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Are particle rest masses variable? Theory and constraints from solar system experiments*

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We consider the possibility that particle rest masses may vary in spacetime. According to arguments originated by Dicke, if this is the case various null experiments indicate that all masses vary in the same way. Their variation relative to the Planck-Wheeler mass defines a universal scalar rest-mass field. We construct the relativistic dynamics for this field based on very general assumptions. In addition, we assume Einstein's equations to be valid in Planck-Wheeler units. A special case of the theory coincides with Dicke's reformulation of Brans-Dicke theory as general relativity with variable rest masses. In the general case the rest-mass field is some power r of a scalar field which obeys an ordinary scalar equation with coupling to the curvature of strength q. The r and q are the only parameters of the theory. Comparison with experiment is facilitated by recasting the theory into units in which rest masses are constant, the Planck-Wheeler mass varies, and the metric satisfies the equations of a small subset of the scalar-tensor theories of gravitation. The results of solar system experiments, usually used to test general relativity, are here used to delimit the acceptable values of r and q. We conclude that if cosmological considerations are not invoked, then the solar-system experiments do *not* rule out the possibility of rest-mass variability. That is, there are theories which agree with all null and solar-system experiments, and yet contradict the strong equivalence principle by allowing rest masses to vary relative to the Planck-Wheeler mass. We show that the field theory of the rest-mass field can be quantized and interpreted in terms of massless scalar quanta which interact very weakly with matter. This explains why they have not turned up in high-energy experiments. In future reports we shall investigate the implications of various cosmological and astrophysical data for the theory of variable rest masses. The ultimate goal is a firm decision on whether rest masses vary or not.

I. INTRODUCTION

The rest mass of a particle is customarily defined in terms of the square of its four-momentum. It is well known that Poincaré invariance guarantees that the rest mass so defined is strictly constant. However, in the presence of gravitation Poincaré invariance is inapplicable, and it is no longer clear that the rest mass cannot vary under such circumstances. Nevertheless, it is generally assumed that elementary-particle rest masses are constant in spacetime. This assumption is part and parcel of the strong equivalence principle which, as stressed by Dicke,[1] is only very partially supported by experiment. Certainly the assumption of the constancy of rest masses has never been tested in strong gravitational fields, or over cosmological time scales. One need only imagine the repercussions that variability of rest masses would have in elementary-particle physics, astrophysics, and cosmology to realize the importance of obtaining a firm answer to the question: "Are elementary-particle rest masses constant in spacetime or not?"

The concept of variable rest masses is not new. It appears, for example, in Dicke's reformulation[2] of Brans-Dicke theory[3] as a theory in which the metric obeys Einstein's equations, but in which rest masses vary in a particular way, in Hoyle and Narlikar's conformally invariant theory of gravitation,[4] and in Malin's cosmological theory of variable rest masses,[5] to mention just a few. Each such theory is based on specific assumptions, and thus cannot provide a general framework for evaluating the hypothesis of the variability of rest masses. What is needed is a general theory of variable rest masses based only on very general assumptions. To construct such a theory and to test it against solar-system experiments are the aims of the present paper.

The outline of this paper is as follows. In Sec. II we follow closely the pioneering analysis of Dicke[1,6,7] to show that the null experiments of Eötvös *et al.*,[8] Dicke *et al.*,[9] Braginsky and Panov,[10] Hughes *et al.*,[11] Sherwin *et al.*,[12] and Drever[13] strongly suggest that rest masses of particles are all proportional to a universal scalar field which we call the rest-mass field. In Sec. III we formulate the dynamics of this field assuming only covariance, that the corresponding field equation is of no higher order than the second, and that the equation includes no constant scale of length. Of the resulting general theory a special case coincides with the reformulation of Brans-Dicke theory[2] mentioned earlier. In the general case the rest-mass field is found to be some power r of a scalar field which obeys a scalar field equation with coupling to the curvature of strength q. In Sec. IV we write down the Einstein gravitational field equations with the rest-mass field contribut-

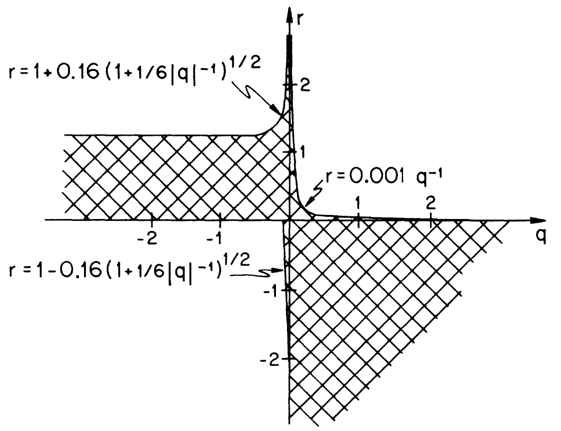

FIG. 1. The theory of variable rest masses is compatible with solar system experiments only for values of r and q in the hatched areas of the rq plane.

ing to the stress-energy tensor. The equations are regarded as valid in Planck-Wheeler units. From Einstein's equations we obtain the equations of motion of matter with variable rest masses. As first noted by Dicke,[2] the transformation to units determined by rods, clocks, and material masses is accomplished by a conformal transformation; in the new units rest masses are constant, but the gravitational "constant" varies. In Sec. V we show that in the new units the gravitational field equations are those of a subset of measure zero of the scalar-tensor theories.[14,15] This representation of the theory is particularly well suited for comparison with experiment. In Sec. VI we compare the theory with the results of measurements of the deflection of electromagnetic waves by the sun, the radar time delay, the Mercury perihelion precession, and the Nordtvedt effect for the moon. We find the acceptable values of r and q to be restricted as shown in Fig. 1. However, without appeal to cosmological considerations one cannot rule out the variability of rest mass from null and solar-system experiments alone. In Sec. VII we show that on the quantum level the theory allows interpretation in terms of positive-energy massless scalar quanta which interact very weakly with matter. This explains why they have not been seen in high energy experiments. In future reports we shall consider the implications for the theory of astrophysical and cosmological data, and of limits on the time dependence of solar-system parameters which might be due to the expansion of the universe. Our ultimate goal is a definite decision on whether rest masses vary or not.

