Impact-parameter analysis of exclusive $\bar{p}p$ (5.7 GeV/c) and $\bar{p}d$ (4.72, 5.55, 9.3, and 14.6 GeV/c) reactions

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Lower bounds for the root-mean-square impact parameter have been obtained for a set of 27 exclusive $\bar{p}$-induced reactions. A comparison between annihilation, production, and coherent reactions is carried out. In order to see whether or not the lower bounds have a real connection with the actual impact parameter we have also analyzed the production processes by means of collective or multiparticle variables evaluated event by event.

I. INTRODUCTION

The impact-parameter concept has been used rather widely in high-energy interactions. As yet, however, not much information exists about the impact-parameter dependence of exclusive reactions. Therefore, following a method proposed recently,[1] we will determine lower bounds for the root-mean-square impact parameters (b_L) for a set of exclusive reactions induced by incident $\bar{p}$. More precisely, we will use our $\bar{p}p$ data at 5.7 GeV/c and our $\bar{p}d$ data at 4.72, 5.55, 9.3, and 14.6 GeV/c. Details on the experiments from which the present data are extracted can be found in our previous works.[2] Although we will only be able to get lower limits, the method used has the advantage of being model-independent. Furthermore, by comparing the results obtained from annihilation, production, and coherent reactions, we expect to learn something about the multiparticle production.

This study will be completed by an analysis of the production processes in terms of some multiparticle or collective variables evaluated event by event. A few attempts to use the distributions of such multiparticle variables for studying production processes have been carried out previously.[3,4] The main idea in this approach was to search for variables built from the single-particle properties of the emitted particles, and which are sensitive to the production mechanisms. The ideal situation will occur when such a variable is distributed according to a δ function, thus leading to a new conservation rule. Of course, the ambition of the present work is much more limited. We will only investigate some collective variables which can be related to the impact-parameter concept or to the peripheral nature of the reactions. This last study is expected to give some indication whether or not the lower bounds we determine have a real connection with the actual impact parameter appearing in high-energy reactions.

II. METHOD FOR CALCULATING b_L

The relation used for calculating the lower bound of the mean-square impact parameter is deduced from the Schwarz inequality, i.e.,

$$\langle J^2\rangle\langle F^2\rangle \geq |\langle \vec{J}\cdot\vec{F}\rangle|^2 .$$

Here $\vec{J}$ and $\vec{F}$ can be any vectorial operators (their moduli are denoted by J and F, respectively), although for the present purpose $\vec{J}$ is taken to be the total spin of the outgoing or incoming system of particles. The average ($\langle\rangle$) will be taken over all the events belonging to a given reaction. The above inequality can also be transformed as

$$\langle J^2\rangle\langle F^2\rangle \geq \tfrac{1}{4}|\langle[\vec{J},\vec{F}]\rangle|^2 ,$$

which will be used throughout in the following way. Assuming that the spins of the individual particles can be neglected, the conservation of the total spin will be replaced by the conservation of the total orbital angular momentum. Since in addition the impact parameter b satisfies the approximate relation $\langle b^2\rangle = \langle J^2\rangle/p^2$, where $\vec{p}$ (modulus p) is the incident c.m. momentum, one has finally

$$\langle b^2\rangle \geq \frac{1}{4p^2}\,\frac{|\langle[\vec{J},\vec{F}]\rangle|^2}{\langle F^2\rangle} ,$$

with the lower limit maximized when $\vec{J}$ and $\vec{F}$ anticommute. In order to have a strong inequality we follow Ref. 5 and choose the form

$$\vec{F} = \sum_{i=1}^{n} w_i(x_i, \ldots, x_n)\vec{p}\times\vec{p}_i ,$$

where $\vec{p}_i$ (modulus p_i) is the c.m. momentum of the ith outgoing particle and the sum runs over all the n outgoing particles. The w_i are arbitrary functions of the Feynman variable $x_i = p_i/p'_i$. Here p'_i is the maximum allowed p_i value calculated separately for each kind of particle emitted in the studied reactions. The w_i functions will be chosen to maximize the lower bound. In the limit where there is no correlation between the transverse momenta of the outgoing particles or between the longitudinal and transverse momenta, an optimal bound is obtained from the relation[5]

$$\langle b^2\rangle \geq b_L{}^2 = \frac{\langle \sum_i^n (x_i^2 - r_i^2/2p_i'^2)\rangle^2}{\langle \sum_{i,j}^n x_i x_j \vec{r}_i \vec{r}_j \rangle},$$

where $\vec{r}_i$ (modulus r_i) is the transverse momentum of the ith outgoing particle. The quantity $\sum(x_i^2 - r_i^2/2p_i'^2)$ gives a measure of the peripherality of the studied reactions[6] whereas the quantity in the denominator is sensitive to the jet structure of the events.[7] Although the correlation between the transverse and longitudinal momenta cannot be neglected in our c.m. energy region, we will use the latter formula as an approximate optimal bound for $\langle b^2\rangle$.

III. EXPERIMENTAL RESULTS

The results for the annihilation, production, and coherent reactions are given in Tables I–III. At our low momenta (~5 GeV/c) one can worry about the validity of the impact-parameter concept, as the de Broglie wavelength associated with the incoming particle is about one quarter of the nucleon radius. Nevertheless, we consider here b_L as a means to characterize high-energy processes; the interpretation of b_L in terms of an impact-parameter limit becomes more evident when the c.m. energy increases. For the analysis of our data we will separate the studied reactions into annihilation (a), production (b), and coherent (c) reactions.

