PHYSICAL REVIEW D VOLUME 15, NUMBER 6 15 MARCH 1977

Divergence identity and the dynamical rearrangement of symmetry in the conventional field theory*

Yasushi Takahashi
Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton, Alberta, Canada
(Received 11 May 1976)

The canonical equation of motion is rewritten in an alternative form, and a divergence identity is derived. It is shown that the generalized Ward identity is a special case of the general divergence identity. The use of the divergence identity together with the Haag-Nishijima-Zimmermann construction theorem is illustrated with the Nambu–Jona-Lasinio model. It is proved without any approximation, within the framework of the conventional field theory, that the Goldstone boson carries the chiral gauge transformation.

I. INTRODUCTION

A number of identities holding between various propagators have widely been utilized to obtain kinematical relations among different physical processes. In particular, the power of such identities is demonstrated in connection with the spontaneous breakdown of symmetries in gauge theories.

In order to derive the identities in a general way, the path-integral formalism has been employed.[1,2] However, such a formalism is not yet very familiar to many quantum physicists. The purpose of this paper is to show that the general identity can be derived in conventional field theory, and to demonstrate that it is an alternative form of the fundamental canonical equation.

In Sec. II, we consider a transformation and calculate the variation of the action integral due to the transformation, which is a familiar identity in Lagrangian field theory [Eq. (2.2)].[3] The canonical equation of motion (2.6) is then rewritten in terms of the chronological product of the field quantity and the generating current. It will be shown that the so-called generalized Ward identity[4] can be derived from this form of the canonical equation of motion. Section III is devoted to some remarks on the identity when the symmetry is spontaneously broken; i.e., there exists a Goldstone boson.[1,2]

When the Goldstone boson exists, it is convenient to rewrite the identity in terms of the interpolating field of the Goldstone boson by using the Haag-Nishijima-Zimmermann construction.[5] This form is particularly useful to investigate the relation between the transformation properties of the Heisenberg fields and the asymptotic fields. In Sec. IV, we take the Nambu–Jona-Lasinio model and prove, without adopting any approximation, that the Goldstone field carries the original chiral gauge transformation: In terms of Umezawa's terminology, this means that the symmetry is *dynamically rearranged*.[6]

II. THE DIVERGENCE IDENTITY

Let us consider the transformation

$$\begin{aligned} x_\mu \to x'_\mu &= x_\mu + \delta x_\mu\,, \\ \phi(x) \to \phi'(x') &= \phi(x) + \delta\phi(x)\,. \end{aligned} \tag{2.1}$$

The change of the action integral due to the transformation (2.1) is given by the Noether identity[3]

$$\begin{aligned} \delta I &= \delta\int_{\sigma_1}^{\sigma_2} d^4x\,\mathcal{L}(x) \\ &= -\int_{\sigma_1}^{\sigma_2} d^4x\,\partial_\mu J_\mu(x) = G[\sigma_1] - G[\sigma_2]\,, \end{aligned} \tag{2.2}$$

where

$$J_\mu(x) \equiv -\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi(x)}\,\delta\phi(x) + \text{H.c.} + T_{\mu\nu}(x)\delta x_\nu\,, \tag{2.3}$$

$$T_{\mu\nu}(x) \equiv \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi(x)}\,\partial_\nu\phi(x) + \text{H.c.} - \delta_{\mu\nu}\mathcal{L}(x)\,. \tag{2.4}$$

The generator of the transformation (2.1) is

$$G[\sigma(x)] = \int_\sigma d\sigma_\mu(x) J_\mu(x) \tag{2.5}$$

and satisfies the canonical equation

$$-i\delta^L\phi(x) = [\phi(x), G[\sigma(x)]]\,, \tag{2.6}$$

where

$$\delta^L\phi(x) \equiv \phi'(x) - \phi(x) = \delta\phi(x) - \delta x_\nu\partial_\nu\phi(x)\,. \tag{2.7}$$

We can derive a certain identity as a consequence of the fundamental relations (2.6) and (2.2). Let us rewrite (2.6) as

$$\begin{aligned} -i\delta^L\phi(x) = \phi(x)&\{G[\sigma(x)] - G[-\infty]\} \\ &+ \{G[\infty] - G[\sigma(x)]\}\phi(x) \\ &- G[\infty]\phi(x) + \phi(x)G[-\infty]\,. \end{aligned} \tag{2.8}$$

Upon using the Noether identity (2.2), we obtain

$$-i\delta^L\phi(x)=\int d^4x' T(\phi(x)\partial'_\mu J_\mu(x'))$$
$$-\int d^4x'\partial'_\mu T(\phi(x)J_\mu(x')), \qquad (2.9)$$

where T stands for the chronological-ordering operation.

