PHYSICAL REVIEW D VOLUME 15, NUMBER 1 1 JANUARY 1977

Sum rules for the single-pion-observed inclusive reactions induced by electromagnetic currents or hadronic weak currents

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(Received 5 April 1976; revised manuscript received 10 September 1976)

By using partial conservation of the axial-vector current and a technique of Dicus, Jackiw, and Teplitz, sum rules for the inclusive reactions induced by electromagnetic currents or hadronic weak currents where the single pion is observed are derived.

I. INTRODUCTION

Recently this author proposed sum rules for the inclusive reactions such as $\gamma_V + N \to \pi^{\pm} + X$ (anything), where N is a stable hadron.[1] The main assumptions are the following: (i) partial conservation of axial-vector currents (PCAC), which is nothing but the assumption of smooth extrapolation of the $q^2 = 0$ amplitude to the on-shell pion amplitude as is usually stated[2]; (ii) the Ward identity[2] (we assume here cancellation between the seagull terms and the Schwinger terms; since the axial-vector current is included, the assumption may not be adequate, but we hope the corrections would be small compared to the leading terms); and (iii) that the currents are composed of spin-$\frac{1}{2}$ quark partons (we use canonical quantization at the null plane). This is the assumption of the algebra of currents or bilocal currents (AC or ABC) restricted at the null plane and abstracted from a free-quark model or a gluon-quark model.[3] Thus, the sum rules will be useful as a test of ABC since we take the connected matrix elements and, further, since the virtual-photon mass is fixed. Though the method is explained in previous papers, we repeat it in Sec. II for the purpose of extending the analysis and to make this article self-contained. In Sec. III the method is applied to the inclusive reactions induced by hadronic weak currents. In the Appendix we discuss the high-energy limit by using the Ward identity and the light-cone Bjorken-Johnson-Low theorem (LCBJL).[4] Throughout this paper we use the SU(4) charm model[5] as an illustration since the discussions concerning the symmetry properties of hadrons or leptons are not decisive.

II. THE DERIVATION OF THE SUM RULES

We consider the inclusive reactions $\gamma_V + N \to \pi^{\pm} + X$ (see Fig. 1). According to a usual technique[2] the hadronic part of the process will be given as

$$T^{\mu\nu} = (m_\pi^2 - q^2)^2 \times \int d^4x\, d^4y\, d^4z \exp[-iq\cdot(x-z) + ik\cdot y] \times \langle p | T^*(\phi_{\pi^+}(x) J^\mu(y)) T(\phi_{\pi^+}(z) J^\nu(0)) | p \rangle \,, \tag{2.1}$$

where $J^\mu(x)$ is the electromagnetic current, completeness of the intermediate states is assumed, and the average over spin is understood. Now we assume the PCAC relation[2]

$$\partial_\mu J_a^{5\mu}(x) = m_\pi^2 F \phi_\pi(x) + O(e) \,, \tag{2.2}$$

where $F = \sqrt{2} f_\pi$ when $a = 1 \pm i2$, $F = f_\pi$ when $a = 3$, and $O(e)$ means the contribution from the vector potential $A^\mu(x)$ as far as the electromagnetic interaction is concerned. We neglect this term whenever the first term in Eq. (2.2) provides the main contribution. Further, there may be an anomaly, but we hope the contribution from an anomaly would be small compared to the leading term if it exists. Therefore, we neglect it and rewrite Eq. (2.1) as

$$\begin{aligned} T^{\mu\nu}_{abcd} = \frac{(m_\pi^2 - q^2)^2}{2 m_\pi^4 f_\pi^2} \int d^4x\, d^4y\, d^4z \exp[-iq\cdot(x-z) + ik\cdot y] \\ \times \{ q_\lambda q_\rho \langle p | [T^*(J_{a'}^{5\lambda}(x), J_b^\mu(y)), T(J_c^{5\rho}(z), J_d^\nu(0))] | p \rangle - iq_\rho \delta(x^+ - y^+) \langle p | [[J_a^{5+}(x), J_b^\mu(y)], T(J_c^{5\rho}(z), J_d^\nu(0))] | p \rangle \\ - iq_\lambda \delta(z^+) \langle p | [T^*(J_{a'}^{5\lambda}(x), J_b^\mu(y)), [J_c^{5+}(z), J_d^\nu(0)]] | p \rangle \\ - \delta(x^+ - y^+) \delta(z^+) \langle p | [[J_a^{5+}(x), J_b^\mu(y)], [J_c^{5+}(z), J_d^\nu(0)]] | p \rangle \} \,, \end{aligned} \tag{2.3}$$

