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Sum rules for $\Delta C = \Delta S$ decays of charmed hadrons

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Relations for the $\Delta C = \Delta S$ decays of the charmed hadrons belonging to the $\bar{3}$ representation of SU(3) are given which provide tests of the isospin and SU(3) transformation properties of the full $\Delta C = \Delta S$ nonleptonic interaction.

The extremely narrow width of the new vector mesons at 3.1 GeV (see Ref. 1) and 3.7 GeV (see Ref. 2) is at present understood on the basis of a new quantum number which implies the existence of a larger symmetry group than SU(3) underlying the strong interactions of hadrons. A popular choice for the larger symmetry is SU(4), in which[3] the new quantum number is called charm. A direct test of the underlying SU(4) symmetry, apart from mass relations, would be relations between strong-interaction coupling constants.[4] However, the low-lying charmed hadrons would be stable with respect to strong decays and would manifest themselves through their weak-interaction decays. Both the lowest baryons and mesons[5,6] carrying charm $C=1$ are expected to belong to the $\bar{3}$ representation of SU(3). We denote the $C=1$, $\frac{1}{2}^+$ baryon, 0^- and 1^- meson antitriplets by $B(\bar{3})$, $P(\bar{3})$, and $V(\bar{3})$, respectively. We show that their $\Delta C = \Delta S$ decays can provide simple tests of the isospin and SU(3) properties of the weak interaction proposed by Glashow, Iliopoulos, and Maiani.[7] The hadronic weak current in the GIM scheme[7] is given by (suppressing space-time structure)

$$J_h = \cos\theta J_2^1 + \sin\theta J_3^1 + \cos\theta J_3^4 - \sin\theta J_2^4 \,, \tag{1}$$

where θ is the Cabibbo angle and the indices indicate the transformation properties under SU(4), etc. J_h belongs to the 15-dimensional representation (1, 0, 1) of SU(4). J_3^4 and J_2^4 satisfy the selection rules $\Delta C = \Delta S = +1$ and $\Delta C = 1$, $\Delta S = 0$, respectively, where S is strangeness. The leading charm-changing term (since θ is small) is

$$H_{CS} = H_- + H_+ \,, \tag{2}$$

$$H_\pm = \tfrac{1}{2}\cos^2\theta\,[\{J_2^1, J_4^3\} \pm \{J_2^3, J_4^1\}] + \text{H.c.} \,, \tag{3}$$

The full H_{CS} satisfies a pure $|\Delta \vec{I}| = 1$ isospin selection rule since clearly both H_+ and H_- individually satisfy it. Further, it is easy to see that (i) H_- transforms as $(\underline{6} + \bar{\underline{6}})$ under SU(3) and as the self-conjugate 20-dimensional representation (0, 2, 0), denoted by $\underline{20}''$, under SU(4); (ii) H_+ transforms as $(\underline{15} + \overline{\underline{15}})$ under SU(3) and belongs to the 84-dimensional representation (2, 0, 2) of SU(4). The 15-dimensional representation of SU(3) which enters in H_+ is (2, 1) and its conjugate (1, 2), and is denoted by $\underline{15}$ and $\overline{\underline{15}}$.

The SU(4) symmetry of the leading terms at short distances suggests[8] that the SU(4) representation $\underline{20}''$, i.e., H_- is enhanced relative to $\underline{84}$, i.e., H_+. The consequences of $\underline{20}''$ dominance have already been considered[5,9-11] for the decays $B(\bar{3}) \to B(8) + P(8)$, $P(\bar{3}) \to P(8) + P(8)$, and $V(\bar{3}) \to P(8) + P(8)$, where $B(8)$ and $P(8)$ are the usual $\frac{1}{2}^+$ and 0^- octets. In particular, it was found[9] that $\underline{20}''$ dominance gives an extra relation for S-wave decay of hyperons, namely $S(\Xi_-^-) = 2S(\Lambda_-^0)$, which is violated by about 50%. Consequently we explore the consequences of the full H_{CS} at the isospin and SU(3) level only and show that it is possible to obtain simple relations between decay amplitudes, the verification of which, in the future, would test the presence of both the H_- and H_+ pieces in H_{CS}. The SU(3) analysis of the $V(\bar{3})$ and $P(\bar{3})$ decays mentioned above, using the full H_{CS}, has also been given by Kingsley *et al.*[11] We present the analysis for the $B(3) \to B(8) + P(9)$ decays as well as the $P(\bar{3}) \to P(9) + V(9)$ decays using the full H_{CS}. We also extend the analysis of Kingsley *et al.*[11] for the decay of $P(\bar{3})$ and $V(\bar{3})$ into two identical nonets with arbitrary mixing angle.

