PHYSICAL REVIEW D VOLUME 15, NUMBER 5 1 MARCH 1977

Reaction rate, weak corrections, and background in $e^+e^- \to \pi^0\gamma, \eta\gamma$

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(Received 13 August 1976; revised manuscript received 16 December 1976)

The $e^+e^- \to \pi^0\gamma, \eta\gamma$ cross sections at high energies are computed on the basis of resonance-pole and quark-loop approximations. A sizable difference is found between the results of these two models; this makes the measurement one of importance especially for detection of charmed, and heavier, quarks. The effects of beam polarization are included, as are the weak (neutral-current) corrections. The weak corrections are, in most weak-interaction models, relatively small. The 3γ background is also computed including the effects of beam polarization. It is large and there does not seem to be an easy way of using beam-polarization effects to suppress it.

I. INTRODUCTION

The pseudoscalar-meson, two-photon vertex has been studied in great detail by a number of physicists. The study of the case where all particles are on the mass shell was begun by Schwinger[1] and by Steinberger[2] in 1949 and, perhaps, concluded with the work on anomalies by Bell and Jackiw[3] and Adler[4] 20 years later. The off-mass-shell case has been studied by considering the reactions $\pi^0 \to e^+e^-\gamma$, $\pi^0 \to e^+e^-$ and the corresponding processes for η mesons. This work was started by Drell[5] and by Berman and Geffen[6] in 1959, continued through Pratap and Smith in 1972,[7] and is still being done.[8]

The off-mass-shell vertex can also be studied in e^+e^- scattering. The reaction $e^+e^- \to e^+e^-\pi^0$ was first proposed by Low[9] in 1960; the rates at storage ring beam energies of 1–3 GeV were discussed by Parisi and Kessler[10] more recently. The reaction rates for $e^+e^- \to \pi^0\gamma$ were calculated by Young[11] for low-energy beams using hard-meson techniques. Our purpose here is to consider this last process for higher energies in order to try to determine whether the counting rates would be large enough to make it feasible at PEP and what conclusions could be drawn from measurements of its cross section.

In the next section we calculate the cross sections for $e^+e^- \to \pi^0\gamma$ and $e^+e^- \to \eta\gamma$ for two models of the pseudoscalar-meson–photon–photon vertex. One model simply assumes a simple pole form factor, the other model takes a quark loop. The two models give surprisingly different answers.

The energies at PEP will be large enough to make weak neutral-current effects significant in many processes. In Sec. III we discuss the neutral-current contribution to these reactions. Section IV gives the background due to the process $e^+e^- \to 3\gamma$.

II. REACTION RATE FOR $e^+e^- \to \pi^0\gamma$ AND $e^+e^- \to \eta\gamma$

The $P \to \gamma\gamma$ matrix element, where $P = \pi^0$ or η, is defined as

$$\langle \gamma(k)\gamma(k') | T | P \rangle = \epsilon_{\mu\nu\alpha\beta}\epsilon^{\mu}(k)\epsilon^{\nu}(k')k^{\alpha}k'^{\beta}F(k^2, k'^2) . \tag{1}$$

In terms of $F(k^2, k'^2)$, the decay rate for $P \to \gamma\gamma$ is

$$\Gamma(P \to \gamma\gamma) = \alpha^2 \frac{\pi}{4} \mu^3 |F(0,0)|^2 , \tag{2}$$

where μ is the mass of P and α is the fine-structure constant. The differential cross section for $e^+e^- \to P\gamma$ is given by

$$\begin{aligned}\frac{d\sigma}{d\Omega_\gamma} = \alpha^3 \frac{\pi}{4}\left(1 - \frac{\mu^2}{4E^2}\right)^3 |F(4E^2, 0)|^2 \\ \times [\tfrac{1}{2}(1+\cos^2\theta)(1 + P_1^L P_2^L - P_1^T P_2^T) \\ + P_1^T P_2^T (1 - \sin^2\theta\cos^2\phi)] .\end{aligned} \tag{3}$$

