PHYSICAL REVIEW D VOLUME 15, NUMBER 5 1 MARCH 1977

Multiplicities in hadronic and current-induced reactions*

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(Received 15 October 1976)

Mean multiplicities of hadrons in hadron-hadron collisions and in current-induced reactions are discussed. It is argued that we have to consider one of three possibilities: (1) A more rapid growth with $\ln s$ of multiplicities in e^+e^- annihilations than in hadronic collisions. (2) Violations of constituent-counting rules for exclusive cross sections. (3) Substantially different clustering properties of hadrons in e^+e^- annihilations than in hadronic collisions.

I. INTRODUCTION

The idea of "universal" hadron multiplicities, which are asymptotically independent of target and beam in hadronic and current-induced reactions, has entered the folklore of particle physics. This possibility was suggested when Bjorken and Kogut[1] formulated a principle of "correspondence" which played an important role in the development of theoretical ideas about final states in current-hadron interactions. This hypothesis suggests that appropriately normalized quantities in current-induced processes should not show any systematic change with Q^2, Correspondence requires that average multiplicities in virtual-photon–hadron interactions depend only on the available center-of-mass (c.m.) energy. By extension it can be hypothesized that hadronic multiplicities in e^+e^- annihilations, deep-inelastic leptoproduction, and hadron-hadron collisions all display the same asymptotic growth of multiplicity with c.m. energy. Brodsky and Gunion[2] have recently argued that this universality arises naturally in a quark-gluon bremmstrahlung model. It also appears that there is some experimental support for universality.[3]

However, like most attractive simplifications, this one needs constant checking from both an experimental and a theoretical viewpoint. For this reason, we would like to point out here that there exist compelling theoretical arguments which suggest that the coefficient of growth of mean multiplicity with the logarithm of energy should be higher by as much as a factor of 2 in e^+e^- annihilations than in hadronic collisions. These arguments are formulated most simply in terms of simple multiperipheral-like models for the final states but can also be seen to be a consequence of general properties of the generating functionals of the different processes. At this point we can claim no compelling experimental support for this pattern of noncorrespondence, but it seems important to bring the arguments out in the open so that the interesting question of hadronic multiplicities can receive further critical examination. Universality of multiplicities would seem to require violations of constituent-counting rules or significantly different clustering properties of the final-state hadrons in the different reactions.

II. COMPARISON OF AVERAGE MULTIPLICITIES IN e^+e^- ANNIHILATIONS AND HADRON-HADRON COLLISIONS

There are several possible approaches to average multiplicties in production processes ranging from a Mueller-Regge approximation for single-particle inclusive spectra to specific models for the exclusive final states. For the purpose of comparing e^+e^- annihilation with hadronic collisions, it is instructive to consider first an exclusive viewpoint. This allows us to focus on both the similarities and the differences of the two processes.

Basic assumptions. The first important requirement is to set down the basic assumptions concerning the final states which are involved in our argument. These are as follows:

(*a*) *Limited transverse momentum*. In both e^+e^- annihilations and hadronic collisions the average momentum of hadrons transverse to some preferred axis is limited. In e^+e^- annihilations this jet axis is distributed on an event-by-event basis according to an approximate $(1+\cos^2\theta)$ distribution and represents the direction of an underlying quark-antiquark pair. In hadronic collisions the jet axis coincides with the beam axis. For our purposes here the limitation of transverse momentum means that, at high energies, the phase space available for the production of hadrons can be treated as being one-dimensional in both types of processes. We will leave open the possibility that the average transverse momentum will be different in the different reactions. There is, at present, good experimental support for the jet hypothesis,[4,5] which we will discuss in more detail later.

(*b*) *Cluster decomposition*. We assume that the final states in both reactions display the amount of short-range order necessary to define a generating functional and obtain the various moments of the multiplicity distributions through the standard manipulations.[6,7] This means that asympotically the mean multiplicity can grow no faster than logarithmically,

$$\langle n\rangle = O(\ln s)\,, \tag{2.1}$$

with the c.m. energy.