A. Annihilation channels

At ~5.5 GeV/c we have a rather large number of $\bar{p}p$ (5.7 GeV/c) and $\bar{p}n$ (5.55 GeV/c) annihilation channels. The calculated b_L values (Table I) are presented in Fig. 1. One notices the decrease of b_L with increasing number of outgoing particles, a feature already noted for $\bar{p}p$ annihilations at 4.6 and 9.1 GeV/c (Ref. 8) and also for other types of interactions.[5,9] The b_L annihilation values are smaller than those obtained from production reactions with the same number of outgoing particles (see Tables I–III). This can be easily understood if b_L is really related to the true impact parameter, since annihilation channels are expected to be produced with smaller impact parameters than the $\bar{p}N$ production reactions.[10] A fact that supports the idea that the b_L can be related to the real impact parameter arises also from the examination of the multiparticle variables $C_1 = (1/n)\sum_i r_i^2/p_i^2$ and $C_2 = (1/n)\sum_i (x_i^2 - r_i^2/2p_i'^2)$ (the latter quantity enters into the calculation of b_L). Indeed, for (central) collisions occurring with small impact parameters on the average, the transverse momenta of the secondaries will be larger than for peripheral reactions. Thus, if b_L really represents a typical quantity related to the average impact parameter, any decrease of b_L must be accompanied by an increase in $\langle C_1\rangle$ and a decrease in $\langle C_2\rangle$. The fact that this is indeed the case [see Table I and also Figs. 2(a) and (b)] gives us some confidence about the validity of using the lower bounds b_L as a tool for analyzing multiparticle production. One can also note that the utilization of C_1 and C_2 is simply an alternative manner of analyzing the production angular distribution. Indeed, if θ_i is the c.m. emission angle of the ith outgoing particle, one has also $C_1 = (\sum_i \sin^2\theta_i)/n$ and $C_2 = (1/n)\sum_i p_i^2/p_i'^2 P_2(\cos\theta_i)$ for which ordinary phase space gives averages of $\langle C_1\rangle = \frac{2}{3}$ and $\langle C_2\rangle = 0$ (P_2 is the second-order Legendre polynomial).

TABLE I. Values of b_L, $\bar{b}_e$, and $\langle C_{1\text{-}3}\rangle$ for $\bar{N}N$ annihilation channels at 5.55 and 5.7 GeV/c.

Channel	Momentum (GeV/c)	b_L (fm)	$\bar{b}_e$ (fm)	$\langle C_1\rangle$	$\langle C_2\rangle$	$10^2\langle C_3\rangle$
$\bar{p}n\to\pi^+2\pi^-\pi^0$	5.55	0.129±0.008	0.251±0.017	0.536±0.009	0.103±0.006	63.8 ±2.6
$\bar{p}n\to 2\pi^+3\pi^-$	5.55	0.116±0.010	0.262±0.018	0.573±0.013	0.050±0.004	18.2 ±1.0
$\bar{p}n\to 2\pi^+3\pi^-\pi^0$	5.55	0.097±0.009	0.300±0.019	0.597±0.008	0.029±0.003	8.53±0.48
$\bar{p}n\to 3\pi^+4\pi^-$	5.55	0.091±0.020	0.247±0.035	0.601±0.013	0.016±0.003	3.14±0.34
$\bar{p}n\to 3\pi^+4\pi^-\pi^0$	5.55	0.058±0.012	0.247±0.030	0.618±0.009	0.008±0.002	2.10±0.17
$\bar{p}n\to 4\pi^+5\pi^-\pi^0$	5.55	0.046±0.034	0.188±0.040	0.658±0.003	0.003±0.002	0.49±0.04
$\bar{p}p\to\pi^+\pi^-\pi^0$	5.7	0.147±0.016	0.315±0.035	0.457±0.021	0.226±0.023	2.38±0.17
$\bar{p}p\to 3\pi^+3\pi^-$	5.7	0.116±0.006	0.268±0.015	0.580±0.006	0.035±0.002	8.79±1.34
$\bar{p}p\to 3\pi^+3\pi^-\pi^0$	5.7	0.075±0.003	0.240±0.008	0.623±0.002	0.015±0.001	4.03±0.08
$\bar{p}p\to 4\pi^+4\pi^-$	5.7	0.092±0.016	0.242±0.025	0.617±0.012	0.013±0.002	2.04±0.22
$\bar{p}p\to 4\pi^+4\pi^-\pi^0$	5.7	0.036±0.010	0.221±0.029	0.654±0.007	0.004±0.001	1.04±0.08

