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Large-p_T π^0 production in hadronic reactions in terms of the parton model*

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We show that the parton model adequately describes π^0 production for Fermilab pp reactions. In addition, we point out that the model directly predicts π^0 production in $\bar{p}p$ reactions providing extensive tests for experimental verification. We remark that, withstanding these tests, the model may be used to extract π, K inelastic form factors from π^0 production data for $\pi p, Kp$ reactions.

Many recent experimental and theoretical developments indicate the existence of a small number of fundamental constituents, in terms of which hadron spectroscopy can be qualitatively understood.[1] These constituents, partons (or quarks) also play an important role in hadron dynamics. It is speculated[2] that the particles with large transverse momenta are produced via parton-parton interactions. Indeed, on the basis of this model,[3] the large $-p_T$ π^0 production data[4] in pp reactions at CERN ISR energies have been satisfactorily accounted for.[5] In this note, we point out that the model has more predictive power than for just describing pp data: (a) It predicts large $-p_T$ π^0 production cross sections for all high-energy $\bar{p}p$ reactions, free of any adjustable parameter; (b) it accounts for the fact that the ratio of π^0 yields in π^-p and the π^+p reactions at the same energy is unity, independent of p_T; (c) it can qualitatively account for relative π^0 yields in π^-p and pp reactions.

According to the parton-parton hard-collision model, the production cross section of a hadron C in a process $A+B\to C+X$ is determined by three factors: (a) the parton content of hadrons A and B, (b) the probability of a parton's materializing into a cloud of hadrons C, and (c) the parton-parton interaction cross section. The single-particle inclusive invariant cross section for rapidity y and transverse momentum $p_T=(\sqrt{s}/2)x_T$ ($\sqrt{s}$ = c.m. energy), $f(y,p_T^2)\equiv(1/\pi)(d^2\sigma/dy\,dp_T^2)$, is given by[3]

$$f(y,p_T^2)=\frac{C}{\pi}\iint F_1(x_1)F_1(x_2)\frac{g(z)}{z}\times\frac{d\sigma(\omega,t,u)}{dt}\,dx_1dx_2\,. \qquad (1)$$

Using $\cot\theta=(p_T^2+m^2)^{1/2}\sinh y/p_T$, and $\alpha=\theta/2$ (m = mass of the produced particle at c.m. angle θ), the quantities in Eq. (1) are defined as

$$z=\frac{x_T}{2}\left(\frac{\tan\alpha}{x_2}+\frac{\cot\alpha}{x_1}\right),\quad \omega=sx_1x_2\,,$$

$$t=-x_1s\tan\alpha\Big/\left(\frac{\tan\alpha}{x_2}+\frac{\cot\alpha}{x_1}\right),\quad u=-(\omega+t)\,.$$

Here $F_1(x)dx$ is the probability that an incident hadron contains a parton with fraction x of its momentum, $g(z)dz$ is the probability that a parton yields a hadron carrying a fraction z of its momentum, and σ is the parton-parton cross section. The values of x_1 and x_2 are bounded by $x_T\cot\alpha/(2-x_T\tan\alpha)\le x_1\le 1$ and $x_Tx_1\tan\alpha/(2x_1-x_T\cot\alpha)\le x_2\le 1$.

For pp reactions, the functions $F_1(x_1)$ and $F_1(x_2)$ are given by $F_1(x)=A[\nu W_2(x)]/x$; using deep-inelastic lepton-proton scattering data Barger and Phillips[6] have parametrized $\nu W_2(x)$, which represents the quark-antiquark content of the proton. For pion production, we use[7] $g(z)=B(1-z)/z$ as derived from e^-e^+ annihilation data. From the analysis of large $-p_T$ π^0 production data at ISR energies,[4] it is found[3,5] that the quark-quark cross section corresponding to spin-$\frac{1}{2}$ partons with vector-gluon interaction yields the best results. For pointlike particles, the quark-quark (qq) cross section is given by[8]

$$\frac{d\sigma}{dt}(qq)=\frac{2\pi\alpha^2}{\omega^2}\left(\frac{\omega^2+u^2}{t^2}+\frac{\omega^2+t^2}{u^2}+\frac{2\omega^2}{ut}\right) \qquad (2)$$

(α = quark-gluon coupling constant). However, to account for the rapid falloff of the π^0 cross section with p_T (p_T^{-8} dependence), Hwa *et al.*[5] have modified the pointlike quark-quark interaction with a form factor showing the existence of quark structure. This form factor is apparently related[5] to the scaling violation in deep-inelastic lepton-proton process. Following these authors, we use the quark form factor of Chanowitz and Drell,[9] $G(t)=(1-t/\Lambda^2)^{-2}$, $\Lambda^2=50$ GeV^2 to modify the parton-parton cross sections. The constants A, B, and α^2 can be absorbed in the normalizing constant C of Eq. (1), which we have determined by comparing the calculated cross section with the experimental ISR data at $\sqrt{s}=23.5$ GeV. The cross sections for π^0 production in pp reaction at other energies are then given by Eq. (1) in absolute scale.