E is the beam energy, θ and ϕ are the scattering angles, and $P_{1,2}^{L,T}$ give the polarization of the initial beams in the following way: If the four-momenta of the initial particles are given by

$$p_1^\mu = (E, 0, 0, p) , \tag{4a}$$

$$p_2^\mu = (E, 0, 0, -p) , \tag{4b}$$

then

$$s_1^\mu = P_1^L(p, 0, 0, E)\frac{1}{m} + P_1^T(0, 1, 0, 0) , \tag{4c}$$

$$s_2^\mu = P_2^L(-p, 0, 0, E)\frac{1}{m} + P_2^T(0, -1, 0, 0) . \tag{4d}$$

Notice that the dependence on the transverse polarization goes away when the ϕ integral is evaluated.

It is convenient to divide (3) by (2), expressing the $e^+e^- \to P\gamma$ differential cross section in terms of the $P \to 2\gamma$ rate, and to integrate over some solid angle. If we choose

$$0 \leq \phi \leq 2\pi, \quad -\frac{1}{\sqrt{2}} \leq \cos\theta \leq \frac{1}{\sqrt{2}},$$

then

$$\Delta\sigma(P\gamma) = \alpha \frac{1}{\mu^3} \Gamma(P \to \gamma\gamma)\left(1 - \frac{\mu^2}{4E^2}\right)^3 \tfrac{7}{6}\sqrt{2}\,\pi(1 + P_1^L P_2^L)\left|\frac{F(4E^2,0)}{F(0,0)}\right|^2 \tag{5}$$

$$= \begin{Bmatrix} 4.3 \pm 0.5 \\ 8.9 \pm 1.7 \end{Bmatrix} \times 10^{-35}\left(1 - \frac{\mu^2}{4E^2}\right)^3 (1 + P_1^L P_2^L)\left|\frac{F(4E^2,0)}{F(0,0)}\right|^2 \text{cm}^2 \text{ for } \begin{Bmatrix} P = \pi^0 \\ P = \eta \end{Bmatrix}. \tag{6}$$

To proceed further we need a model for $F(4E^2,0)/F(0,0)$. Two possibilities immediately come to mind; one is simply to choose a pole form, such as

$$\frac{F(k_1^2,k_2^2)}{F(0,0)} = \frac{1}{1 - (k_1^2 + k_2^2)/m^2} \tag{7a}$$

or

$$\frac{F(k_1^2,k_2^2)}{F(0,0)} = \frac{a}{1 - k_1^2/m_1^2} + \frac{(1-a)}{1 - k_2^2/m_2^2}, \tag{7b}$$

where m_1 and m_2 are some unknown masses. Such forms go as $1/k^2$ as k_1^2 and k_2^2 get large with k_1^2/k_2^2 fixed. This large-k behavior was derived by Cornwall[12] and by Gross and Treiman[13] by assuming quark-model commutation relations (equal time or light cone) and the Bjorken-Johnson-Low (BJL) theorem. In fact, if one assumes that the BJL light-cone theorem is finite, then it is possible to show that $F(k^2,0)$ should also go as $1/k^2$, which would rule out (7b). This is the same as assuming that $F(k^2,0)$ scales and is a stronger assumption than is probably warranted; however, we will not consider (7b) further because of the intuitive feeling that $F(k_1^2,0)$ should decrease for large k^2. Plots of (7a) are shown in Fig. 1 for m equal to one nucleon mass and m equal to two nucleon masses.

A second possibility is to take a quark-loop model for $F(k^2,0)$. Such a model gives the correct answer for $\pi^0 \to \gamma\gamma$ and $\pi^0 \to e^+e^-\gamma$, but of course that is thought to be an "accident," related to the fact that only the bare quark loop has an anomaly. Nevertheless, this is an interesting model because of the possibility that there is some validity in doing quark-model calculations. In this model

$$\frac{F(k^2,k'^2)}{F(0,0)} = \frac{4m^2}{(2\pi)^2}\left[\int d^4s \frac{1}{s^2 - m^2}\frac{1}{(s-k)^2 - m^2}\frac{1}{(s+k')^2 - m^2} + (k \leftrightarrow k')\right], \tag{8}$$

where m is the quark mass.