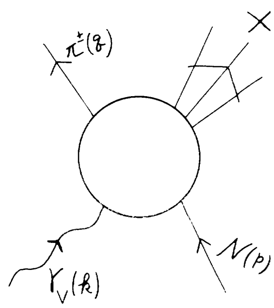

FIG. 1. The inclusive inelastic scattering where the single pion is observed: $\gamma_\nu + N \rightarrow \pi^{\pm} + X$.

where $a^* = a' = c = 1 - i2$, $b = d$ specify electromagnetic currents, and a spectral condition is used to obtain the commutator. Since $T^{\mu\nu}_{abcd}$ satisfies gauge invariance and the average over spin is taken, it will be given as

$$T^{\mu\nu} = G^{\mu\nu} V_L + \left[-k^2 P^\mu P^\nu + \frac{(p\cdot k)^2}{k^2} G^{\mu\nu} \right] V_2 + (P^\mu K^\nu + P^\nu K^\mu) V_3 + K^\mu K^\nu V_4 , \tag{2.4}$$

where

$$G^{\mu\nu} = k^\mu k^\nu - k^2 g^{\mu\nu} ,$$
$$P^\mu = p^\mu - \frac{p\cdot k}{k^2} k^\mu , \tag{2.5}$$

and

$$K^\mu = q\cdot k\, k^\mu - k^2 q^\mu ,$$

and V_i ($i = 1, \ldots, 4$) is a function of $p\cdot q$, q^2, $q\cdot k$, k^2, and $p\cdot k$. According to a usual technique we can investigate the singular terms as $q^\mu \rightarrow 0$ on the right-hand side of Eq. (2.3) and find that we may set $q^+ = 0$, $\vec{q}_\perp = 0$ safely. At this limit $q^2 = 2q^+q^- - \vec{q}_\perp^{\,2} = 0$, $p\cdot q = p^+ q^-$, $q\cdot k = q^- k^+$, and q^- remains as an arbitrary parameter. We discuss the fact by using a simple Ward identity in the Appendix. Now we continue to discuss the case $q^- \rightarrow 0$.
We take the currents as

$$J^\mu_a(x) = \bar{q}(x)\gamma^\mu \tfrac{1}{2}\lambda_a q(x) ,$$
$$J^{5\mu}_a = \bar{q}(x)\gamma^\mu\gamma^5 \tfrac{1}{2}\lambda_a q(x) . \tag{2.6}$$

Here we can apply the method of Adler's consistency conditions.[2] Taking the discontinuity over X and using completeness, we get the following as $q^- \rightarrow 0$:

$$G^{\mu\nu} V_L + \left[-k^2 P^\mu P^\nu + \frac{(p\cdot k)^2}{k^2} G^{\mu\nu} \right] V_2$$
$$= A^{\mu\nu} - \frac{1}{2f_\pi^{\,2}} \int d^4x\, d^4y\, d^4z \exp(ik\cdot y)\, \delta(x^+ - y^+)\, \delta(z^+) \langle p | [[J^{5+}_a(x), J^\mu_b(y)], [J^{5+}_c(z), J^\nu_d(0)]] | p \rangle , \tag{2.7}$$

where

$$\begin{aligned} A^{\mu\nu} = {} & \frac{1}{8f_\pi^{\,2}(p^+)^2} \int d^4y \exp(ik\cdot y) [\langle p | J^{5+}_a(0) | N(p)\rangle \langle N(p) | J^\mu_b(y) J^\nu_d(0) | N(p)\rangle \langle N(p) | J^{5+}_c(0) | p\rangle \\ & - \langle p | J^{5+}_c(0) | N(p)\rangle \langle N(p) | J^\nu_d(0) J^\mu_b(y) | N(p)\rangle \langle N(p) | J^{5+}_a(0) | p\rangle] \\ & - \frac{1}{4f_\pi^{\,2} p^+} \int d^4x\, d^4y \exp(ik\cdot y)\, \delta(x^+ - y^+) \{ \langle p | [J^{5+}_a(x), J^\mu_b(y)] J^\nu_d(0) | N(p)\rangle \langle N(p) | J^{5+}_c(0) | p\rangle \\ & + \langle p | J^{5+}_c(0) | N(p)\rangle \langle N(p) | J^\nu_d(0) [J^{5+}_a(x), J^\mu_b(y)] | p\rangle \} \\ & + \frac{1}{4f_\pi^{\,2} p^+} \int d^4y\, d^4z \exp(ik\cdot y)\, \delta(z^+) \{ \langle p | J^{5+}_a(0) | N(p)\rangle \langle N(p) | J^\mu_b(y) [J^{5+}_c(z), J^\nu_d(0)] | p\rangle \\ & + \langle p | [J^{5+}_c(z), J^\nu_d(0)] J^\mu_b(y) | N(p)\rangle \langle N(p) | J^{5+}_a(0) | p\rangle \} , \end{aligned} \tag{2.8}$$