(*c*) *Energy dependence.* One important difference between e^+e^- annihilations and hadronic collisions involves the energy dependence of the exclusive and total cross sections. We will assume that the exclusive cross sections in e^+e^- annihilation follow constituent-counting rules.[8] That is, we assume

$$\sigma(e^+e^- \to MM, s) \sim C/s^3\,, \tag{2.2}$$

while the total cross section displays scaling behavior

$$\sigma_{\text{tot}}(e^+e^- \to \text{hadrons}, s) \sim R/s\,. \tag{2.3}$$

Recent evidence for the validity of the constituent-counting rules has been discussed by Brodsky and Gunion.[9] To compare with the energy behavior of cross sections in e^+e^- annihilations, we can isolate two contributions to a typical exclusive hadronic cross section,

$$\sigma(hh \to hh, s) \cong \sigma_D(s) + \sigma_M(s)\,. \tag{2.4}$$

The diffractive component, σ_D, of the 2-2 elastic or pseudoelastic cross section can be considered the shadow of the many-body inelastic channels. It displays the energy dependence

$$\sigma_D(s) \sim \text{const}(\text{mod}\ln s)\,. \tag{2.5}$$

The nondiffractive exclusive cross sections are assumed to have the energy dependence

$$\sigma_M(s) \sim cs^{2\alpha_M - 2}(\text{mod}\ln s)\,, \tag{2.6}$$

where $\alpha_M \cong \frac{1}{2}$ is the intercept of the leading nondiffractive Regge singularity. The total cross section also has a similar decomposition

$$\sigma_{\text{tot}}(hh, s) \sim \sigma^D_{\text{tot}}(s) + \sigma^M_{\text{tot}}(s)\,, \tag{2.7}$$

with

$$\sigma^D_{\text{tot}}(s) \sim \text{const}(\text{mod}\ln s)\,, \tag{2.8}$$

$$\sigma^M_{\text{tot}}(s) \sim cs^{\alpha_M - 1}\,. \tag{2.9}$$

The energy dependence of the exclusive and inclusive cross sections in hadronic collisions has been the subject of considerable theoretical and phenomenological attention associated with the precise nature of the leading, Pomeranchuk, singularity and the way it is "generated" through unitarity.[10] Without going into the details of the various schemes, we acknowledge that the energy dependence indicated by (2.5)–(2.9) may be modified slightly if, for example, there exists a "bare" Pomeron with intercept not precisely unity. What is important for our considerations here is that the different relative energy dependence displayed by exclusive and total cross sections in e^+e^- annihilation (2.2) and (2.3) and hadron-hadron collisions (2.4)–(2.9) suggests that

$$\langle n\rangle_{e^+e^-} = \frac{\sum_{n=2}^{\infty} n\sigma(e^+e^- \to nh; s)}{\sigma_{\text{tot}}(e^+e^- \to \text{hadrons}; s)}\,, \tag{2.10}$$

$$\langle n\rangle_{hh} = \frac{\sum_{n=2}^{\infty} n\sigma(hh \to nh; s)}{\sigma_{\text{tot}}(hh; s)} \tag{2.11}$$

need not be intimately related. In fact we will demonstrate a simple model in which the e^+e^- mean multiplicity is higher than that in hadron-hadron collisions and show that this type of noncorrespondence is strongly suggested by the constraints that (2.2)–(2.9) impose on the different generating functionals.

The simple one-dimensional model. We assume the independent production of hadronic clusters in e^+e^- annihilation. The cross section for the production of $n+2$ of these clusters in e^+e^- annihilation will be approximated by

$$\sigma(e^+e^- \to (n+2)h, Y) \simeq C\exp(-3Y)\frac{[g^2(Y - nb_{e^+e^-})]^n}{n!} \times \theta(Y - nb_{e^+e^-})\,, \tag{2.12}$$

where $Y = \ln s$ and g^2 is an effective coupling constant and $b_{e^+e^-}$ is a "threshold" parameter. The fact that all the exclusive cross sections display the same power behavior is suggested by the derivations of the counting rules, while the logarithmic factors in (2.12) are simple estimates of one-dimensional phase-space integrations. From these cross sections we form the generating functional

$$Q^{e^+e^-}(z, Y) = \sum_{n=0}^{\infty} z^n \sigma(e^+e^- \to (n+2)h; Y)\,. \tag{2.13}$$