II. THE REST-MASS FIELD

We take as our basic units the Planck-Wheeler mass, length, and time:

$$M_{\rm PW}=(\hbar c/G)^{1/2},\quad L_{\rm PW}=\hbar M_{\rm PW}{}^{-1}c^{-1},\quad T_{\rm PW}=L_{\rm PW}c^{-1}\,. \tag{1}$$

In these Planck-Wheeler units the speed of light c, the quantum of action $\hbar$, and the gravitational coupling constant G are all constant (in fact unity) *by definition*. By contrast the rest mass m of a particle may not be constant, i.e., the ratio $m/M_{\rm PW}$ may vary in spacetime.[6] It is precisely such an eventuality, a violation of the strong equivalence principle, which is of interest here.

In considering the way in which rest masses may vary we follow closely the pioneering analysis of Dicke.[1,6,7] Since we are questioning the constancy of rest mass we may as well go all the way and question also its isotropy. That is, we assume the most general covariant linear relation between the four-momentum of a particle (assumed structureless) and its four-velocity:

$$p_\alpha=\tilde{m}f_{\alpha\beta}dx^\beta/d\tau\,, \tag{2}$$

where $\tilde{m}$ is a constant with dimensions of mass, and $f_{\alpha\beta}$ is a dimensionless "mass tensor" which may vary along the particle's world line. As seen by an observer with four-velocity U^α, the energy of the particle is $-p_\alpha U^\alpha$. Clearly, as seen by the particle itself, this energy must be non-negative. Thus, for any possible particle motion dx^α, $-f_{\alpha\beta}dx^\alpha dx^\beta$ must be non-negative. The parameter and coordinate invariant action which yields the relation (2) by the usual prescription is[7]

$$S=-\tilde{m}c\int\left(-f_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}\right)^{1/2}d\lambda\,, \tag{3}$$

where λ is an arbitrary parameter along the world line, and one defines

$$d\tau=c^{-1}(-f_{\alpha\beta}dx^\alpha dx^\beta)^{1/2}\,. \tag{4}$$

Variation of S with respect to the world line and use of (2) and (4) give

$$\frac{d}{d\tau}p_\alpha=\tfrac{1}{2}\tilde{m}f_{\beta\gamma,\alpha}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau}\,. \tag{5}$$

This is just the sort of equation we should have expected. It says that the momentum of the particle changes because of a "force" which derives from the variation of $f_{\alpha\beta}$; thus the mass tensor is the "potential" in which the motion takes place.

Substituting (2) into (5) and differentiating gives

$$f_{\alpha\beta}\frac{d^2x^\beta}{d\tau^2}+\tfrac{1}{2}(f_{\alpha\beta,\gamma}+f_{\gamma\alpha,\beta}-f_{\beta\gamma,\alpha})\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau}=0\,, \tag{6}$$

which is just the equation for geodesic motion in the metric $f_{\alpha\beta}$. The relation (4) is also the usual one between coordinate and proper-time intervals for the metric $f_{\alpha\beta}$. Thus the particle with variable anisotropic mass behaves as a free particle in a geometry with metric $f_{\alpha\beta}$.[7] The quantum equations for the particle (Dirac equation, for instance) will have to be written in the geometry with metric $f_{\alpha\beta}$ so as to yield (6) as a classical limit. We conclude that all physics of the particle takes place in the geometry represented by $f_{\alpha\beta}$.[7]

A priori there could be a different $f_{\alpha\beta}$ for each type of particle. However, the null experiments of Hughes *et al.*,[11] Sherwin *et al.*,[12] and Drever[13] severely restrict the possibilities. These experiments employing magnetic resonance[11,13] and Mössbauer spectroscopy[12] showed that the behavior of nuclei such as Li^7 and Fe^{57} is unaffected by spatial rotation of the system to high accuracy.[11-13] Likewise, the behavior of orbital electrons, such as those of chlorine, is unaffected by spatial rotation.[11] Rotational effects might have been expected, owing to the anisotropic mass distribution of the earth's galactic environment, if particle rest masses were tensorial. There has been some discussion as to the precise implications of the null results. The best view seems to be that of Dicke[7] that the experiments show any spatial mass anisotropy to be universal, the same for all particles. This means that the spatial parts of the $f_{\alpha\beta}$ for various types of particles can differ at most by rotationally invariant conformal factors. If this property is to be unaffected by a Lorentz boost, then clearly the full $f_{\alpha\beta}$ for different particles must be identical up to scalar conformal factors. The experiments are so accurate that this conclusion may be regarded as rather firm for particles such as the electron, nucleons, pions, and the photon[16] which occur either actually, or virtually, in the atoms and nuclei used, but not for neutrinos,[16] the muon, and hyperons which do not.

Further information is provided by the very precise null experiments of Eötvös *et al.*,[8] Dicke *et al.*,[9] and Braginsky and Panov,[10] which established that the world line of a freely falling composite body is universal: It is independent of composition (gold or aluminum, for example[9]) to great accuracy. This implies that the different kinds of particles involved must follow universal world lines when free.[1] But according to (6) the world line of a particle depends on the gradient of $f_{\alpha\beta}$ so that world lines can be universal only if the conformal factors connecting the different $f_{\alpha\beta}$ are constant. In view of the great accuracy of the experiments this conclusion may be regarded as firm. By absorbing the constant factors into the $\tilde{m}$'s [see (2)] one can make all the $f_{\alpha\beta}$ identical. We must here stress again that this conclusion is established only for particles occurring actually, or virtually, in ordinary matter (electron, nucleons, pions, photon), but not for those which do not occur (neutrinos, muon, hyperons). Nevertheless, we shall assume, in the absence of evidence to the contrary, that the $f_{\alpha\beta}$ is the same for all types of particles.

The universal $f_{\alpha\beta}$ may now be regarded as defining the geometry in which all particles and fields evolve. The $f_{\alpha\beta}$ is the metric with respect to which all classical and quantum equations of matter take their standard forms. Any other metric defined *a priori* is disconnected from physics[6,7] and will be ignored henceforth. If $f_{\alpha\beta}$ is indeed the metric, it would appear from (2) that four-momentum is proportional to four-velocity, i.e., that the rest mass is a constant scalar $\tilde{m}$ for each type of particle. This would seem to dispose of the possibility that rest masses vary. But we have here a subtle point. The $f_{\alpha\beta}dx^\alpha dx^\beta$ is the line element in units defined by particle properties since $f_{\alpha\beta}$ was defined from particle dynamics (see also Sec. VI). In such "particle units" rest masses are constant scalars as shown by our argument. However, Planck-Wheeler units are independent of particle properties, and thus may differ from particle units in a spacetime-dependent way. In particular, the ratio of the particle mass unit to the Planck-Wheeler mass M_{PW} may be a scalar function of the coordinates which we shall call χ. Thus, if in particle units the rest mass of a particle is $\tilde{m}$, in Planck-Wheeler units it is $m=\tilde{m}\chi$. Therefore rest masses may vary in Planck-Wheeler units; if they do, they must all vary in the same way, and the variation defines a universal dimensionless "rest-mass field" χ. Since the particle unit of mass may always be changed, the overall scale of χ is devoid of physical significance; only relative changes in χ from event to event matter physically.