$B(\bar{3}) \to B(8) + P(9)$ decays. For the charmed baryons we will use the notation $B^Q(N_3, I)$, where N_3 denotes its SU(3) representation, I its isospin, and charge Q its I_3 value. Thus $B(\bar{3})$ with charm $C=+1$ has the isodoublet $B^+(\bar{3}, \frac{1}{2})$, $B^0(\bar{3}, \frac{1}{2})$ with strangeness $S=-1$ and $B^+(\bar{3}, 0)$ with $I=0$ and $S=0$. There are 16 decay amplitudes $A_1, \ldots, A_{16}$ which satisfy $\Delta C = \Delta S$ and are defined in Table I. The A_i depend on seven parameters which can be counted by considering $B(\bar{3}) + H_\pm \to B(8) + P(9)$ and treating $H_\pm$ as spurions. Now only the $\underline{6}$ part of H_- having $C=-1$ contributes giving the three parameters $g_{10} = (\overline{10} \to \overline{10}(BP))$, $g_S = (8 \to 8_S(BP))$, and $g_A = (8 \to 8_A(BP))$, where $8_S(BP)$ means the symmetric octet made out of $B(8)$ and $P(8)$, etc. Similarly, only the $\underline{15}$ part of H_+ contributes and gives rise to four amplitudes, namely, $g'_{27} = (27 \to 27(BP))$, $g'_{10} = (10 \to 10(BP))$, and $g'_{S,A} = (8 \to 8_{S,A}(BP))$. The physical states in the SU(3) nonet $P(9)$ are defined by (mixing angle θ_P)

TABLE I. The sixteen $\Delta C = \Delta S$ decay amplitudes for $B(\bar{3}) \to B(8) + P(9)$.

$A_1 = A(B^+(\bar{3}, \frac{1}{2}) \to \pi^+ \Xi^0)$	$A_9 = A(B^+(\bar{3}, 0) \to \pi^0 \Sigma^+)$
$A_2 = A(B^+(\bar{3}, \frac{1}{2}) \to \bar{K}^0 \Sigma^+)$	$A_{10} = A(B^+(\bar{3}, 0) \to \pi^+ \Sigma^0)$
$A_3 = A(B^0(\bar{3}, \frac{1}{2}) \to \pi^0 \Xi^0)$	$A_{11} = A(B^+(\bar{3}, 0) \to \eta \Sigma^+)$
$A_4 = A(B^0(\bar{3}, \frac{1}{2}) \to \eta \Xi^0)$	$A_{12} = A(B^+(\bar{3}, 0) \to \pi^+ \Lambda)$
$A_5 = A(B^0(\bar{3}, \frac{1}{2}) \to \pi^+ \Xi^-)$	$A_{13} = A(B^+(\bar{3}, 0) \to K^+ \Xi^0)$
$A_6 = A(B^0(\bar{3}, \frac{1}{2}) \to K^- \Sigma^+)$	$A_{14} = A(B^+(\bar{3}, 0) \to \bar{K}^0 p)$
$A_7 = A(B^0(\bar{3}, \frac{1}{2}) \to \bar{K}^0 \Sigma^0)$	$A_{15} = A(B^0(\bar{3}, \frac{1}{2}) \to \eta' \Xi^0)$
$A_8 = A(B^0(\bar{3}, \frac{1}{2}) \to \bar{K}^0 \Lambda)$	$A_{16} = A(B^+(\bar{3}, 0) \to \eta' \Sigma^+)$

$$\eta = \sin\theta_P P_1 - \cos\theta_P P_8 , \quad (4a)$$

$$\eta' = \cos\theta_P P_1 + \sin\theta_P P_8 , \quad (4b)$$

where P_1 is a pure SU(3) singlet which mixes with P_8, the eighth component of the octet. For convenience define the amplitudes

$$A_4' = A(B^0(\bar{3}, \tfrac{1}{2}) \to P_8 \Xi^0), \quad A_{15}' = A(B^0(\bar{3}, \tfrac{1}{2}) \to P_1 \Xi^0) , \quad (5a)$$

$$A_{11}' = A(B^+(\bar{3}, 0) \to P_8 \Sigma^+), \quad A_{16}' = A(B^+(\bar{3}, 0) \to P_1 \Sigma^+) . \quad (5b)$$

Using Eqs. (4) and (5) one can express A_4, A_{11}, A_{15}, and A_{16} in terms of A_4', A_{15}', A_{11}', and A_{16}'. Note that A_{15}' and A_{16}' receive contributions only from g_S and g_S'.

