Calculating hadronic cross sections in specific symmetry schemes: An application to the ψp total cross section*

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We construct a model for calculating ratios of cross sections. We apply the model to SU(3), calculating $\sigma_{Kp}/\sigma_{\pi p}$, and then to SU(4), in order to estimate the ψp cross section. Results are obtained in both "pure" and broken SU(4). The energy dependence of $\sigma_{\psi p}$ and the slope of the ψp differential cross section are qualitatively discussed. Kinematic effects arising from mass splittings and $t_{\min}$ effects are crucial.

I. INTRODUCTION

The newly discovered particles, $\psi(3100)$ etc.,[1] are being subjected to a number of theoretical interpretations.[2] One might hope to use the growing body[3] of experimental information on the "strong" interactions of the ψ to test these various interpretations. Unfortunately, of course, there does not exist any well-defined prescription for calculating the hadronic properties of particles. Nonetheless, there do now exist some widely accepted, phenomenologically motivated "pictures" for production processes—in particular, pictures based on the multiperipheral model.[4] In this paper we shall develop such a phenomenologically motivated (and, we believe, realistic) model, and shall use it to obtain the ψp total cross section at Fermilab energies in the picture where the ψ is taken to be built from a charmed quark-antiquark pair.

The model will be developed in the next section. We then test it in the context of SU(3) by calculating the ratio of πp and Kp total cross sections, and comparing this to the experimental value. The results of this comparison then motivate us to extend the model to a systematic calculation of cross-section ratios in a full SU(4) scheme (with Zweig's rule[2]), using both pure and broken couplings. The masses of course are always kept broken. We then estimate the effect of taking one ψ leg to zero mass so as to be able to compare our results to the photoproduction results.[3]

While it is true that the charmed picture of the ψ has the desirable properties of being both interesting and sufficiently detailed for us to be able to carry out a systematic analysis of total-cross-section ratios within it, its physical status remains uncertain. Accordingly, we shall (in Appendix A) take a brief excursion beyond SU(4) and show how, in a quite general situation, a cross section of the order of 1 mb is to be expected from the kinematic effects that seem universal in strong production processes and are explicit in our model.

The questions of the energy dependence of the ψp total cross section and the slope of its differential cross section will be briefly touched upon. The size and energy dependence of the inclusive cross section will be discussed elsewhere.

II. THE MODEL[4,5]

Consider the total cross section for some particle A on protons. We represent it as a sum over intermediate states, assuming, as shown in Fig. 1, a peripheral structure at the "top vertex."

We shall be interested only in *ratios* of cross sections such as $\sigma_{\pi p}/\sigma_{Kp}$. We shall find that such ratios are primarily determined by the fact that strange mesons are generally heavier than the corresponding nonstrange mesons. Since this seems to be the case for all multiplets, in order to calculate *ratios* of cross sections it should be sufficient to pick out a representative pair of multiplets B and C in the summation in Fig. 1. We shall follow Ref. (6) and specify B and C as follows: When A is a pseudoscalar (π, K, etc.) we choose B to be the vectors (ρ, K^*, etc.) and C the pseudoscalars.

Thus, in the case of SU(3), A and C will be the pseudoscalars π, K, etc. and B will be the vectors ρ, K^*, etc. Since the lower blob in Fig. 1 is just the Cp total cross section, we obtain a set of coupled equations for the ratios of the various pseudoscalar-proton total cross sections, which can then be solved. We shall do this for both SU(3) and SU(4).

How general can we expect our results to be? Our cluster multiperipheral model is consistent

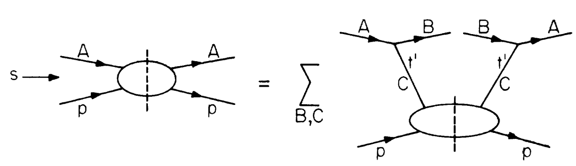

FIG. 1. Peripheral model for the Ap total cross section.