If $f_{\alpha\beta}dx^\alpha dx^\beta$ is the line element in particle units, which is the line element in Planck-Wheeler units? To answer this we must know the relation between the units of length and time in both sets of units. The particle unit of length may be defined as the Compton length of the particle mass unit, while the unit of time can be taken as the unit of length divided by the speed of light. We recall that in Planck-Wheeler units c and $\hbar$ are constants while the particle unit of mass is just χ. Thus the particle units of length and time will both vary as χ^{-1} relative to the corresponding Planck-Wheeler units. It follows that in Planck-Wheeler units the line element is

$$g_{\alpha\beta}dx^\alpha dx^\beta = \chi^{-2} f_{\alpha\beta}dx^\alpha dx^\beta . \qquad (7)$$

For convenience we shall henceforth regard the metric $g_{\alpha\beta}$, rather than the coordinates, as carrying the dimensions of length and time.

III. DYNAMICS OF THE REST-MASS FIELD

If the rest mass of a particle varies, there would seem to be a violation of the conservation of energy. This paradox is explained when one realizes that the particle is not free; it is coupled to an external field—the rest-mass field. The particle's changing mass energy is as natural as the changing energy of a charge moving in an external electromagnetic field. And just as the electromagnetic field carries energy and has dynamics of its own, so should the rest-mass field carry energy and have dynamics of its own. Let us investigate these dynamics. We shall endeavor to avoid specific models; therefore, we only make some general assumptions and leave it to experiment to narrow down the possibilities (Sec. VII).

Our assumptions are the following: (a) We assume that the dynamics of χ are derivable from an action S_χ constructed out of χ and its derivatives, and $g_{\alpha\beta}$ and its derivatives. (b) We assume that S_χ is coordinate independent so as to obtain covariant dynamics. (c) We assume that the dynamical equation for χ is of no higher order than the second, since higher-order equations are known to suffer from causal anomalies related to the initial-value problem. (d) We assume that S_χ contains no constant scale of length. Such a length could only be a Compton length, the universal length L_{PW}, or some new constant of nature. It is inappropriate to introduce a constant Compton length because of the implication that it associates a constant rest mass with the field responsible for the variability of all other rest masses. The scheme is clearly forced. We hesitate to introduce L_{PW}, an intrinsically quantum length, into the dynamics of χ already at the classical level. Moreover, L_{PW} is the characteristic length of quantum gravitation; were we to introduce it in S_χ, we would for self-consistency have to introduce it also into the gravitational dynamics. Since it is not known how this should be done, we exclude L_{PW} altogether. Finally, it is premature to invent a new constant of nature with units of length just for the problem at hand. On these grounds we exclude a constant scale of length from S_χ. It must be admitted that of all our assumptions, (d) is the least firmly grounded, and there may be room for reevaluating it in the future.

In constructing S_χ we shall work in Planck-Wheeler units. The most general action conforming to assumptions (a)–(d) and having the correct dimensions (those of $\hbar c$) is

$$S_\chi = -\tfrac{1}{2}G^{-1}c^4\int[E(\chi)\chi_{,\alpha}\chi^{,\alpha}+F(\chi)R](-g)^{1/2}d^4x\,, \tag{8}$$

where the constants G and c are introduced for dimensional reasons, R is the scalar curvature, and E and F are two arbitrary dimensionless functions of χ. A term such as $H(\chi)\chi_{,\alpha}{}^{;\alpha}$ could also be included; however, it differs from the first term in (8) only by a perfect divergence (which is dynamically inconsequential) and so would be superfluous. In the absence of a constant scale of length, a function of χ alone and a function of $\chi_{,\alpha}\chi^{,\alpha}$ (other than $\chi_{,\alpha}\chi^{,\alpha}$ itself) do not have the appropriate dimensions to be included in (8). Terms involving $R^\alpha{}_{\beta\gamma\delta}$, $R_{\alpha\beta}$, or derivatives of R contracted with derivatives of χ lead to third derivatives of χ in the gravitational equations and also, via the curvature, indirectly in the equation for χ. Thus we rule them out. For similar reasons we may rule out contractions of second- or higher-order derivatives of χ with themselves or lower-order derivatives.

We mentioned in Sec. II that the overall scale of χ is devoid of physical significance. Thus the dynamics of χ should not determine its overall scale. This means that S_χ should be invariant under multiplication of χ by an arbitrary positive constant a. A necessary condition for this to be true is that

$$E(\chi)=A\chi^s\,, \tag{9}$$

$$F(\chi)=B\chi^{s+2}\,, \tag{10}$$

where A, B, and s are arbitrary real constants. If $s=-2$, S_χ is automatically invariant; if $s\neq-2$, S_χ will get multiplied by a constant unless we assume that $A\to Aa^{-(s+2)}$ and $B\to Ba^{-(s+2)}$. Thus for $s\neq-2$, A and B separately have no physical significance; only their ratio has. The inelegant appearance of A and B in S_χ will be remedied presently.

Let us consider first the special case $s=-2$. Then $F=$const and the second term in (8) has the same form as the gravitational action [see (17)]. Thus without loss of generality we may take $B=0$. Defining $\lambda=\chi^{-2}$ and $\omega=-\frac{3}{2}+\frac{1}{8}A$ we have

$$S_\chi=-(\omega+\tfrac{3}{2})G^{-1}c^4\int\lambda^{-2}\lambda_{,\alpha}\lambda^{,\alpha}(-g)^{1/2}d^4x\,. \tag{11}$$

This may be recognized as the action for the scalar field in Dicke's[2] reformulation of Brans-Dicke theory[3] as a theory in which gravitation is governed by Einstein's equations, but in which rest masses vary as $\lambda^{-1/2}$. Since we shall adopt Einstein's equations (Sec. IV), and since $\lambda^{-1/2}=\chi$, we see that the special case $s=-2$ of the theory of variable rest masses is physically equivalent to Brans-Dicke theory. The latest results from solar-system experiments suggest with fair confidence that either $\omega > 60$ or that $\omega < -33$ (see Sec. VI). Thus $A > 492$ or $A < -252$. The large values of $|A|$ required to avoid conflict with experiment make the special case $s = -2$ seem forced; we shall thus not consider it further.