Equation (8) gives[7]

$$\frac{F(4E^2,0)}{F(0,0)} = \frac{1}{1 - \mu^2/4E^2}\frac{m^2}{4E^2}$$

times, if $E/m \leq 1$,

$$4[(\sin^{-1}E/m)^2 - (\sin^{-1}\mu/2m)^2],$$

or, if $E/m \geq 1$, times

$$\pi^2 - \ln^2\left[\frac{1 + (1 - m^2/E^2)^{1/2}}{1 - (1 - m^2/E^2)^{1/2}}\right] + 2i\pi \ln\left[\frac{1 + (1 - m^2/E^2)^{1/2}}{1 - (1 - m^2/E^2)^{1/2}}\right] - 4(\sin^{-1}\mu/2m)^2, \tag{9}$$

where μ is the pseudoscalar mass, $\mu^2 = (k + k')^2$.

The square of (9) is shown in Fig. 1. Unlike the pole form of (7a) which decreases rapidly as soon as $E > m/2$, eventually going as $1/E^4$, this form continues to increase until $E = m$ and then decreases as

$$\left(\frac{m}{2E}\right)^4 \ln^4\left(\frac{2E}{m}\right)$$

when $E \to \infty$. Thus, if the form-factor mass in (7a) is equal to the quark mass in (9), the quark model gives a much larger cross section.

III. NEUTRAL CURRENTS

The amplitude for the photon-exchange contribution to $e^+e^- \to P\gamma$ goes as E^{-2} because of the photon propagator. The amplitude for neutral-current exchange is weak, but does not decrease at PEP energies as E gets large, since E is still much less than the effective mass of the exchange. Thus the possibility of a significant contribution from the neutral current increases as the energy increases.

Let us write the effective weak neutral interaction in the canonical form

$$\mathcal{H}_{\text{eff}} = \bar{\psi}_e \gamma^\alpha (g_V + g_A\gamma_5)\psi_e Z_\alpha + G_V V^\alpha Z_\alpha + G_A A^\alpha Z_\alpha , \quad (10)$$

where V^α and A^α are the hadronic vector and axial-vector currents. The axial vector will not contribute to our reaction because of charge conjugation, so this reaction offers a chance to see the weak vector current alone. Including the cross terms between (10) and the one-photon exchange changes (3) into

$$\frac{d\sigma}{d\Omega} = \alpha^3 \frac{\pi}{4}\left(1-\frac{\mu^2}{4E^2}\right)^3 |F(4E^2,0)|^2 \left\{\left(1+\frac{2g_V G_V}{e^2}\frac{4E^2}{m_Z{}^2}f\right)\left[\tfrac{1}{2}(1+\cos^2\theta)(1+P_1^L P_2^L - P_1^T P_2^T) + P_1^T P_2^T(1-\sin^2\theta\cos^2\phi)\right] + \frac{g_A G_V}{e^2}\frac{4E^2}{m_Z{}^2} f(1+\cos^2\theta)(P_1^L+P_2^L)\right\}, \quad (11)$$

where f is the ratio of $F_W(4E^2,0)$, defined with one weak and one electromagnetic current, to the $F(4E^2,0)$ defined in (1) and in general depends on the strong interactions.

