Extended partially conserved axial-vector current hypothesis and chiral-symmetry breaking*

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An extended partially conserved axial-vector current (PCAC) hypothesis that incorporates a family of heavy bosons in a model-independent way is proposed. This is motivated by the impossibility of accounting for the corrections to Goldberger-Treiman relations, both in $SU(2)\times SU(2)$ and $SU(3)\times SU(3)$, by means of ordinary dynamical mechanisms (many-particle intermediate states). This new hypothesis coupled with an assumption on the strong-coupling constants of the heavy bosons leads to the following results: (i) A universality among the corrections to Goldberger-Treiman relations for $\Delta S=0$ decays, Δ_π, on the one hand and for $\Delta S\neq 0$ decays, Δ_K, on the other. (ii) From this universality there follow two sets of sum rules involving masses and strong and weak coupling constants. These sum rules become identities in the chiral as well as in the SU(3) limit and although a definite check has to await for the advent of accurate hyperon data, there are indications that they might be saturated. (iii) By studying the Dashen-Weinstein sum rules, new sets of sum rules involving only strong coupling constants are predicted as well as an expression for Δ_π/Δ_K in good agreement with present data. (iv) It is found that Δ_π and Δ_K, which are a measure of chiral-symmetry breaking, determine completely the on-mass-shell corrections to soft-meson theorems. Since both Δ_π and Δ_K are known experimentally, a calculation is made of the on-mass-shell amplitudes for $\pi^0\to\gamma\gamma$, $\eta\to\gamma\gamma$, $\eta\to\pi\pi\gamma$, $\gamma\to\pi\pi\pi$, $\gamma\gamma\to\pi\pi\pi$ starting from the zero-mass limits implied by anomalous Ward identities. In particular, it is found that the results for the radiative η decays are in agreement with present experimental data without the need for invoking, η-η' mixing. Finally, the corrections to the soft-pion and soft-kaon theorems on K_{l3} decay are also obtained and it turns out that in the last case the ratio f_K/f_π comes into agreement with present data after the derived chiral-symmetry-breaking correction is performed. In summary the proposed extended PCAC hypothesis links many chiral-symmetry-breaking problems together in a unified fashion.

I. INTRODUCTION

Despite all efforts devoted to it, the problem of the reliability of the PCAC (partially conserved axial-vector current) hypothesis still remains an open question.[1,2]. Clearly, the usefulness of the low-energy or soft-meson theorems depends crucially on the validity of the PCAC hypothesis. Since these low-energy theorems are a consequence of symmetry,[3] there is a link between PCAC and current-algebra and the chiral limit theorems.[2] Thus, a proper understanding of the corrections to PCAC can shed light on the nature of the deviations from chiral symmetry. One can obtain an estimate of these deviations by looking at the corrections to the Goldberger-Treiman relations (GTR).[4] In the case of neutron β decay these corrections are defined by

$$\Delta_{\pi N}=1-\frac{(m_p+m_n)g_A}{\sqrt{2}\,g_{np\pi^+}f_\pi}. \qquad (1)$$

Inserting recent experimental values[5] it turns out that

$$\Delta_{\pi N}=0.06\pm 0.02\,. \qquad (2)$$

Since in the chiral limit $\Delta_{\pi N}=0$, one concludes that $SU(2)\times SU(2)$ is a good symmetry to 6%.[6] However, a problem arises when one attempts to calculate $\Delta_{\pi N}$ theoretically. In fact, as has been shown by Pagels and Zepeda,[7] theoretical estimates fail by one or two orders of magnitude.

Turning to $SU(3)\times SU(3)$, one can derive GTR for strangeness-changing ($\Delta S\neq 0$) hyperon β decays and define corrections in a similar fashion as for $\Delta S=0$ decays. For instance, in the case of Λ β decay one has

$$\Lambda_{K\Lambda}=1-\frac{(m_\Lambda+m_p)g_\Lambda^A}{\sqrt{2}\,g_{\Lambda pK}f_K}. \qquad (3)$$

Experiment gives in this case[8]

$$\Delta_{K\Lambda}=0.30\pm 0.15\,, \qquad (4)$$

while theoretical estimates give[9] $\Delta_{K\Lambda}\simeq 0.051$.

Therefore in both $SU(2)\times SU(2)$ and $SU(3)\times SU(3)$ we are faced with the situation that ordinary dynamical continua[7,9] are unable to account for the corrections to the GTR. One is then led to expect that the corrections to soft-meson theorems (the extrapolations to on-mass-shell mesons) estimated on the basis of ordinary dynamical cuts might also give wrong results. Some examples of this are $\pi^0\to\gamma\gamma$, $\gamma\to\pi\pi\pi$, $\gamma\gamma\to\pi\pi\pi$, $\eta\to\gamma\gamma$, $\eta\to\pi\pi\gamma$, and the soft-pion and soft-kaon theorems in K_{l3} decay. Then, a proper understanding of the dynamical origin of the corrections to the GTR might also lead us to a correct prediction for the on-mass-shell extrapolations of the soft-meson theorems.

It has been suggested[7,10] that if one wishes to preserve a no-subtraction hypothesis in the dispersion relation for the matrix element of the divergence of the axial-vector current, $D(q^2)$, then there might exist a heavy pion and a heavy kaon which could account for the corrections to the GTR. Such a heavy kaon, by the way, seems to have been detected recently at SLAC.[11]

Shortly after this suggestion was made, a specific model was proposed[12] in which an infinite number of pions of the Veneziano type was used to saturate the dispersion relation for $D(q^2)$. The result for $\Delta_{\pi N}$ was 2–3%, which is fairly close to the present experimental value. Later on, Drell[13] constructed a model with a single heavy pion which had the feature that in the chiral-symmetry limit the axial-vector current was not conserved. The corrections to PCAC due to such a heavy pion were also discussed later by other authors.[14]

In the present paper we want to elaborate further on the idea of modifying the standard PCAC hypothesis in order to accommodate a family of heavy bosons (π', K', η'). We shall work in the framework of the so-called strong PCAC, i.e., we shall preserve axial-vector current conservation in the chiral-symmetry limit, and our heavy bosons shall not become Goldstone bosons in that limit. If there are one or more boson daughters in the game, it becomes difficult to accept that they might turn out to be Goldstone bosons because this will require a larger symmetry group with a larger breaking.

Our generalization of the PCAC hypothesis, which we name extended PCAC (EPCAC), is defined by[15]

$$\partial^\mu A_\mu^+ = \sum_{n=0}^{N} f_{\pi_n} \mu_{\pi_n}^2 \phi_{\pi_n}^+ . \tag{5}$$

For the purposes of the present paper we do not need to commit ourselves to any particular model for the mass spectrum or the decay rates. Therefore, we shall leave N $(N \geq 1)$, $\mu_{\pi_n}^2$, $\phi_{\mu_n}^+$, and f_{π_n} unspecified. However, the Goldstone mechanism we just mentioned poses some restrictions on f_{π_n}, as we immediately discuss.

Identifying the axial-vector current with the axial-vector current in the weak interactions, its coupling to pions is defined by

$$\langle 0 | A_\mu^\pm(0) | \pi_n^\mp(q) \rangle = i f_{\pi_n} q_\mu . \tag{6}$$

Taking the divergence in Eq. (6) we have that in the chiral limit

$$f_{\pi_n} \mu_{\pi_n}^2 = 0 . \tag{7}$$

Since we wish to prevent the heavy pions $(n \geq 1)$ from becoming Goldstone bosons, f_{π_n} has to have the following form:

$$f_{\pi_n} = \frac{\mu_\pi^2}{\mu_{\pi_n}^2} \tilde{f}_{\pi_n} , \tag{8}$$

where μ_π is the mass of the ground-state pion and $\tilde{f}_{\pi_n}$ is a certain (unknown) function of n.