The series can be summed by forming the Laplace transform

$$Q^{e^+e^-}(z, \beta) = \int_0^\infty e^{-\beta Y} Q^{e^+e^-}(z, Y) \tag{2.14}$$

and by noting that the leading behavior of the inverse transform is governed by the pole at

$$3 + \beta - g^2 z \exp[-(3+\beta)b_{e^+e^-}] = 0\,. \tag{2.15}$$

In order for this to be compatible with scaling for the total cross section, (2.3), it must have the solution

$$\beta_0(z)|_{z=+1} = -1\,. \tag{2.16}$$

The mean cluster multiplicity is then given by

$$\frac{\langle n\rangle^c_{e^+e^-}}{Y}=\frac{\partial\beta_0(z)}{\partial z}\bigg|_{z=1}=\frac{2}{1+2b_{e^+e^-}}. \tag{2.17}$$

If $\langle m\rangle_{e^+e^-}$ is the mean number of stable particles in the decay of the cluster we have the hadronic multiplicity

$$\frac{\langle n\rangle_{e^+e^-}}{Y}=\frac{2\langle m\rangle_{e^+e^-}}{1+2b_{e^+e^-}}. \tag{2.18}$$

The value 2 in (2.18) is merely the difference in the energy dependence of the typical exclusive and the total annihilation cross sections modulo logarithms. If we give up scaling and/or constituent counting and write

$$\frac{\sigma_{\text{tot}}(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to hh)}\sim Cs^{\Delta\beta}, \tag{2.19}$$

then (2.18) could be written in the form

$$\frac{\langle n\rangle_{e^+e^-}}{Y}=\frac{(\Delta\beta)\langle m\rangle_{e^+e^-}}{1+(\Delta\beta)b_{e^+e^-}}. \tag{2.20}$$

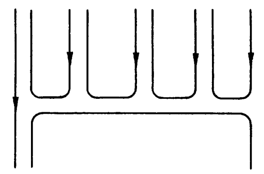
Analogous arguments can be made for the production of particles in hadronic collisions. The cross sections are subject to the more complicated restrictions of unitarity and the existence of both Regge and diffractive components. One general approach which has had considerable success in dealing with this type of question goes under the labels of "dual unitarization scheme" (DUS)[11] or "dual topological expansion" (DTE).[12] It is not appropriate here to develop the entire framework of DUS and DTE, but it is instructive to consider the generating functional for particle production in a simplified form similar to what was done in e^+e^- and apply some of the ideas from this approach.

One important piece of input involves the concept of a planar diagram. As shown in Fig. 1, we distinguish two types of multiperipheral diagrams, planar diagrams and those with twists allowed on exchanged links. Neglecting interference terms we can write the "planar" cross section,

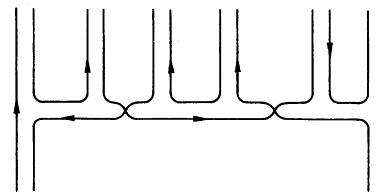
$$\sigma(hh\to(n+2)h;Y)\big|_{\text{planar}}\cong C_h\exp[(2\alpha_M-2)Y]\times\frac{[g_h^2(Y-nb_h)]^n}{n!}\,\theta(Y-nb_h), \tag{2.21}$$

in analogy to (2.12), where we have used the nondiffractive Regge exchange (2.6). Since there are approximately 2^n diagrams containing twists we can also write

$$\sigma(hh\to(n+2)h;Y)\big|_{\text{twists}}\cong C_h\exp[(2\alpha_M-2)Y]\times\frac{[2g_h^2(Y-nb_h)]^n}{n!}\,\theta(Y-nb_h). \tag{2.22}$$



(a)
PLANAR DIAGRAM



(b)
TWISTS ALLOWED ON EXCHANGED LINKS

FIG. 1. Two types of multiperipheral diagrams discussed in the text.