and $q^2 = 0$, $q\cdot k = 0$, $p\cdot q = 0$. Hereafter we write $V_i = V_i(k^2, \nu = p\cdot k)$ and assume it to be smoothly continued to the on-shell pion form factor at threshold. Since Eq. (2.7) is odd under exchange of $a \leftrightarrow c$, $b \leftrightarrow d$, $\mu \leftrightarrow \nu$, and $k \leftrightarrow -k$, we get the same crossing property for V_L or V_2 as in $eP \rightarrow X$. $A^{\mu\nu}$ is the contribution from pole terms which are nonzero only when the attachment of the proper vertex of $J^{5\lambda}_a$ in the initial target does not change its mass and is not forbidden by parity or isospin. The experimentally interesting case is that of the nucleon target. Thus we evaluate $A^{\mu\nu}$ only in that case, and N in Eq. (2.8) will be a suitable nucleon state which will be determined by the particular process we consider. By using a technique to get the fixed-mass sum rules[6] we integrate over k^-, change the variable from k^- to $\nu = p\cdot k$, assume we can interchange setting $k^+ = 0$ and performing the ν integration,[7] and get

$$\frac{1}{2\pi p^+}\int d\nu\left\{G^{\mu\nu}V_L+\left[-k^2P^\mu P^\nu+\frac{(p\cdot k)^2}{k^2}G^{\mu\nu}\right]V_2\right\}$$

$$=\frac{1}{2\pi}\int dk^- A^{\mu\nu}\Big|_{k^+=0}-\frac{1}{2f_\pi^2}\int d^4x\,d^4y\,d^4z\,\exp(-i\vec{k}_\perp\cdot\vec{y}_\perp)\delta(x^+-y^+)\delta(z^+)\delta(y^+)\langle p|[[J_a^{5+}(x),J_b^\mu(y)],[J_c^{5+}(z),J_d^\nu(0)]]|p\rangle\,. \quad (2.9)$$

The right-hand side of Eq. (2.9) is restricted at the null plane $x^+=y^+=z^+=0$. Then we use the canonical quantization at the null plane[8]

$$\{q_{(+)}^\dagger(x),q_{(+)}(0)\}|_{x^+=0}=\frac{1}{\sqrt{2}}\Lambda_{(+)}\delta(x^-)\delta^2(\vec{x}_\perp),\ \text{etc.} \quad (2.10)$$

By taking $\mu=\nu=+$, we get

$$\int_0^\infty d\nu\,\vec{k}_\perp^{\,2}[V_2^+(\nu,-\vec{k}_\perp^{\,2})-V_2^-(\nu,-\vec{k}_\perp^{\,2})]=\frac{2\pi}{f_\pi^2}\Gamma_3\,, \quad (2.11)$$

where $\pm$ means $\gamma_V+N\to\pi^\pm+X$, respectively, and

$$\langle p|J_a^\mu(0)|p\rangle=p^\mu\Gamma_a\,. \quad (2.12)$$

As already pointed out, we get the additional contribution $-4\pi g_A^2(0)\Gamma_3/f_\pi^2$ on the right-hand side of Eq. (2.11) in the case of the nucleon target. As in the case of the Cabibbo-Radicati sum rule,[2,9] the sum rule (2.11) will be transformed into the more useful photoproduction sum rule: Consider the case when N is the proton. By use of current conservation we rewrite $T^{++}=[(p^+)^2/\nu^2]k_ik_jT^{ij}$, take the frame $\vec{p}_\perp\cdot\vec{k}_\perp=0$, separate the neutron Born term, take the derivative with respect to $\vec{k}_\perp^{\,2}$, set $\vec{k}_\perp^{\,2}=0$, and get finally

$$2g_A(0)g_A'(0)+\frac{f_\pi^2}{2\pi^2\alpha}\int\frac{d\omega}{\omega}[\sigma^{\pi^+}(\omega)-\sigma^{\pi^-}(\omega)]=0\,, \quad (2.13)$$

where $g_A(0)$ is the nucleon axial-vector coupling constant, $g_A'(0)=(d/dt)g_A(t)|_{t=0}$, α is the fine-structure constant, ω is the laboratory energy of the photon, and σ^π is defined for the reaction $\gamma+p\to\pi^\pm+X$ as