When taking matrix elements of Eq. (5) between hadronic states A and B (we shall not be concerned here with composite systems such as nuclei) we have to specify the coupling constants $g_{\pi_n AB}$. Since the heavy pions do not belong to the same multiplet of the ground-state pion, there is no *a priori* relation for the coupling constants. However, we know that even within a multiplet the couplings are related by Clebsch-Gordan coefficients and, therefore, are all of the same order of magnitude. This is still true for all measured strong-coupling constants of hadrons whether they belong to the same multiplet or not. But since we know nothing about the couplings of the heavy bosons, it is at the level of a simplifying assumption that we postulate them to be approximately constant, i.e.,

$$g_{\pi_n AB} \simeq g_{\pi AB} \ (n = 0, 1, \ldots) . \tag{9}$$

This assumption might look too drastic and with almost no justification. However, in Sec. II we present alternative and more plausible arguments which lead to the same results as Eq. (9).

Equations (5) and (9) (and their equivalents for K and η) are then our working hypothesis of EPCAC. The main motivation is the need for an additional dynamical origin of the corrections to the GTR. It is obvious, though, that with such general hypothesis it will not be possible to produce a definite number for those corrections. However, it is known that within certain models[12] the family of heavy pions do give the right answer for $\Delta_{\pi N}$. Therefore, we take that EPCAC when complemented by additional assumptions provides the correct answer for the corrections to the GTR. Granting this, Eqs. (5) and (9), despite their quite general nature, are enough to allow us to derive a series of sum rules and relations among different processes. One of the main results is that the corrections to the GTR determine completely the extrapolations to on-mass-shell bosons of the soft-meson theorems. Therefore, we obtain a unified picture in which the corrections to the GTR (which in turn measure the deviations from chiral-symmetry limits) play the central role. Since these corrections are known experimentally we are able to give a set of numerical predictions in a number of cases.

In Sec. II we discuss the corrections to the GTR for $\Delta S = 0$ decays, Δ_π, and obtain a sum rule involving masses and strong and weak coupling constants. In Sec. III we derive a relation between $\pi^0 \to \gamma\gamma$ and Δ_π. In Sec. IV we discuss the corrections to the GTR for $\Delta S \neq 0$ decays, Δ_K, and find another sum rule analogous to the one obtained for $\Delta S = 0$ decays. Section V is devoted to the Dashen-Weinstein sum rules[16] and we derive there new sum rules and a prediction for Δ_π/Δ_K. In Sec. VI we obtain the corrections to the soft-pion and soft-kaon theorems for K_{l3} in terms of Δ_π and Δ_K, respectively. In Sec. VII we discuss $\eta \to \gamma\gamma$ and $\eta \to \pi\pi\gamma$ decays and give numerical predictions which agree with experiment without the need of introducing singlet-octet mixing at all. In Sec. VIII we consider $\gamma \to \pi\pi\pi$ and $\gamma\gamma \to \pi\pi\pi$ processes, and finally Sec. IX is devoted to the discussion and conclusions.

II. GOLDBERGER-TREIMAN RELATIONS FOR $\Delta S = 0$ DECAYS

Let us consider two different $\Delta S = 0$ β decays: $B \to B' + l\bar{l}$ and $C \to C' + l\bar{l}$. The matrix elements of the axial-vector current are defined as

$$\langle B'(P_f)|A_\mu^+(0)|B(p_i)\rangle = \bar{u}(P_f)\tau^+[\gamma_5\gamma_\mu g_B^A(q^2) + \gamma_5 q_\mu h_B^A(q^2)]u(p_i)\,, \quad q = P_f - P_i\,. \tag{10}$$

and similarly for the other decay. Taking the divergence in Eq. (10) we have

$$\langle B'(P_f)|i\partial^\mu A_\mu^+(0)|B(P_i)\rangle = \bar{u}(P_f)\gamma_5\tau^+ D_B(q^2)u(P_i)\,, \tag{11}$$

where

$$D_B(q^2) = (m_B + m_{B'})g_B^A(q^2) + q^2 h_B^A(q^2)\,. \tag{12}$$

Substituting Eqs. (5) and (9) in Eq. (11) we find

$$D_B(q^2) = \sqrt{2}\, g_{B'B\pi^+} \sum_{n=0}^{N} \frac{f_{\pi_n}\mu_{\pi_n}^2}{\mu_{\pi_n}^2 - q^2} \tag{13}$$

and equivalently

$$D_C(q^2) = \sqrt{2}\, g_{C'C\pi^+} \sum_{n=0}^{N} \frac{f_{\pi_n}\mu_{\pi_n}^2}{\mu_{\pi_n}^2 - q^2}\,, \tag{14}$$

where $\sqrt{2}\, g_{B'B\pi^+}$ and $\sqrt{2}\, g_{C'C\pi^+}$ are the strong coupling constants, and we have ignored the normal continuum because it is negligible.[7] The first term of the sum in Eqs. (13) and (14) is clearly the pion pole and, therefore, the result of standard PCAC. If we consider just this term and take $q^2 = 0$ we obtain the GTR, i.e.,

$$\sqrt{2}\, g_{B'B\pi^+} f_\pi = (m_{B'} + m_B) g_B^A \tag{15}$$

and

$$\sqrt{2}\, g_{C'C\pi^+} f_\pi = (m_{C'} + m_C) g_C^A\,. \tag{16}$$

We stress that in the chiral-symmetry limit the contributions of the heavy pions ($n \geq 1$) to Eqs. (13) and (14) vanish, so that the GTR are a result of symmetry and not of pion pole dominance.[2] Since SU(2) × SU(2) is not exact we expect these GTR to show deviations, and therefore we define the corrections as

$$\Delta_{\pi B} = 1 - \frac{(m_{B'} + m_B)g_B^A}{\sqrt{2}\, g_{B'B\pi^+} f_\pi}\,, \tag{17}$$

and

$$\Delta_{\pi C} = 1 - \frac{(m_{C'} + m_C)g_C^A}{\sqrt{2}\, g_{C'C\pi^+} f_\pi}\,. \tag{18}$$

Using Eqs. (12), (13), and (14) at $q^2 = 0$, we find

$$\Delta_{\pi B} = 1 - \frac{1}{f_\pi}\sum_{n=0}^{N} f_{\pi_n} = -\frac{1}{f_\pi}\sum_{n=1}^{N} f_{\pi_n} \tag{19}$$

and

$$\Delta_{\pi C} = 1 - \frac{1}{f_\pi}\sum_{n=0}^{N} f_{\pi_n} = -\frac{1}{f_\pi}\sum_{n=1}^{N} f_{\pi_n}\,, \tag{20}$$

Hence,

$$\Delta_{\pi B} = \Delta_{\pi C} \equiv \Delta_\pi\,. \tag{21}$$

In other words, EPCAC as defined by Eqs. (5) and (9), predicts a universality among all the corrections to GTR for $\Delta S = 0$ decays. Equation (21) leads immediately to the following sum rule:

$$\frac{m_{B'} + m_B}{m_{C'} + m_C} = \frac{g_C^A}{g_B^A}\frac{g_{B'B\pi^+}}{g_{C'C\pi^+}}\,. \tag{22}$$

We remark that in the chiral-symmetry limit $\Delta_{\pi B}$ and $\Delta_{\pi C}$ strictly vanish, as they do if we substitute Eq. (8) in Eqs. (19) and (20) and take the limit $\mu_\pi^2 \to 0$. Therefore, Eq. (21) as well as Eq. (22) become trivial identities in this limit, although away from it, i.e., in the real world, they acquire the status of meaningful sum rules. Another interesting feature of the sum rule Eq. (22) is that it becomes an identity in the SU(3) limit (if all four particles belong to the same multiplet).