Since the Lie derivative (2.7) does not upset the chronological ordering,[7] we can generalize (2.9) and obtain a general divergence identity

$$-i\delta^L T(\phi(x_1)\cdots)$$
$$=-i\sum_{i=1} T(\phi(x_1)\cdots\delta^L\phi(x_i)\cdots)$$
$$=\int d^4x' T(\phi(x_1)\cdots\partial'_\mu J_\mu(x'))$$
$$-\int d^4x'\partial'_\mu T(\phi(x_1)\cdots J_\mu(x')). \qquad (2.10)$$

This is obviously an alternative form of the canonical equation (2.6). Note that the region of the integration in (2.10) does not have to be extended over the entire space: As long as all the points $x_1,\ldots$ are contained, we may take a 4-dimensional finite volume. To see this and the fact that (2.9) is the generalized Ward relation,[4] let us calculate

$$\partial'_\mu T(\phi(x)J_\mu(x'))=i\delta(x_0-x'_0)[\phi(x),J_4(x')]$$
$$+T(\phi(x)\partial'_\mu J_\mu(x')), \qquad (2.11)$$

which is the generalized Ward relation. The first term on the right-hand side of (2.11) contains the δ function $\delta(\vec{x}-\vec{x}')$ and its derivatives in the local field theory. Now, on integrating (2.11) over a volume V_4 including the point x, and on using the relation (2.6), we arrive at

$$-i\delta^L\phi(x)=\int_{V_4} d^4x' T(\phi(x)\partial'_\mu J_\mu(x'))$$
$$-\int_{V_4} d^4x'\partial'_\mu T(\phi(x)J_\mu(x')). \qquad (2.12)$$

However, to derive the identity in differential form, the transformation on the constraint variables is required.

It goes without saying that if the transformation (2.1) leaves the action integral invariant, we have

$$\partial_\mu J_\mu(x)=0; \qquad (2.13)$$

therefore the first term in the right-hand side of (2.9) and (2.10) vanishes.

Examples

(*a*) *The space-time translation.*

$$\delta x_\mu=\epsilon_\mu,$$
$$\delta^L\phi(x)=-\epsilon_\nu\partial_\nu\phi(x), \qquad (2.14)$$
$$J_\mu(x)=T_{\mu\nu}(x)\epsilon_\nu.$$

Hence,

$$-i\partial_\nu\phi(x)=\int d^4x'\partial'_\mu T(\phi(x)T_{\mu\nu}(x')). \qquad (2.15)$$

This is an alternative form of the Heisenberg equation of motion.

(*b*) *Chiral gauge transformation.* Take

$$\mathcal{L}(x)=-\bar\psi(x)(\gamma\cdot\partial+M)\psi(x) \qquad (2.16)$$

and consider the transformation

$$\delta x_\mu=0,$$
$$\delta^L\psi(x)=i\alpha\gamma_5\psi(x), \qquad (2.17)$$
$$\delta^L\bar\psi(x)=i\alpha\bar\psi(x)\gamma_5,$$

with the infinitesimal parameter α. Then, we have

$$J_\mu(x)=i\alpha\bar\psi(x)\gamma_\mu\gamma_5\psi(x)\equiv\alpha j_{\mu 5}(x), \qquad (2.18)$$
$$\partial_\mu j_{\mu 5}(x)=2iM\bar\psi(x)\gamma_5\psi(x)\equiv 2M\rho_5(x). \qquad (2.19)$$

We thus obtain from (2.9)

$$\gamma_5\psi(x)=2M\int d^4x' T(\psi(x)\rho_5(x'))$$
$$-\int d^4x'\partial'_\mu T(\psi(x)j_{\mu 5}(x')), \qquad (2.20)$$

and from (2.10)

$$T(\gamma_5\psi(x)\bar\psi(y))+T(\psi(x)\bar\psi(y)\gamma_5)$$
$$=2M\int d^4x' T(\psi(x)\bar\psi(y)\rho_5(x'))$$
$$-\int d^4x'\partial'_\mu T(\psi(x)\bar\psi(y)j_{\mu 5}(x')). \qquad (2.21)$$