When we take the summation over all planar topologies

$$Q_{\text{planar}}(z,Y)=\sum_{n=0}^{\infty}z^n\sigma(hh\to(n+2)h;Y)\big|_{\text{planar}}, \tag{2.23}$$

we are supposed to generate the nondiffractive cross section

$$Q_{\text{planar}}(z,Y)\big|_{z=1}\sim C_1\exp[(\alpha_M-1)Y], \tag{2.24}$$

while the summation over all diagrams containing twists

$$Q_T(z,Y)=\sum_{n=0}^{\infty}z^n\sigma(hh\to(n+2)h;Y)\big|_{\text{twists}} \tag{2.25}$$

leads to the diffractive component of the total cross section

$$Q_T(z,Y)\big|_{z=1}\sim C_2\exp[(\alpha_D-1)Y], \tag{2.26}$$

where $\alpha_D\cong1$ is the intercept of the Pomeron. We therefore have two equations:

$$(1-\alpha_M)\exp[-(1-\alpha_M)b_h]\cong g_h^2, \tag{2.27}$$

$$(1+\alpha_D-2\alpha_M)\exp[-(1+\alpha_D-2\alpha_M)b_h]\cong2g_h^2. \tag{2.28}$$

For small b_h, these equations are approximately consistent with the usual assignment of the singularities $\alpha_D\cong1$, $\alpha_M\cong\frac{1}{2}$. This result, due to Lee and Veneziano,[13] is an important check on the validity of the DUS and DTE. This means we have some confidence in the overall validity of the result

$$\frac{\langle n\rangle^c_{hh}}{Y} = \frac{(1+\alpha_D - 2\alpha_M)}{1+(1+\alpha_D - 2\alpha_M)b_H}, \tag{2.29}$$

which is to be compared with (2.17). If $\langle m\rangle_h$ is the mean number of stable particles in the decay of the cluster we can write

$$\frac{\langle n\rangle_{hh}}{Y} \cong \frac{(1+\alpha_D - 2\alpha_M)\langle m\rangle_h}{1+(1+\alpha_D - 2\alpha_M)b_h} \cong \frac{\langle m\rangle_h}{1+b_h}. \tag{2.30}$$

In order for the average multiplicities in e^+e^- annihilations and *hh* collisions to have the same high-energy behavior we would need, comparing (2.18) and (2.30), either different clustering properties among the hadrons or different values of *b*. These possibilities are discussed later.

At this point it is interesting to note that this result follows from the general properties (a)–(c) and is not specific to our simple model. If we write the generating functionals (2.13) and (2.25) in the form

$$\begin{aligned} &Q^{e^+e^-}(z,Y) \sim C_e(z)\exp[\beta^{e^+e^-}(z)Y],\\ &Q_T(z,Y) \sim C_h(z)\exp[\beta^{hh}(z)Y], \end{aligned} \tag{2.31}$$

we can see that our assumptions about the energy behavior imply

$$\begin{aligned} &\beta^{e^+e^-}(0) = -3, \quad \beta^{e^+e^-}(1) = -1,\\ &\beta^{hh}(0) = -1, \quad \beta^{hh}(1) = 0. \end{aligned} \tag{2.32}$$

The cluster decomposition properties ensure that the $\beta(z)$ are well defined and that

$$\lim_{Y\to\infty} \frac{\langle n\rangle}{Y} = \frac{\partial\beta(z)}{\partial z}\bigg|_{z=1}. \tag{2.33}$$

The different boundary conditions indicated in Fig. 2 make it highly unlikely that the slopes of the curves are the same at $z=1$. If we artificially enforce universal multiplicities then we *must* have that higher derivatives of the curves be different so that $\langle n(n-1)\rangle = f_2$ or f_3 are not universal. This means, of course, that the clustering properties of hadrons are different in the two types of reactions.

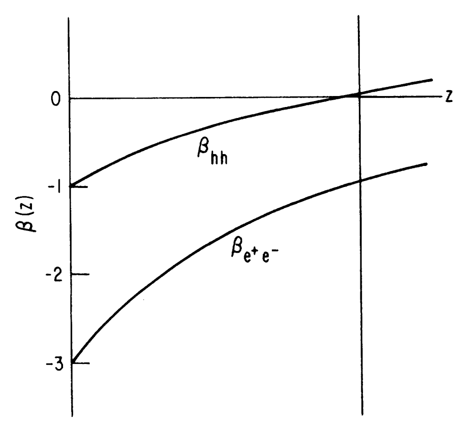


FIG. 2. Sketch of possible leading behavior of generating functional $Q(z,Y) \sim \exp[\beta(z)Y]$ in e^+e^- annihilations and hadronic collisions.