It must be pointed out that Eq. (22) is a well-known result in the chiral-symmetry limit. The new point here is that even when that symmetry is broken, the sum rule remains unchanged. Therefore, it becomes useful to study the interrelation between chiral- and SU(3)-symmetry-breaking effects.

As an example we consider $n \to p + l\bar{l}$ and $\Sigma^- \to \Lambda + l\bar{l}$ and obtain from Eq. (22) the following:

$$\frac{m_p + m_n}{m_\Lambda + m_{\Sigma^-}} = \frac{g_\Sigma^A}{g_A}\frac{g_{np\pi^+}}{g_{\Sigma^-\Lambda\pi^+}}\,. \tag{23}$$

This relation cannot yet be tested because g_Σ^A is very poorly known. However, we can get a feeling of its validity by computing g_{Ξ}^{A} from Eq. (23) and comparing the result with the prediction of the Cabibbo theory. We wish to emphasize that although this is not a legitimate procedure [a real test should be performed by inserting *all* experimental values in Eq. (23)], its only purpose is to see whether the sum rule makes sense or not. Using: $g_{\Xi^-\Lambda\pi^+}{}^2/4\pi=12\pm2$ (see Pilkuhn *et al.*, Ref. 5) and taking the phases from the SU(3) limits,[5] we find

$$g_{\Xi}^{A}=-(0.64\pm0.07)\,, \qquad (24)$$

whereas the result of the Cabibbo theory is[5] $g_{\Xi}^{A}=-0.68$. Final tests of the sum rule, Eq. (22), will have to await the advent of hyperon-beam facilities.

Before closing this section we wish to discuss alternative arguments to Eq. (9). We start from Eq. (5) when $\langle 0|\phi_{\pi_n}|\pi\rangle=0$ (which is not a dynamical assumption). Since ϕ_{π_n} carries the same quantum numbers as the pion, there exist diagrams such that of Fig. 1. It seems reasonable to assume that such diagrams together with that of Fig. 2 dominate for q^2 small. Denoting the "blob," Fig. 3, by $h_{\pi_n}(q^2)$ and $h_{\pi_n}(0)\equiv h_{\pi_n}$, and assuming that the meson-baryon coupling is smooth, i.e., $g_{B'B\pi}(\mu_\pi{}^2)=g_{B'B\pi}(0)$, we find the following GTR:

$$\sqrt{2}\,f_\pi g_{B'B\pi^+}+\sqrt{2}\,g_{B'B\pi^+}\sum_{n=1}^{N}f_{\pi_n}h_{\pi_n}=(m_B+m'_B)g_B^A\,.$$

Therefore,

$$\Delta_{\pi B}=-\frac{1}{f_\pi}\sum_{n=1}^{N}f_{\pi_n}h_{\pi_n}\,,$$

which is independent of B,B'. Since we are not attempting to predict the numerical value of $\Delta_{\pi B}$, the above result is, for all practical purposes, equivalent to Eq. (19) obtained by means of Eq. (9). None of the results of present paper will be affected by a choice between the above equation and Eq. (19). Finally, the same arguments just presented can be easily extended to the case of K and η discussed in later sections.

III. $\pi^0\to\gamma\gamma$ DECAY

The $\pi^0\to\gamma\gamma$ decay amplitude, $F_\pi(q^2)$, is defined by

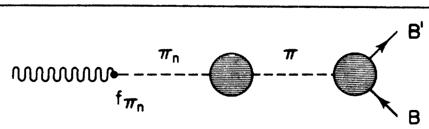

FIG. 1. Off-diagonal contribution to $D_B(q^2)$, Eq. (13).

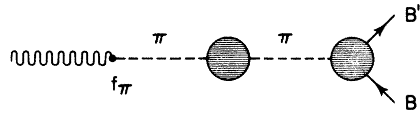

FIG. 2. Diagonal contribution to $D_B(q^2)$, Eq. (13).

$$\frac{\mu_\pi{}^2-q^2}{f_\pi\mu_\pi{}^2}\langle\gamma(k_1,\epsilon_1)\gamma(k_2,\epsilon_2)|D_3|0\rangle=\epsilon_{\mu\nu\alpha\beta}k_1^\mu k_2^\nu\epsilon_1^\alpha\epsilon_2^\beta F_\pi(q^2)\,, \qquad (25)$$

where $q=k_1+k_2$ and $D_3=\partial_\mu A_3^\mu$ is the divergence of the (neutral) axial-vector current. As is well known from the theory of the PCAC anomaly[17] for triangle graphs, $F_\pi(q^2)$ satisfies the following low-energy theorem:

$$F_\pi(0)=-\frac{2\alpha}{\pi}\frac{1}{f_\pi}S\,, \qquad (26)$$

where to any finite order in renormalized perturbation theory, S is fixed by the pointlike constituents that circulate around the triangle loop. For instance, in the three triplet models,[18] $S=\frac{1}{2}$ and in the single-triplet-quark model,[19] $S=\frac{1}{6}$.

Since we need to know $F_\pi(\mu_\pi{}^2)$ in order to compute the decay rate, the question arises as to what is the value of the extrapolation from the zero-point limit, i.e., the value of E_π defined by

$$\frac{F_\pi(\mu_\pi{}^2)}{F_\pi(0)}\equiv E_\pi\,. \qquad (27)$$

According to the strong PCAC philosophy, $E_\pi\simeq1$ and using $S=\frac{1}{2}$ one obtains a decay rate in agreement with experiment. It has been estimated[20] that the deviation of E_π from unity due to normal continuum contributions is approximately 5×10^{-3}. Such a small contribution reminds us of the smallness of Δ_π when calculated using ordinary dynamical cuts. In other versions of PCAC,[13] however, E_π could be much larger than 1 and may accommodate the value $S=\frac{1}{6}$.

We shall show now that in the framework of EPCAC the extraplation factor E_π is fixed entirely by the corrections to the GTR for $\Delta S=0$ β decays, and therefore it can be computed from the experimental value for $\Delta_{\pi N}$, Eq. (1).

Coupling the pion family to the circulating bare constituents in the triangle loop with γ_5 coupling, one obtains

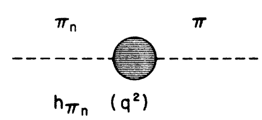

FIG. 3. The $\pi_n-\pi$ coupling, $h\pi_n(q^2)$.

$$F_\pi(q^2) = Cg\frac{\mu_\pi^2 - q^2}{\mu_\pi^2} \sum_{n=0}^{N} \frac{f_{\pi_n}\mu_{\pi_n}^2}{\mu_{\pi_n}^2 - q^2}, \tag{28}$$

where C is a constant and g the coupling of the π^0 to the pointlike constituents, which has been assumed to be independent of n as in Eq. (9). We remark that if $F_\pi(q^2)$ develops eventually a q^2 dependence, it is due entirely to the pion family propagators and not to quark structure.