with the view of a predominant short-range component in inelastic data mediated by clusters. Here clusters are defined as being separated by nondual 0^- exchanges. The model has been shown to be capable of accounting for a variety of multiparticle distributions at least semiquantitatively.[6,7] These include mass distributions, correlations, etc., and have been explicitly checked by computer calculation with exact phase space in specific exclusive reactions near average multiplicities. Such a model is also capable of resolving difficulties associated with the calculation of the overlap function when cluster-spin effects are taken into account (at least in the energy range $E_{\rm lab} < 30$ GeV/c where it has been applied). Note especially that we treat the kinematics realistically without resorting to large-subenergy strong-ordering assumptions that neglect important $t_{\rm min}$ effects. A restrictive assumption might seem to be that of pseudoscalar exchange. There are several points that mitigate this assumption. First, since we include cluster production, which might be regarded in some sense as dual to vector-tensor exchanges, we are in fact implicitly including some of these effects. Secondly, low-mass (e.g., π, K) exchange cannot actually easily be distinguished from ordinary Reggeon exchange in multiparticle amplitudes owing to a kinematic $t_{\rm min}$ effect which leads to nearly identical subenergy dependences in the two cases[8] ("low mass" means small compared to cluster masses). Next we shall wind up using an off-shell ABC (0^-0^- cluster) vertex function $g_{ABC}(t')$ that rises for moderate values of the off-shell mass t' before falling to zero. (Actually we shall examine several versions of this form factor to test the stability of the calculation.) A form factor of this type is required within the context of this model for consistency with both inclusive and certain exclusive data.[7,9] It should be noted that even with this choice of g_{ABC} the particle–particle–output-Reggeon form factor $\beta_{CC}(t)$—which is closer to an ordinary form factor than is g_{ABC}—falls uniformly for spacelike off-shell particle mass, $M_C^2 = t'$ [cf. Eq. (2.5)]. $\beta_{CC}(t)$ also falls uniformly for spacelike Reggeon t at a rate consistent with experiment as shown in the overlap-function calculation reported in the last paper of Ref. 7. We conclude that there is no theoretical objection to our choice of the form of $g_{ABC}(t')$ of Eq. (2.4) at moderate t'. We repeat that we expect $g_{ABC}(t')$ to damp out at large t'—the details of which will not matter much here. Now the reader is entirely free to reinterpret the enhancement in $g_{ABC}(t')$ to mean that we are qualitatively including ordinary Reggeon exchange as well as pseudoscalar exchange. If that is so, claims of model generality would have to rely on the pure- and broken-SU(N) couplings being relatively stable under interchange of multiplets. As stated earlier we would expect that taking cross-section ratios would help eliminate this effect. Finally, we use elementary exchanges. This differs of course from the popular idea of dual Reggeized models, but is similar in the sense that heavier-mass exchanges replace lower-lying Reggeons. However, it should not make much difference for the SU(3) calculations, given the small subenergies involved (on the order of 1 GeV^2), and might even be preferred in view of this fact. For SU(4) this assumption is crucial. Naive Reggeization with respect to ordinary hadronic scales [i.e., choosing $s_0 = 1 = 1/\alpha'$ in the formula $(s/s_0)^\alpha$ corresponding to iterated clusters of mass $\sqrt{s_0}$] leads to a value of $\sigma_{\psi p}$ many orders of magnitude too small.[10] It is important to note that the choice of these scales provides an inherent arbitrariness in any attempt to construct a Reggeized model. Our assumption is closer to Reggeization with a scale set by heavier clusters (e.g., containing charmed quarks).

We close this introduction by stating that of course we can only test the combined consistency of the model and the symmetry schemes. We believe that the results we shall obtain in SU(3) will persist in more realistic models. The extension to SU(4) is of course much more speculative and, as far as $\sigma_{\psi p}$ is concerned, only the qualitative result that it is on the order of 1 mb should be taken seriously.

In accord with the above discussion, we write

$$\frac{\sigma_{Ap}}{\sigma_{A'p}} = \frac{\sum_{B,C} \int \sigma_{Cp} \, \frac{M^2 dM^2 dt'}{(t' - m_C{}^2)^2} \, g_{ABC}{}^2(t') F_{Cp}(t')}{\sum_{B,C} \int \sigma_{Cp} \, \frac{M^2 dM^2 dt'}{(t' - m_C{}^2)^2} \, g_{A'BC}{}^2(t') F_{Cp}(t')} . \tag{2.1}$$

We shall assume a constant total Cp cross section (a reasonable approximation—again improved by taking ratios). $F_{Cp}(t)$ is the off-shell dependence of the Cp amplitude in Fig. 1, and $g_{ABC}(t')$ is the off-shell ABC vertex coupling.

The simplest choice is to take both g_{ABC} and F_{Cp} independent of t'. This is referred to as SFF (simple form factors). The second simplest choice is to keep g_{ABC} independent of t' but to use a form for $F_{Cp}(t')$ that one obtains if one calculates the full "box" diagram in Fig. 1, assuming no *internal* off-shell dependence. This is referred to as AFF (analytic form factors). Thus AFF is a first approximation to the off-shell dependence one obtains in a multiperipheral model with point vertices. That it is indeed a *first* approximation follows from the fact that as we vary an external mass on a ladder the effects damp out as we move down the ladder away from the relevant external leg. We shall use the AFF off-shell dependence calculated for asymptotic energies—which is reasonable since we calculate all our cross-section ratios at $s=400\ \mathrm{GeV}^2$. This analytic off-shell dependence is then

$$F_{Cp}(t')=f(t')/f(m_C^2)\ , \tag{2.2}$$

where

$$f(t')=\frac{1}{s^2}\int_{t_{\min}}^{s}\int\frac{M^2dt''dM^2}{(t''-m_{B'}^2)^2} \tag{2.3}$$

and B' is the first particle produced within the Cp discontinuity. Using the approximation in Eq. (A3) for $t_{\min}$, Eq. (2.3) may be evaluated analytically (the result is exact in the $s\to\infty$ limit).