$$\sigma^\pi=2(2\pi)^3q^0\frac{d\sigma}{d^3q} \quad (2.14)$$

where the right-hand side is evaluated at threshold for the single-pion emission. Next we take $\mu=+,\nu=i$ and get

$$\int_0^\infty d\nu\,\nu[V_2^+(\nu,-\vec{k}_\perp^{\,2})+V_2^-(\nu,-\vec{k}_\perp^{\,2})]$$
$$=\frac{\pi}{4f_\pi^2}\int_{-\infty}^\infty d\alpha\,\epsilon(\alpha)\left[\sqrt{2}A_0(\alpha,0)+\tfrac{2}{3}\sqrt{3}A_8(\alpha,0)+\frac{\sqrt{6}}{3}A_{15}(\alpha,0)\right], \quad (2.15)$$

where

$$\left\langle p\left|\frac{1}{2i}[\bar{q}(x)\gamma^\mu\tfrac{1}{2}\lambda_a q(0)-\bar{q}(0)\gamma^\mu\tfrac{1}{2}\lambda_a q(x)]\right|p\right\rangle=p^\mu A_a(p\cdot x,x^2)+x^\mu\bar{A}_a(p\cdot x,x^2)\,. \quad (2.16)$$

Further, in the case of the nucleon target we get the additional contribution

$$-\frac{(5-2I_3)\pi g_A^2(0)}{6f_\pi^2}\int_{-\infty}^\infty d\alpha\,\epsilon(\alpha)A(\alpha,0)+\frac{\pi g_A(0)I_3}{3f_\pi^2}\int_{-\infty}^\infty d\alpha\,\epsilon(\alpha)A^5(\alpha,0)$$

on the right-hand side of Eq. (2.15), where $A_a^5(\alpha,0)$ is defined as in Eq. (2.16) for the axial-vector bilocal currents.[10] The sum rule (2.15) will be useful as a test of ABC since $\int_{-\infty}^\infty d\alpha\,\epsilon(\alpha)A(\alpha,0)$ will be measured by the deep-inelastic electron scattering.[6] We get, for example, the following when N is the nucleon:

$$\int_0^\infty d\nu\,\nu\{\textstyle\sum V_2^+(\nu,-\vec{k}_\perp^{\,2})+\sum V_2^-(\nu,-\vec{k}_\perp^{\,2})\}$$
$$=\frac{1}{3f_\pi^2}[9-5g_A^2(0)]\int_0^\infty d\nu\frac{\nu}{\vec{k}_\perp^{\,2}}W_2^{ep}(\nu,-\vec{k}_\perp^{\,2})$$
$$\to\frac{1}{3f_\pi^2}[9-5g_A^2(0)]\int_0^1\frac{d\omega}{2\omega^2}F_2^{ep}(\omega)\,. \quad (2.17)$$

(if the scaling holds), where $\omega=-k^2/2pk$ and $F_2^{ep}(\omega)$ is the famous scaling function in the deep-inelastic electron scattering. Finally we take $\mu=+\nu=-$ and get

$$\int_0^\infty d\nu\,\vec{k}_\perp^{\,2}[V_L^+(\nu,-\vec{k}_\perp^{\,2})-V_L^-(\nu,-\vec{k}_\perp^{\,2})]=\frac{\pi}{f_\pi^2}\int_{-\infty}^\infty d\alpha\,\epsilon(\alpha)\bar{S}_3(\alpha,0)\,, \quad (2.18)$$

where

$$\langle p|\tfrac{1}{2}[\bar{q}(x)\gamma^\mu\tfrac{1}{2}\lambda_a q(0)+\bar{q}(0)\gamma^\mu\tfrac{1}{2}\lambda_a q(x)]|p\rangle=p^\mu S_a(p\cdot x,x^2)+x^\mu\bar{S}_a(p\cdot x,x^2)\,. \quad (2.19)$$

The sum rule (2.18) will also be useful as a test of ABC, but will be more model-dependent than the sum rule (2.15). Further, since the measurement of $\bar{S}(\alpha,0)$ will be done through the scaling function $G_L^{[ab]}(\omega)$ (see Ref. 6) which is defined as

$$\tilde{F}_L^{ab}(\omega, q^2) \xrightarrow[q^2\to\infty]{} F_L^{ab}(\omega) + \frac{1}{q^2} G_L^{ab}(\omega)\,, \tag{2.20}$$

an experimental check of the sum rule (2.18) will be more difficult than the sum rule (2.15). Therefore, we investigate the sum rules derived from cases $\mu = \nu = +$ and $\mu = +, \nu = i$ in other reactions.