III. REMARKS ON SPONTANEOUS BREAKDOWN OF SYMMETRY

For an invariant transformation, we have

$$i\delta^L\phi(x)=\int d^4x'\partial'_\mu T(\phi(x)J_\mu(x')). \qquad (3.1)$$

Let us suppose that the vacuum expectation value of $\delta^L\phi(x)$ is nonzero, i.e.,

$$\langle 0|\delta^L\phi(x)|0\rangle\equiv a\neq 0, \qquad (3.2)$$

then

$$a=-i\int d^4x'\partial'_\mu\langle 0|T(\phi(x)J_\mu(x'))|0\rangle\,. \qquad (3.3)$$

To evaluate the right-hand side, we recall the spectral representation

$$\langle 0|\phi(x)J_\mu(x')|0\rangle = -i\int_0^\infty d\kappa^2\rho_J(\kappa^2)\partial_\mu\Delta^{(+)}(x-x';\kappa^2), \quad (3.4)$$

which gives the current-conservation condition

$$\kappa^2\rho_J(\kappa^2)=0\,. \quad (3.5)$$

On the other hand,

$$\begin{aligned}\partial'_\mu\langle 0|T(\phi(x)J_\mu(x'))|0\rangle &= i\int_0^\infty d\kappa^2\rho_J(\kappa^2)\Box\Delta_c(x-x';\kappa^2)\\ &= i\int_0^\infty d\kappa^2\rho_J(\kappa^2)\kappa^2\Delta_c(x-x';\kappa^2)\\ &\quad + i\int_0^\infty d\kappa^2\rho_J(\kappa^2)\delta^{(4)}(x-x')\,. \end{aligned} \quad (3.6)$$

Substituting (3.6) into (3.3), we have

$$a=\int_0^\infty d\kappa^2\rho_J(\kappa^2)\neq 0 \quad (3.7)$$

on account of (3.5) and (3.2). The relations (3.5) and (3.7) then imply the existence of a massless boson. This is essentially the proof of the Goldstone theorem given by Goldstone, Salam, and Weinberg.[8]

As can be seen in the above proof, the right-hand side of (3.3) contains only the contribution of the massless boson, if it exists. In other words, the operation

$$\int d^4x'\partial'_\mu\langle 0|\cdots|0\rangle \quad (3.8)$$

singles out the massless boson from the quantity

$$T(\phi(x)J_\mu(x'))\,. \quad (3.9)$$

This fact suggests that on calculating (3.3), we can separate out the contribution of the massless boson by putting

$$\begin{aligned}\partial'_\mu\langle 0|T(\phi(x)J_\mu(x'))|0\rangle &= \eta_B\Box'\langle 0|T(\phi(x)B(x'))|0\rangle\\ &\quad + \text{term vanishing when integrated}, \end{aligned} \quad (3.10)$$

where η_B is a constant to be determined by the dynamics of the system, and the operator $B(x)$ is the interpolating field of the massless boson. The choice of $B(x)$ depends on the model. In fact, the relation (3.10) can be justified by the argument by Haag, Nishijima, and Zimmermann (the HNZ construction).[5] Of course, there is no reason why the vector field with zero mass does not contribute to (3.10). We shall economize our argument, however, by considering only the scalar (or pseudoscalar) boson. The D'Alembertian in (3.10) is to separate out the contribution of the massless field.

The HNZ construction makes it possible to generalize (3.10) as

$$\begin{aligned}\partial'_\mu\langle 0|T(\phi(x_1)\cdots J_\mu(x'))|0\rangle &= \eta_B\Box'\langle 0|T(\phi(x_1)\cdots B(x'))|0\rangle\\ &\quad + \text{term vanishing when integrated}\,. \end{aligned} \quad (3.11)$$

The substitution of (3.11) into (2.10) gives

$$i\langle 0|\delta^L T(\phi(x_1)\cdots)|0\rangle = \eta_B\int d^4x'\Box'\langle 0|T(\phi(x_1)\cdots B(x'))|0\rangle\,. \quad (3.12)$$

This form is much more useful for the discussion of spontaneous breakdown of symmetry, as will be seen in the next section.