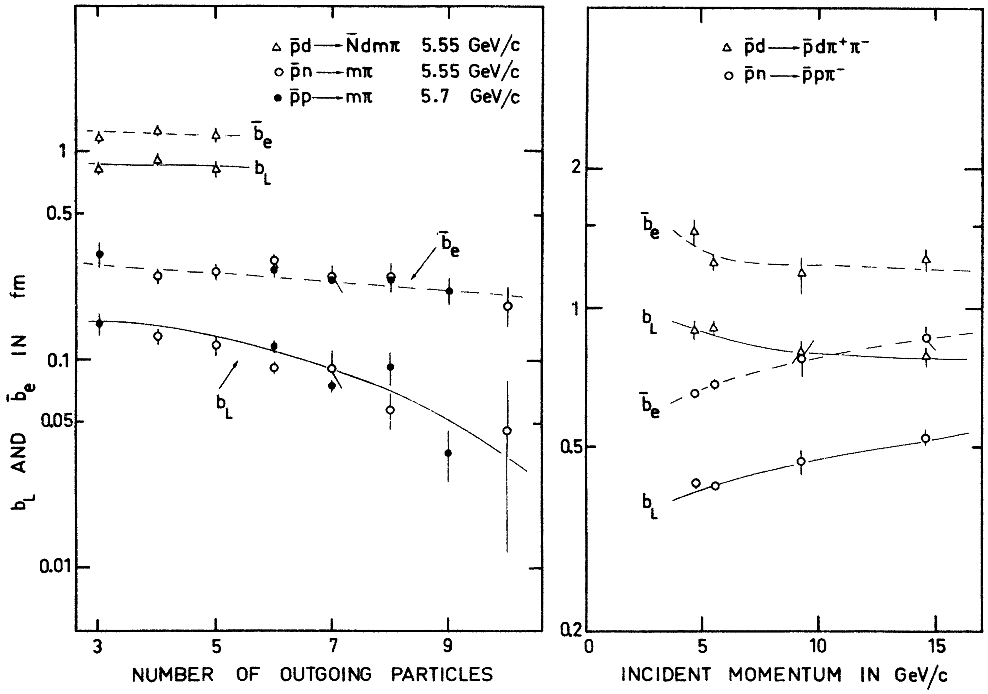

FIG. 1. Values of b_L and $\bar{b}_e$ for some exclusive reactions as a function of the number of outgoing particles (left-hand side) and as a function of the incident momentum (right-hand side), m being the number of emitted π. Here b_L is the lower limit of the rms impact parameter while $\bar{b}_e$ is the average of the b_e multiparticle variable distribution (see text). The lines are drawn to guide the eye.

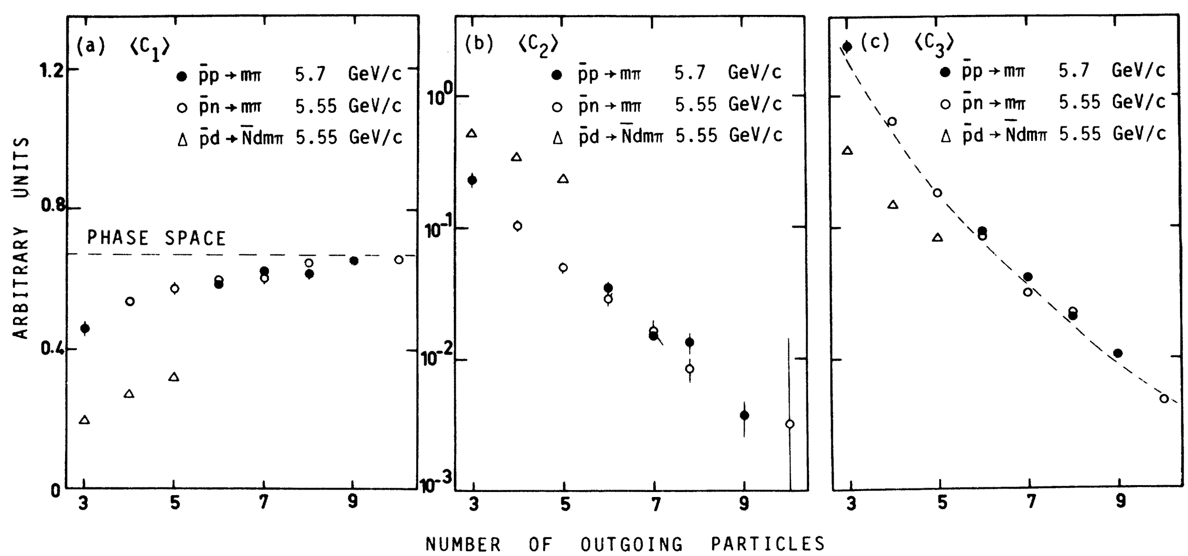

FIG. 2. Distribution of the average $\langle C_1 \rangle = \langle \sum_i r_i^2/p_i^2 \rangle / n$, $\langle C_2 \rangle = \langle \sum_i (x_i^2 - r_i^2/2p_i'^2) \rangle / n$, and $\langle C_3 \rangle = \langle \sum_{i,j} x_i x_j \vec{r}_i \vec{r}_j \rangle / n^2$ as function of the outgoing particles for the annihilation and coherent reactions at ~ 5.5 GeV/c. In (a) and (c) the dashed line represents the phase-space prediction while in (b) phase space gives $\langle C_2 \rangle = 0$.

TABLE II. Values of b_L, $\bar{b}_e$, and $\langle C_{1\text{-}3}\rangle$ for the production $\bar{p}N$ reactions.