In the general case $s \neq -2$ we define a new real scalar field ψ by

$$\psi^2 = 4G^{-1}c^4|A|(s+2)^{-2}\chi^{s+2}. \qquad (12)$$

We notice that ψ is invariant when χ is rescaled by a positive constant a since $A \to Aa^{-(s+2)}$. Thus ψ may be regarded as the more fundamental field. In terms of it (11) takes the form

$$S_\chi = -\tfrac{1}{2}\int (\psi_{,\alpha}\psi^{,\alpha} + qR\psi^2)(-g)^{1/2}d^4x \qquad (13)$$

for $A > 0$, and with the opposite sign for $A < 0$. In (13) $q = \frac{1}{4}BA^{-1}(s+2)^2$. We notice that q is unaffected by a rescaling of χ since A and B scale in the same way. As we shall see in Sec. VII, it is possible to construct a consistent quantum theory of the χ (or ψ) field with positive-energy quanta only if the sign of S_χ is as given in (13). We must thus assume that nature has chosen $A > 0$. The relation (12) may be written as

$$\chi \propto \psi^r, \qquad (14)$$

where $r = 2(s+2)^{-1}$, and where the proportionality constant depends on the scale chosen for χ. The action (13) and the relation (14) summarize the dynamics of the general theory of variable rest masses. It is surprisingly that the theory has only two free parameters, r and q, despite the very general assumptions used in its construction.

The dynamical equation for χ may be obtained by setting equal to zero the variation of $S_\chi + S_m$ with respect to ψ, where S_m is the action for all matter. We may write

$$S_m = \int \mathcal{L}_m(-g)^{1/2}d^4x, \qquad (15)$$

where the Lagrangian $\mathcal{L}_m$ contains χ in the rest-mass terms. From (13) and (14) it follows that

$$\psi_{,\alpha}{}^{;\alpha} - qR\psi = -r\psi^{-1}\chi\,\partial\mathcal{L}_m/\partial\chi. \qquad (16)$$

This is a scalar equation with coupling to the curvature of strength q and with the rest-mass terms as its source. The equation is unaffected by a rescaling of χ since $\mathcal{L}_m$ contains χ only in combinations such as $\tilde{m}\chi$ which are unaffected by the change in particle units which generates the rescaling. We notice a Machian feature of the theory. According to (16), all rest masses in the universe serve as sources for ψ, which determines χ, which in turn determines the rest masses of particles (their "inertia"). In an empty universe (no sources), ψ would vanish identically. With the choice $r > 0$ χ would also vanish, and all test particles would have to be massless (without "inertia").

IV. EINSTEIN'S GRAVITATIONAL FIELD EQUATIONS

We continue to use Planck-Wheeler units in which c and G are constant. The simplest gravitational action which is coordinate invariant, and which is built out of $g_{\alpha\beta}$ and its derivatives only, is the traditional one from Einstein's general relativity,

$$S_G = c^4(16\pi G)^{-1}\int R(-g)^{1/2}d^4x, \qquad (17)$$

which we adopt. Einstein's equations are obtained by setting the variation of $S_G + S_\chi + S_m$ with respect to $g^{\alpha\beta}$ equal to zero. They are

$$G_{\alpha\beta} = 8\pi Gc^{-4}(\tau_{\alpha\beta} + \psi_{,\alpha}\psi_{,\beta} - \tfrac{1}{2}g_{\alpha\beta}\psi_{,\gamma}\psi^{,\gamma} - q\psi^2{}_{,\alpha;\beta} + qg_{\alpha\beta}\psi^2{}_{,\gamma}{}^{;\gamma} + qG_{\alpha\beta}\psi^2), \qquad (18)$$

where the stress-energy tensor for the ψ field enters on the same footing as that for the matter fields, $\tau_{\alpha\beta}$.

We may use these equations to obtain the equation of motion of matter with variable rest masses. Taking the divergence of (18) we have, after some rearrangement and cancellation,

$$\tau_\alpha{}^\beta{}_{;\beta} = -\psi_{,\alpha}\psi_{,\beta}{}^{;\beta} + q(\psi^{2,\beta}{}_{;\alpha;\beta} - \psi^{2,\beta}{}_{;\beta;\alpha}) - qG_\alpha{}^\beta\psi^2{}_{,\beta}. \qquad (19)$$

By the Ricci identity[17] the term in parentheses is just $R_\alpha{}^\beta\psi^2{}_{,\beta}$ so that

$$\tau_\alpha{}^\beta{}_{;\beta} = -(\psi_{,\beta}{}^{;\beta} - qR\psi)\psi_{,\alpha}. \qquad (20)$$

Substituting from (16) we have

$$\begin{aligned}\tau_\alpha{}^\beta{}_{;\beta} &= r\chi(\partial\mathcal{L}_m/\partial\chi)\psi^{-1}\psi_{,\alpha} \\ &= (\partial\mathcal{L}_m/\partial\chi)\chi_{,\alpha}. \end{aligned} \qquad (21)$$

We see that if rest masses vary, matter is subject to an anomalous force $(\partial\mathcal{L}_m/\partial\chi)\chi_{,\alpha}$ due to the variability. Those fields such as the electromagnetic or neutrino ones which are massless (and hence do not feel the variability) do not contribute to this anomalous force.

V. PHYSICS IN PARTICLE UNITS

The theory of variable rest masses is now complete. It is a theory with two free parameters only, $-\infty < r < \infty$ and $-\infty < q < \infty$. The case $r = 0$ corresponds to constant rest masses ($\chi \propto \psi^0$), while the case $q = 0$, $r = \infty$ ($s = -2$) corresponds to Brans-Dicke theory in Planck-Wheeler units. In principle one could determine the r and q in nature by comparing predictions of the theory as a function of r and q with experiment. However, Planck-Wheeler units are not convenient for this purpose. The reason is that the results of experiments are obtained in particle units such as the atomic time second and the length of a meterstick which are determined by the dynamics of atomic electrons, and the mass of the standard kilogram which is determined by the rest masses of nucleons and electrons. It is thus convenient to recast the theory in particle units. Such a transformation from Planck-Wheeler units to particle units was first considered by Dicke.[2]

As mentioned in Sec. II the relation between rest mass in Planck-Wheeler units m and rest mass in particle units $\tilde{m}$ is $m=\tilde{m}\chi$. In particle units the line element was found to be $f_{\alpha\beta}dx^\alpha dx^\beta$, where $f_{\alpha\beta}$ is dimensionless. It is convenient instead to consider a metric $\tilde{g}_{\alpha\beta}$ differing from $f_{\alpha\beta}$ only in that it, rather than the coordinates, carries the dimensions of length and time. It follows immediately from (7) that

$$\tilde{g}_{\alpha\beta}=\chi^2 g_{\alpha\beta}\,, \tag{22}$$

where $g_{\alpha\beta}$ is the metric in Planck-Wheeler units. Thus we transform the units of the metric by a scale transformation with scale factor χ^2. One can similarly transform the units of any material field (i.e., meson or electron field) by multiplying it by an appropriate power of χ in accordance with its dimensions.