The third and most sophisticated choice of form factors we shall use is taken from a phenomenological multiperipheral model.[9] This is referred to as PFF (phenomenological form factors). For moderate spacelike t' it is defined as

$$g_{ABC}^2(t')=g_{ABC}^2(1-t'/m_B^2)^{\delta}\ . \tag{2.4}$$

In this case it follows from strong-coupling solutions to the multiperipheral model that

$$F_{Cp}(t')\approx(1-t'/m_{B'}^2)^{-\alpha-1+\delta}\ , \tag{2.5}$$

where B' is the first produced particle in the lower blob in Fig. 1 (i.e., in σ_{Cp}), and α is the intercept of the leading trajectory—in our case $\alpha\simeq1$. Equation (2.5) is only a reliable approximation for $m_C^2 \lesssim m_{B'}^2$ and $t'\leqslant 0$, which will usually be the case in our calculations. There are a few instances in which this is not the case—these generally give small contributions—but we shall use Eq. (2.5) there anyway. We choose the normalization of Eq. (2.5) in reasonable ways [recall that Eq. (2.5) is valid for $t'\leqslant0$ so that it is not clear how to normalize it]. We shall state the normalization as we present the results in Secs. III and IV.

We use $\delta=1$ (this is the enhancing form factor referred to earlier). This implies that the form factor rises as $(-t')$ increases. In practice, of course, one expects such a rise to turn into a falloff as $|t'|$ becomes large; since, however, our integrals will be dominated by $|t'|$ not large (although not small), this need not be incorporated explicitly. The reason for choosing $\delta=1$ is phenomenological: In order to obtain total cross sections from the ABFST (Amati-Bertocchi-Fubini-Stanghellini-Tonin) multiperipheral model that do not fall with energy it is necessary to increase average couplings above their physical on-shell values.[8,9]

Note that the choice of $\delta=1$ and $m_{B'}^2=m_B^2=\langle m_{\text{cluster}}\rangle^2$ leads back to our first (SFF) choice (suggesting that it may not be as naive as it first appeared).

The remarks above define the variations on our model as used here. In the succeeding sections we employ this model on a variety of couplings: pure SU(3), broken SU(3), pure SU(4), and broken SU(4). All of these calculations will be performed numerically using exact kinematics. In Appendix A we present an analytic (approximate) calculation of $\sigma_{\psi p}$ in which we hope the physics of the model will be clarified. In particular, the nature of the suppression of integrals due to important $t_{\min}$ effects will be emphasized.

III. SU(3) AND $\sigma_{\pi p}/\sigma_{Kp}$

To begin we use the model within the SU(3) context to calculate $\sigma_{\pi p}/\sigma_{Kp}$ and $\sigma_{\eta p}/\sigma_{Kp}$, since $\sigma_{\pi p}/\sigma_{Kp}$ at any rate is well known experimentally, having a value

$$\sigma_{\pi p}/\sigma_{Kp}\approx1.3 \tag{3.1}$$

for both total and inelastic cross sections. Such a calculation is, of course, of interest in itself.

The appropriate equations derived from Eq. (2.1) are

$$\frac{\sigma_{\pi p}}{\sigma_{Kp}}=\frac{4I_{\pi\rho\pi}\sigma_{\pi p}+2I_{\pi K^*K}\sigma_{Kp}}{\frac{3}{2}I_{KK^*\pi}\sigma_{\pi p}+(\frac{3}{2}I_{K\rho K}+I_{K\phi K}+\frac{1}{2}I_{K\omega K})\sigma_{Kp}+\frac{3}{2}I_{KK^*\eta}\sigma_{\eta p}}\ ,$$
$$\frac{\sigma_{\eta p}}{\sigma_{\pi p}}=\frac{6I_{\eta K^*K}\sigma_{Kp}}{4I_{\pi\rho\pi}\sigma_{\pi p}+2I_{\pi K^*K}\sigma_{Kp}}\ , \tag{3.2}$$

where

$$I_{ABC}=\frac{1}{s^2}\int_{t_{\min}}^{(\sqrt{s}-m_B)^2}\int\frac{M^2dM^2dt'}{(t'-m_C^2)^2} \tag{3.3}$$

and $t_{\min}$ is the minimum momentum transfer for the appropriate process.

In Eq. (3.2) we have used pure-SU(3) couplings [since the SU(3) coupling breaking is small]. Note that we have used our first simple-minded choice for all off-shell dependences (SFF). We find

$$\frac{\sigma_{\pi p}}{\sigma_{Kp}}\approx1.7,\quad\frac{\sigma_{\eta p}}{\sigma_{Kp}}\approx0.7\ . \tag{3.4}$$

If we now solve Eq. (3.2) with conventionally broken couplings (about 5%), we obtained instead

$$\frac{\sigma_{\pi p}}{\sigma_{Kp}}\approx1.4,\quad\frac{\sigma_{\eta p}}{\sigma_{Kp}}\approx0.8\ . \tag{3.5}$$

Notice that the broken- and pure-SU(3) results differ from each other by around 20%.