Channel	Momentum (GeV/c)	b_L (fm)	$\bar{b}_e$ (fm)	$\langle C_1\rangle$	$\langle C_2\rangle$	$10^2\langle C_3\rangle$
$\bar{p}n\to\bar{p}p\pi^-$	4.72	0.416±0.0019	0.650±0.0010	0.211±0.004	0.362±0.006	75.8 ±2.6
$\bar{p}n\to\bar{p}p\pi^-$	5.55	0.406±0.005	0.683±0.013	0.290±0.003	0.379±0.003	87.0 ±1.7
$\bar{p}p\to\bar{p}p\pi^-\pi^0$	5.55	0.314±0.006	0.558±0.017	0.437±0.005	0.158±0.003	25.2 ±0.6
$\bar{p}p\to\bar{p}n\pi^+\pi^-$	5.55	0.247±0.005	0.466±0.011	0.483±0.004	0.135±0.003	30.1 ±0.6
$\bar{p}p\to\bar{n}p2\pi^-$	5.55	0.239±0.009	0.482±0.022	0.500±0.007	0.124±0.004	27.0 ±1.0
$\bar{p}p\to\bar{p}p\pi^0$	5.7	0.376±0.005	0.686±0.018	0.313±0.004	0.345±0.003	84.4 ±1.8
$\bar{p}p\to\bar{p}n\pi^+$ $\bar{p}n\pi^-$	5.7	0.323±0.004	0.634±0.018	0.357±0.003	0.314±0.002	93.9 ±1.1
$\bar{p}p\to\bar{p}p2\pi^+2\pi^-$	5.7	0.135±0.013	0.366±0.033	0.594±0.008	0.029±0.003	4.48±0.26
$\bar{p}n\to\bar{p}p\pi^-$	9.3	0.462±0.026	0.784±0.073	0.223±0.012	0.452±0.010	96 ±11
$\bar{p}n\to\bar{p}p\pi^-$	14.6	0.821±0.019	0.858±0.048	0.160±0.008	0.482±0.005	85.7 ±5.9

Figures 2(a) and (b) show clearly that the more n increases, the more the data resemble phase space. For completeness we also examined the average of the $C_3=\sum_{i,j}x_i x_j\vec{r}_i\vec{r}_j/n^2$ quantity used to calculate b_L. The average $\langle C_3\rangle$ is presented in Fig. 2(c) as a function of n, the number of outgoing particles. Here n^2 represents the number of terms entering into the calculation of C_3. As can be seen from this figure, the $\langle C_3\rangle$ quantity is not very sensitive to the production mechanisms as the $\langle C_3\rangle$ distribution follows the phase-space predictions rather well.

The present b_L values have the same order of magnitude as those found for the annihilation channels at 4.6 and 9.1 GeV/c (Ref. 8) when the same method of calculation is used. Furthermore, for a given n the b_L increases when the incident momentum is changed from ~5.5 to 9.1 GeV/c. No such a clear tendency is observed by comparing the 4.6 and ~5.5 GeV/c data.

B. Production reactions

Table II presents the b_L and the $\langle C_{1\text{-}3}\rangle$ values obtained from the production reactions. Although the data are not so numerous as for the annihilation case, some conclusions can nevertheless be drawn. Also, here b_L appears to decrease with an increasing number of outgoing particles. In the $\bar{p}n\to\bar{p}p\pi^-$ reaction for which we have four data points, i.e., 4.72, 5.55, 9.3, and 14.6 GeV/c, b_L increases with increasing c.m. energy (see also Fig. 1). The number of production channels available in the present work is not sufficient to detect any influence of the charge partition in the final state on b_L.

C. Coherent reactions

For our coherent channels (Table III) one obtains b_L which are systematically higher than for the production and annihilation channels. These large values appear to be a general feature of diffractive dissociation processes[11] which contribute predominantly to the coherent $\bar{p}d$ production.[12] One may also note that the present values have the same order of magnitude as those found in coherent production on nuclei.[13] We observe that, similarly to the part of the $pp\to pp\pi^+\pi^-$ reaction occurring via diffractive dissociation,[11] the b_L decrease with increasing incident laboratory momentum (see Fig. 1). Also here the decrease of b_L with the incident momentum is accompanied by a tendency of $\langle C_1\rangle$ ($\langle C_2\rangle$) to increase (decrease). In Fig. 1 we have also presented b_L as a function

TABLE III. Values of b_L, $\bar{b}_e$, and $\langle C_{1\text{-}3}\rangle$ for the $\bar{p}d$ coherent production.

Channel	Momentum (GeV/c)	b_L (fm)	$\bar{b}_e$ (fm)	$\langle C_1\rangle$	$\langle C_2\rangle$	$10^2\langle C_3\rangle$
$\bar{p}d\to\bar{p}d\pi^+\pi^-$	4.72	0.888±0.037	1.453±0.091	0.280±0.009	0.338±0.004	14.5±1.2
$\bar{p}d\to\bar{n}d\pi^-$	5.55	0.819±0.010	1.152±0.031	0.140±0.005	0.510±0.003	38.7±1.8
$\bar{p}d\to\bar{p}d\pi^+\pi^-$	5.55	0.900±0.024	1.240±0.038	0.270±0.006	0.342±0.003	14.4±0.7
$\bar{p}d\to\bar{p}d\pi^+\pi^-\pi^0$ $\bar{n}d\pi^+2\pi^-$	5.55	0.808±0.050	1.184±0.056	0.321±0.009	0.232±0.003	8.3±0.7
$\bar{p}d\to\bar{p}d\pi^+\pi^-$	9.3	0.796±0.038	1.175±0.108	0.221±0.012	0.361±0.005	20.6±1.9
$\bar{p}d\to\bar{p}d\pi^+\pi^-$	14.6	0.782±0.034	1.271±0.067	0.198±0.008	0.362±0.003	21.5±1.8

of n. The number of data points is, however, too small for observing a clear trend for the variation of b_L with n.