The size of the neutral-current corrections varies greatly from model to model. For example, in the Weinberg-Salam model

$$\frac{g_V G_V}{m_Z{}^2} = -\frac{G}{\sqrt{2}}(1-4\sin^2\theta_W), \quad (12a)$$

$$\frac{g_A G_V}{m_Z{}^2} = -\frac{G}{\sqrt{2}}, \quad (12b)$$

where G is the weak Fermi coupling and θ_W is the Weinberg angle. In this model, the neutral vector current is

$$V_3^\mu - 2\sin^2\theta_W V_{\text{em}}^\mu \quad (13)$$

and since, by isospin invariance,

$$\langle \pi^0 | (V_3^\mu(x) V_{\text{em}}^\nu(0))_+ | 0\rangle = \tfrac{1}{2}\langle \pi^0 | (V_{\text{em}}^\mu(x) V_{\text{em}}^\nu(0))_+ | 0\rangle \quad (14)$$

we have

$$f = \tfrac{1}{2} - 2\sin^2\theta_W \quad (15)$$

for the $\pi^0\gamma$ final state, independent of the structure of the currents. In the quark model

$$f = \tfrac{3}{2} - 2\sin^2\theta_W \quad (16)$$

for the $\eta\gamma$ final state. Thus

$$1+\frac{8g_V G_V}{e^2}\frac{E^2}{m_Z{}^2}f = 1-\frac{8E^2}{e^2}\frac{G}{\sqrt{2}}(1-4\sin^2\theta_W)\times\begin{cases}\tfrac{1}{2}-2\sin^2\theta_W, & \text{for } \pi^0\gamma \text{ final state,}\\ \tfrac{3}{2}-2\sin^2\theta_W, & \text{for } \eta\gamma \text{ final state,}\end{cases} \quad (17a)$$

$$\frac{8g_A G_V}{e^2}\frac{E^2}{m_Z{}^2}f = -\frac{8E^2}{e^2}\frac{G}{\sqrt{2}}\times\begin{cases}\tfrac{1}{2}-2\sin^2\theta_W, & \text{for } \pi^0\gamma \text{ final state,}\\ \tfrac{3}{2}-2\sin^2\theta_W, & \text{for } \eta\gamma \text{ final state.}\end{cases} \quad (17b)$$

At $\sin^2\theta_W = \frac{1}{2}$ Eq. (17a) gives

$$1 \pm 3.5\times 10^{-4}E^2,\ E \text{ in GeV}, \quad (18a)$$

while (17b) is

$$\mp 3.5\times 10^{-4}E^2,\ E \text{ in GeV}. \quad (18b)$$

The upper sign is for π^0, the lower sign is for η. Thus the corrections are quite small, even for $E = 15$ GeV (~8%) where the photon-exchange part of the cross section has decreased considerably.

A model in which the weak corrections are sometimes larger is the scalar-exchange model.[14,15] There the weak interaction proceeds through the exchange of two scalar mesons in the presence of heavy leptons. If the heavy leptons are charged, as in Ref. 14 and Ref. 15, then the effective neutral-current coupling to electrons is purely axial-vector and there is no contribution to $e^+e^- \to P\gamma$. However, the heavy leptons could be neutral, in which case the effective neutral-current interaction is

$$-\frac{G}{\sqrt{2}}\xi\bar{\psi}_e\gamma^\alpha(1-\gamma_5)\psi_e(V_\alpha - A_\alpha), \quad (19)$$

where the vector current is purely isovector so that $f = \frac{1}{2}$ for $\pi^0\gamma$.

ξ depends on the particular version of the theory; it can have the values 3.2, 4.5, 6.3, or 18.6 in special versions. Thus in this model, the corrections in (18) would be multiplied by one of these numbers for $\pi^0\gamma$ or by approximately three times these numbers for $\eta\gamma$ and could be very large even at low E.