Encouraged by these results we shall now proceed to calculate all the ratios in a full SU(4) scheme.

IV. SU(4) AND $\sigma_{\psi p}/\sigma_{\pi p}$

To solve the equations which result from the model of Fig. 1 in SU(4), several pseudoscalar mesons must be added to the conventional SU(3) mesons. We follow here the notation of Gaillard *et al.*[11] and call the singlets $F^{\pm}$ and the doublets D and $\bar{D}$. The $I_3=0$, $Y=0$ physical mesons now include the η_c. The masses are determined to be $m_D=m_F=2.2$ GeV and $m_{\eta_c}=2.7$ GeV. This latter value results from the solution to the mixing problem[12]; this solution is also used in determining the appropriate physical coupling constants. The coupling-constant determination is outlined in Appendix B.

We now proceed along the same lines as in Sec. III. Taking into account the various possible charge states, the model of Fig. 1 leads to a somewhat more complicated set of equations than in Eq. (3.2). The SU(3) equations become modified by charmed terms (for example, $\sigma_{\pi p}$ picks up a πD^*D term) and similar equations for σ_{Dp}, σ_{Fp}, and $\sigma_{\eta_c p}$ are obtained. We shall not write these down, as they are long and unedifying. We shall, however, keep all terms and allow for SU(4) coupling symmetry breaking which was not done in Ref. 4. We also investigate changes due to the various form factors described previously. This introduces the following complication. Consider Eq. (2.1). The off-shell behavior of σ_{Cp}, namely $F_{Cp}(t')$, depends on the vector meson D and exchanged pseudoscalar E at the *CDE* vertex inside σ_{Cp}. Thus we should calculate σ_{Ap} from Eq. (2.1) using the form of σ_{Cp} iterated so as to exhibit this behavior. We have thus solved the coupled equations using a numerical iteration procedure. All integrals were evaluated at $s=400$ GeV^2.

We thus obtain results for $\sigma_{Ap}/\sigma_{\pi p}$ with $A=D,F,\eta_c$ as well as modified results with $A=K,\eta,X_0$. One final assumption (surely reasonable) was to take η_c as effectively pure $c\bar{c}$ with zero $s\bar{s}$ mixing, since in the broken coupling schemes used this mixing turned out to be very small.

We now obtain $\sigma_{\psi p}$ by simply assuming

$$\sigma_{\psi p}=\sigma_{\eta_c p}. \tag{4.1}$$

The major corrections for Eq. (4.1) should come from the nonzero spin of the ψ, but we take comfort from the fact that $\sigma_{\rho p}\simeq\sigma_{\pi p}$ and ignore this point. We have also estimated the deviations from Eq. (4.1) due to SU(4) mass and coupling-constant variations along the same lines as our previous calculation (ignoring spin); the results are not appreciably changed.

With Eq. (4.1) we obtain the final results shown in Table I. We see that the results are relatively insensitive to the choice of the form factor but exclude one of the sets of broken couplings [set b]. (See Appendix B for a definition and derivation of the couplings referred to here.) We prefer set (a) of broken-SU(4) couplings over the pure-SU(4) results, partly because of the improved $\sigma_{Kp}/\sigma_{\pi p}$ value. [In Ref. 4 where pure SU(4) was used, the SU(3) cross sections were assumed to be unmodified.]

In comparing the results for $\sigma_{\psi p}$ with that obtained in $\gamma p\to\psi p$ photoproduction at Fermilab we should emphasize that the model prediction for $\sigma_{\psi p}$ must be modified to let one of the ψ's have zero mass. This decreases the value of $\sigma_{\psi p}$ owing to unequal-mass $t_{\min}$ effects. A numerical calculation of this effect has been performed with the result

$$(\sigma_{\psi p})_{\substack{\text{one leg}\\ \text{off-shell}}}\approx 0.6\sigma_{\psi p}. \tag{4.2}$$

We see that our results for $\sigma_{\psi p}$ including the factor 0.6 are on the order of the Fermilab values[3,13] for pure SU(4) and the set (a) of broken couplings, with set (a) again preferred. [In Ref. 4 the off-shell extrapolation in Eq. (4.2) was not included.]

V. ENERGY DEPENDENCE; ELASTIC SLOPES

A. Energy dependence

Our evaluation of $\sigma_{\psi p}$ was carried out at $s=400$ GeV^2. Since the final states involve massive particles, one might wonder as to the energy depen-

TABLE I. Ratios of cross sections $\sigma_{Ap}/\sigma_{\pi p}$, at $s=400$ GeV^2, where A are the SU(4) pseudoscalars. Also included is $\sigma_{\psi^{\text{off}}p}/\sigma_{\pi p}$ (which has been corrected for by taking one of the ψ legs to zero mass).