In order to see how close the limits b_L are to the actual root-mean-square impact parameter, we estimated $\langle b^2\rangle$ for the $\bar{p}p$ inelastic reactions. To this end we assumed that the elastic $\bar{p}p$ scattering is the shadow of the inelastic reaction and that the distribution of the elastic four-momentum transfer (t) has an e^{Bt} form. Taking then the B values as well as the total $\bar{p}p$ cross sections from the literature, we find that $(\langle b^2\rangle)^{1/2}$ varies from 1.09 to 1.04 fm in the 5.5 to 14.6 GeV/c range.[14] Of course, these values are only estimates, as the e^{Bt} form is not valid for large $|t|$. This means, in particular, that the shape of the impact-parameter profile function is not well known for small b. Nevertheless we see that the b_L values for our production reactions fall below the calculated $\langle b^2\rangle$ quantities by a factor of about 2 to 3. Using the same kind of approximation one can also estimate the root mean square for elastic scattering, which is $(\langle b^2\rangle_{el})^{1/2} \sim 0.7$ fm. As all our b_L values are below $(\langle b^2\rangle_{el})^{1/2}$, nothing can be said here about the comparison of the production and elastic-scattering processes.

IV. MULTIPARTICLE OR COLLECTIVE VARIABLES

In the last section we considered some multiparticle variables (C_{1-3}) which have a rather clear connection with the peripherality of the reaction and hence with the impact parameter entering into the reactions. The more the reaction is peripheral, the more b will tend to increase; this fact is reflected in the behavior of b_L.

As an additional variable we may also consider[13] the expression

$$b_e = \frac{|\sum(x_i^2 - r_i^2/2p_i'^2)|}{(\sum x_i x_j \vec{r}_i \vec{r}_j)^{1/2}} = \frac{|C_2|}{C_3^{1/2}}$$

built from the quantities entering into the calculation of the lower bound b_L. This variable b_e contains information about the peripherality (numerator) and the jet structure (denominator) of the events. The average values $\bar{b}_e$ for all our reactions are given in Tables I–III (see also Fig. 1), while some typical b_e distributions are given in Figs. 3 and 4.