What happens to the fundamental constants under the transformation of units? From (1) we see that $c=L_{\mathrm{PW}}T_{\mathrm{PW}}^{-1}$. But we know (Sec. II) that the Planck-Wheeler length and time both vary as χ with respect to the corresponding particle units. Hence the speed of light is still the constant c in particle units. From (1) it also follows that $\hbar=M_{\mathrm{PW}}L_{\mathrm{PW}}^2T_{\mathrm{PW}}^{-1}$. Since the Planck-Wheeler mass varies as χ^{-1} with respect to the particle mass unit, it follows that in particle units the quantum of action is still the constant $\hbar$. According to (1) $G=L_{\mathrm{PW}}^3T_{\mathrm{PW}}^{-2}M_{\mathrm{PW}}^{-1}$. It follows that in particle units the gravitational coupling "constant" is no longer constant, but varies as χ^2. Thus one can speak of variable rest masses and constant G (Planck-Wheeler units), or alternatively of constant rest masses and a variable gravitational constant (particle units). Of course, the dimensionless coupling constant $Gm^2\hbar^{-1}c^{-1}$ is independent of the system of units used; it varies as χ^2.

Action has the dimensions of $c\hbar$, and since ch is not modified in passing from Planck-Wheeler units to particle units, neither is the action. Thus the action of the theory is still $S_G+S_\chi+S_m$; the dynamical equations in particle units will be obtained by setting to zero the variation of this action with respect to the appropriate variables in particle units. To this end it is convenient to express the action in terms of fields in particle units. In accordance with the discussion in Sec. II, S_m must take the standard form appropriate to the fields present; rest masses in it must be constants $\tilde{m}$, and the metric used must be $\tilde{g}_{\alpha\beta}$. In S_χ and S_G given by (13) and (17) one must replace $g_{\alpha\beta}$ everywhere by $\chi^{-2}\tilde{g}_{\alpha\beta}$ in order to make the dependence on the metric in particle units explicit. It is not necessary to replace ψ in S_χ in terms of that field in particle units since we shall not need the corresponding dynamical equation. By the same token we leave G as it is since we shall never have to vary with respect to it. Thus wherever it appears henceforth, G is still a constant.

When rewriting S_G+S_χ it is useful to use the relation[18]

$$R=\tilde{R}\chi^2+6\chi^{-1}(-g)^{-1/2}[(-g)^{1/2}g^{\alpha\beta}\chi_{,\beta}]_{,\alpha} \tag{23}$$

between the scalar curvatures R and $\tilde{R}$ computed from $g_{\alpha\beta}$ and $\tilde{g}_{\alpha\beta}$, respectively. The calculations, which involve an integration by parts with neglect of surface terms, are tedious. They give

$$S_G+S_\chi=c^4(16\pi G)^{-1}\int \chi^{-2}\{\tilde{R}(1-qf)-\tfrac{1}{4}[(1-12qr+6qr^2)f-6r^2]\tilde{g}^{\alpha\beta}f_{,\alpha}f_{,\beta}f^{-2}\}(-g)^{1/2}d^4x\,, \tag{24}$$

where

$$f=8\pi Gc^{-4}\psi^2\,. \tag{25}$$

We now define a new field ϕ by

$$\phi=\chi^{-2}(1-qf)\,, \tag{26}$$

so that

$$\phi_{,\alpha}=-\chi^{-2}[r+(1-r)qf]\,f_{,\alpha}f^{-1}\,. \tag{27}$$

Eliminating $1-qf$ and $f_{,\alpha}f^{-1}$ from (24) with the help of (26) and (27) gives

$$S_G+S_\chi=c^4(16\pi G)^{-1}\Big[\int \phi\tilde{R}(-g)^{1/2}d^4x - \int \omega\phi^{-1}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}(-g)^{1/2}d^4x\Big]\,, \tag{28}$$

where

$$\omega=-\tfrac{3}{2}-\tfrac{1}{4}[(q-6q^2)f^2-f][r+(1-r)qf]^{-2}\,. \tag{29}$$

The action (28) has the same form as that for the metric and scalar field in the scalar-tensor theories of gravitation[14,15] with vanishing cosmological term. In these theories ω is an arbitrary function of ϕ, the scalar field; the case $\omega=\text{const}$ is the Brans-Dicke theory. In our case ω is a very specific function of ϕ (through f) with only two free parameters. Further, it is clear from (27) that when r and $(1-r)q$ have opposite sign, $\partial\phi/\partial f$ vanishes for some positive f. Thus f is double-valued in ϕ and so is ω. We thus conclude that general relativity with variable masses (Planck-Wheeler units) is physically equivalent to a two-parameter subset of measure zero of the one- and two-valued scalar-tensor theories with constant masses (particle units). This latter representation is more adapted to comparison with experiment.

Before we proceed to confront experiment we wish to mention the conservation law for the theory. The stress-energy tensor in particle units, $\tilde{T}_{\alpha\beta}$, is obtained by functionally differentiating S_m with respect to $\tilde{g}_{\alpha\beta}$.[17] It then follows from coordinate invariance of S_m,[17] or from direct differentiation of the scalar-tensor theory field equations, that

$$\tilde{T}_{\alpha}{}^{\beta}{}_{;\beta}=0\,. \tag{30}$$

Thus in particle units there is no sign of the anomalous force [see (21)] owing to the variability of rest masses. This is reasonable since in particle units rest masses are constant. One consequence of (30) is that free test particles follow geodesics of $\tilde{g}_{\alpha\beta}$. This is in agreement with the conclusions of Sec. II.