	A	K	η_c	ψ^{off}	η	X^0	D	F	Comments
SFF	pure SU(3)	0.59			0.41				...
SFF	broken SU(3)	0.71			0.57				
PFF	pure SU(4)	0.52	0.01	0.006	0.34	0.02	0.15	0.03	improbable
AFF	pure SU(4)	0.46	0.02	0.012	0.26	0.01	0.21	0.05	
PFF	broken SU(4)	0.59	0.05	0.03	0.40	0.03	0.35	0.17	preferred
AFF	couplings (a)	0.51	0.11	0.07	0.31	0.02	0.38	0.12	
PFF	broken SU(4)	0.88	5.4	3.2	0.77	0.05	14	11	excluded
AFF	couplings (b)	0.80	7.2	4.3	0.72	0.06	7.1	6.1	

dence of $\sigma_{\psi p}$. Will it involve a slow rise to the asymptotic limit, or will the threshold rise be very rapid? We shall address ourselves here to this qualitative question.

In Fig. 2 we see the numerically calculated energy dependence of $\sigma_{\psi p}$ using the exact flux factor. (The graph is in fact for $\sigma_{\eta_c p}$, but $\sigma_{\psi p}$ will be very similar.) Note that the cross section rapidly approaches its asymptotic value; the cross section reaches 75% of its asymptotic value at

$$s \approx 1.8 s_{\text{th}} = 1.8(2m_D + m_p)^2. \tag{5.1}$$

In practice we might expect the experimental threshold, s_{th}, to equal $m_{B_c} + m_D$ rather than the larger value $2m_D + m_p$, where B_c is a charmed baryon. Since the cross section for $\psi p \to B_c D$ is not expected to be small, just as $pp \to N\Delta$ is not small in the pp context, this would just involve an appropriate shift of the curve in Fig. 2 to lower energies. This is an important fact to keep in mind if one wishes to use the experimental threshold of $\sigma_{\psi p}$ to deduce the mass of the D's. Using $\sqrt{s_{\text{th}}} = 2m_D + m_p$ one would underestimate m_D [and perhaps be led to expect a broad $\psi'(3700) \to 2D$ decay width].

B. Elastic ψp slopes

In this subsection we shall present a qualitative argument that the t slope of $d\sigma/dt$ for ψp scattering should be flatter than that of ordinary cross sections.

Consider the decomposition of the ψp elastic amplitude as in Fig. 3(a) for $t \neq 0$. The t slope, $b_{\psi p}$, will have three contributions: $b_{\mu p}$ from the lower μp amplitude, ζ from the integration over t' and t'', and b' from the upper ($D\bar{D}$ production) blob. So,

$$b_{\psi p} = b_{\mu p} + b' + \zeta. \tag{5.2}$$

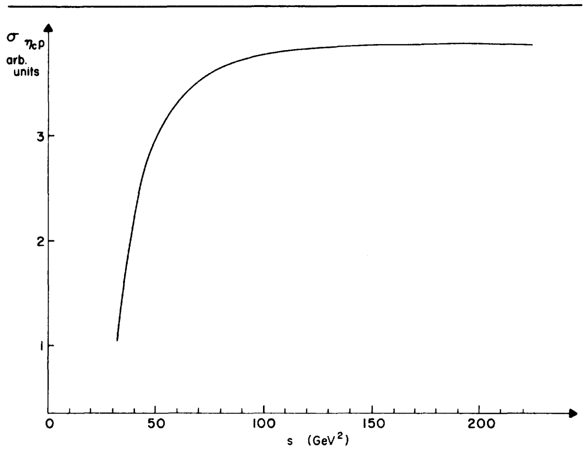

FIG. 2. Energy dependence of $\sigma_{\eta_c p}$, with an effective threshold given by $s_{\text{th}} = (2M_D + M_p)^2$.

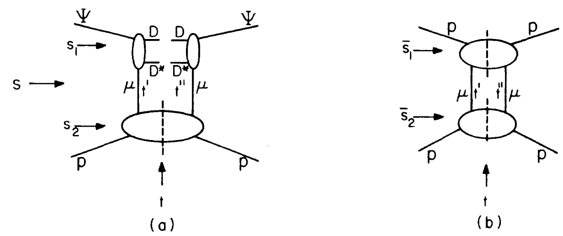

FIG. 3. (a) Peripheral decomposition of the ψp amplitude. s_1 is the invariant mass of the DD^* pair, s_2 that of the lower blob. (b) Peripheral decomposition of the pp amplitude.