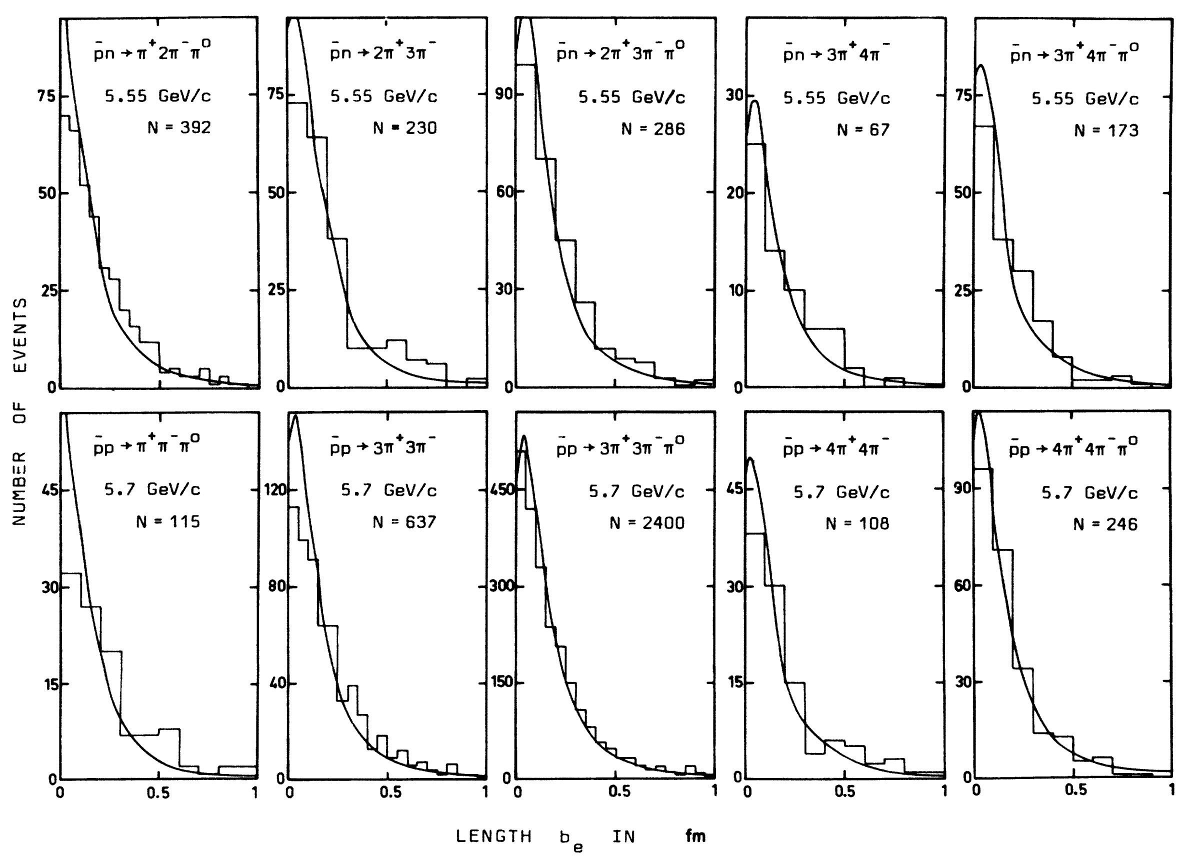

FIG. 3. Distributions of the collective $b_e = |\sum_i (x_i^2 - r_i^2/2p_i'^2)| / (\sum_{i,j} x_i x_j \vec{r}_i \vec{r}_j)^{1/2}$ variable for the annihilation channels at ~ 5.5 GeV/c. The curves represent the phase-space predictions.

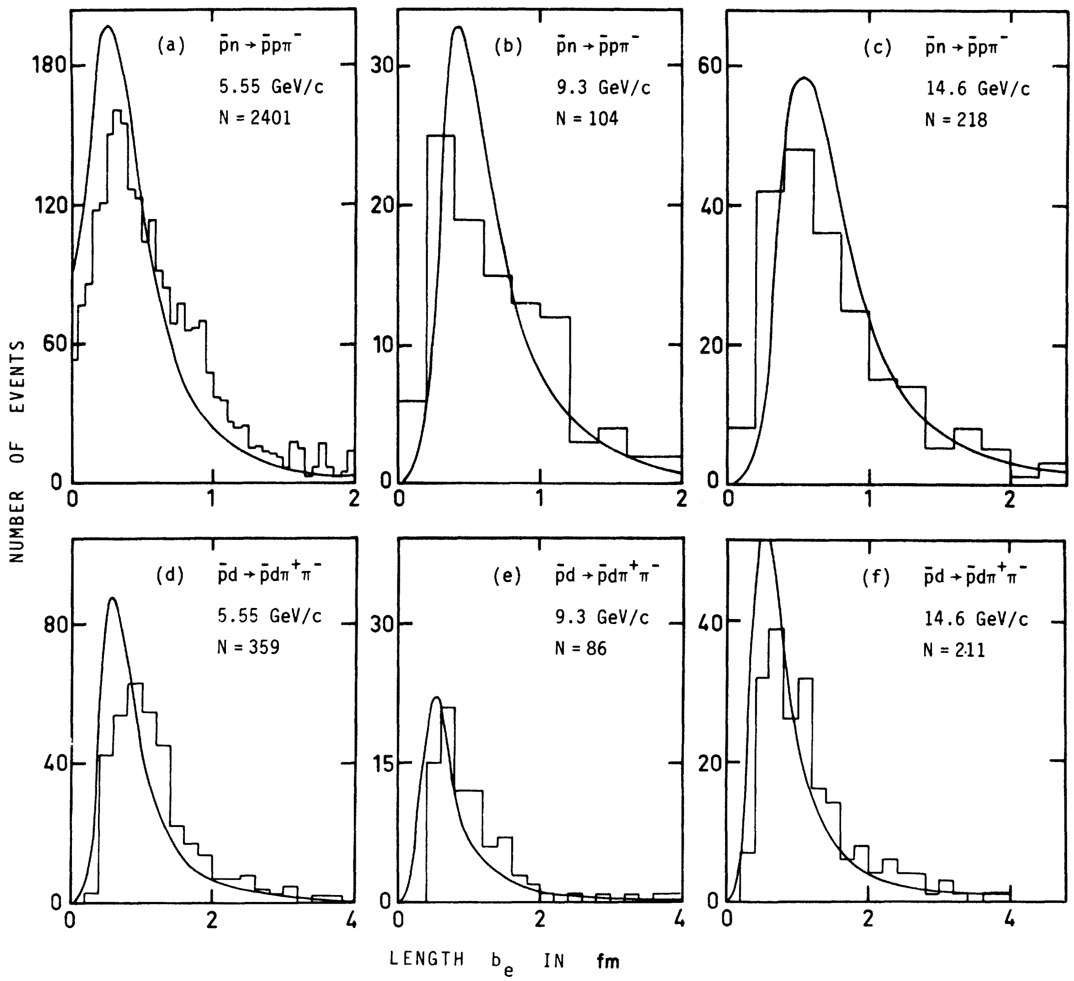

FIG. 4. Distributions of the $b_e = |\sum_i (x_i^2 - r_i^2/2p_i'^2)| / (\sum_i x_i x_j \vec{r}_i \vec{r}_j)$ variable for the $\bar{p}n \to \bar{p}p\pi^-$ and $\bar{p}d \to \bar{p}d\pi^+\pi^-$ reactions at 5.55, 9.3, and 14.6 GeV/c. The curves represent the peripheral phase-space predictions (see text).

For the annihilation reactions one sees from Fig. 3 that the phase-space predictions (full curves) give a rather acceptable description of the b_e distribution. In the case of production reactions, big discrepancies appear as the phase-space curves peak at too low a b_e value. By using peripheral phase space[15] one obtains a better agreement with the data, although it is still not very good as can be seen from the examples displayed in Fig. 4. This simply shows that the b_e distribution is sensitive to the production mechanism since by imposing some peripheral structure into the phase space one is able to reproduce some features of the production process.