The difficulty in observing neutral currents in

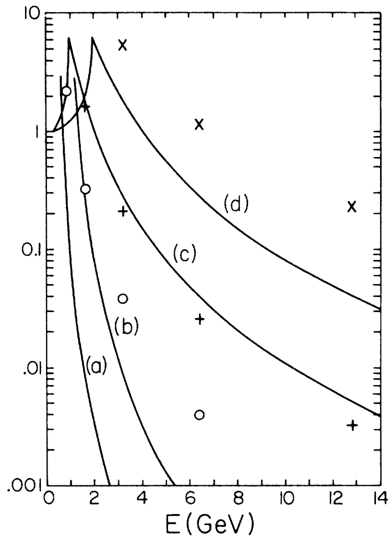

FIG. 1. $|F(4E^2,0)/F(0,0)|^2$ as a function of the beam energy, E. (a) and (b) show the dipole form, while (c) and (d) are for the quark loop. (a) and (c) use a mass equal to the nucleon mass, while (b) and (d) have a mass of twice the nucleon mass. The curves are drawn for the $\pi^0\gamma$ final state but also hold for $\eta\gamma$ if $E \geq 0.5$ GeV. The points without a line through them give the $e^+e^- \to 3\gamma$ background for various counter conditions normalized to the $\pi^0\gamma$ cross section, Eq. (6). The X correspond to case (i) in Sec. IV, while + and O give the results of cases (ii) and (iii). See Fig. 2 for more detail of $e^+e^- \to 3\gamma$.

e^+e^- collisions is the background. For example, the most popular e^+e^- process in which to look for neutral-current corrections is $e^+e^- \to \mu^+\mu^-$, but there the two-photon exchange simulates a neutral current and must be carefully subtracted out. Our reactions have no problem from two-photon exchange but they do have severe background problems from $e^+e^- \to 3\gamma$. These turn out to be severe to the point of destroying the utility of the process and we discuss them next.

IV. BACKGROUND DUE TO $e^+e^- \to 3\gamma$

The $\pi^0\gamma$ or $\eta\gamma$ final states will have to be detected by detecting the three photons. Young[11] pointed out that the background due to $e^+e^- \to 3\gamma$ could be suppressed by requiring that one of the photons have the correct energy, $\omega = E - \mu^2/4E$, to within some $\Delta\omega$ and by taking *only* events with the three photons near the plane perpendicular to the beam axis. The background problem becomes worse for higher energies since $\sigma(3\gamma)$ goes roughly like

$$\int_{E-\Delta\omega}^{E+\Delta\omega} d\omega_1 \frac{\omega_1}{E^4} \sim \frac{\Delta\omega}{E^3}, \tag{20}$$

and thus falls off only as E^{-3} for fixed $\Delta\omega$, whereas $\sigma(P\gamma)$ falls as E^{-4}. Thus higher E may require even smaller $\Delta\omega$ and other restrictions on the events accepted. Therefore we have calculated the cross section for $e^+e^- \to 3\gamma$.

This cross section has been calculated before but not, to our knowledge, including the effects of beam polarization. The matrix element squared is

$$X_{\text{total}} = X_1(1 + P_1^L P_2^L) + X_2 P_1^T P_2^T, \tag{21}$$

where $P_{1,2}^{L,T}$ are defined in (4). X_1 is given in Jauch and Rohrlich.[16] If we call the photon momenta k_1, k_2, and k_3 then in terms of the definitions

$$\kappa_1 = \omega_1(1-\cos\theta_1), \tag{22a}$$

$$\kappa_2 = \omega_2(1-\cos\theta_2), \tag{22b}$$

$$\kappa_3 = \omega_3(1-\cos\theta_3), \tag{22c}$$

$$\kappa_1' = \omega_1(1+\cos\theta_1), \tag{22d}$$

$$\kappa_2' = \omega_2(1+\cos\theta_2), \tag{22e}$$

$$\kappa_3' = \omega_3(1+\cos\theta_3), \tag{22f}$$

X_1 is given by

$$X_1 = \frac{1}{\kappa_1^2\kappa_2^2\kappa_3^2}\left(4E^2 - \sum_i \kappa_i^2\right)^2 + \frac{1}{\kappa_1'^2\kappa_2'^2\kappa_3'^2}\left(4E^2 - \sum_i \kappa_i'^2\right)^2 - 16E\left(\frac{\kappa_1\kappa_2\kappa_3 + \kappa_1'\kappa_2'\kappa_3'}{\kappa_1\kappa_2\kappa_3\kappa_1'\kappa_2'\kappa_3'}\right)$$
$$+ 8\frac{1}{\kappa_1\kappa_2\kappa_3\kappa_1'\kappa_2'\kappa_3'}\sum_i \omega_i^2(1-z_i^2)\sum_i \omega_i^2(1+z_i^2) - 8\sum_i \frac{1}{\omega_i^2(1-z_i^2)}\sum_i \frac{1+z_i^2}{1-z_i^2}, \tag{23}$$