III. CORRESPONDENCE AND COMPARISON WITH DATA

Although our discussion in Sec. II dealt only with mean multiplicities in e^+e^- annihilations and in hadron-hadron collisions, with a few additional assumptions we can extend the results to other reactions. Within the context of the parton model[14] and in a Mueller-Regge approach,[15] inclusive hadron production in deep-inelastic electroproduction has been shown to display the structure indicated schematically in Fig. 3. With form factors given by constituent-counting rules and with Bjorken scaling for the deep-inelastic cross section the height of the current plateau in rapidity space should be the same as that in e^+e^- annihilations. This leads to the approximate expression

$$\langle n\rangle_{\gamma^* p} \sim B_{e^+e^-} \ln\frac{Q^2}{\langle m_T^2\rangle} + B_h \ln\omega, \tag{3.1}$$

where $B_{e^+e^-}$ is, in our scheme, given by (2.18) and B_h by (2.30). In (3.1) Q^2 is the $|\text{mass}^2|$ of the spacelike photon and ω is the usual scaling variable.

Given the existence of scaling in deep-inelastic neutrino collisions and assuming the validity of constituent-counting rules for weak form factors, expression (3.1) should be asymptotically valid in these reactions as well.

Existing data do *not* support the idea that the growth of multiplicities is higher in e^+e^- annihilations than in ordinary hadronic collisions, nor do they support (3.1) with $B_{e^+e^-} > B_h$. Fits to multiplicities in several different reactions have been performed by Albini *et al.*[3] For e^+e^- they give the charged multiplicity

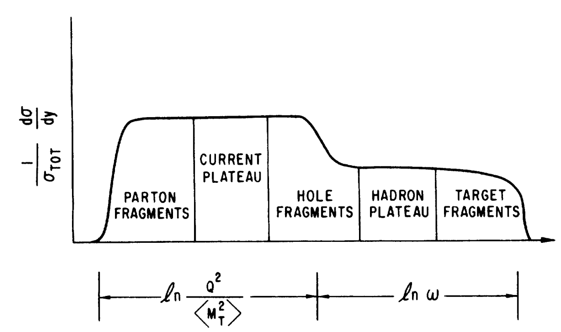


FIG. 3. Inclusive production in deep-inelastic scattering.

$$\langle n_{\rm ch}\rangle_{e^+e^-} \sim (1.93 \pm 0.12) + (0.75 \pm 0.04) \ln s \quad (\sqrt{s} = 2\text{–}8 \text{ GeV}), \tag{3.2}$$

which can be compared to the hadronic reactions

$$\langle n_{\rm ch}\rangle_{pp} \sim (-3.19 \pm 0.30) + (1.84 \pm 0.05) \ln s \quad (\sqrt{s} = 10\text{–}152 \text{ GeV}) \tag{3.3}$$

or

$$\langle n_{\rm ch}\rangle_{\pi^- p} \sim (-0.72 \pm 0.06) + (1.45 \pm 0.02) \ln s \quad (\sqrt{s} = 3\text{–}23 \text{ GeV}). \tag{3.4}$$

The energy ranges over which the data are collected in the different interactions are, of course, quite different. It is also significant that the data on multiplicities in inelastic leptoproduction show no systematic variation with Q^2 (see Ref. 16), so that the picture of Fig. 3 and Eq. (3.1) cannot be considered valid with $B_{e^+e^-} \gg B_h$.

In addition, preliminary data from νN collisions in the 15-ft bubble chamber at energies where $\langle Q^2\rangle \cong 20$ GeV^2 suggest that neutrino multiplicities are consistent with those of ordinary hadronic processes.[17] This fact also argues against a large variation with Q^2 of the mean multiplicity.