From the values given in Tables I–III one sees that a systematic trend appears in the sense that for each reaction one has always $\bar{b}_e > b_L$. For the annihilation channels, $\bar{b}_e$ appears to be a more slowly decreasing function of n than is b_L. As a function of the incident momentum, we do not have enough data points to observe systematic differences between the $\bar{b}_e$ and b_L behavior, apart from the fact already mentioned that $\bar{b}_e > b_L$.

If the impact parameter is considered to be a useful concept for studying multiparticle production, one can ask if the emitted particles will be aligned in a preferred direction in the transverse plane containing the impact-parameter vector. In other words one may wonder if the produced particles have some memory concerning the impact-parameter vector. Several methods were proposed to measure the degree of alignment of the outgoing particles in the transverse plane. We used the collective variable[3]

$$C_4 = \frac{|\sum_i r_{xi} r_{yi}|}{[(\sum_i r_{xi}^2)(\sum_i r_{yi}^2)]^{1/2}},$$

which will attain the value 1 if all the outgoing particles are aligned in the transverse plane. Here r_{xi} and r_{yi} represent the components of the transverse momentum $\vec{r}_i$ defined with respect to a fixed coordinate system. The distributions for our annihilation channel at 5.55 GeV/c are shown in Fig. 5. As can be seen from this figure no accumulation of events is seen in the $C_4 \sim 1$ region when the data are compared with phase-space predictions. Figure 6 presents the C_4 distributions for the $\bar{p}n \to \bar{p}p\pi^-$ and $\bar{p}d \to \bar{p}d\pi^+\pi^-$ reactions at 5.55, 9.3, and 14.6 GeV/c. Also here we do not observe any alignment of the outgoing particles in the transverse plane.

V. CONCLUSIONS

In this work we have determined lower bounds for the root-mean-square impact parameter (b_L) for 27 $\bar{p}$-induced reactions in the 5–15 GeV/c region. The results indicate that at a given incident momentum, the b_L decrease with the number of outgoing particles for the annihilation, production, and coherent reactions. We have also observed that for a given multiplicity and incident momentum the coherent reactions have larger b_L values than the production and annihilation reactions. This means that, similarly to other types of reactions, b_L tend to be large for inelastic diffractive dissociation processes, which are expected to give an important contribution to coherent production. For the annihilation reactions at 5.55 GeV/c we found b_L values which are much smaller than for the production and the coherent reaction.

The fact that we are dealing with lower bounds b_L makes it somewhat difficult to obtain firm conclusions unless these lower bounds are of comparable strength for all the reactions under study. That this may be the case is supported by the fact that b_L varies from channel to channel as expected for a real impact parameter; this latter behavior is determined from the peripheral properties of the reactions.

The study of the peripherality of the reactions was made by using collective variables evaluated event by event. We also attempted to see whether or not the outgoing particles are aligned in the transverse plane. The proposed multiparticle variables used for this study did not allow us to observe such an alignment.

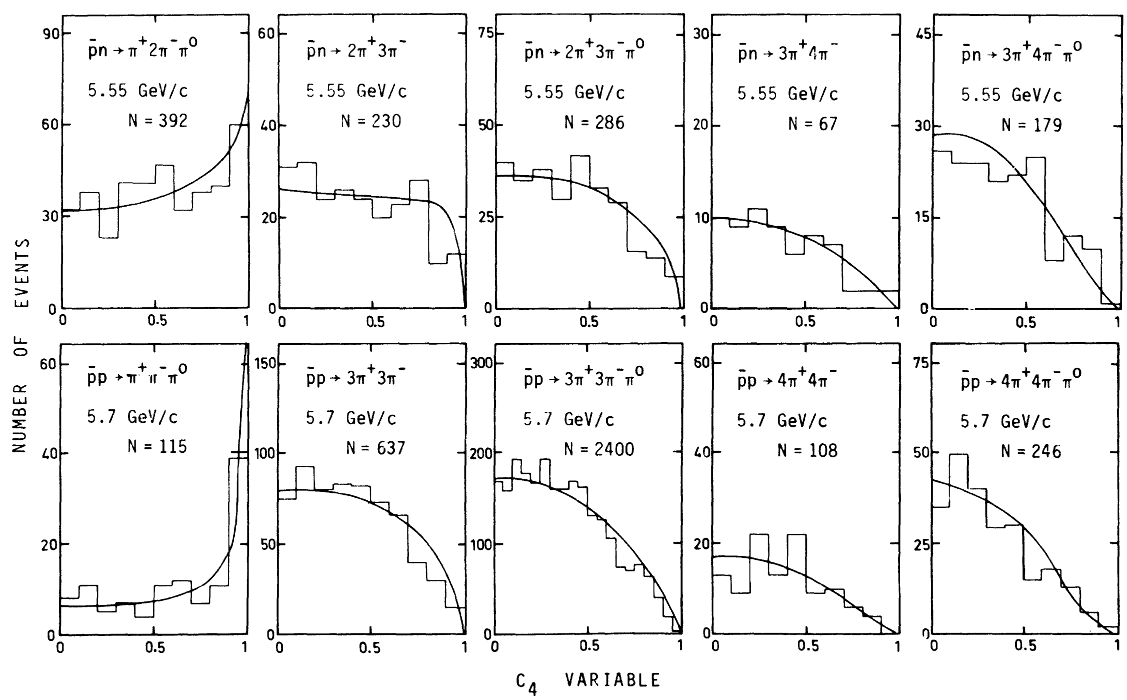

FIG. 5. Distributions of the collective or multiparticle variable $C_4 = |\sum_i r_{xi} r_{yi}| / [(\sum_i r_{xi}^2)(\sum_i r_{yi}^2)]^{1/2}$ for the annihilation channels at ~ 5.5 GeV/c. The curves represent the phase-space predictions.