III. REACTIONS INDUCED BY HADRONIC WEAK CURRENTS

Since the derivation is the same as in the case of electromagnetic currents, we only write the different points and the results. Further, we do not write the contributions from the pole terms $A^{\mu\nu}$ in A or B or C, since we explicitly discuss the case of the nucleon target in D.

A. $\nu_l + N \to l + \pi^0 + X$ or $\bar{\nu}_l + N \to \bar{l} + \pi^0 + X$

The hadronic part of the reaction will be given as $q^- \to 0$,

$$T^{\mu\nu} = -\frac{1}{f_\pi^2}\int d^4x\, d^4y\, d^4z\, \exp(ik\cdot y)\delta(x^+ - y^+)\delta(z^+)\langle p|[[J_a^{5+}(x), V_b^{\mu}(y)], [J_c^{5+}(z), V_d^{\nu}(0)]]|p\rangle\,, \tag{3.1}$$

where $a = c = 3$, $b^* = d$ specify charged weak currents, and $V_d^{\mu}(x)$ will be given as

$$V_\alpha^{\mu}(x) = \bar{q}(x)\gamma^{\mu}(1-\gamma_5)C_+ q(x) \tag{3.2}$$

$$C_+ = \begin{bmatrix} 0 & \cos\theta_C & \sin\theta_C & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\sin\theta_C & \cos\theta_C & 0 \end{bmatrix}, \tag{3.3}$$

and θ_C is the Cabibbo angle.[2,5] Since the current is not conserved $T^{\mu\nu}$ will be given as $q^- \to 0$,

$$T^{\mu\nu} = -g^{\mu\nu}W_1 + p^{\mu}p^{\nu}W_2 - i\epsilon^{\mu\nu\alpha\beta}p_\alpha k_\beta W_3 + (p^{\mu}k^{\nu} + p^{\nu}k^{\mu})W_4 + (p^{\mu}k^{\nu} - p^{\nu}k^{\mu})W_5 + k^{\mu}k^{\nu}W_6\,, \tag{3.4}$$

and W_i is a function of $q^2 = 0$, $q\cdot k = 0$, $p\cdot q = 0$, $p\cdot k$, and k^2. Then we write $W_i = W_i(p\cdot k, k^2)$ and assume it to be smoothly continued to the on-shell pion form factor at threshold; further, we neglect W_4 to W_6 since they do not contribute to the cross section in the zero-mass approximation. By taking $\mu = \nu = +$, we get

$$\int_0^\infty d\nu\,[W_2^+(\nu, -\vec{k}_\perp^{\,2}) - W_2^-(\nu, -\vec{k}_\perp^{\,2})] = -\frac{8\pi}{f_\pi^2}\left[\cos^2\theta_C\,\Gamma_3 + \tfrac{1}{4}\sin^2\theta_C\left(\Gamma_3 + \frac{\sqrt{3}}{3}\Gamma_8 - \frac{\sqrt{6}}{3}\Gamma_{15}\right) - \tfrac{1}{2}\sin\theta_C\cos\theta_C(\Gamma_6 + \Gamma_9)\right], \tag{3.5}$$

where + or − means the reaction $\nu_l + N \to l + \pi^0 + X$ or $\bar{\nu}_l + N \to \bar{l} + \pi^0 + X$, respectively. By taking $\mu = +, \nu = i$, we get

$$\begin{aligned}\int_0^\infty d\nu\,[W_3^+(\nu, -\vec{k}_\perp^{\,2}) + W_3^-(\nu, \vec{k}_\perp^{\,2})] &= -\frac{\pi}{f_\pi^2}\left\{\cos^2\theta_C\int_{-\infty}^\infty d\alpha\,\epsilon(\alpha)\left[\sqrt{2}A_0(\alpha,0) + \tfrac{2}{3}\sqrt{3}A_8(\alpha,0) + \frac{\sqrt{6}}{3}A_{15}(\alpha,0)\right]\right.\\ &\quad\left. + \frac{\sqrt{2}}{2}\sin^2\theta_C\int_{-\infty}^\infty d\alpha\,\epsilon(\alpha)A_0(\alpha,0) + \sin\theta_C\cos\theta_C\int_{-\infty}^\infty d\alpha\,\epsilon(\alpha)[A_6(\alpha,0) - A_9(\alpha,0)]\right\}.\end{aligned} \tag{3.6}$$