Multiplicities in deep-inelastic muon experiments at Fermilab and at the CERN SPS (Super Proton Synchrotron) can provide a sensitive test for (3.1) as can further data from neutrino collisions. If the arguments presented here are correct there should be some growth of mean hadronic multiplicity with Q^2. Although existing experimental data are consistent with no Q^2 dependence, it is not clear that they have yet ruled out the kind of variation considered here.

If multiplicities are shown experimentally to be universal, one possibility which is suggested by comparison of (2.18) and (2.30) is that the relevant values of b and/or $\langle m\rangle$ might be different in e^+e^- and hadron-hadron collisions. From our point of view this would be an artificial way of achieving equal multiplicities, but it must be considered.

The value of b in our one-dimensional model represents the mean amount of rapidity space taken up by a hadronic cluster. This is, of course, related to the p_T distribution, and the fact that[4]

$$\langle p_T\rangle_{e^+e^-} \simeq 0.315 \pm 0.02 \text{ GeV}/c \quad (\text{at } \sqrt{s} = 7.4 \text{ GeV}) \tag{3.5}$$

is very close to the comparable value in pp collisions[5] argues against any big effect. It should be noted that the experimental numbers are not exactly comparable since $\langle p_T\rangle$ in e^+e^- annihilations is measured by minimizing the sphericity tensor event-by-event and should be less than the value of p_T relative to the unknown jet axis. Strictly within the context of hadronic collisions, the success of the DUS and DTE argue against b_h large since the Pomeron and meson trajectories would not have the proper intercepts. If the arguments are consistent the value (3.5) would also rule out a large $b_{e^+e^-}$.

The clustering properties of hadrons represent a different picture. Naively, there is no reason to suppose that we should have significantly fewer hadrons per cluster in an e^+e^- event than in a hadronic collision, but it is hard to formulate an argument which would rule out this possibility. There may be some suggestion for differences in clustering from data on neutral vs charged energies, and it may be possible to settle this question experimentally. The mean-square charge fluctuations, $\langle u^2\rangle$, are sensitive to the number of charged particles per hadronic cluster. The existence of jets in e^+e^- makes it feasible to measure this quantity in this reaction in order to compare with hadronic data and see if there is any evidence for different types of clusters.[18] At this point all we can say is that it would seem a bit artificial if the clusters in e^+e^- and hadron-hadron collisions differed in just the right amount to ensure equal mean multiplicities.

If we are to question any of the assumptions in (a)–(c) it would seem that the necessity for the power behavior (2.2) of exclusive e^+e^- cross sections is the most open to modification. The experimental determination of the pion form factor has shown overall consistency with the $1/s$ asymptotic behavior favored by constituent-counting laws,[19] but this may be a special case which, for some reason, does not apply to other hadrons. It is notable that a strict application of Bjorken-Kogut correspondence, which assumes that the process $\gamma^* h \to hh$ have Regge behavior

$$\frac{\sigma(\gamma^* h \to hh)}{\sigma_{\rm tot}(\gamma^* h \to \text{all})} \sim s^{2\alpha_M - 2} \tag{3.6}$$

independently of Q^2, would require the form factor to have the behavior[1]

$$F_h(Q^2) \sim (Q^2)^{-1+\alpha_M}. \tag{3.7}$$

If we adopt the energy behavior suggested by (3.7) for typical cross sections instead of (2.1) it can be seen from (2.20) and (2.30) that the difference between e^+e^- and hadron-hadron multiplicities vanishes. It may seem to be asking too much of these type of arguments to distinguish between (3.7) and constituent counting, but it is clear that if we are to understand the significance of the apparent universality of mean multiplicities in different reactions it is important that we pin down as precisely as possible the energy behavior of the relevant exclusive cross sections.

If we confine attention to only hadron-hadron collisions, the idea that these are a universal growth of all mean multiplicities with Y is not completely well founded. An approximately factorizable Pomeron would lead to a universal Y behavior in all diffractive processes, but Dias de Deus[20] has argued that it is possible to isolate the Reggeon "component" of multiplicities by taking the difference between Ap and $\bar{A}p$ topological cross sections. He claims some experimental support for the pattern of multiplicities suggested by (2.27) and (2.28) with

$$\frac{\langle n\rangle_D}{Y} \cong \frac{2\langle n\rangle_M}{Y}. \tag{3.8}$$

He also argues from DUS and DTE concepts for a mean multiplicity in baryon-antibaryon annihilations of the form

$$\frac{\langle n\rangle_{B\bar{B}}}{Y} \cong \frac{3}{2}\frac{\langle n\rangle_D}{Y}, \tag{3.9}$$

and claims that this is also consistent with data. Figure 4 from his paper shows the comparison of (3.8) and (3.9) with data.