From Eq. (28) one readily obtains

$$\frac{F_\pi(\mu_\pi^2)}{F_\pi(0)} = \frac{f_\pi}{\sum_{n=0}^{N} f_{\pi_n}} = \frac{1}{1 + \frac{1}{f_\pi}\sum_{n=1}^{N} f_{\pi_n}}, \tag{29}$$

and comparing with Eqs. (19), (20), and (21), it follows that

$$E_\pi \equiv \frac{F_\pi(\mu_\pi^2)}{F_\pi(0)} = \frac{1}{1-\Delta_\pi} \simeq 1 + \Delta_\pi. \tag{30}$$

Therefore, the corrections to the GTR, which are a measure of chiral-symmetry breaking, determine completely the on-mass-shell extrapolation factor in $\pi^0 \to \gamma\gamma$. That this is a matter of general nature, within the framework of EPCAC, can be seen from the structure of $D(q^2)$, Eq. (13). Apart from an over-all coupling constant, $D(q^2)$ is given by a sum of pion propagators times $f_{\pi_n}\mu_{\pi_n}^2$ which comes from the divergence of the axial-vector current, Eq. (5). Thus, the sum in Eq. (13) is a universal feature of the divergence; the only difference between different matrix elements being the over-all coupling constant. Since at $q^2 = 0$ the sum in Eq. (13) is related to Δ_π, we expect that the corrections to the GTR shall determine the on-mass-shell extrapolations from the soft-meson limits. We shall encounter more examples of this as we proceed.

Numerical results for the $\pi^0 \to \gamma\gamma$ decay width are as follows: Using $S = \frac{1}{2}$ (three-triplet quark model) we find $\Gamma(\pi^0 \to \gamma\gamma) = 7.8$ eV for $E_\pi = 1$ and $\Gamma(\pi^0 \to \gamma\gamma) = 8.7 \pm 0.3$ eV for $E_\pi = 1.06 \pm 0.02$ as determined by Eq. (30) with Δ_π given by Eq. (2). The present experimental value is $\Gamma(\pi^0 \to \gamma\gamma) = 7.8 \pm 0.9$ eV. Although there is agreement within errors, it could improve if Δ_π turns out to be smaller than its present value (we recall that Δ_π has been decreasing with time; a few years ago it used to be three times bigger than what it is now).

IV. GOLDBERGER-TREIMAN RELATIONS FOR $\Delta S \neq 0$ DECAYS

In this section we discuss GTR for strangeness-changing hyperon β decays. Therefore, one has to replace everywhere A_μ^{1+i2} by A_μ^{4+i5}, f_{π_n} by f_{K_n}, $\mu_{\pi_n}^2$ by $m_{K_n}^2$, and $g_{\pi_n AB}$ by $g_{K_n AB}$. Once these changes are made, the considerations of Sec. II are identical for the present case, except that now we are dealing with SU(3) × SU(3) instead of SU(2) × SU(2).

Considering two different $\Delta S \neq 0$ β decays, $E \to E' + l\bar{l}$ and $H \to H' + l\bar{l}$, one can derive GTR analogous to Eqs. (15) and (16). The corrections to these GTR are, as in Eqs. (17) and (18),

$$\Delta_{KE} = 1 - \frac{(m_{E'} + m_E)g_E^A}{\sqrt{2} f_K g_{E'EK^+}} \tag{31}$$

and

$$\Delta_{KH} = 1 - \frac{(m_{H'} + m_H)g_H^A}{\sqrt{2} f_K g_{H'HK^+}}. \tag{32}$$

The expressions for the divergence, Eqs. (13) and (14), in the present case are

$$D_{KE}(q^2) = \sqrt{2}\, g_{E'EK^+} \sum_{n=0}^{N} \frac{f_{K_n} m_{K_n}^2}{m_{K_n}^2 - q^2} \tag{33}$$

and

$$D_{KH}(q^2) = \sqrt{2}\, g_{H'HK^+} \sum_{n=0}^{N} \frac{f_{K_n} m_{K_n}^2}{m_{K_n}^2 - q^2}. \tag{34}$$

Upon taking $q^2 = 0$ one readily finds, as in Eqs. (19) and (20), that

$$\Delta_{KE} = -\frac{1}{f_K} \sum_{n=1}^{N} f_{K_n} = \Delta_{KH} \equiv \Delta_K, \tag{35}$$

i.e., the corrections to $\Delta S \neq 0$ GTR are universal. Using Eqs. (31) and (32) one obtains the following sum rule:

$$\frac{m_{E'} + m_E}{m_{H'} + m_H} = \frac{g_H^A}{g_E^A} \frac{g_{E'EK^+}}{g_{H'HK^+}}, \tag{36}$$

which is the analogous of Eq. (22). Once again, Eqs. (35) and (36) are trivial identities in the SU(3) × SU(3) limit because of the vanishing of Δ_{KE} and Δ_{KH}; and in the SU(3) limit Eq. (36) also becomes an identity (if the four particles belong to the same multiplet).

As an example we consider $\Lambda \to p + l\bar{l}$ and $\Sigma^- \to n + l\bar{l}$ decays in which case the sum rule Eq. (36) reads

$$\frac{m_\Lambda + m_p}{m_{\Sigma^-} + m_n} = \frac{g_{\Lambda p K^+}}{g_{\Sigma^- n K^+}} \frac{\tilde{g}_\Sigma^A}{g_\Lambda^A}. \tag{37}$$

Since $g_{\Sigma^- n K^+}$ is very poorly known[5] one cannot yet test this relation. However, for the sake of an "order-of-magnitude" verification we can compute $g_{\Sigma^- n K^+}$ from Eq. (37) and compare it with its SU(3) value. Using $\tilde{g}_\Sigma^A = 0.335 \pm 0.032$ as given in Ref. 21 and the other values quoted in Ref. 8, we find $g_{\Sigma^- n K^+}^2/4\pi = 3.3 \pm 0.8$, as compared to the SU(3) limit[5]: $g_{\Sigma^- n K^+}^2/4\pi = 3.2$.

V. GENERALIZED GOLDBERGER-TREIMAN RELATIONS AND SUM RULES

Some time ago, Dashen and Weinstein[16] derived a set of sum rules for the hadronic corrections to generalized GTR. These sum rules were obtained by noting that those corrections are proportional to $\mu^2\langle r^2\rangle$, where μ^2 is the mass squared of a ground-state meson, and $\langle r^2\rangle$ the meson-baryon-interaction mean square radius. Since $\langle r^2\rangle$ is finite in the SU(3) × SU(3) limit, it can be parametrized in terms of f and d couplings.

In this section we discuss the consequences of EPCAC, for the Dashen-Weinstein sum rules. The universality among the corrections to the GTR for $\Delta S=0$ decays on one side and for $\Delta S\neq 0$ decays on the other, enable us to derive new sets of sum rules. Furthermore, we obtain a relation for Δ_π/Δ_K which in the SU(3) limit reduces to a very simple expression.