VI. CONSTRAINTS FROM SOLAR SYSTEM EXPERIMENTS

To check our theory one has to compare the predictions, for various solar system experiments, of a scalar-tensor theory having an ω given by (29) with the experimental results, to see what values of r and q are acceptable. Predictions of a broad class of gravitational theories for solar-system experiments have been computed in the framework of the parametrized post-Newtonian (PPN) formalism of Will and Nordtvedt,[15,19] and have been expressed in terms of a set of "PPN parameters." Each theory has its own set of such parameters; those for the scalar-tensor theories have been computed by Will[15] for arbitrary $\omega(\phi)$. He assumes that the field ϕ is nearly constant, at a value ϕ_0 determined by the universe as a whole, but suffers a small perturbation ϕ_1 due to the solar system: $\phi=\phi_0+\phi_1$. Likewise, in appropriate coordinates $\tilde{g}_{\alpha\beta}=\eta_{\alpha\beta}+h_{\alpha\beta}$, where $h_{\alpha\beta}$ is the perturbation from the Minkowski metric due to the solar system. Will expands $\omega(\phi)=\omega_0+\omega'\phi_1$, where $\omega_0=\omega(\phi_0)$ and $\omega'=\partial\omega/\partial\phi\,|_{\phi_0}$, and calculates ϕ_1 and $h_{\alpha\beta}$ to post-Newtonian order from the field equations for the theory. The PPN parameters appear as coefficients in the results. Only two of them are nonvanishing:

$$\gamma=1-(2+\omega_0)^{-1} \tag{31}$$

and

$$\beta=1+\omega'(3+2\omega_0)^{-2}(4+2\omega_0)^{-1}. \tag{32}$$

In addition, the local Newtonian gravitational constant that would be measured by a Cavendish balance experiment turns out to be

$$G_N=2G\phi_0{}^{-1}(1+\gamma)^{-1}\,. \tag{33}$$

It must be emphasized that ω_0, ω', and ϕ_0 are to be evaluated asymptotically far from the solar system—they are cosmological boundary values. For comparison with (31)–(33) we note that for general relativity $\gamma=\beta=1$ and $G_N=G$.[15]

For the scalar-tensor theories the first-order gravitational red-shift equals c^{-2} times the difference in Newtonian potential between the absorber and the emitter, *independent* of γ or β.[15] This result is the same as in general relativity, and it has been verified experimentally to an accuracy of 1% by Pound and Rebka, and Pound and Snyder.[20] The experiment clearly says nothing about r and q. However, its results can be interpreted as showing that the universal world lines of freely falling bodies (Eötvös-Dicke-Braginski experiments) are, in fact, geodesics of the metric which measures lengths and times in units of material rods and clocks.[21] Thus the red-shift experiments support the identification of the tensor $f_{\alpha\beta}$ as the metric of spacetime in particle units (Sec. II), and are consistent with the result (30).

The deflection of electromagnetic waves by the sun's gravitational field is nowadays most accurately measured by using signals from extragalactic radio sources. To date, the most precise determination is that of Fomalont and Sramek[22] from which they deduce that $\frac{1}{2}(1+\gamma)=1.015\pm0.011$. A closely related effect is the relativistic time delay for radar signals sent past the sun to a planet or spacecraft, and reflected back by the same route. Using Mercury as a reflector, Shapiro and his group have measured the effect; the latest measurements give[23] $\frac{1}{2}(1+\gamma)=0.993\pm0.014$. Using the Mariner 6 and 7 spacecraft as reflectors Anderson *et al.*[24] deduce that $\frac{1}{2}(1+\gamma)=1.00\pm0.03$.

The perihelion precession of Mercury's orbit can now be determined by radar ranging as well as from optical observations. In view of the low solar oblateness determined by Hill and Stebbins[25] there is no longer reason to believe in a large solar mass quadrupole due to a rapidly rotating core which might be responsible for part of the precession.[26] The latest determinations of the precession by Shapiro's group,[27] reduced under the assumption that the solar quadrupole is exclusively due to uniform rotation, gives $\frac{1}{3}(2+2\gamma-\beta)=1.003\pm0.005$.

According to the scalar-tensor theories, the orbit of the moon should suffer an anomalous "polarization" in the direction of the sun (Nordtvedt effect) due to the breakdown of the equivalence principle for self-gravitating bodies in these theories.[15] This effect has been searched for by laser ranging to the lunar reflectors left by the Apollo astronauts. One analysis of the results, by Shapiro, Counselman, and King,[28] yields $4\beta-\gamma-3=-0.001\pm0.015$, while an independent analysis by Williams *et al.*[29] arrives at $4\beta-\gamma-3=0.00\pm0.03$.

Shapiro, Counselman, and King[28] have combined all available results for the four solar-system experiments, and have arrived at the presently most accurate values $\gamma=1.008\pm0.008$ and $\beta=1.003\pm0.005$. All our considerations will be based on these, except that to be on the safe side we shall not exclude the possibility that γ and β may fall up to 3σ away from their mean values. Thus $0.984<\gamma<1.032$ and $0.988<\beta<1.018$. It follows from (31) that

$$\omega_0<-33 \text{ or } \omega_0>60\,, \tag{34}$$

while from (32) we have

$$-0.012<\omega'(3+2\omega_0)^{-2}(4+2\omega_0)^{-1}<0.018\,. \tag{35}$$

Our task is to determine when the ω given by (29) can satisfy both (34) and (35).

It must be emphasized that the determination of f_0, the asymptotic value of f, can be made only in the context of a cosmological model, and is thus beyond the scope of the present paper. We shall only determine the physical range of f. First, by definition, $f\geq0$. Now consider (33). From everyday experience $G_N>0$ while from first principles[17] $G>0$. Since $\gamma\approx1$ experimentally we see that $\phi_0>0$. It then follows from (26) that $1-qf>0$. Thus, for $q>0$, $0<f<q^{-1}$, while for $q\leq0$, $0<f<\infty$. From the point of view of this paper any f_0 in these ranges is a good choice.

Consider the function $\omega(f)$. For the nontrivial case $r\neq0$ we have $\omega(0)=-\frac{3}{2}$ and $\omega(q^{-1})=0$. Thus condition (34) can be satisfied for a physical f_0 only if (a) $\omega(f)$ becomes unbounded in the physical range, (b) $\omega(f)$ has a maximum larger than 60 or a minimum less than -33 in the physical range, or (c) for $q\leq0$ the limit of ω as $f\to\infty$ exceeds 60 or is less than -33. We see from (29) that for $q\neq0$, $r\neq0$, and $r\neq1$, $\omega(f)$ does become unbounded at $f=rq^{-1}(r-1)^{-1}$. This f is in the physical range only when $q>0$, $r<0$ or when $q<0$, $0<r<1$. For parameters in these regions of the rq plane condition (34) is satisfied for some physical f_0.