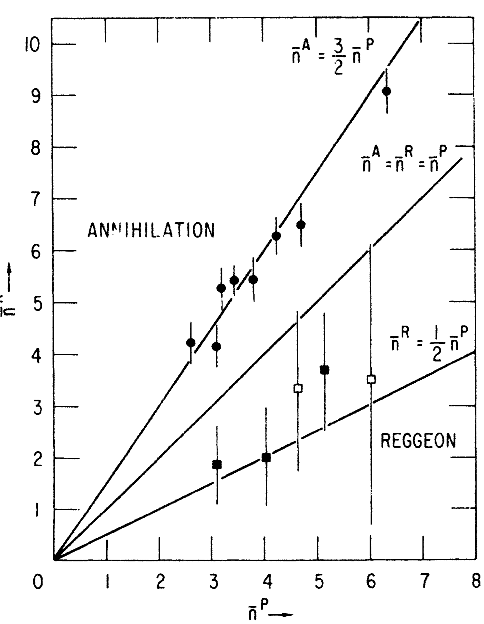


FIG. 4. Figure from Ref. 20. Comparison of mean multiplicities in $\sigma(K^+p) - \sigma(K^-p)$ and in $\bar{p}p$ annihilations with pp multiplicities. The curves are predictions of DUS or DTE.

IV. HADRON MULTIPLICITIES IN COLOR GAUGE GLUON BREMMSTRAHLUNG MODELS

Brodsky and Gunion[2] have proposed a simple, attractive explanation of hadronic multiplicities in different processes in terms of the color confinement mechanism in non-Abelian gauge theories. They point out that each type of fundamental production process can be considered as producing a separation or flow in momentum space of color quantum numbers. Just as the acceleration of charge in QED produces a bremmstrahlung spectrum of soft photons, the flow of color charge will produce emission of colored gluons. Although the details of the mechanism by which the emission of gluons is subsequently converted into hardons is not worked out at all, this way of looking at the problem makes it plausible that the mean multiplicity of hadrons should be proportional to the mean multiplicity of colored gluons. Just as the multiplicity of photons in QED depends on the charges of the particles being accelerated, it is clear that the multiplicity color gauge gluons will depend on the color charges of the accelerated constituents. Brodsky and Gunion argue for a picture of e^+e^- annihilation, γ^*p scattering, and hadron-hadron collisions in which the quantum-number separation is always 3-$\bar{3}$ (quark-antiquark) and hence the multiplicity is universal,

$$\langle n(s)\rangle\big|_{e^+e^-,\,hh,\,\gamma^*h} \sim n_{3\bar{3}}(s). \tag{4.1}$$

Since our arguments here have shown that universal multiplicities are not favored on general theoretical grounds, it is important to point out that this does not conflict with the attractive connection between the confinement mechanism and the gluon multiplicity hypothesized by Brodsky and Gunion. This is desirable since it has been hoped for a long time that the basic notions of dual models can be made compatible with gauge theories for the strong interactions. (Veneziano[21] has taken

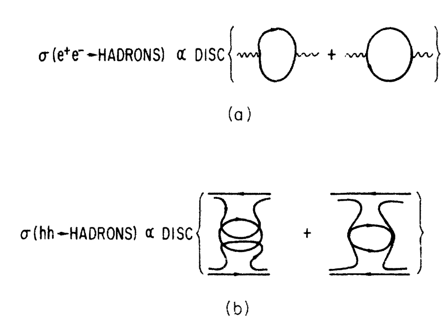


FIG. 5. (a) Parton-model diagram for $\sigma_{tot}(e^+e^- \to \text{hadrons})$. (b) Dual model "cylinder" diagram for $\sigma_{tot}(hh)$.