In our notation, Eq. (10), the Dashen-Weinstein sum rules read

$$\sqrt{2}\, g_{B'Bi} f_i = (m_{B'}+m_B) g^A_{B'Bi} + \delta_{B'Bi}(0) \quad (i=\pi^+, K^+)\,, \tag{38}$$

where $\sqrt{2}\, g_{B'Bi}$ is the meson-baryon strong coupling constant and $g^A_{B'Bi}$ the axial-vector-current-baryon coupling. The $\delta_{B'Bi}$ are the (hadronic) corrections to the GTR which can be parametrized in terms of f and d couplings as follows:

$$\delta_{B'Bi}(0) = c[(1-a) f_{B'Bi} + iad_{B'Bi}] m_i^2 f_i\,. \tag{39}$$

Recalling the definitions of Δ_π and Δ_K, Eqs. (17) and (31), Eq. (38) can be rewritten as

$$\sqrt{2}\, g_{B'Bi} f_i \Delta_{iB} = \delta_{B'Bi}(0) \quad (i=\pi^+, K^+)\,. \tag{40}$$

As an example, if one considers $n\to p$, $\Sigma^-\to n$, and $\Lambda\to p$ β decays it follows from Eqs. (39) and (40) that

$$\sqrt{2}\, g_{np\pi^+} \Delta_{\pi N} = c\mu_\pi^2\,, \tag{41}$$

$$\sqrt{2}\, g_{\Sigma^-\Lambda\pi^+} \Delta_{\pi\Sigma} = c\frac{2a}{\sqrt{6}}\mu_\pi^2\,, \tag{42}$$

$$\sqrt{2}\, g_{\Sigma^- nK^+} \Delta_{K\Sigma} = c(1-2a) m_K^2\,, \tag{43}$$

$$\sqrt{2}\, g_{\Lambda pK^+} \Delta_{K\Lambda} = c\,\frac{3-2a}{\sqrt{6}}\, m_K^2\,, \tag{44}$$

where our conventions for the coupling constants and SU(3) phases follow those of Pilkuhn *et al.*[5]

We proceed now to exploit the universality among the corrections to the GTR, Δ_π and Δ_K. Considering $B'\to B$ and $C'\to C$ β decays with $\Delta S=0$ and using Eqs. (21) we find from Eq. (40) that

$$\sqrt{2}\, g_{B'B\pi^+} f_\pi \Delta_\pi = \delta_{B'B\pi^+}(0) \tag{45}$$

and

$$\sqrt{2}\, g_{C'C\pi^+} f_\pi \Delta_\pi = \delta_{C'C\pi^+}(0)\,. \tag{46}$$

Taking the ratio of the preceding two equations the following sum rule results:

$$\frac{g_{B'B\pi^+}}{g_{C'C\pi^+}} = \frac{\delta_{B'B\pi^+}(0)}{\delta_{C'C\pi^+}(0)}\,. \tag{47}$$

Since μ_π^2 and f_π cancel in the right-hand side [see Eq. (39)], Eq. (47) is a relation involving only strong-coupling constants.

In a similar fashion, considering $E'\to E$ and $H'\to H$ β decays with $\Delta S\neq 0$ and using Eq. (35), one finds

$$\sqrt{2}\, g_{E'EK^+} f_K \Delta_K = \delta_{E'EK^+}(0)\,, \tag{48}$$

$$\sqrt{2}\, g_{H'HK^+} f_K \Delta_K = \delta_{H'HK^+}(0)\,, \tag{49}$$

and

$$\frac{g_{E'EK^+}}{g_{H'HK^+}} = \frac{\delta_{E'EK^+}(0)}{\delta_{H'HK^+}(0)}\,. \tag{50}$$

Taking the ratio between Eqs. (45) and Eq. (48) it follows that

$$\frac{\Delta_\pi}{\Delta_K} = \frac{f_K}{f_\pi}\,\frac{\delta_{B'B\pi^+}(0)}{\delta_{E'EK^+}(0)}\,\frac{g_{E'EK^+}}{g_{B'B\pi^+}}\,. \tag{51}$$

Recalling the definitions of $\delta_{B'B\pi^+}(0)$ and $\delta_{E'EK^+}(0)$, Eq. (39), Eq. (51) becomes

$$\frac{\Delta_\pi}{\Delta_K} = \frac{\mu_\pi^2}{m_K^2}\,\frac{\lambda_1}{\lambda_2}\,\frac{g_{E'EK^+}}{g_{B'B\pi^+}}\,, \tag{52}$$

where λ_1 and λ_2 stand for the parametrizations in terms of f and d couplings [the terms inside the square brackets in Eq. (39)]. If one takes the SU(3) limit for the coupling constants (all baryons belonging to the same multiplet) then Eq. (52) reduces to the following simple expression:

$$\frac{\Delta_\pi}{\Delta_K} = \frac{\mu_\pi^2}{m_K^2}\,. \tag{53}$$

It should be stressed that when comparing with experimental data, it is Eq. (52) and not Eq. (53) which should be used. In fact Eq. (53) does not agree with experiment owing to SU(3)-symmetry breaking.

As an example of the preceding sum rules we consider the four decays that led to Eqs. (41)–(44). After some manipulation we obtain

$$g_{\Sigma^- nK^+} = \frac{\Delta_\pi}{\Delta_K}\,\frac{m_K^2}{\mu_\pi^2}\,(g_{np\pi^+} - \sqrt{6}\, g_{\Sigma^-\Lambda\pi^+})\,, \tag{54}$$

$$g_{\Lambda pK^+} = \frac{\Delta_\pi}{\Delta_K}\,\frac{m_K^2}{\mu_\pi^2}\,[(\tfrac{3}{2})^{1/2} g_{np\pi^+} - g_{\Sigma^-\Lambda\pi^+}]\,, \tag{55}$$

and

$$\frac{g_{\Lambda pK^+}}{g_{\Sigma^- nK^+}} = \frac{(\frac{3}{2})^{1/2} g_{np\pi^+} - g_{\Sigma^-\Lambda\pi^+}}{g_{np\pi^+} - \sqrt{6}\, g_{\Sigma^-\Lambda\pi^+}} . \tag{56}$$

In the SU(3) limit for the meson-baryon coupling constants, Eqs. (54) and (55) reduce to Eq. (53) and Eq. (56) becomes an identity. Equations (54) and (56) cannot yet be tested because of a lack of information on $g_{\Sigma^- nK^+}$, but Eq. (55) can be used to compute Δ_π/Δ_K. The result is $\Delta_\pi/\Delta_K = 0.09 \pm 0.01$ as compared to the present experimental value of $\Delta_\pi/\Delta_K = 0.2 \pm 0.1$ [the SU(3) limit, Eq. (53), gives $\Delta_\pi/\Delta_K = 0.078$]. This is a good indication that the sum rules just derived might be well satisfied by the data, once available from hyperon-beam facilities.

VI. CORRECTIONS TO SOFT-MESON THEOREMS IN K_{l3}

It is well known that the soft-pion[22] and soft-kaon[23] theorems in K_{l3} decay[24] are a consequence of chiral symmetry.[2] Therefore, we expect corrections to these soft-meson theorems due to chiral-symmetry breaking. Since this breaking is approximately gauged by the corrections to the GTR, Δ_π and Δ_K, we would expect these parameters to play a role in determining the deviations from the soft-meson limits. In this section we show how EPCAC allows us to derive the explicit form of these corrections entirely in terms of Δ_π and Δ_K. Moreover, in the case of the soft-kaon theorem, the magnitude of the correction we find brings the ratio f_K/f_π into perfect agreement with experiment.

We consider the decay $K^+(k) \to \pi^0(p) + l\bar{l}$, and define the K_{l3} form factors, $f_\pm(q^2)$, as

$$\begin{aligned} F_\mu(q^2, p_\mu, q_\mu) &= \langle \pi^0 | V_\mu^{K^-} | K^+ \rangle \\ &= \frac{i}{\sqrt{2}} [(k+p)_\mu f_+(q^2) + q_\mu f_-(q^2)] , \end{aligned} \tag{57}$$

where $q = k - p$ is the momentum carried away by the lepton-antilepton pair.