The function $\omega(f)$ has a single extremum at $f=rq^{-1}(r+1-12rq)^{-1}$ which is a maximum (minimum) when $q(r+1-12rq)$ is positive (negative). At the extremum $\omega=-\frac{3}{2}-[96rq(rq-\frac{1}{6})]^{-1}$. If the extremum is a maximum, we must have $r>0$ so that it may be situated at $f>0$. The maximum value exceeds 60 for $0<rq<0.001$ or for $-0.001<rq-\frac{1}{6}<0$. Since $r>0$, $q>0$ in both cases. For $0<rq<0.001$ the maximum falls in the physical range $0<f<q^{-1}$. However, for $-0.001<rq-\frac{1}{6}<0$ the maximum falls at $f>q^{-1}$, so we must exclude this case. If the extremum is a minimum, we must have $r<0$ in order that it may fall at $f>0$. The minimum value is less than -33 for $-0.002<rq<0$ or for $0<rq-\frac{1}{6}<0.002$. Since $r<0$, $q>0$ in the first case, but if so the minimum occurs for $f>q^{-1}$, so this case is excluded. In the second case $1-12rq\approx-1$. If indeed $r<0$, then $q>0$ from the condition for a minimum. But this is inconsistent with $rq\approx\frac{1}{6}$, so this case is also excluded. To summarize, for $r>0$ and $0<rq<0.001$ condition (34) is satisfied for some physical f_0.

For $r\neq1$ and $q<0$ ω has the asymptotic value

$$\omega(\infty)=\tfrac{3}{2}[-1+(1+\tfrac{1}{6}|q|^{-1})(1-r)^{-2}].$$

This exceeds 60 for a strip in the rq plane bounded by the curves $r=1\pm0.016(1+\frac{1}{6}|q|^{-1})^{1/2}$ for $q<0$. For values of r and q in this strip condition (34) is satisfied for sufficiently large f_0. In the special cases $r=1$, $q<0$ and $q=0$, $r\neq0$ which have not yet been treated, $\omega\to\infty$ as $f\to\infty$. Thus for such values of r and q condition (34) is also satisfied for sufficiently large f_0. Figure 1 summarized our conclusions. For values of r and q in the hatched areas condition (34) is satisfied for some physical f_0.

We now turn to condition (35). Computing ω' we have

$$\omega'=\tfrac{1}{4}\chi_0^{\,2}[q(1+r-12rq)f_0^{\,2}-rf_0]\times[r+(1-r)qf_0]^{-4}\,. \tag{36}$$

We now need to know the constant of proportionality between χ and ψ^r which is fixed by the choice of particle units. In arriving at formulas (31)–(33) Will[15] chose units for which at the present cosmological era $G_N/G=1$. According to (33) and (26) this fixes the proportionality constant so that at the present time $\chi_0^{\,2}=1-qf_0$ since $\gamma\approx1$. We see that ω' becomes unbounded as $\omega_0^{\,2}$ for $f_0\to rq^{-1}(r-1)^{-1}$. Sufficiently near this point (35) is clearly satisfied as is (34). We know that ω' vanishes at an extremum of ω. Thus if ω has a maximum or a minimum satisfying (34), then sufficiently near this point, both (34) and (35) will be satisfied. Now consider (36). If $q<0$, $r\neq 1$ we see that as $f_0\to\infty$, $\omega'\to 0$. Thus whenever asymptotically ω satisfies (34), (35) will also be satisfied for sufficiently large f_0. If $q=0$, $r\neq 0$ ($r=1$, $q<0$) then ω' diverges asymptotically as f_0 (as f_0^3) while ω_0 diverges as f_0 (as f_0^2). Thus both (34) and (35) are satisfied for sufficiently large f_0. What we have shown is that if the parameters r and q are such that (34) is satisfied for some physical range of f_0, condition (35) is automatically satisfied inside this range.

Thus if we do not appeal to cosmological considerations to determine f_0, all values of r and q in the hatched areas of Fig. 1 are compatible with solar-system experiments to date. It is interesting that were the experiments to improve in accuracy so much as to agree with the canonical predictions of general relativity with constant masses to arbitrary precision, we would still be left with the following cases: $q=0$, any r; $q>0$, $r\leq 0$ and $q<0$, $0\leq r\leq 1$. Thus a decision on the question of rest-mass variability will apparently be possible only after the value of f_0 has been determined within the framework of a cosmological model. This will be the subject of a future report. We shall also consider the influence of the time dependence of f_0 due to the expansion of the universe on the evolution of the solar system, stars, and pulsars. In addition we shall investigate the properties of collapsed objects from the viewpoint of the present theory.

VII. QUANTUM THEORY OF THE REST-MASS FIELD

Since the rest-mass field is a dynamical field, it must ultimately be quantized. Here we consider some consequences of such quantization. It is best to focus on the ψ field since it, unlike the χ field, is free of scaling ambiguity, and since its free-field action (13) has the standard quadratic form. Since field quantization in curved spacetime is not completely understood, we shall confine our remarks to Minkowski spacetime, where the metric is $\eta_{\alpha\beta}$ and the curvature vanishes. To begin with we work in Planck-Wheeler units.

As with any field,[30] we can decompose ψ as $\psi=\psi_C+\psi_Q$, where ψ_C is the classical (vacuum or ground-state) part, and ψ_Q is a Hermitian quantum field operator which may be expanded in creation and annihilation operators. For simplicity we assume that ψ_C is just the constant (cosmological) background field ψ_0. From (18) we infer the free-field stress-energy tensor in the limit of flat spacetime

$$\theta_{\alpha\beta}=\psi_{Q,\alpha}\psi_{Q,\beta}-\tfrac{1}{2}\eta_{\alpha\beta}\psi_{Q,\gamma}\psi_Q{}^{,\gamma} - q(\psi^2{}_{,\alpha,\beta}-\eta_{\alpha\beta}\psi^2{}_{,\gamma}{}^{,\gamma})\,. \tag{37}$$

The total energy is given by $-\int\theta_0^0\,d^3x$. The contribution to θ_0^0 of the terms proportional to q is the divergence $q\psi^2{}_{,i}{}^{,i}$ which integrates out to zero (periodic boundary conditions). The total momentum of the field in the i direction is $-\int\theta_0{}^i d^3x$. The contribution to $\theta_0{}^i$ of the terms proportional to q is $-q\psi^2{}_{,0}{}^{,i}$ which also integrates out to zero with periodic boundary conditions. Hence the ψ field has the same total energy and momentum operators as an ordinary massless scalar field. In exact analogy with that field, ψ_Q allows an interpretation in terms of massless scalar quanta carrying positive energy $\hbar\omega$ and momentum $\hbar\vec{k}$. It is relevant to note that had the constant A introduced in (9) been negative, the sign of S_χ would have been the opposite of the choice in (13), and as a consequence the quanta would have carried negative energy. Hence the choice $A>0$.