where $z_i = \cos\theta_i$. We have calculated the coefficient of the transverse polarization using the algebraic-manipulation program SCHOONSCHIP[17] to do the traces. The coefficient can be expressed in a simple form by making use of the definitions

$$X = E\sum_{i=1}^{3}\kappa_i, \quad Z = E^2\sum_{i=1}^{3}\kappa_i\kappa_i', \tag{24a}$$

$$A = E^3\kappa_1\kappa_2\kappa_3, \quad B = E^3\kappa_1'\kappa_2'\kappa_3', \quad C = E^2\kappa_1\kappa_1', \quad D = E^2\kappa_2\kappa_2', \quad F = E^2\kappa_3\kappa_3', \tag{24b}$$

as

$$X_2=\frac{8x}{D}(\cos 2\phi_1+\cos 2\phi_3)+\frac{4xz}{D}\left(\frac{\cos 2\phi_1}{F}+\frac{\cos 2\phi_3}{C}\right)-\frac{2x^2}{AB}(D-3C-3E)\omega_1\omega_3\sin\theta_1\sin\theta_3\cos(\phi_1+\phi_3), \tag{25}$$

where we have used four-momentum conservation to eliminate the k_2 dependence. ϕ_1 and ϕ_3 are the azimuthal angles of k_1 and k_3.

The phase-space integrals were evaluated twice, in two different ways. The δ function was used to integrate over d^3k_2. The first method used the remaining δ function to evaluate the ω_3 integral leaving

$$d\sigma=\frac{\alpha^3}{8\pi}\frac{1}{E}\frac{(E-\omega_1)X_{\text{total}}}{[2E-\omega_1(1-\cos\phi_{13})]^2}\omega_1\,d\omega_1\,d(\cos\phi_1)d(\cos\phi_3)d\phi_3\,d\phi_1. \tag{26}$$

θ_{13} is the angle between $\hat{k}_1$ and $\hat{k}_3$. ω_2 and ω_3 are given by

$$\omega_3=\frac{2E(E-\omega_1)}{2E-\omega_1(1-\cos\theta_{13})}, \tag{27a}$$

$$\omega_2=2E-\omega_1-\omega_3. \tag{27b}$$

The second method used the remaining δ function for the azimuthal integral. This method facilitates an analytic proof that the coefficient of the transverse polarization is zero when all values of ϕ_1 and ϕ_3 are integrated over. In this case

$$d\sigma=\frac{\alpha^3}{32\pi^2}\frac{1}{E^2}\frac{X_{\text{total}}\delta(\cos(\phi_1-\phi_3)-Q)}{\sin\theta_1\sin\theta_3}d\omega_1\,d\omega_3\,d(\cos\theta_1)d(\cos\theta_3)d\phi_1\,d\phi_3, \tag{28}$$

where

$$Q=\frac{2E(E-\omega_1-\omega_3)+\omega_1\omega_3(1-\cos\theta_1\cos\theta_3)}{\omega_1\omega_3\sin\phi_1\sin\phi_3}. \tag{29}$$

To show that $\int_0^{2\pi}d\phi_1\int_0^{2\pi}d\phi_3X_2$ is zero, it is convenient to change variables to

$$\phi_+=\tfrac{1}{2}(\phi_1+\phi_3),\qquad \phi_-=\phi_1-\phi_3. \tag{30}$$

Then X_2 can be written (X_1 is independent of ϕ)

$$X_2=(U+V)\cos 2\phi_+\cos\phi_-+(U-V)\sin 2\phi_+\sin\phi_- +W\cos 2\phi_+, \tag{31}$$

where, from (25),

$$U=\frac{8x}{D}+\frac{4xz}{FD}, \tag{32a}$$

$$V=\frac{8x}{D}+\frac{4xz}{CD}, \tag{32b}$$

$$W=-\frac{2x^2}{AB}(D-3C-3F)\omega_1\omega_3\sin\theta_1\sin\theta_3. \tag{32c}$$