B. $\nu_l + N \to l + \pi^+ + X$ or $\bar{\nu}_l + N + \bar{l} + \pi^- + X$

In Eq. (3.1) we only change $f_\pi^2 \to 2f_\pi^2$ and $a = c = 3 \to a^* = c = 1 - i2$. By taking $\mu = \nu = +$, we get

$$\int_0^\infty d\nu\,[W_2^{++}(\nu, -\vec{k}_\perp^{\,2}) - W_2^{--}(\nu, -\vec{k}_\perp^{\,2})] = \frac{4\pi}{f_\pi^2}\left[\cos\theta_C\sin\theta_C(\Gamma_6 + \Gamma_9) + \sin^2\theta_C\left(\Gamma_3 - \frac{\sqrt{3}}{3}\Gamma_8 + \frac{\sqrt{6}}{3}\Gamma_{15}\right)\right], \tag{3.7}$$

where ++ or −− means $\nu_l+N\to l+\pi^+ +X$ or $\bar\nu_l+N\to\bar l+\pi^- +X$, respectively. By taking $\mu=+$, $\nu=i$, we get

$$\int_0^\infty d\nu\,[W_3^{++}(\nu,-\vec{k}_\perp^{\,2})+W_3^{--}(\nu,-\vec{k}_\perp^{\,2})]$$
$$=-\frac{\pi}{f_\pi^2}\Big\{\cos^2\theta_C\int_{-\infty}^{\infty}d\alpha\,\epsilon(\alpha)\Big[\sqrt2 A_0(\alpha,0)+\tfrac{2}{3}\sqrt3 A_8(\alpha,0)+\frac{\sqrt6}{3}A_{15}(\alpha,0)\Big]$$
$$+\sqrt2\sin^2\theta_C\int_{-\infty}^{\infty}d\alpha\,\epsilon(\alpha)A_0(\alpha,0)+\sin\theta_C\cos\theta_C\int_{-\infty}^{\infty}d\alpha\,\epsilon(\alpha)[A_6(\alpha,0)-A_9(\alpha,0)]\Big\}\,. \tag{3.8}$$

C. $\nu+N\to\nu+\pi^\pm+X$

In Eq. (3.1) we change $f_\pi^2\to 2f_\pi^2$, $a=c=3\to a^*=c=1-i2$, and $V_a^\mu(x)\to V_{\text{neutral}}^\mu(x)$, where

$$V_{\text{neutral}}^\mu(x)=\bar q(x)\gamma^\mu C_V q(x)+\bar q(x)\gamma^\mu\gamma^5 C_A q(x)\,, \tag{3.9}$$

$$C_V=\begin{bmatrix}\frac12-\frac43\sin^2\theta_W & 0 & 0 & \\ 0 & -\frac12+\frac23\sin^2\theta_W & 0 & \\ 0 & 0 & -\frac12+\frac23\sin^2\theta_W & \\ 0 & 0 & 0 & \frac12-\frac43\sin^2\theta_W\end{bmatrix}\,, \tag{3.10}$$

$$C_A=\begin{bmatrix}-\frac12 & 0 & 0 & 0\\ 0 & \frac12 & 0 & 0\\ 0 & 0 & \frac12 & 0\\ 0 & 0 & 0 & -\frac12\end{bmatrix}\,,$$

and θ_W is the Weinberg angle.[5] By taking $\mu=\nu=+$, we get

$$\int_0^\infty d\nu\,[W_2^{0+}(\nu,-\vec{k}_\perp^{\,2})-W_2^{0-}(\nu,-\vec{k}_\perp^{\,2})]=\frac{2\pi}{f_\pi^2}[(1-2\sin^2\theta_W)^2+1]\,\Gamma_3\,, \tag{3.11}$$

where 0+ or 0− means $\nu+N\to\nu+\pi^+ +X$ or $\nu+N\to\nu+\pi^- +X$, respectively. By taking $\mu=+$, $\nu=i$, we get

$$\int_0^\infty d\nu\,[W_3^{0+}(\nu,-\vec{k}_\perp^{\,2})+W_3^{0-}(\nu,-\vec{k}_\perp^{\,2})]=-\frac{\pi}{2f_\pi^2}(1-2\sin^2\theta_W)\int_{-\infty}^{\infty}d\alpha\,\epsilon(\alpha)\Big[\sqrt2 A_0(\alpha,0)+\tfrac23\sqrt3 A_8(\alpha,0)+\frac{\sqrt6}{3}A_{15}(\alpha,0)\Big]\,. \tag{3.12}$$