IV. THE NAMBU-JONA-LASINIO MODEL

In order to see the effective use of the relation (3.12) in the case of spontaneous breakdown of symmetry, let us take the Nambu-Jona-Lasinio model. The Lagrangian

$$\begin{aligned}\mathcal{L}(x) &= -\bar\psi(x)\gamma\cdot\partial\psi(x)\\ &\quad + g\{[\bar\psi(x)\psi(x)]^2+[i\bar\psi(x)\gamma_5\psi(x)]^2\} \end{aligned} \quad (4.1)$$

is invariant under the infinitesimal chiral gauge transformation

$$\begin{aligned}\psi(x)&\to\psi'(x)=(1+i\alpha\gamma_5)\psi(x)\,,\\ \bar\psi(x)&\to\bar\psi'(x)=\bar\psi(x)(1+i\alpha\gamma_5)\,. \end{aligned} \quad (4.2)$$

The generator of (4.2) is

$$G\equiv\int d\sigma_\mu j_{\mu 5}(x)\,, \quad (4.3)$$

with

$$j_{\mu 5}(x)=i\bar\psi(x)\gamma_\mu\gamma_5\psi(x)\,. \quad (4.4)$$

The generalized Ward relation (2.10) then turns out to be

$$\begin{aligned}&T(\gamma_5\psi(x_1)\psi(x_2)\cdots\bar\psi(y_1)\cdots)+T(\psi(x_1)\gamma_5\psi(x_2)\cdots\bar\psi(y_1)\cdots)+\cdots+T(\psi(x_1)\psi(x_2)\cdots\bar\psi(y_1)\gamma_5\cdots)+\cdots\\ &\qquad = -\int d^4x'\partial'_\mu T(\psi(x_1)\psi(x_2)\cdots\bar\psi(y_1)\cdots j_{\mu 5}(x'))\,. \end{aligned} \quad (4.5)$$

Upon taking the vacuum expectation value of (4.5) and using (3.12), we obtain

$$\langle 0|T(\gamma_5\psi(x_1)\psi(x_2)\cdots\bar\psi(y_1)\cdots)|0\rangle+\langle 0|T(\psi(x_1)\gamma_5\psi(x_2)\cdots\bar\psi(y_1)\cdots)|0\rangle+\cdots$$
$$+\langle 0|T(\psi(x_1)\psi(x_2)\cdots\bar\psi(y_1)\gamma_5\cdots)|0\rangle+\cdots$$
$$=-\eta_B\int d^4x'\Box'\langle 0|T(\psi(x_1)\psi(x_2)\cdots\bar\psi(y_1)\cdots B(x'))|0\rangle. \quad (4.6)$$

In this particular model, the interpolating field $B(x)$ may be taken as

$$B(x)=i\bar\psi(x)\gamma_5\psi(x)\equiv\rho_5(x). \quad (4.7)$$

[The normalization of the interpolating field is adjusted by η_B, which can be determined for example by the relation

$$\langle 0|j_{\mu5}(x)|q\rangle=\eta_B\partial_\mu\langle 0|\rho_5(x)|q\rangle,$$

with $q^2=0$.] From the general relation (4.6) it follows that

$$\langle 0|\bar\psi(x)\psi(x)|0\rangle$$
$$=-\tfrac{1}{2}\eta_B\int d^4x'\Box'\langle 0|T(\bar\psi(x)\gamma_5\psi(x)B(x'))|0\rangle$$
$$=\tfrac{1}{2}i\eta_B\int d^4x'\Box'\langle 0|T(B(x)B(x'))|0\rangle. \quad (4.8)$$

If we demand, following Nambu and Jona-Lasinio, that the left-hand side of (4.8) is nonzero, i.e.,

$$\langle 0|\bar\psi(x)\psi(x)|0\rangle=-\frac{1}{2}\frac{M}{g}\neq 0, \quad (4.9)$$

we have

$$-\frac{M}{g}=i\eta_B\int d^4x'\Box'\langle 0|T(B(x)B(x'))|0\rangle. \quad (4.10)$$