The limits on the $\phi_\pm$ integrals are

$$\int_0^{2\pi}d\phi_1\int_0^{2\pi}d\phi_3=\int_0^{\pi}d\phi_+\int_{-2\phi_+}^{2\phi_+}d\phi_- +\int_{\pi}^{2\pi}d\phi_+\int_{-4\pi+2\phi_+}^{4\pi-2\phi_+}d\phi_-. \tag{33}$$

Because of the δ function in (28), the ϕ_- integral is independent of the ϕ_+ and appears in (31) in the form $\cos 2\phi_+$ or $\sin 2\phi_+$ which integrate to zero in both of the terms in (33). Alternatively, one sees that since the δ function in (28) is even in ϕ_- any terms added in ϕ_- will vanish and (33) becomes

$$2\left(\int_0^{\pi}d\phi_+\int_0^{2\pi}d\phi_- +\int_{\pi}^{2\pi}d\phi_+\int_0^{4\pi-2\phi_+}d\phi\right) =2\int_0^{2\pi}d\phi_-\int_{\phi_-/2}^{2\pi-\phi_-/2}d\phi_+.$$

Performing the ϕ_+ integration we are left with

$$\int_0^{2\pi}d\phi_-\delta(\cos\phi_- -Q)[(U+V)\cos\phi_-\sin\phi_-+W\sin\phi_-]=\frac{1}{|\sin(\cos^{-1}Q)|}[(U+V)\cos\phi_-\sin\phi_-+W\sin\phi_-]\Big|_{\phi_-=\pm\cos^{-1}Q},$$

which is zero.[18]

After the ϕ integrals were evaluated we evaluated the remaining four-dimensional integral over X_1 by means of straightforward Monte Carlo integration. We used about 150 000 points at each energy E and considered a range of energies from $E=0.1$ GeV to $E=20$ GeV. (We checked our program against the exact answer given in Ref. 16 for the nonrelativistic limit. In addition, the integrals were evaluated using the two different methods above completely independently.) We considered the case of restricted solid angle for the γ detectors, requiring (a) $45°<\theta_i<135°$, $i=1,2,3$, and also took into account finite-energy resolution, requiring (b) $|\omega_1-\omega|<\Delta\omega$ with $\omega=E-\mu^2/4E$.

We evaluated (26) for three cases:

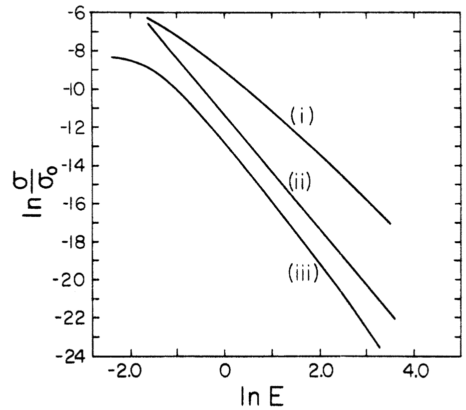

FIG. 2. $\ln(\sigma/\sigma_0)$ vs $\ln E$ for the $e^+e^- \to 3\gamma$ background process where E is in GeV and $\sigma_0 = \alpha^3/8\pi m_e^{\,2}\beta = 2.31 \times 10^{-29}\ \mathrm{cm}^2/\beta$. σ is given by Eq. (21) and the three curves (i), (ii), and (iii) are the cases (i)–(iii) described in the text following Eq. (21).

(i) (a) and (b) obtain with $\Delta\omega = 20$ MeV;

(ii) We require in addition [the largest contribution to (26) is from the small ω_2, ω_3 region] (c) $\omega_2, \omega_3 > 0.1E$;

(iii) (a), (b), and (c) obtain with $\Delta\omega = 2$ MeV.