D. The case when N is the nucleon

Now we list the sum rules when N is the nucleon:

$$\int_0^\infty d\nu\,[W_2^+-W_2^-]=\Big\{-\frac{4\pi}{f_\pi^2}[4\cos^2\theta_C+\sin^2\theta_C+g_A^2(0)]+\frac{24}{f_\pi^2}g_A(0)(1+\cos^2\theta_C)\,\mathrm{P}\!\int d\alpha\,\frac{1}{\alpha}A^5(\alpha,0)\Big\}I_3\,, \tag{3.13}$$

$$\int_0^\infty d\nu(W_2^{++}-W_2^{--})=-\frac{8\pi}{f_\pi^2}[3g_A^2(0)-\sin^2\theta_C]\,, \tag{3.14}$$

$$\int_0^\infty d\nu(W_2^{0+}-W_2^{0-})=\frac{4\pi}{f_\pi^2}[1-2g_A^2(0)][(1-2\sin^2\theta_W)^2+1]I_3\,, \tag{3.15}$$

$$\int_0^\infty d\nu(W_3^{++}+W_3^{--})=-\frac{3}{f_\pi^2}(2\cos^2\theta_C+\sin^2\theta_C)\int_0^\infty d\nu\,\frac{\nu}{\vec{k}_\perp^{\,2}}W_2^{ep}(\nu,-\vec{k}_\perp^{\,2})\to-\frac{3}{f_\pi^2}(2\cos^2\theta_C+\sin^2\theta_C)\int_0^1\frac{d\omega}{2\omega^2}F_2^{ep}(\omega)\,, \tag{3.16}$$

$$\int_0^\infty d\nu(W_3^+ + W_3^-) = -\frac{1}{2f_\pi^2}(12\cos^2\theta_C + 3\sin^2\theta_C)\int_0^\infty d\nu\,\frac{\nu}{\vec{k}_\perp^2}W_2^{ep}(\nu, -\vec{k}_\perp^2)$$
$$\to -\frac{1}{2f_\pi^2}(12\cos^2\theta_C + 3\sin^2\theta_C)\int_0^1 \frac{d\omega}{2\omega^2}F_2^{ep}(\omega)\,, \tag{3.17}$$

$$\int_0^\infty d\nu(W_3^{0+} + W_3^{0-}) = -\frac{3}{f_\pi^2}(1 - 2\sin^2\theta_W)\int_0^\infty d\nu\,\frac{\nu}{\vec{k}_\perp^2}W_2^{ep}(\nu, -\vec{k}_\perp^2) \to -\frac{3}{f_\pi^2}(1-2\sin^2\theta_W)\int_0^\infty \frac{d\omega}{2\omega^2}F_2^{ep}(\omega)\,. \tag{3.18}$$

We define

$$\Delta W_i = W_{ip} - W_{in} \quad\text{and}\quad \Sigma W_i = W_{ip} + W_{in}\,, \tag{3.19}$$

where p or n denotes when N is the proton or the neutron, respectively:

$$\int_0^\infty d\nu(\Sigma W_2^+ - \Sigma W_2^-) = 0\,, \tag{3.20}$$

$$\int_0^\infty d\nu(\Sigma W_2^{++} - \Sigma W_2^{--}) = 0\,, \tag{3.21}$$

$$\int_0^\infty d\nu(\Sigma W_2^{0+} - \Sigma W_2^{0-}) = 0\,, \tag{3.22}$$

$$\int_0^\infty d\nu(\Delta W_3^+ + \Delta W_3^-) = 0\,, \tag{3.23}$$

$$\int_0^\infty d\nu(\Delta W_3^{++} + \Delta W_3^{--}) = 0\,, \tag{3.24}$$

$$\int_0^\infty d\nu(\Delta W_3^{0+} + \Delta W_3^{0-}) = 0\,. \tag{3.25}$$

Note added in proof. I have been informed of papers by Sakai and Yamada.[11] They already treated the soft-pion limit in inclusive reactions. I thank T. Oka for bringing these papers to my attention. According to these works, my treatment of pole terms as $q^- \to 0$ is not sufficient. I have neglected the pole terms due to bremsstrahlung pion from one of the nucleons in the final state. It is straightforward to include them but difficult to estimate them generally. Therefore I comment about them as follows.

(1) Pion target will be useful if possible. In this case we can neglect all the pole terms as $q^- \to 0$, since the probability to find a nucleon in the final state is strongly suppressed because it must be produced by $N\bar{N}$ production. Thus the sum rules in case of the nucleon target will soon be rewritten to those of the pion target.