This shows the relation between the mass M of the fermion and the residue of the propagator of the Goldstone particle, which are determined, respectively, by dynamics of the model. If we assume

$$\langle 0|T(B(x)B(x'))|0\rangle=\frac{i}{(2\pi)^4}\int d^4k\Delta'_{CB}(k^2)e^{ik\cdot(x-x')}, \quad (4.11)$$

then Eq. (4.10) gives

$$\frac{M}{g}=\eta_B Z_B, \quad (4.12)$$

where

$$Z_B=\lim_{k\to 0}(-k^2)\Delta'_{CB}(k^2). \quad (4.13)$$

The dynamical calculation carried out by Aurilia, Takahashi, and Umezawa[9] indeed confirms the relation (4.12). [According to Ref. 9, the chain approximation gives

$$Z_B{}^{1/2}=\frac{1}{2g}\frac{1}{[I(0)]^{1/2}},$$
$$\eta_B=4gMI(0),$$

with

$$I(0)\equiv\int_{4M^2}^{\Lambda^2}\frac{d\kappa^2}{\kappa^2}\frac{1}{8\pi^2}\left(1-4\frac{M^2}{\kappa^2}\right)^{1/2},$$

where Λ is the cutoff.] A judicious use of the relation (4.6) yields a number of interesting relations.[2]

V. DYNAMICAL REARRANGEMENT

The power of the relation (4.6) is appreciated when we investigate the dynamical rearrangement of the transformation (4.2).[2] Let us denote the asymptotic fields by $\psi^{\text{in}}(x)$ and $B^{\text{in}}(x)$, which satisfy

$$(\gamma\cdot\partial+M)\psi^{\text{in}}(x)=0, \quad (5.1)$$

$$\Box B^{\text{in}}(x)=0, \quad (5.2)$$

and investigate what transformation on $\psi^{\text{in}}(x)$ and $B^{\text{in}}(x)$ will correctly reproduce the original transformation (4.2). For this purpose, we need some relation between the Heisenberg operators and the asymptotic fields. According to the Lehmann-Symanzik-Zimmermann (LSZ) formalism, we have

$$S\psi(x)=\sum_{l,m,n=0}^{\infty}\frac{1}{l!\,m!\,n!}\int:\prod_{i=1}^{l}d^4x_i\bar\eta(x_i)S(x,x_1,\ldots,x_l,y_1,\ldots,y_m,z_1,\ldots,z_n)\prod_{j=1}^{m}\eta(y_j)d^4y_j\prod_{k=1}^{n}J(z_k)d^4z_k:, \quad (5.3)$$

with

$$\bar\eta(x)=-iZ_2{}^{-1/2}\bar\psi^{\text{in}}(x)(\gamma\cdot\partial+M),\quad \eta(x)=-iZ_2{}^{-1/2}(-\gamma\cdot\overleftarrow{\partial}+M)\psi^{\text{in}}(x),\quad J(x)=-iZ_B{}^{-1/2}\overleftarrow{\Box}B^{\text{in}}(x), \quad (5.4)$$

$$\mathcal{S}(x, x_1, \ldots) = \langle 0 | T(\psi(x)\psi(x_1) \cdots \psi(x_l)\bar{\psi}(y_1) \cdots \bar{\psi}(y_m)B(z_1) \cdots B(z_n)) | 0 \rangle , \tag{5.5}$$

where the S matrix is given by

$$S = \sum_{l,m,n=0}^{\infty} \frac{1}{l!m!n!} \int : \prod_{i=1}^{l} d^4x_i \bar{\eta}(x_i) \mathcal{S}(x_1, \ldots, x_l, y_1, \ldots, y_m, z_1, \ldots, z_n) \prod_{j=1}^{m} \eta(y_j) d^4y_j \prod_{k=1}^{n} J(z_k) d^4z_k : , \tag{5.6}$$

with

$$\mathcal{S}(x_1, \ldots, z_n) = \langle 0 | T(\psi(x_1) \cdots \psi(x_l)\bar{\psi}(y_1) \cdots \bar{\psi}(y_m)B(z_1) \cdots B(z_n)) | 0 \rangle . \tag{5.7}$$

We shall first prove that the transformation

$$\psi^{\text{in}}(x) \to \psi^{\text{in}}(x), \quad B^{\text{in}}(x) \to B^{\text{in}}(x) + \alpha b , \tag{5.8}$$

with an arbitrary c number b, leaves the S matrix (5.6) invariant, and then using this property we show that the original transformation

$$\psi(x) \to \psi'(x) = (1 + i\alpha\gamma_5)\psi(x) \tag{5.9}$$

can be induced by the choice

$$b = \eta_B Z_B^{1/2} . \tag{5.10}$$

Thus, the transformation (5.9) is rearranged to (5.8) with (5.10) at the level of asymptotic fields. Note that the Goldstone field $B^{\text{in}}(x)$ carries the entire transformation. In fact, the fundamental function of the Goldstone boson is to recover the symmetry lost by the asymptotic field $\psi^{\text{in}}(x)$.