Let us now consider the interaction of the ψ field with matter, say, with an electron field Ψ. The mass term in the action of the (Dirac) electron field is $\int m\bar{\Psi}\Psi(-\eta)^{1/2}d^4x$. Since $m=\tilde{m}\chi$ and $\chi=b\psi^r$, where $\tilde{m}$ and b are constants, the interaction action is

$$S_I=b\tilde{m}\psi_0{}^r\int(1+\psi_0{}^{-1}\psi_Q)^r\bar{\Psi}\Psi(-\eta)^{1/2}d^4x. \tag{38}$$

The operator $(1+\psi_0{}^{-1}\psi_Q)^r$ is a polynomial in ψ_Q only if r is a non-negative integer. Thus unless $r=0,1,2,\ldots$ the quantum field theory of the mass field interacting with an electron field is nonpolynomial. However, it is not clear whether the values $r=0,1,2,\ldots$ are to be preferred on this account.

In the classical theory one passes from Planck-Wheeler units to particle units by multiplying the metric by χ^2, and the fields by appropriate powers of χ in accordance with their dimensions. In particle units there is no interaction between χ and the matter (masses constant). As we shall see, the story is different in the quantum theory. The conformal factor to be used in changing units must clearly be a classical quantity; the only obvious candidate is the normal-ordered vacuum expectation value of χ, $\langle\chi\rangle$. Since $\chi=b\psi^r$ we clearly have $\langle\chi\rangle=b\psi_0{}^r$. Denoting by the symbol ~ fields in particle units, we have $(-\eta)^{1/2}=\langle\chi\rangle^{-4}(-\tilde{\eta})^{1/2}$ and $\Psi=\langle\chi\rangle^{3/2}\tilde{\Psi}$ (since $\bar{\Psi}\Psi$ is a volume density). Thus (38) can be written as

$$S_I=\tilde{m}\int(1+r\psi_0{}^{-1}\psi_Q+\cdots)\bar{\tilde{\Psi}}\tilde{\Psi}(-\tilde{\eta})^{1/2}d^4x\,. \tag{39}$$

We see that in particle units there is still a quantum interaction between ψ_Q and Ψ, although classically the interaction is removed by the change in units. In (39) the term

$$m\int\bar{\tilde{\Psi}}\tilde{\Psi}(-\tilde{\eta})^{1/2}d^4x$$

is the standard mass term. To lowest order the interaction is

$$r\tilde{m}\psi_0^{-1}\int \psi_Q \bar{\tilde{\Psi}}\tilde{\Psi}(-\tilde{\eta})^{1/2}d^4x.$$

The strength of this interaction is measured by the dimensionless coupling constant

$$\begin{aligned} g_\chi &= (\tilde{m}r\psi_0^{-1})^2\hbar^{-1}c^3 \\ &= 8\pi r^2 f_0^{-1}(G\tilde{m}^2/\hbar c). \end{aligned} \tag{40}$$

Thus in rough order of magnitude the interaction is as strong as the gravitational one (more precisely, $8\pi r^2 f_0^{-1}$ times as strong). It follows that the probability that a scattered electron emits a ψ_Q quantum is of the same order as the probability that it emits a graviton of the same energy. The extreme weakness of the ψ_Q-matter interaction explains why the massless scalar meson, if it exists, has not turned up in high-energy experiments.

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APPENDIX

We pointed out in Sec. V that in the scalar-tensor representation of the theory, $\omega(\phi)$ can be double-valued. In practice this causes no problem because by referring to f as an independent variable (Sec. VI), one chooses one of the two branches of $\omega(\phi)$ in any given calculation. The only problematical point might be where the two branches meet, i.e., where $\partial\phi/\partial f=0$. By comparing (27) with (29) we see that at such a point $\omega\to\infty$. This implies that $\gamma=\beta=1$ [see (31) and (32)], so that at this point the theory becomes degenerate with general relativity. Could this be an artifact of the scalar-tensor representation due to the meeting of the branches of $\omega(\phi)$?

To show that this is not so we return to the action in particle units (24) which is not in scalar-tensor form. Here there is no double-valuedness. By varying f we obtain the field equation

$$Pf_{,\alpha}{}^{;\alpha}+\tfrac{1}{2}P'f_{,\alpha}f^{,\alpha}+\tilde{R}\partial\phi/\partial f=0, \tag{A1}$$

where

$$P(f)=\tfrac{1}{2}\chi^{-2}[(1-12qr+6qr^2)f^{-1}-6r^2f^{-2}] \tag{A2}$$

and $\partial\phi/\partial f$ may be inferred from (27). There are no contributions to (A1) from S_m because in particle units rest masses are constant. In the neighborhood of $f=f_c\equiv rq^{-1}(r-1)^{-1}$, we write $f=f_c+f'$, where f' is a small spacetime-dependent field. Since $\partial\phi/\partial f=0$ at $f=f_c$, to first order in f' (A1) reduces to

$$P(f_c)(\tilde{g}^{\alpha\beta}f'_{,\beta})_{;\alpha}+\tilde{R}\partial^2\phi/\partial f^2|_{f=f_c}f'=0\,, \tag{A3}$$

Because $\partial\phi/\partial f$ vanishes at f_c, this equation is homogeneous in f'—it is *sourceless everywhere*. Therefore the physical solution must vanish identically. We conclude that if the value of f asymptotically far from the solar system is f_c, then f is constant within the solar system. The action (24) then reduces to that of pure general relativity with $G\chi_c^2(1-qf_c)^{-1}$ playing the role of gravitational constant [compare with (33)]. Thus the fact that for $f_0=f_c$ our theory degenerates to general relativity is *not* an artifact of the double-valuedness of the scalar-tensor representation.

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