likely that the coupling of the undiscovered resonance to the $\bar{K}N$ state is small. In fact, it is plausible that the couplings of this resonance to the $\pi\Lambda$ and $\pi\Sigma$ states are also not large, so that their absence does not affect the sums in Tables I and II very much.

The decay modes included in these tables are those for which the experimental numbers are meaningful. Some PB_{10} modes are omitted because the experimental errors are very large. In the spin-$\frac{1}{2}$ case, the $\Lambda \to \bar{K}N$ decays are omitted because the $\Lambda(1405)$ is below $\bar{K}N$ threshold, so that the coupling constant for this decay cannot be computed in the same manner as the others. The spin-$\frac{3}{2}$ Σ's are omitted because only two of the predicted particles are seen, and the errors in their decays are appreciable.

III. DETERMINATION OF THE EXPERIMENTAL NUMBERS

Most of the information used for the experimental values and errors of the masses, widths, and branching fractions was taken from Ref. 6. However, the experiments involved are different so there was no simple formula for determining all the numbers. The values and errors of these three quantities for the resonance decays involved are shown in Table III. The subscript following the resonance symbol is twice the resonance spin, and the number in parentheses is the mass in MeV. Although the fractional errors in the masses are small, these lead in some cases to fractional errors in the D-wave phase-space factors p^5/M^2 that are comparable with the fractional errors in the widths and branching fractions. For example, the percentage errors in p^5/M^2 for the decays $\Delta_3(1660) \to \pi N$ and $\Delta_1(1634) \to \pi\Delta$ are 16 and 38, respectively. (This source of error is omitted in most analyses.) However, the phase error is not the largest fractional error in any of the cases considered. The error in mass is omitted in Table III in cases for which it would have a negligible relative effect on all decays from any resonances.

If average values of the experimental quantities are given in Ref. 6, these have been used. If such averages are not given, my procedure in most cases was to average the measurements that are listed and considered suitable for averaging in Ref. 6. In the cases of the $\pi\Delta$ and $\pi\Sigma^*$ decays of the $\frac{3}{2}^-$ resonances, one must know how much of the partial width corresponds to the S wave and how much to the D wave. The amplitudes of Hey, Litchfield, and Cashmore were used for this purpose.[3]

The fractional error used in the experimental value of $g_{ri}{}^2$ for a particular resonance r is the square root of the sum of the squares of the fractional errors of the width, branching ratio, and phase-space factor (p^5/M^2 for D waves). Fractional errors larger than one are not listed. If the fractional error is one for a $g_{ri}{}^2$ that makes a large contribution to the sum $G_i{}^2 = \sum_r g_{ri}{}^2$, this sum is only given within a certain range in Table I or II. In those cases where the fractional errors in the $g_{ri}{}^2$ are significantly smaller than one, the error listed for $G_i{}^2$ in Table I or II is the square root of the sum of the squares of the errors in the $g_{ri}{}^2$ in the sum.

IV. DISCUSSION OF RESULTS

I consider the D-wave results first. It is seen from Table I that the agreement is generally good, and is better in the case of angular momentum $\frac{3}{2}$, where the results are summed over several resonances, than it is for $j=\frac{5}{2}$, where no sum is taken. SU(3) analyses of the $\frac{5}{2}^-$ octet show that the fit is best with an f/d ratio of about -0.14,[7] rather than $-\frac{1}{3}$ as predicted by $\text{SU}(6)_W$.

The general agreement for PB_8 decays of the $\frac{3}{2}^-$ resonances supports $\text{SU}(6)_W$ in several ways. A large coupling to PB_8 states is predicted for the $^2 1$ multiplet of the SU(6) $\underline{70}$, and a small coupling is predicted for the $^2 10$ multiplet.[9] These predictions are verified by the observed $\Lambda \to \bar{K}N$, $\Lambda \to \pi\Sigma$, and $\Delta \to \pi N$ decays. The quark-spin $\frac{1}{2}$ octet is predicted to be coupled strongly to PB_8 states with a large positive f/d ratio ($\frac{5}{3}$), while the quark-spin $\frac{3}{2}$ octet is predicted coupled weakly, with $f/d = -\frac{1}{3}$.[9] This leads to predicted G^2 values for $N \to \pi N$ and $\Sigma \to \pi\Sigma$ decays larger than those for $\Sigma \to \pi\Lambda$ and $\Sigma \to \bar{K}N$ decays, in agreement with experiment. The results imply that the π-K, N-Λ, and N-Σ mass differences do not make a large effect in the coupling-constant symmetry.