If we consider energies in the region of $s \approx 400$ GeV^2, then the energies s_1 and s_2 through the upper and lower blobs, respectively, will be about equal. Consider now the pp elastic amplitude decomposed as in Fig. 3(b) so that $\bar{s}_1 \approx s_1$ and hence, at $s \sim 400$ GeV^2, $\bar{s}_2 \approx \bar{s}_1 \approx s_1$. Then

$$b_{pp} \approx 2b_{\mu p} + \zeta', \tag{5.3}$$

where $\zeta \approx \zeta'$ owing to similar kinematics in the loop integrals. Explicit numerical calculations suggest $\zeta \leq 0.5$ GeV^{-2}. Now, we expect $b' \approx 0$ both because the D exchanges are very massive and because the spin (vector in our model) carried by the produced D^* produces negligible effects because its mass is so large. This statement has been checked by explicit calculation. So

$$b_{\psi p} \approx \tfrac{1}{2}(b_{pp} + \tfrac{1}{2}). \tag{5.4}$$

Inserting a value $b_{pp} \simeq 5$ for $|t| > 0.2$ GeV^2 we obtain

$$\frac{d\sigma_{\psi p}}{dt} \underset{\substack{|t|>0.2 \\ s\sim 400\ \text{GeV}^2}}{\sim} e^{5t}. \tag{5.5}$$

Since in this energy range the corresponding exponent for πp and Kp scattering is around 8, we conclude that the elastic ψp cross section should be appreciably flatter in t than one finds with the usual mesons.

A detailed calculation has now been carried out by Jones within the context of the model (with PFF vertices) we presented earlier, leading to similar conclusions for a range of processes.[14]

VI. CONCLUSIONS

This paper involved the construction of a dynamical model specifically adapted to the calculation of relative cross sections within particular symmetry schemes. The results of the calculation are summarized in Table I for pure SU(4) and for two sets of broken-SU(4) couplings with two choices of off-shell behaviors (PFF, AFF) described in the text. The results for pure SU(4) and a third (SFF) off-shell behavior were given in Ref. 4, along with

SU(3)+SFF results.

The peripheral model (which is known to provide an adequate account of the bulk of production processes) appears to reasonably reproduce the response of production amplitudes to momentum transfers and masses. While this determines the kind of model we use, our confidence in the fine details of the model is less certain. Since one of the major sources of uncertainty concerns off-shell dependences, we have employed several such form factors. The results of Table I are reassuring in that they do not seem overly sensitive to such details.

We are forced by lack of experimental knowledge to accede to a theoretical picture of the particle spectrum. For better or worse, this is supplied here by the charm hypothesis. The SU(4) *mass* spectrum is probably essentially correct (see Appendix A). The number of mass-degenerate particles is specific to SU(4). A more serious problem concerns the calculation of the SU(4) couplings. The large mass breaking requires the use of broken couplings. Given the experimental uncertainties in the known decay widths, the range of possible values of the couplings is quite broad (compare, for example, our (a) and (b) sets of couplings). Naturally such difficulties will arise in any symmetry scheme we should choose to accommodate the ψ.

The completely dissimilar results from the not too dissimilar sets of broken coupling constants suggests that here we may have a practical tool for choosing between sets of couplings that appear *a priori* equally probable.

The "experimental" value for $\sigma_{\psi p}$ at Fermilab energies[1] is about 1 mb. This value is obtained assuming that the zero-mass $\gamma\psi$ coupling equals that at the ψ mass. Theoretical estimates suggest that if anything it will be smaller. Consequently, one expects $\sigma_{\psi p} \gtrsim 1$ mb. This is with one ψ leg at zero mass. If we compare this with Table I we see that the (b) set of broken couplings is obviously excluded, and the (a) set is preferred over the unbroken-SU(4) results within the context of the model.

Many of the uncertainties described above do not apply within a purely SU(3) context. Using the known masses and conventionally broken couplings we obtained a value for $\sigma_{\eta p}/\sigma_{Kp}$ within 15% of the experimental value. It should, however, be noted that within a full SU(4) calculation the agreement worsened.

In conclusion, a peripheral treatment of hadronic processes shows how the mass breakings in SU(3) or SU(4) multiplets lead to varying total cross sections. Superimposed on this is the effect of broken couplings, and we have seen how this can help to select out the "correct" set of couplings. A value of $\sigma_{\psi p} \sim 1$ mb is to be expected from the *mass* spectrum alone. Moreover, a relatively flat ψp differential cross section is to be expected.

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APPENDIX A: $\sigma_{\psi p}$ WITHOUT SU(4)—AN APPROXIMATE ANALYTIC CALCULATION

In the body of this paper we carried out a detailed analysis of $\sigma_{\psi p}$ within an SU(4) context. However, the relevance of charm is far from experimental vindication. Do we know enough about the "new particles" to carry out a more model-independent analysis? In this appendix we carry out a "minimal" such analysis; that is, we shall, where necessary, make the simplest assumptions consistent with the data. We also perform approximate analytic rather than numerical calculations so as to clarify what is important in determining the result. We hope this will make more transparent the reasons for our previous results also.