Assuming an unsubtracted dispersion relation for F_μ we have

$$\begin{aligned} F_\mu(p^2, q^2, p_\mu, q_\mu) \\ = \frac{\mu_\pi^2 - p^2}{f_\pi \mu_\pi^2} \frac{1}{\pi} \int \frac{dp'^2}{p'^2 - p^2} \tilde{F}_\mu (p'^2, q^2, p_\mu, q_\mu) , \end{aligned} \tag{58}$$

where

$$\begin{aligned} \tilde{F}_\mu(p'^2, q^2, p_\mu, q_\mu) \\ = \int d^4x\, e^{ip'\cdot x} \langle 0 | [D_\pi^3(x), V_\mu^K(0)] | K \rangle \\ = (2\pi)^4 \sum_n \delta^4(p' - p_n) \langle 0 | D_\pi^3(0) | n \rangle \langle n | V_\mu^K(0) | K \rangle . \end{aligned} \tag{59}$$

Using the EPCAC hypothesis, Eq. (5), and the assumption Eq. (9), i.e., $g_{\pi_n KK^*} = g_{\pi KK^*}$, we obtain

$$\begin{aligned} F_\mu(p^2, q^2, p_\mu, q_\mu) = \frac{\mu_\pi^2 - p^2}{f_\pi \mu_\pi^2} \frac{i}{\sqrt{2}} \\ \times [(k+p)_\mu f_+(q^2) + q_\mu f_-(q^2)] \\ \times \sum_{n=0}^{N} \frac{f_{\pi_n} \mu_{\pi_n}^2}{\mu_{\pi_n}^2 - p^2} . \end{aligned} \tag{60}$$

Taking the soft-pion limit, $p \to 0$, and using Eqs. (19)–(21) we find

$$\frac{f_K}{f_\pi} = [f_+(m_K^2) + f_-(m_K^2)](1 - \Delta_\pi) . \tag{61}$$

In the chiral limit $\Delta_\pi = 0$, and we recover the soft-pion theorem. As expected, the correction term is small, although the interesting result is that, as in $\pi^0 \to \gamma\gamma$, Δ_π is measuring the deviation from the soft-pion limit.

The derivation of the soft-kaon limit is performed by considering $\langle \pi | [V_\mu^K(0), D_K^3(0)] | 0 \rangle$ instead of $\langle 0 | [D_\pi^3(x), V_\mu^K(0)] | K \rangle$ in Eq. (59). It is straightforward to show that EPCAC gives in this case

$$\frac{f_\pi}{f_K} = [f_+(\mu_\pi^2) - f_-(\mu_\pi^2)](1 - \Delta_K). \tag{62}$$

Once again one recovers the standard result[23] by taking the chiral limit in which $\Delta_K = 0$. However, in contrast to the soft-pion case the correction here is large. In fact, replacing experimental values[6] for the form factors one has $f_K/f_\pi f_+(0) = 0.93$, if $\Delta_K = 0$. This is to be compared with the experimental ratio $f_K/f_\pi f_+(0) = 1.26 \pm 0.02$. On the other hand, for $\Delta_K \neq 0$ we find from Eqs. (62) and (4) that

$$\frac{f_K}{f_\pi f_+(0)} = 1.3 \pm 0.2 , \tag{63}$$

in perfect agreement with experiment.

VII. DECAYS $\eta \to \gamma\gamma$ AND $\eta \to \pi\pi\gamma$

It is known[25] that anomalous Ward identities imply low-energy theorems for $\eta \to \gamma\gamma$ and $\eta \to \pi\pi\gamma$ decays. In the first case one has a triangle anomaly similar to the one for $\pi^0 \to \gamma\gamma$, while for $\eta \to \pi\pi\gamma$ both triangle and square anomalies make a contribution. If one attempts to calculate the decay rates starting from the low-energy theorems, two problems, among others, arise. The first one is the η-η' mixing, and the second the large value of the η mass. It is generally assumed[25–26] that, despite the largeness of m_η^2, the amplitudes on mass shell are still dominated by the anomaly. It then follows that the η-η' mixing becomes essential to bring the decay rates into agreement with experiment. However, we see no reason to expect the extrapolation of the low-energy theorem for $\eta \to \gamma\gamma$ to be small. In fact, we have seen that in the case of the soft-kaon theorem the K_{l3}, the smoothness assumption gives a wrong result for f_K/f_π. It is only when we introduced the chiral-symmetry-breaking correction that agreement was found.

In the following we study the consequences of EPCAC for $\eta \to \gamma\gamma$ and $\eta \to \pi\pi\gamma$ and show that when proper account for the extrapolations is taken, the decay rates come out in agreement with experiment without further need for any η-η' mixing.

We start by defining the η decay amplitudes as

$$M(\eta \to \gamma\gamma) = \epsilon_{\mu\nu\alpha\beta} k_1^\mu k_2^\nu \epsilon_1^\alpha \epsilon_2^\beta F_\eta(q^2), \tag{64}$$

$$M(\eta \to \pi^+\pi^-\gamma) = \epsilon_{\mu\alpha\beta\gamma} \epsilon^\mu k^\alpha p_+^\beta p_-^\gamma G_\eta(p_+\cdot k, p_-\cdot k, \dots), \tag{65}$$

where ϵ and k are the polarizations and momenta of the photons, $q = k_1 + k_2$ is the η momentum, and $p_\pm$ the $\pi^\pm$ momenta.

The low-energy theorems implied by the anomalous Ward identities are[25]

$$F_\eta(0) = -\frac{1}{\sqrt{3}} \frac{2\alpha}{\pi} \frac{1}{f_\eta} S \tag{66}$$

and

$$G_\eta(0,0,\dots) = -\frac{1}{\sqrt{3}} \frac{\sqrt{\pi\alpha}}{\pi^2} \frac{1}{f_\pi^2 f_\eta} S, \tag{67}$$

where S has been already defined in Eq. (26). The extrapolation factor for the $\eta \to \gamma\gamma$ amplitude is defined as

$$E_\eta \equiv \frac{F_\eta(m_\eta^2)}{F_\eta(0)}. \tag{68}$$

It can be shown[25] that for $\eta \to \pi\pi\gamma$ one has

$$\frac{G_\eta(m_\eta^2, \mu_\pi^2, \mu_\pi^2, 0, \dots)}{G_\eta(0,0,0,0,\dots)} = E_\pi^2 E_\eta, \tag{69}$$

where E_π is the extrapolation factor for $\pi^0 \to \gamma\gamma$, Eq. (27). Since we already know from Eq. (30), what E_π is, i.e., $E_\pi = (1-\Delta_\pi)^{-1}$, we can compute the ratio $\Gamma(\eta\to\pi\pi\gamma)/\Gamma(\eta\to\gamma\gamma)$, which is independent of f_η and E_η. After performing the phase-space integration[25] one has

$$\frac{\Gamma(\eta\to\pi^+\pi^-\gamma)}{\Gamma(\eta\to\gamma\gamma)} = E_\pi^4 \frac{1}{f_\pi^4} \frac{1}{4\pi\alpha}\phi = \left(\frac{1}{1-\Delta_\pi}\right)^4 \frac{1}{f_\pi^4} \frac{1}{4\pi\alpha} \phi, \tag{70}$$

where

$$\phi = \frac{m_\eta^4}{96\pi^2}(7.48\times10^{-3}), \tag{71}$$

is the phase-space factor. The EPCAC correction in Eq. (70), i.e., the term $(1-\Delta_\pi)^{-4}$, amounts to 26%, which certainly is not negligible. In fact, we find for the ratio, Eq. (70),

$$\frac{\Gamma(\eta\to\pi^+\pi^-\gamma)}{\Gamma(\eta\to\gamma\gamma)} = 0.14 \pm 0.01, \tag{72}$$

while for $\Delta_\pi = 0$ the result is 0.109. This is to be compared with the experimental value[5,27] of 0.132 $\pm$ 0.004. Therefore, the chiral-symmetry-breaking factor is enough to get agreement with experiment and there is no need to invoke η-η' mixing.