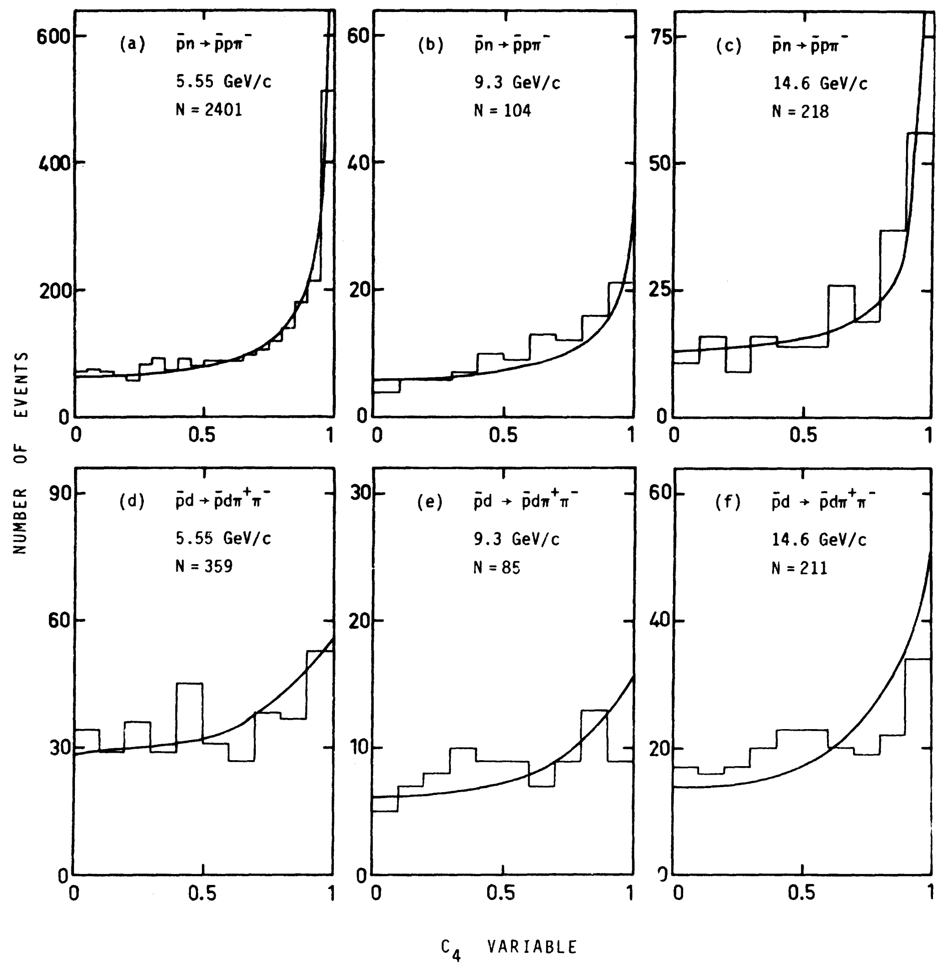

FIG. 6. Distributions of the $C_4 = |\sum_i r_{xi} r_{yi}| / [(\sum_i r_{yi}^2)]^{1/2}$ for the $\bar{p}n \to \bar{p}p\pi^-$ and $\bar{p}d \to \bar{p}d\pi^+\pi^-$ reactions at 5.55, 9.3, and 14.6 GeV/*c*. The curves represent the peripheral phase-space predictions (see text).

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[6]The numerator will vanish whenever the outgoing particles are emitted isotropically in the c.m., while it will be maximized when there are strong leading-particle effects.

[7]When all the particles are emitted in a jet structure, i.e., in the forward or backward c.m. within an angle α, the quantities $\sum_{i,j} x_i x_j \vec{r}_i \vec{r}_j$ will be proportional to $\sin^2 2\alpha$.

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[14]That $\langle b^2 \rangle$ decreases somewhat when the incident momentum increases is simply due to the fact that the diffraction $\bar{p}p$ peak presents an expanding behavior in the 5.5–15 GeV/*c* region.

[15]For the production reaction the peripheral phase-space predictions were obtained by generating Monte Carlo events weighted by an $e^{b_1 t_1} \times e^{b_2 t_2}$ factor. Here t_1 (t_2) is the four-momentum transfer between the incident and outgoing $\bar{N}$ (N), the $b_{1,2}$ being determined from the experimental data. For the coherent reactions we used only an $e^{b_d t_d}$ factor, t_d being the four-momentum transfer between the incident and outgoing deuteron. In the latter case we also took into account in our calculation the experimental losses occurring for events having small $|t_d|$ values.