In order to pursue the above argument, let us first note the kinematical relations

$$\int d^4x \bar{\psi}^{\text{in}}(x)(\gamma \cdot \partial + M)\langle 0 | T(\cdots \gamma_5\psi(x) \cdots) | 0 \rangle = 0 , \tag{5.11}$$

$$\int d^4y \langle 0 | T(\cdots \bar{\psi}(y)\gamma_5 \cdots) | 0 \rangle (-\gamma \cdot \overleftarrow{\partial}_y + M)\psi^{\text{in}}(y) = 0 , \tag{5.12}$$

$$\int d^4z \langle 0 | T(\cdots \bar{\psi}(z)\psi(z) \cdots) | 0 \rangle \overleftrightarrow{\Box}_z B^{\text{in}}(z) = 0 , \tag{5.13}$$

where it is assumed that no scalar massless mode exists. Applying (5.11)–(5.13) to (4.6), we obtain

$$\begin{aligned} &\int : \prod_{i=1}^{l} d^4x_i \bar{\eta}(x_i) \langle 0 | T(\gamma_5\psi(x)\psi(x_1) \cdots B(z_n)) | 0 \rangle \prod_{j=1}^{m} \eta(y_j) d^4y_j \prod_{k=1}^{n} J(z_k) d^4z_k : \\ &\qquad = -\eta_B \int : \prod_{i=1}^{l} d^4x_i \bar{\eta}(x_i) \langle 0 | T(\psi(x)\psi(x_1) \cdots B(z_n)B(x')) | 0 \rangle \overleftrightarrow{\Box}' d^4x' \prod_{j=1}^{m} \eta(y_j) d^4y_j \prod_{k=1}^{n} J(z_k) d^4z_k : . \end{aligned} \tag{5.14}$$

Let us now calculate the change of the S matrix (5.6) induced by (5.8). Since $\mathcal{S}(x_1, \ldots, z_n)$ is c number, the total change is

$$\delta S = \sum_{l,m,n=0}^{\infty} \frac{1}{l!m!n!} \int : \prod_{i=1}^{l} d^4x_i \bar{\eta}(x_i) \mathcal{S}(x_1, \ldots, z_n, x') \prod_{j=1}^{m} \eta(y_j) d^4y_j \prod_{k=1}^{n} J(z_k) d^4 \, \delta J(x') d^4x' : , \tag{5.15}$$

with

$$\mathcal{S}(x_1, \ldots, z_n, x') = \langle 0 | T(\psi(x_1) \cdots B(z_n)B(x')) | 0 \rangle , \tag{5.16}$$

$$\delta J(x') = -iZ_B^{-1/2} \overleftrightarrow{\Box}' \alpha b . \tag{5.17}$$

Using (4.6) and (5.11)–(5.13), we obtain

$$\delta S = 0 . \tag{5.18}$$

Thus, the S matrix is invariant under the transformation (5.8) for an arbitrary b.

Our next task is to show that the choice (5.10) reproduces (5.9). On account of (5.18), we have

$$\begin{aligned} \delta(S\psi(x)) &= S\delta\psi(x) \\ &= \sum_{l,m,n=0}^{\infty} \frac{1}{l!m!n!} \int : \prod_{i=1}^{l} d^4x_i \bar{\eta}(x_i) \mathcal{S}(x, x_1, \ldots, z_n, x') \prod_{j=1}^{m} \eta(y_j) d^4y_j \prod_{k=1}^{n} J(z_k) d^4z_k \delta J(x') d^4x' : , \end{aligned} \tag{5.19}$$

with

$$S(x, x_1, \ldots, z_n, x') = \langle 0 | T(\psi(x)\psi(x_1) \cdots B(z_n)B(x')) | 0 \rangle . \tag{5.20}$$

On account of the relation (5.14), the right-hand side of (5.19) becomes

$$S\delta\psi(x) = i\eta_B^{-1} Z_B^{-1/2} b\alpha S\gamma_5\psi(x) = i\alpha S\gamma_5\psi(x) . \tag{5.21}$$

Hence, the relation

$$\delta\psi(x) = i\alpha\gamma_5\psi(x) \tag{5.22}$$

can be reproduced.