Since 16 amplitudes are given in terms of seven parameters one expects nine sum rules; of these three follow directly from the $|\Delta \vec{I}| = 1$ property of H_{CS} and the other six are a consequence of its SU(3) transformation properties. The $|\Delta \vec{I}| = 1$ relations are

$$A_1 = A_5 + \sqrt{2}\, A_3 , \quad (6a)$$

$$A_2 = A_6 + \sqrt{2}\, A_7 , \quad (6b)$$

$$A_9 = -A_{10} . \quad (6c)$$

The SU(3) sum rules are

$$A_3 + A_7 = \sqrt{3}\,(A_4' + A_8) , \quad (7a)$$

$$\sqrt{3}\,(A_{11}' - A_{12}) = \sqrt{2}\,(A_5 - A_6) + (A_3 - \sqrt{3}\, A_4') , \quad (7b)$$

$$-\sqrt{6}\,(A_{11}' + A_{12}) = 2(A_1 + A_2) + 2(A_{13} + A_{14}) , \quad (7c)$$

$$2A_9 + \sqrt{2}\,(A_{13} - A_{14}) = -2A_3 + A_7 + \sqrt{3}\, A_8 , \quad (7d)$$

$$-\sqrt{3}\, A_{15}' = \tfrac{2}{5}(A_1 + A_2) + (A_{13} + A_{14}) , \quad (7e)$$

$$-\sqrt{3}\, A_{16}' = \tfrac{2}{5}(A_1 + A_2) - (A_5 + A_6) . \quad (7f)$$

The $|\Delta \vec{I}| = 1$ sum rules are simple and may be checked. Further, they are independent of any mixing of P_1 and P_8 as well as mixing of $B(\bar{3}, \frac{1}{2})$ with the $B(6, \frac{1}{2})$ baryon states.[9] Of the SU(3) sum rules one may hope to check (7a), (7e), and (7f) which involve only five decay amplitudes. However, for the pseudoscalar mesons one may neglect the small mixing and have, e.g.,

$$A_3 + A_7 \simeq \sqrt{3}\,(A_4 + A_8) , \quad (7a')$$

and this may be amenable to experimental verification in the near future.

In the case of H_- enhancement, one would have four additional SU(3) relations because H_+ does not contribute so that $g_{27}' = g_{10}' = g_S' = g_A' = 0$. This gives the 10 SU(3) relations

$$A_1 = -A_2, \quad A_5 = A_{14}, \quad A_6 = A_{13} , \quad (8a)$$

$$A_9 + \sqrt{3}\, A_{11}' = -\sqrt{2}\, A_6 = A_3 + \sqrt{3}\, A_4' , \quad (8b)$$

$$-A_9 + \sqrt{3}\, A_{12} = -\sqrt{2}\, A_5 = A_7 + \sqrt{3}\, A_8 , \quad (8c)$$

$$-\sqrt{2}\, A_9 = A_6 + \sqrt{2}\, A_3 , \quad (8d)$$

$$\sqrt{3}\, A_{15}' = -\sqrt{3}\, A_{16}' = (A_5 + A_6) . \quad (8e)$$

The 10 relations in (8) generalize those given earlier[9] to the case of arbitrary mixing of η and η'. Thus the sum rules in Eqs. (8) are valid, with appropriate changes ($\eta \to \phi$, $\eta' \to \omega$, etc.), for the decays $B(\bar{3}) \to B(8) + V(9)$, where $V(9)$ is the usual 1^- nonet with ideal mixing. It is clear that all the above relations are valid for both S- and P-wave decays.