Although the errors are large for the $\pi\Delta$ and $\pi\Sigma_{1382}$ decays, these modes provide significant comparisons. The p^5/M^2 phase-space factors are much smaller for $\pi\Delta$ than for πN decays. For example, the ratio of these factors for the $\pi\Delta$ and πN decays of the $N_{5/2}(1665)$ is about $\frac{1}{10}$. Because of this, large experimental values of G^2 result for several D-wave modes for which the $\pi\Delta$ branching fractions are appreciable. It is seen from Table I that the predicted G^2 are also large for these cases.

We consider next the S-wave decays. It is seen from Table II that there are not enough measured G^2 values for a good test of the theory for S waves.

TABLE III. Values and percentage errors for resonance masses, widths, and branching percentages. All branching percentages not denoted with a subscript S refer to D-wave modes.

Decay mode	% error in mass	Width in MeV	% error in width	Branching percentage	% error in branching percentage
$N_5(1665)\to\pi N$	0.80	155	8	45	9
ηN				<1	
$\pi\Delta$				50	33
$\Lambda_5(1827)\to\bar{K}N$	0.58	95	28	6	50
$\pi\Sigma$				55	36
$\Sigma_5(1768)\to\bar{K}N$	0.33	117	5.8	41	4.4
$\pi\Lambda$				14	12
$\pi\Sigma$				1	35
$\pi\Sigma_{1382}$				10	22
$\Delta_3(1660)\to\pi N$	1.96	200	15	15	20
$\pi\Delta$				<6.7	
$\pi\Delta$				32_S	52
$N_3(1512)\to\pi N$	0.67	125	13	55	8.7
$\pi\Delta$				12.6	53
$\pi\Delta$				11_S	58
$N_3(1710)\to\pi N$	...	200	50	8.9	55
$\pi\Delta$				16.9	100
$\pi\Delta$				32_S	83
$\Lambda_3(1519)\to\bar{K}N$	0.13	15	13.3	46	2.2
$\pi\Sigma$				42	2.4
$\Lambda_3(1690)\to\bar{K}N$	0.59	60	32	25	20
$\pi\Sigma$				27	45
$\Sigma_3(1582)\to\bar{K}N$	...	11	36	3	33
$\pi\Lambda$				33	55
$\pi\Sigma$				3	100
$\Sigma_3(1670)\to\bar{K}N$	0.60	50	27	12	55
$\pi\Lambda$				<20	
$\pi\Sigma$				40	60
$\Sigma_3(1940)\to\bar{K}N$	...	220	32	<20	
$\pi\Lambda$				4	100
$\pi\Sigma$				7	100
$\Delta_1(1634)\to\pi N$	2.01	140	20	35_S	12
$\pi\Delta$				50	40
$N_1(1516)\to\pi N$	0.60	100	33	30_S	23
ηN				65_S	19
$\pi\Delta$				<3	
$N_1(1668)\to\pi N$	0.90	150	33	55_S	14
$\pi\Delta$				4	50
$\Lambda_1(1405)\to\pi\Sigma$	...	40	20	100_S	0
$\Lambda_1(1672\to\pi\Sigma$	...	40	33	35_S	35
$\eta\Sigma$				25_S	40
$\Lambda_1(1827)\to\pi\Sigma$	...	150	53	2_S	100

Nevertheless, the table suggests that the predictions are not as accurate as the D-wave predictions. This is not surprising, since in many quark-model calculations involving exact $SU(6)_W$, the phase-space factor is proportional to p^5 at small p for both the S and D waves.[10] This is a normal p dependence only for the D waves.

The main conclusion is that l-broken $SU(6)_W$ works as well for comparing π and K modes, and comparing N and Δ modes, as it does for comparing different πN, $\pi\Lambda$, and $\pi\Sigma$ modes. Meson and baryon mass difference do not play an important role for the interaction symmetry. A secondary conclusion is that the procedure of summing g^2 over resonances of the same spin, parity, hypercharge, and isotopic spin is very useful.

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[8]In Refs. 1, 3, and 7 the phase-space factor differs from the one used here only that the M_r in the denominator is not squared. This makes little difference to the results.

[9]Convenient tables for calculating ratios of coupling constants from l-broken $SU(6)_W$ are given in Ref. 1.

[10]See, for example, D. Faiman, Nucl. Phys. B32, 573 (1971).