The enormous suppression of ψ decay and the strong suppression of $\psi' \to \psi +$ anything decays argue that the direct interaction between any number of ψ and the usual particles is strongly suppressed. However, the ψp cross section is of the order of mb rather that of μb (unlike $\sigma_{\psi p \to \psi X}$ which *is* of the order of μb). This implies the existence of a set of yet unobserved particles that couple "strongly" to both ψ and ordinary particles. Let us call them D's. The minimal assumption is that ψ couples to a pair of such D's. In that case both the threshold behavior of $\sigma_{\psi p}$ and the fact that $\psi(4100)$ is broad suggest that

$$m_D \approx 2 \text{ GeV}. \tag{A1}$$

The simplest further assumption is that the couplings $\{\psi DD\}$ and $\{DD(\text{ordinary particles})\}$ are typical strong couplings. So a ψp collision is just like any other except that it involves the production of a pair of massive particles. What we need then is a model that will treat more or less realistically the way production amplitudes respond to the masses of the particles involved. The best current candidate is the multiperipheral model, hence its use in the body of the paper and here. We use the same type of model as used previously; details should not be too important to the results.

A typical contribution to $\sigma_{\psi p}$ is then as in Fig. 1

with $A=\psi$, $B=D$, $C=D$. This compares with a typical contribution to $\sigma_{\pi p}$ of $A=\pi$, $B=\rho$, $C=\pi$. Assuming SFF off-shell dependences

$$\frac{\sigma_{\psi p}}{\sigma_{\pi p}} \approx \frac{\sigma_{Dp}\int M^2 dM^2 dt/(t-m_D{}^2)^2}{\sigma_{\pi p}\int M^2 dM^2 dt/(t-m_\pi{}^2)^2}$$

$$\approx \frac{\sigma_{Dp}}{\sigma_{\pi p}} \frac{\displaystyle\int_0^1 \frac{x\,dx}{-m_D{}^2 x/(1-x)+m_\psi{}^2 x-m_D{}^2}}{\displaystyle\int_0^1 \frac{x\,dx}{-m_\rho{}^2 x/(1-x)+m_\pi{}^2 x-m_\pi{}^2}} \tag{A2}$$

where $x=M^2/s$ and we have used the usual (here excellent) approximation for $t_{\min}$

$$t_{\min} \approx -\frac{m_B{}^2 x}{1-x}+m_A{}^2 x. \tag{A3}$$

A typical value in the integrands of Eq. (A2) is $x \approx \frac{1}{2}$, giving

$$\frac{\sigma_{\psi p}}{\sigma_{\pi p}} = \frac{\sigma_{Dp}}{\sigma_{\pi p}} \frac{m_\rho{}^2}{m_D{}^2}. \tag{A4}$$

Similarly,

$$\frac{\sigma_{Dp}}{\sigma_{\pi p}} \approx \frac{\sigma_{\pi p}\displaystyle\int_0^1 \frac{x\,dx}{-m_D{}^2 x/(1-x)+m_D{}^2 x-m_\pi{}^2}}{\sigma_{\pi p}\displaystyle\int_0^1 \frac{x\,dx}{-m_\rho{}^2 x/(1-x)+m_\pi{}^2 x-m_\pi{}^2}}$$

$$\approx \frac{2m_\rho{}^2}{m_D{}^2}. \tag{A5}$$

(A4) and (A5) give

$$\frac{\sigma_{\psi p}}{\sigma_{\pi p}} \approx 2\left(\frac{m_\rho{}^2}{m_D{}^2}\right)^2 \approx \tfrac{1}{25} \Rightarrow \sigma_{\psi p} \approx 1 \text{ mb}. \tag{A6}$$

Note how the masses come in to determine the values of the integrals; in Eq. (A2) we see that both the exchanged and produced masses are important. It is clear now why $\sigma_{\pi p} > \sigma_{Kp}$. With an incident strange particle one more often has strange particles produced and exchanged, and strange particles are heavier than corresponding nonstrange particles.

Clearly the value of $\sigma_{\psi p}$ in Eq. (A6) will alter one way or another with modifications in our "minimal" assumptions. The fact, however, that the experimental cross section is of this order strongly suggests that the strong interactions of the ψ—whatever its basic nature—are entirely conventional.