Turning to $\eta \to \gamma\gamma$, we would need to know f_η and E_η in order to calculate the decay rate. Since both quantities are unknown, the most we can do is to invoke SU(3) symmetry. In this case one has $f_\eta \simeq f_\pi$. Regarding E_η we expect, as in $\pi^0 \to \gamma\gamma$, that $E_\eta = 1/(1-\Delta_\eta)$, where Δ_η is a hypothetical correction to a GTR. By SU(3) symmetry we would have $\Delta_\eta \simeq \Delta_K$ and, therefore, E_η would be approximately equal to the soft-kaon-limit correction in K_{l3}, i.e.,

$$E_\eta \simeq \frac{1}{1-\Delta_K}. \tag{73}$$

In actual fact, E_η should be slightly larger owing to the η-K mass difference.

Under these assumptions we obtain ($S = \frac{1}{2}$)

$$\Gamma(\eta\to\gamma\gamma) = \frac{m_\eta^3}{64\pi} E_\eta^2 F_\eta^2(0) = 0.35 \pm 0.14 \text{ keV}, \tag{74}$$

while the experimental value[27] is 0.324 ± 0.046 keV. If we had used, instead, $\Delta_K = 0$ or $E_\eta = 1$ the result would have been $\Gamma(\eta\to\gamma\gamma) = 0.175$ keV, i.e., a factor of two smaller.

In conclusion, the smoothness assumption, i.e., $E_\pi = E_\eta = 1$, is found to give wrong results for the radiative η decays. However, EPCAC correctly accounts for the on-mass-shell extrapolations of the low-energy theorems without the need for an introduction of η-η' mixing.

VIII. $\gamma \to \pi\pi\pi$ AND $\gamma\gamma \to \pi\pi\pi$

As a final application of EPCAC we discuss in this section the $\gamma \to \pi\pi\pi$ and $\gamma\gamma \to \pi\pi\pi$ processes. Here, the predictions of standard strong PCAC[25] differ from those of weak PCAC[13] by at least an order of magnitude, the later enhancing the amplitudes by that amount. The experimental possibilities of determining which prediction is correct have been extensively discussed in the literature,[25,28] so that we shall concentrate in deriving the results of EPCAC. As is to be expected from our results for $\pi^0 \to \gamma\gamma$, the predictions of EPCAC will turn out to be close to those of standard strong PCAC.

For the process $\gamma \to \pi^+\pi^-\pi^0$ there is a low-energy theorem,[25] implied by anomalous Ward identies, similar to the one for $\eta \to \pi^+\pi^-\gamma$, i.e.,

$$F_{\pi^+\pi^-\pi^0}(0,0,0,0,\ldots) = -\frac{\sqrt{\pi\alpha}}{\pi^2}\frac{1}{f_\pi^3}S. \qquad (75)$$

The on-mass-shell amplitude is given by[25]

$$\begin{aligned} F_{\pi^+\pi^-\pi^0}(\mu_\pi^2, \mu_\pi^2, \mu_\pi^2, \ldots) &= E_\pi^3 F_{\pi^+\pi^-\pi^0}(0,0,0,0,\ldots) \\ &= \left(\frac{1}{1-\Delta_\pi}\right)^3 F_{\pi^+\pi^-\pi^0}(0,0,0,0\ldots), \end{aligned} \qquad (76)$$

where we have used Eq. (30) to fix E_π. Therefore, according to EPCAC, we would expect an enhancement of approximately 20% with respect to strong PCAC ($E_\pi \simeq 1$). On the other hand, weak PCAC gives[13] $E_\pi = 3$ and consequently a much larger extrapolation in (76).

Turning to $\gamma\gamma \to \pi^0\pi^0\pi^0$ (similar remarks apply to $\gamma\gamma \to \pi^+\pi^-\pi^0$), it can be shown[29] that the low-energy theorem is proportional to $F_\pi(0)$, the off-mass-shell amplitude for $\pi^0 \to \gamma\gamma$. Since the relevant diagram for $\gamma\gamma \to \pi\pi\pi$ is a pole diagram with a π^0 as intermediate state,[29] one is concerned with the value of $F_\pi(q^2)$, where q^2 is the invariant mass of that intermediate π^0. Choosing $q^2 = \mu_\pi^2$, as suggested by dispersion theory, we find then that EPCAC predicts an enhancement in the amplitude of (6 ± 2)% [see Eq. (30)] over the standard strong PCAC result, $E_\pi \simeq 1$, while weak PCAC[13] predicts in this case a result three times larger. It is obvious that here the difference between standard PCAC and EPCAC is experimentally undetectable.

IX. DISCUSSION AND CONCLUSIONS

The impossibility of accounting for the corrections to GTR, both in SU(2) × SU(2) and SU(3) × SU(3), led to the suggestion[7,10] that there might exist heavy bosons. These heavy bosons would, we hope, provide an additional dynamical origin responsible for the rather large magnitude of the corrections to GTR. That this might be the case was shown later[12] in the context of a specific model incorporating a family of heavy pions of the Veneziano type. However, the problem does not end there because there are a number of instances in which those heavy bosons can make a contribution. Probably the most important situation pertains to the soft-meson theorems which rely heavily on the validity of the PCAC hypothesis. Questions such as how large are the extrapolations to on-mass-shell bosons of the decay amplitudes having anomalous Ward identities, or simply, how reliable are the soft-meson theorems, e.g., those of K_{l3} have been answered in the past only qualitatively. In fact, one generally believes that, e.g., soft-pion theorems are more reliable than soft-kaon theorems and that one should not expect meson-pole dominance to give the correct answer for radiative-η-decay rates. One could attempt to calculate the contribution of several-particle intermediate states to the dispersion relation for the matrix element of the divergence of the axial-vector current, such as was done, e.g., for $\pi^0 \to \gamma\gamma$.[20] However, one can raise the question as to how meaningful those results would be, since we know that that is not the dynamical origin of the corrections to GTR. Therefore, it is only after we have correctly understood those corrections that we can turn our attention to the soft-meson theorems.

In this respect we have been motivated by the fact that the suggestion[7,10] of the existence of heavy bosons has proven to be successful[12] in accounting for the magnitude of Δ_π.[30] With this in mind we have defined an extended PCAC hypothesis by Eq. (5) in a model-independent fashion. This hypothesis coupled with the assumption Eq. (9) was the starting point in the study of the soft-meson theorems and generalized GTR carried out in this paper. In Sec. II and IV we showed that as a consequence of EPCAC there is a universality among the corrections to GTR for $\Delta S = 0$ decays on one hand and for $\Delta S \neq 0$ decays on the other. An immediate result is a set of sum rules, Eqs. (22) and (36), valid in the presence of chiral-symmetry breaking and which become identities in the chiral as well as in the SU(3) limits. Although a true test of these sum rules has to await the advent of hyperon-beam facilities, we showed in two instances that they are expected to be correct.

The above-mentioned universality also enabled us to derive new sets of sum rules, Eqs. (47) and (50), as well as a relation for Δ_π/Δ_K, Eq. (52), starting from the Dashen-Weinstein sum rules for generalized GTR. The prediction for Δ_π/Δ_K is in good agreement with present data, indicating that we might expect good saturation of the sum rules once the hyperon strong-coupling constants become available.