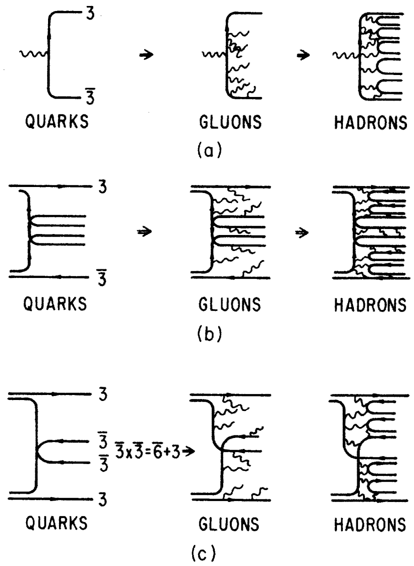


FIG. 6. (a) Unfolding Fig. 5(a) to examine hadronic multiplicities in a quark-gluon model. (b) Similar unfolding of dual "cylinder" with one quark orientation. (c) Unfolding of dual "cylinder" with opposite quark orientation.

some preliminary steps toward formulating a connection.) One basic point where we differ with the view of Brodsky and Gunion is that, in considering the topological structure of quark-line diagrams consistent with the constraints of duality, it does not seem to be true that hadronic collisions always involve the separation of $3\bar{3}$ color charges over a rapidity gap $Y = \ln s$.

To illustrate, consider the structure of a diagram for e^+e^- annihilation in Fig. 5(a). This does indeed represent the flow of quark quantum numbers over a distance in rapidity $Y \simeq \ln s$. In a hadron-hadron collision the analogous diagram in the context of dual models, shown in Fig. 5(b) is the simple cylinder approximation to the Pomeranchuk singularity. When we "decompose" the diagrams as in Fig. 6 we see that in hh there is a component with separation of $3\bar{3}$ quantum numbers by the full rapidity gap $Y = \ln s$, but in the dual cylinder, in order to be consistent with the charge-conjugation properties of the Pomeron, we must also have a contribution where 33 quantum numbers are separated. There is then a region of rapidity space which contains the diquark quantum numbers; $\bar{3} \otimes \bar{3} = \bar{6} + 3$ in this second piece.

Viewed another way, the separation of quantum numbers in hadronic collisions can be partially screened by hadronic matter with the quark quantum numbers not traveling the entire rapidity gap. There is therefore no reason to believe that the entire hadron acts as a coherent radiator of colored gluons in a collision.

It should also be mentioned that in the simple bremmstrahlung model proposed by Brodsky and Gunion the multiplicities are hypothesized to behave asymptotically as

$$\langle n \rangle \sim C \ln^2 s \,, \tag{4.2}$$

and we do not have the cluster decomposition properties necessary to use generating-functional techniques and the arguments of Sec. II do not apply. Although fits to mean multiplicities at accelerator energies can accommodate an $\ln^2 s$, there may be some evidence from cosmic-ray data at very high energies which rules this out.[3] This is comforting because there does seem to be some evidence for short-range-order properties in hadronic production, and it would be very unfortunate to have to forego generating-functional methods in dealing with multiplicity distributions.

V. CONCLUSIONS

Mean hadronic multiplicities can be seen to provide an important test of theoretical ideas. The possibility of asymptotically "universal" multiplicities, which seems favored by experiment and many theorists, can be shown to be in conflict with some widely accepted concepts. Experimental examination of the final states in electroproduction and e^+e^- annihilation, including good measurements of exclusive cross sections, mean multiplicities, and of charge fluctuations, will undoubtedly provide valuable new information. It seems probable that we must either have a growth of mean multiplicities with increasing Q^2, significantly different clustering properties of hadrons produced in current-induced reactions from those in hadron-hadron collisions, or violations of constituent-counting rules for exclusive cross sections. Any one of these possibilities would be extremely interesting.

ACKNOWLEDGMENTS

The author is grateful for valuable and informative discussions with S. J. Brodsky, Chan Hong-Mo, J. Dias de Deus, and G. H. Thomas.

*Work performed under the auspices of the United States Energy Research and Development Administration.

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