APPENDIX B: CALCULATION OF THE VECTOR-PSEUDOSCALAR-PSEUDOSCALAR COUPLING CONSTANTS IN BROKEN SU(4)

We base this calculation on the use of current algebra, the partially conserved axial-vector current (PCAC) hypothesis, and a specific form of the SU(4)-symmetry-breaking Hamiltonian. We suppose that the total Hamiltonian density can be written as

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}',$$

where $\mathcal{H}_0$ conserves the SU(4) symmetry, while $\mathcal{H}'$ breaks it. In terms of the scalar densities, $\mathcal{H}_0 = a_0 u^0$ and $\mathcal{H}' = a_8 u^8 + a_{15} u^{15}$. Considering the process $V_i(p) \to P_j(q) + P_k(q')$, where i,j,k are the SU(4) indices and p,q,q' the four-momenta, we find that

$$\lim_{q\to 0}[2q_0(2\pi)^3]^{1/2}\langle P_j(q), P_k(q')|\mathcal{H}|V_i(p)\rangle = i f_j^{-1}\left(\frac{a_0}{\sqrt{2}} f_{jki} + a_8 d_{j8l} f_{lki} + a_{15} d_{j\,15\,l} f_{lki}\right) G, \tag{B1}$$

where f_j is the decay constant for the pseudoscalar P_j and G is the SU(4) reduced matrix element which contains the Lorentz structure also. In the derivation of Eq. (B1), we have used the PCAC condition and the equal-time commutation relations of the axial-vector current with the scalar densities. If we denote by V and P the 4×4 vector and pseudoscalar matrices of SU(4), respectively, then it is easily checked that the vector-pseudoscalar-pseudoscalar interaction Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{\rm int}(VPP) = {} & iG_0 \mathrm{Tr}(P\overleftrightarrow{\partial}_\mu P V^\mu) \\ & + iG_8 \mathrm{Tr}(\{P,\lambda_l\}\lambda_8)\overleftrightarrow{\partial}_\mu \mathrm{Tr}([P,\lambda_l]V^\mu) \\ & + iG_{15}\mathrm{Tr}(\{P,\lambda_l\}\lambda_{15})\overleftrightarrow{\partial}_\mu \mathrm{Tr}([P,\lambda_l]V^\mu).\end{aligned} \tag{B2}$$

More specifically, the matrices V and P are given by

$$V = \begin{bmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & F^{*-} \\ D^{*0} & D^{*+} & F^{*+} & \phi_c \end{bmatrix}, \quad P = \begin{bmatrix} \frac{\eta_0}{2}+\frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}}+\frac{\eta_{15}}{2\sqrt{3}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & \frac{\eta_0}{2}-\frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}}+\frac{\eta_{15}}{2\sqrt{3}} & K^0 & D^- \\ K^- & \bar{K}^0 & \frac{\eta_0}{2}-\frac{2\eta_8}{\sqrt{6}}+\frac{\eta_{15}}{2\sqrt{3}} & F^- \\ D^0 & D^+ & F^+ & \frac{\eta_0}{2}-\frac{3\eta_{15}}{2\sqrt{3}} \end{bmatrix}.$$

The particle notation (excepting the η_i's) is that of the paper of Gaillard *et al.*[11] Note that in writing (B2), we have set $f_\pi = f_K = f_{\eta_8} = f_D = f_F = f_{\eta_{15}}$, which follows on extending the Gell-Mann–Oakes–Renner treatment to the breaking of SU(4). By evaluating the traces in (B2), we obtain expressions for the coupling constants in terms of G_0, G_8, and G_{15}. The fact that we have exactly three pieces of data (decay widths of ρ, K^*, and ϕ) does not determine the three G's separately. To do so, we have to employ the result according to which the vector-mass-mixing formula fixes the ratio G_{15}/G_8.[12] This result may be used in conjunction with the width equations, taken two at a time, and we end up with three sets of solutions for the unknowns G_0^V, G_8, and G_{15}. Actually, because of the errors on the experimental values of the ρ, K^*, and ϕ widths, one can introduce a type of χ^2 minimization procedure. For example, starting with the ρ and K^* equations and the above values of G_0, G_8, and G_{15}, we seek the values of G_0, G_8, and G_{15} in the vicinity which minimize the quantity

$$\chi^2 = \sum_{(i)=\rho\pi\pi, K^*K\pi, \phi K\bar{K}} [(g_{(i)}^{\text{exp}} - g_{(i)}^{\text{cal}})/\Delta g_{(i)}^{\text{exp}}].$$

Doing this for the pairs (K^*, ϕ) and (ϕ, ρ) as well, we obtain the three solutions

(a) $G_0 = 1.6$, $G_8 = 0.005$, $G_{15} = 0.11$, $\chi^2 = 3.5$;

(b) $G_0 = 2.9$, $G_8 = 0.022$, $G_{15} = 0.48$, $\chi^2 = 5.2$;

(c) $G_0 = 0.6$, $G_8 = 0.025$, $G_{15} = 0.55$, $\chi^2 = 19$.

In the text we consider solutions (a) and (b) only since the solution (c), in addition to having the largest χ^2, gives comparable values for the symmetry-conserving and symmetry-breaking pieces, conflicting with the notion of regarding the symmetry-breaking piece as a relatively small perturbation on the symmetry-conserving one.

Note that the particles η_0, η_8, and η_{15} are essentially mathematical objects. The physical particles X^0, η, and η_c used in the text are suitable linear combinations of the η_i ($i = 0, 8, 15$) and are obtained by diagonalization of the mass matrix.[12]

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