In Fig. 1, we compare theoretical predictions with π^0 production data for pp reactions, recently measured[10] at Fermilab for $p_L = 100$ and 200 GeV/c. (The model calculations, normalized to the ISR data at $\sqrt{s} = 23.5$ GeV, are also shown in this figure.) Using ISR data, Hwa *et al.*[5] have shown that Eq. (1) indeed reproduce π^0 production data at very large p_T ($p_T \geq 3$ GeV/c). Applying the model to low-energy data (Fig. 1), we find (a) that the model continues to be valid at Fermilab energies and (b) that it can account for the data at p_T of medium range (as low as $p_T = 0.5$ GeV/c).

The striking success of the parton model in pp reaction must, however, be borne out by its agreement with data of other reactions, in particular, for $\bar{p}p$ reaction. For π^0 production in $\bar{p}p$ reaction, the calculation of $f(y, p_T^2)$ becomes rigidly prescribed, although the physical situation is quite different. This difference arises because now the function $F_1(x_1)$ in Eq. (1) represents dominantly the antiquark content of the incident particle ($\bar{p}$). Accordingly, the antiquark-quark cross section $\sigma(\bar{q}q)$ enters in the calculation. This cross section,[8] for pointlike spin-$\frac{1}{2}$ partons with vector-gluon interaction, is given by[11]

$$\frac{d\sigma}{dt}(\bar{q}q) = \frac{2\pi\alpha^2}{\omega^2}\left(\frac{\omega^2 + u^2}{t^2} + \frac{u^2 + t^2}{\omega^2} + \frac{2u^2}{\omega t}\right). \tag{3}$$

Thus, all quantities in Eq. (1) are determined; for high enough energy, even the normalization constant C for $\bar{p}p$ is the same[12] as that for pp reaction.

The ratio $R = \sigma(\bar{p}/\pi^0)/\sigma(p/\pi^0)$ of π^0 production cross section at $\theta = 90°$ for $\bar{p}p$ reaction to that for pp reaction as calculated using this model is shown in Fig. 2(a). The model predicts that, for Fermilab energies, (a) R is equal to the ratio of inelastic $\bar{p}p$ and pp cross sections at low p_T, (b) R is less than unity and p_T-dependent at high p_T, and (c) at a given p_T, R increases as energy increases from Fermilab to ISR range. It is worth noting at this point that, using a statistical model, it has recently been argued[13] that R should be unity and independent of p_T. The parton model differs drastically from such a prediction. We believe that a crucial test in discriminating between these two models lies in the experimental determination of R at large p_T. At present experimental data for high-energy $\bar{p}p$ reactions are not available for a direct comparison with our predictions. However, the bubble-chamb-

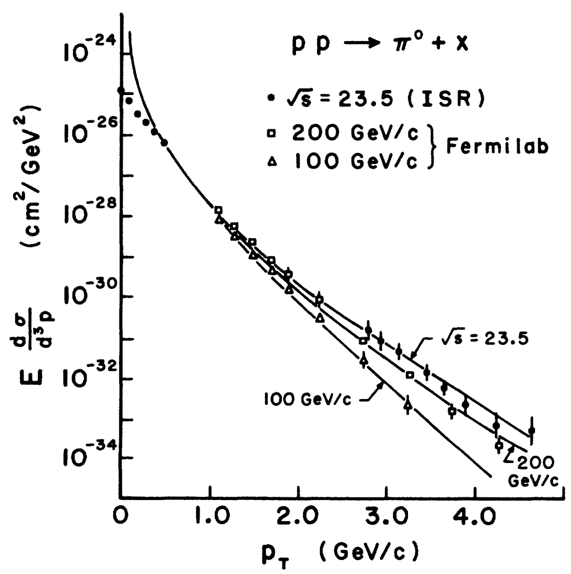

FIG. 1. Inclusive π^0 production cross section at $\theta = 90°$ for pp reaction as a function of p_T. The curves correspond to the model calculation (see text).