VI. DISCUSSION

We have obtained the divergence identity in operator form by rewriting the fundamental canonical equation at equal time in terms of the chronological product. The vacuum expectation of the identity is nothing but the familiar Ward-Takahashi relation. It should be emphasized that the divergence identity holds true for any transformation which may or may not leave the Lagrangian invariant.

One of the advantages of the derivation given in this paper is that the transformation property of the field operator is explicitly involved in the identity. Hence, we can write down the relevant identity without explicit use of the equal-time canonical commutation relations. In fact, the equal-time canonical commutation relations can be derived from (2.10) if we consider the transformation

$$\phi(x) \rightarrow \phi(x) + f(x) ,$$

with the arbitrary c-number test function $f(x)$.

The derivation of the divergence identity given in this paper is of course purely formal. The underlying assumption is that all the operations are mathematically meaningful. In order to ensure that this is the case, a certain regularization procedure must be employed. The Pauli-Villars regulator method seens to be particularly well suited for this purpose. We can add to the original Lagrangian terms involving regularizing fields with various masses, and derive the identity in exactly the same way. The subtraction procedure for renormalization can be performed with the identity as a guide, as was done by Nishijima.[10]

In applying the identity to the spontaneous-symmetry-breaking process, it is convenient to employ the Haag-Nishijima-Zimmermann construction method to separate out the Goldstone boson. A judicious use of the identity then show without any approximation that the Goldstone boson carries the lost symmetry. Umezawa and his collaborators investigated the role of the Goldstone boson in great detail by using the path-integral technique and the Ward-Takahashi identity, and proved that the fundamental function of the Goldstone boson is to maintain the original symmetry. We have shown in this paper that the proof carried out by Umezawa and others by the path-integral method can be equally performed in the usual canonical formalism.

ACKNOWLEDGMENT

The author thanks Dr. Umezawa and Dr. Matsumoto for valuable discussions.

---

*This work is supported by the National Research Council of Canada.

[1]L. D. Faddeev and V. N. Popov, Phys. Lett. 25B, 29 (1967); B. W. Lee, Phys. Rev. D 5, 823 (1972); B. W. Lee and J. Zinn-Justin, *ibid.* 5, 3121 (1972); 5, 3137 (1972); 8, 4654(E) (1973); 5, 3155 (1972); A. Salam and J. Strathdee, Nuovo Cimento 11A, 397 (1972); G. 't Hooft, Nucl. Phys. B35, 167 (1971).

[2]H. Matsumoto, N. J. Papastamatiou, and H. Umezawa, Nucl. Phys. B68, 236 (1974); B82, 45 (1974); B97, 61 (1975); B97, 90 (1975).

[3]J. Schwinger, Phys. Rev. 82, 914 (1951).

[4]H. S. Green, Proc. Phys. Soc. London A66, 873 (1953); Y. Takahashi, Nuovo Cimento 6, 371 (1957); K. Nishijima, Phys. Rev. 119, 485 (1960); E. S. Fradkin, Zh. Eksp. Teor. Fiz. 29, 258 (1955) [Sov. Phys.-JETP 2, 361 (1956)]; E. Kazes, Nuovo Cimento 13, 1226 (1959).

[5]R. Haag, Phys. Rev. 112, 669 (1958); K. Nishijima, *ibid.* 111, 995 (1958); W. Zimmermann, Nuovo Cimento 10, 597 (1958).

[6]H. Umezawa, Nuovo Cimento 40, 450 (1965). See also Ref. 2.

[7]We have used the naive T product throughout this paper. The obvious advantage of doing so is that the naive-T-product operation commutes with the Lie derivative defined by (2.7). Hence the generalization (2.10) immediately follows. As is well known, it is more convenient to use the T^* product from the viewpoint of relativistic covariance. However, it is not obvious that the T^* operation commutes with the Lie derivative. Our preliminary investigation indicates that if we treat the constraint variables as if they were independent, then the relation (2.10) holds true for the T^* product.

This point will be discussed in our subsequent paper.

[8]J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).
[9]A. Aurilia, Y. Takahashi, and H. Umezawa, Prog. Theor. Phys. 48, 290 (1972).
[10]K. Nishijima, Phys. Rev. 119, 485 (1960); 122, 298 (1961); K. Nishijima and R. Sasaki, Izvestia Vusov, Physica (USSR), December (1974).