(2) The sum rules in case of the single-π^0-observed inclusive reaction (reaction A in Sec. III) will be estimated even in the case of nucleon target. When $\mu = \nu = +$, we find correction $-4\pi g_A{}^2(0)I_3/f_\pi{}^2$ and the contribution from low-energy regions. The latter can be neglected if we take $|k^2|$ to be sufficiently large, for example, above 5 GeV^2. When $\mu = +, \nu = i$, we find only the contribution from low-energy region to W_3, which can be neglected also by taking $|k^2|$ large.

ACKNOWLEDGMENT

The author would like to thank Professor T. Nakano, Dr. S. Naito, M. J. Hayashi, and T. Oka. He is also thankful to his wife for discussions and encouragement.

APPENDIX

Let us consider the exclusive reaction $A + \Lambda \to B + \pi$. The matrix element is

$$\langle B\pi|\Lambda|A\rangle \propto (m_\pi{}^2 - q^2)\int d^4x\,\exp(iq\cdot x)\langle B|\,T(\phi_\pi(x), \Lambda(0))|A\rangle = \frac{(m_\pi{}^2 - q^2)}{m_\pi{}^2 F}T\,, \tag{A1}$$

where

$$T = \int d^4x\,\exp(iq\cdot x)\langle B|\,T(\partial_\mu J^{5\mu}(x), \Lambda(0))|A\rangle\,, \tag{A2}$$

and the PCAC relation is used. T can be rewritten as

$$T = -iq_\mu\int d^4x\,\exp(iq\cdot x)\langle B|\,T(J^{5\mu}(x), \Lambda(0))|A\rangle - \int d^4x\,\exp(iq\cdot x)\delta(x^+)\langle B|\,[J^{5+}(x), \Lambda(0)]\,|A\rangle\,, \tag{A3}$$

where the cancellation between the seagull terms and the Schwinger terms is assumed. According to a usual technique we can investigate the behavior of the amplitude T as $q^\mu \to 0$, and find that q^+ and $\vec{q}_\perp$ will be set equal to zero safely. Thus Eq.

(A3) becomes

$$T=-iq^- \int d^4x \exp(iq^-x^+)\langle B|\, T(J^{5+}(x),\Lambda(0))|A\rangle$$

$$-\int d^4x\, \delta(x^+)\langle B|\,[J^{5+}(x),\Lambda(0)]\,|A\rangle\,. \qquad \text{(A4)}$$

Equation (A4) holds at an arbitrary value of q^- and $q^2=2q^+q^- - \vec{q}_\perp{}^2=0$, $p\cdot q=p^+q^-$. Though the usual low-energy theorem will be obtained as $q^-\to 0$, we can set $q^-\to\infty$. In this case $pq\to\infty$, and the assumption of PCAC will be that the $q^2=0, \nu\to\infty$ amplitude is smoothly continued to the $q^2=m_\pi{}^2$, $\nu\to\infty$ amplitude. Thus we get as $q^-\to\infty$ by using the LCBJL theorem[4] the following:

$$T=O\left(\frac{1}{q^-}\right)=O\left(\frac{1}{pq}\right). \qquad \text{(A5)}$$

[1] S. Koretune, Osaka City University report (unpublished).
[2] S. L. Adler and R. F. Dashen, *Current Algebra and Applications to Particle Physics* (Benjamin, New York, 1968); S. B. Treiman, R. Jackiw, and D. J. Gross, *Lectures on Current Algebra and Its Applications* (Princeton Univ. Press, Princeton, New Jersey, 1972).
[3] H. Fritzsch and M. Gell-Mann, Talk presented at the XVI International Conference on High Energy Physics, Chicago, 1972 (unpublished); D. J. Gross and S. B. Treiman, Phys. Rev. D 4, 1059 (1971).
[4] J. M. Cornwall and R. Jackiw, Phys. Rev. D 4, 367 (1971).
[5] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).
[6] D. A. Dicus, R. Jackiw, and V. L. Teplitz, Phys. Rev. D 4, 1733 (1971).
[7] The assumption here will be to neglect the contribution from the class II graph. This is nothing but the assumption of the superconvergence relation for a semistrong or a strong amplitude.
[8] J. B. Kogut and D. E. Soper, Phys. Rev. D 1, 2901 (1970).
[9] N. Cabibbo and L. A. Radicati, Phys. Lett. 19, 697 (1966).
[10] D. A. Dicus and V. L. Teplitz, Phys. Rev. D 8, 4532 (1973); D. A. Dicus, *ibid.* 12, 277 (1975).
[11] N. Sakai and M. Yamada, Phys. Lett. 37B, 505 (1971); N. Sakai, Nucl. Phys. B39, 119 (1972).