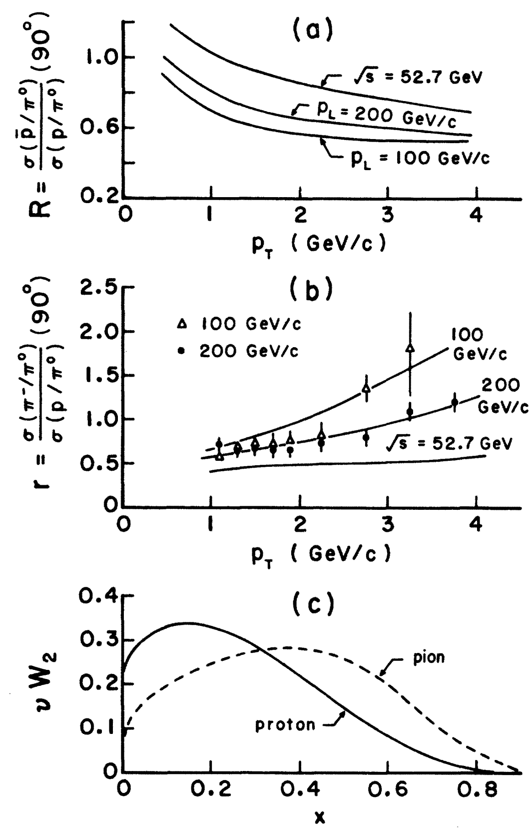

FIG. 2. (a) Ratio of π^0 production at $\theta = 90°$ for $\bar{p}p$ reaction to that for pp reaction as a function of p_T. The calculated curves are obtained from the model (see text). (b) Ratio of π^0 production at $\theta = 90°$ for $\pi^- p$ reaction to that for pp reaction as a function of p_T. The curves correspond to the model calculation (see text). (c) Comparision of the shapes of the inelastic proton and pion form factors (see text).

er π^0 production data[14] at 100 GeV/c yield $R = 1.10 \pm 0.10$, which is consistent with the calculated value of R at low p_T.

We now turn to the discussion of π^0 production in $\pi^- p$ and $\pi^+ p$ reactions. The cross sections for these processes could be explicitly calculated if the inelastic pion form factor $F_1(x_1)$ were known. Even without this knowledge, one can predict that the ratio of π^0 yield in $\pi^- p$ reaction to that in $\pi^+ p$ reaction must be unity, independent of p_T since $F_1(x_1)$ for π^- and π^+ must be the same. This result is indeed supported by Fermilab data.[10]

It is of more interest to compare π^0 yields in $\pi^- p$ and pp reactions. It is experimentally known[10] that the ratio $r = \sigma(\pi^-/\pi^0)/\sigma(p/\pi^0)$ at low p_T is equal to the ratio of the inelastic $\pi^- p$ and pp cross sections and that r increases as p_T increases [Fig. 2(b)]. The increase of r with increasing p_T is expected for simple reasons. If one considers the p as containing three partons and the π as containing two, then the average momentum of a parton in the π should be larger than that in the p. At large p_T one would then expect higher probability for production of a π^0 for πp reaction than for pp reaction.

To put these ideas in quantitative terms, let us assume that the pion and proton form factors are related as $[\nu W_2(x)]_\pi \propto [\nu W_2(x)]_p e^{ax}$, where $a > 0$. The function e^{ax} is introduced to shift the x distribution of partons in the π towards a higher value of x as compared to that in the proton. Using equal amounts of q and $\bar{q}$ contents for the π and the cross sections (2) and (3), we have calculated π^0 yield in $\pi^- p$ reactions for beam momentum, $p_L = 100$ and 200 GeV/c. We have adjusted the parameter a and the normalizing constant C of Eq. (1) in order to reproduce the 200-GeV/c data. The values of r for $p_L = 100$ GeV/c are then calculated without any free parameter. The experimental data and the model calculations for 100- and 200-GeV/c $\pi^- p$ reaction are compared in Fig. 2(b). The model satisfactorily explains the p_T dependence for relative π^0 yields in $\pi^- p$ and pp reactions. In Fig. 2(b), we also show the predictions for r on p_T at ISR energy $\sqrt{s} = 52.7$ GeV.

The curves shown in Fig. 2(b) are obtained with the value $\alpha = 3.0$. In Fig. 2(c), we show the proton form factor $\nu W_2(x)$ and the π form factor, obtained by multiplying the p form factor by e^{3x}. This figure gives a rough idea about the π form factor as compared to that for the p.

In summary, we stress that using the quark-quark interaction derived from inclusive π^0 production in pp reaction, the parton model directly yields the π^0 cross section in $\bar{p}p$ reactions. We predict the p_T and energy dependence of large-p_T π^0 production in $\bar{p}p$ reactions. If these predictions are borne out by experiments, the model will indeed be on a sound footing in describing hadron production dynamics. One may then feel confident enough to rely on this model to extract information about inelastic form factors of unstable particles, such as π, K from π^0 production data in πp, Kp reactions.

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