$P(\bar{3}) \to P(9) + V(9)$ decays. The charm $C = +1, 0^-$ mesons which transform as $\bar{3}$ under SU(3) are denoted by D^+, D^0, and F^+. The (D^+, D^0) form an isodoublet with $S = 0$ while F^+ has $I = 0$ and $S = -1$. It is obvious that the analysis for $B(\bar{3})$ decays applies for the decay of any $\underline{\bar{3}} \to \underline{8} + \underline{9}$ via H_{CS}, so that with appropriate changes it applies to the decay $P(\bar{3}) \to P(8) + V(9)$ if one neglects η and η' mixing. However, to take into account this mixing, that is, to extend the analysis to $P(\bar{3}) \to P(9) + V(9)$, one has to define two extra amplitudes. Let

$$A_{17} = A(D^0 \to \eta' \bar{K}^{0*}), \quad A_{18} = (F^+ \to \eta' \rho^+) , \quad (9a)$$

$$A_8' = A(D^0 \to P_8 \bar{K}^{0*}), \quad A_{12}' = A(F^+ \to P_8 \rho^+) , \quad (9b)$$

$$A_{17}' = A(D^0 \to P_1 \bar{K}^{0*}), \quad A_{18}' = A(F^+ \to P_1 \rho^+) . \quad (9c)$$

Then the earlier sum rules are valid with $A_8 \to A_8'$ and $A_{12} \to A_{12}'$; it is understood that A_i now refer to the appropriate $P(\bar{3}) \to P(9) + V(9)$ decay. In addition one obtains two more relations:

$$A_{17}' = A_{15}', \quad A_{18}' = A_{16}' . \quad (10)$$

The relations obtained above can be specialized to the case of the decay of a $\bar{3}$ into two identical nonets with arbitrary mixing angle. We indicate the specialization for the $P(\bar{3}) \to P(9) + P(9)$ and $V(\bar{3}) \to P(9) + P(9)$ decays.

$P(\bar{3}) \to P(9) + P(9)$ decays. In this case there are only nine decays which are given in terms of the

three symmetric amplitudes g_S, g_S', and g_{27}'. The two isospin and four SU(3) sum rules (with A_i referring to the present case, see Table II) for the full H_{CS} can be obtained by setting $A_1=A_2$, $A_3=A_7$, $A_5=A_6$, $A_4'=A_8'$, $A_9=A_{10}$, $A_{11}'=A_{12}'$, and $A_{13}=A_{14}$ in the relations in (6) and (7). Note that the two relations in (10) become mere identities.

$V(\bar{3})\to P(9)+P(9)$ decays. The nine decay amplitudes (Table II) in this case are given in terms of the four antisymmetric parameters g_{10}, g_A, g_{10}', and g_A'. The one isospin and four SU(3) sum rules for the full H_{CS} can be obtained from (6) and (7) by setting $A_1=-A_2$, $A_3=-A_7$, $A_5=-A_6$, $A_4'=-A_8'$, $A_9=-A_{10}$, $A_{11}'=-A_{12}'$ and $A_{13}=-A_{14}$. Note that the relations in (10) and (6c) are mere identities in this case.

It should be noted that nearly all the SU(3) relations for the full H_{CS}, obtained for the meson decays into identical nonets, involve at most three to four decay amplitudes. Thus these relations together with the relations noted earlier for the $B(\bar{3})$ decays will provide tests for the presence of both the H_- and H_+ pieces in H_{CS}, the $\Delta C=\Delta S$ interaction in the GIM scheme. Finally, it is of interest to note that the relations obtained above are independent of (i) whether the current is pure $(V-A)$ or not and (ii) the addition[12] of a $(V+A)$ piece transforming like J_2^4 as this does not contribute to the $\Delta C=\Delta S$ decays.

TABLE II. The $\Delta C=\Delta S$ decay amplitudes for $P(\bar{3})\to P(9)+P(9)$. The four primed amplitudes A_4', A_{11}', A_{15}', and A_{16}' have been included for the purpose of handy reference. The amplitudes for the $V(\bar{3})\to P(9)+P(9)$ decays are obtained by the replacement $(D^+,D^0,F^+)\to(D^{+*},D^{0*},F^{+*})$.

$A_1=A(D^+\to\pi^+\bar{K}^0)$	$A_{15}=A(D^0\to\eta'\bar{K}^0)$
$A_3=A(D^0\to\pi^0\bar{K}^0)$	$A_{16}=A(F^+\to\eta'\pi^+)$
$A_4=A(D^0\to\eta\bar{K}^0)$	$A_4'=A(D^0\to P_8\bar{K}^0)$
$A_5=A(D^0\to\pi^+K^-)$	$A_{11}'=A(F^+\to P_8\pi^+)$
$A_9=A(F^+\to\pi^0\pi^+)$	$A_{15}'=A(D^0\to P_1\bar{K}^0)$
$A_{11}=A(F^+\to\eta\pi^+)$	$A_{16}'=A(F^+\to P_1\pi^+)$
$A_{13}=A(F^+\to K^+\bar{K}^0)$	

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