Regarding the soft-meson theorems, the interesting result of EPCAC is that Δ_π and Δ_K, which are a measure of chiral-symmetry breaking, determine completely the extrapolation factors in the amplitudes. Although the result for E_π in $\pi^0 \to \gamma\gamma$, Eqs. (27) and (30), turned out to be small as expected, its precise value turned out to be of crucial importance in the ratio $\Gamma(\eta \to \pi\pi\gamma)/\Gamma(\eta \to \gamma\gamma)$. In fact, the rate $\Gamma(\pi^0 \to \gamma\gamma)$ does not change significantly if $E_\pi = 1$ or if it is given by Eq. (30), but since it is

E_π[4] which enters in the above ratio of radiative η decays, the results there can be quite different. In particular, we have seen in Sec. VII that there is no need to invoke η-η' mixing when E_π is taken from Eq. (30). Furthermore, we have also seen that the smoothness assumption usually invoked[25] for $\eta \to \gamma\gamma$ is far from being correct.

Finally, regarding the soft-kaon theorem in K_{l3}, we have seen in Sec. VI that the EPCAC extrapolation factor brings the ratio f_π/f_K into agreement with experiment.

In summary, EPCAC provides a working explanation for the dynamical origin of the corrections to GTR and at the same time predicts expressions for the extrapolation factors in the soft-meson theorems. These expressions are such that in the chiral-symmetry limits one recovers the results of standard PCAC or meson-pole dominance.

There are a number of issues that we deliberately left out in this paper, but that we believe deserve future consideration. Among these we mention the following:

(i) What are the implications of EPCAC for the different existing approaches to chiral-symmetry breaking,[31] e.g., chiral perturbation theory, etc.?

(ii) How does EPCAC influence the results of PCAC for scattering processes, e.g., Adler-Weisberger relations, forward neutrino-nucleon scattering, etc.?

(iii) Specific models that may implement EPCAC should be studied. A good starting point could be the Veneziano model, which has worked well for Δ_π.[30]

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[1]S. Adler and R. Dashen, *Current Algebra* (Benjamin, New York, 1968); S. Treiman, R. Jackiw, and D. Gross, *Lectures on Current Algebra and Its Application* (Princeton University Press, Princeton, N.J., 1972).

[2]H. Pagels, Phys. Rep. 16C, 219 (1975).

[3]S. Weinberg, Phys. Rev. Lett. 16, 169 (1966); Phys. Rev. 166, 1568 (1967); R. Dashen, *ibid.* 183, 1245 (1969); R. Dashen and M. Weinstein, *ibid.* 183, 1291 (1969).

[4]M. L. Goldberger and S. B. Treiman, Phys. Rev. 111, 354 (1958).

[5]We have used $g_A = -(1.25 \pm 0.09)$ as given in Particle Data Group, Rev. Mod. Phys. 48, S1 (1976); $g_{np\pi^+} = -\sqrt{2}\, g_{\pi NN}$, where $g_{\pi NN}{}^2/4\pi = 14.64^{+0.54}_{-0.72}$ given by H. Pilkuhn *et al.*, Nucl. Phys. B65, 460 (1973); and $\sqrt{2}\, f_\pi = (0.932\,51 \pm 0.001\,44)\mu_{\pi^+}$, from R. J. Blin-Stoyle and J. M. Freeman, Nucl. Phys. A150, 369 (1970).

[6]Recent precision results on K_{l3} indicate that there is no longer a conflict with SU(2) × SU(2). G. Donaldson *et al.*, Phys. Rev. Lett. 31, 337 (1973).

[7]H. Pagels, Phys. Rev. 179, 1337 (1968); H. Pagels and A. Zepeda, Phys. Rev. D 5, 3262 (1972).

[8]We have used $g_\Lambda^A/g_\Lambda^V = -(0.66 \pm 0.05)$, $f_K/f_+(0)f_\pi = 1.26 \pm 0.02$, $g_{\Lambda pK}{}^2/4\pi = 13.5 \pm 2.5$ and the renormalizations $f_+(0) = 0.97$, $g_\Lambda^V = (\frac{3}{2})^{1/2}$ (0.87). The SU(3) phases given by the following SU(3) limits were used in calculating (4): $g_{\Lambda pK} = -g_{\pi NN}(3-2\alpha)/\sqrt{3}$, $g_\Lambda^A = g_A(3-2\alpha)/\sqrt{6}$, $g_\Lambda^V = (\frac{3}{2})^{1/2}\alpha \approx \frac{2}{3}$. See Refs. 5 and 2, and also T. W. Rodgers *et al.*, Phys. Rev. 178, 2478 (1969).

[9]J. Chen, Phys. Rev. D 5, 1143 (1972).

[10]C. Michael, Phys. Rev. 166, 1826 (1968).

[11]G. W. Brandenburg *et al.*, Phys. Rev. Lett. 36, 1239 (1976).

[12]C. A. Dominguez, Phys. Rev. D 7, 1252 (1973). For another approach to the corrections to the Goldberger-Treiman relation see H. J. Braathen, Nucl. Phys. B44, 93 (1972); H. F. Jones and M. D. Scadron, Phys. Rev. D 11, 174 (1975).

[13]S. D. Drell, Phys. Rev. D 7, 2190 (1973); J. L. Newmeyer and S. D. Drell, *ibid.* 8, 4070 (1973).

[14]I. Bars and M. B. Halpern, Phys. Rev. D 7, 3043 (1973).

[15]All the following considerations can be translated for the other bosons (K, η).

[16]R. Dashen and M. Weinstein, Phys. Rev. 188, 2330 (1969).

[17]J. S. Bell and R. Jackiw, Nuovo Cimento 60A, 47 (1969); S. L. Adler, Phys. Rev. 177, 2426 (1969).

[18]M. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965); W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, CERN Report No. CERN-TH-1583 (unpublished).

[19]M. Gell-Mann, Phys. Lett. 8, 214 (1964); G. Zweig, CERN Reports No. CERN-TH-401, 1964 and No. CERN-TH-412, 1964 (unpublished).

[20]H. Pagels, private communication.

[21]A. García and E. C. Swallow, Phys. Rev. Lett. 35, 467 (1975); A. García, Phys. Rev. D 12, 2692 (1975). Note that our SU(3) phase convention is opposite to the one used by these authors.

[22]C. G. Callan and S. Treiman, Phys. Rev. Lett. 16, 153 (1966); M. Suzuki, *ibid.* 16, 212 (1966); V. S. Mathur, S. Okubo, and L. K. Pandit, *ibid.* 16, 371

(1966).

[23]R. Oehme, Phys. Rev. Lett. 16, 215 (1966); N. H. Fuchs and T. K. Kuo, Nuovo Cimento 64A, 382 (1969).

[24]L. M. Chounet, J. M. Gaillard, and M. K. Gaillard, Phys. Rep. 4C, 199 (1972).

[25]M. S. Chanowitz, M. S. Chen, and L. F. Li, Phys. Rev. D 7, 3104 (1973).

[26]M. S. Chanowitz, Phys. Rev. Lett. 35, 977 (1975). See also, A. Bramón and M. Greco, Nuovo Cimento 14A, 323 (1973); Phys. Lett. 48B, 137 (1974).

[27]A. Browman *et al.*, Phys. Rev. Lett. 19, 1067 (1974).

[28]R. Aviv and A. Zee, Phys. Rev. D 5, 2372 (1972); A. Zee, *ibid.* 6, 900 (1972).

[29]S. L. Adler *et al.*, Phys. Rev. D 4, 3497 (1971); M. V. Terent'ev, Phys. Lett. 38B, 419 (1972).

[30]The same model used in Ref. 12 also accounts for Δ_K. This as well as other model-dependent results are the subject of a forthcoming publication. C. A. Dominguez (unpublished).

[31]C. A. Dominguez and A. García (unpublished).