Since the pseudoscalar meson has zero spin, its decay is isotropic in its center of mass and thus linear in γ energies measured in the center-of-mass system. Imposing condition (c) therefore rejects only 10% of the $P\gamma$ events while cutting the 3γ background by a factor of 20 or more. The resolution $\Delta\omega$ of 2 MeV for GeV-range photons is perhaps overly optimistic. On the other hand, for an actual experiment, further restrictions on events accepted are possible.

The results for the background are given in Fig. 2 and also, in part, by the points in Fig. 1. It should be noted that our results are larger, where comparable, than those of the appendix of Ref. 11 by a factor of about 10^4. We are rather confident that the present results are (unfortunately) the correct ones.[19]

One sees from Fig. 1 that the 3γ background is larger than the process of interest if the pole form factor obtains (in the region of its validity). If, on the other hand, the larger quark-loop cross section governs and the quark masses are fairly large then the 3γ background is probably manageable, allowing reasonable measurement of $e^+e^- \to \pi^0\gamma$ or $e^+e^- \to \eta\gamma$. It is also clear that the heavier the quark the better these processes are for detecting it; in models with quarks still heavier than the charmed quark, counting rates should be above background. Detecting the contribution of such quarks to this process could be a valuable alternative to observing directly the 1^- bound state for ascertaining the existence of such quarks.

Since the terms proportional to $P_1^T P_2^T$ vanish there is no obvious way to use the beam polarization to reduce the 3γ process. We gave considerable thought to how the polarization might be utilized for this purpose, such as comparing events in one hemisphere with those in a different hemisphere for example, but no method emerged. Observation of the small weak effects of Sec. III seems unlikely.

V. CONCLUSIONS

The process $e^+e^- \to P\gamma$ ($P = \pi^0, \eta$) at PEP energies is, as seen in Sec. II, dramatically different according to whether it is described by a pole-dominated form factor or by the anomalous quark-loop diagram. The rapid falloff of the pole form factor is not unexpected. It is almost impossible at these energies to see any two-particle final state involving hadrons. What is surprising is that the quark-loop form factor is so much larger. Since it is so large, the quark loop may well dominate any other strong-interaction structure of the pseudoscalar meson and then, assuming projected PEP luminosities, it should be possible to see this process at the rate of several events per hour.

The result of Sec. III is that the process is not a good place to look for weak neutral-current effects except for the possibility of checking versions of models, such as that of the neutral-heavy-lepton version of the Kummer-Segré model, that have especially large weak-interaction neutral currents. The result of Sec. IV is that the most important contribution of the background, $e^+e^- \to 3\gamma$, is quite large, and probably precludes observing directly the rapid fall predicted by the pole-dominated form-factor model. This 3γ process is rather interesting in its own right.

ACKNOWLEDGMENTS

We are grateful to Professor John Ficenec and Professor Louis Osborne for several helpful conversations, and to Professor B. L. Young for discussions about Sec. IV.

*Work supported in part by the Energy Research and Development Administration, Contract No. E(40-1)3992.
†Work supported in part by the National Science Foundation.

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[18]The fact that the coefficient of the transverse-beam polarization is zero when integrated over all azimuthal angles is not particularly surprising. It is certainly true for two-body final states. For example, the cross section for $e^+e^- \to \gamma\gamma$, including beam polarization, is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8}\sin^2\theta\left\{\left[\frac{1}{(E-pz)^2}+\frac{1}{(E+pz)^2}\right](1+P_1^L P_2^L) - 2\frac{P_1^T P_2^T}{E^2-p^2z^2}\cos 2\phi\right\},$$

where $z=\cos\theta$.

[19]After this paper was completed we were informed of another recent calculation of $e^+e^- \to 3\gamma$ by E. Pelaquier and F. Renard, Nuovo Cimento 32A, 421 (1976). They take a different range of photon energies but, after allowing for this difference, their values seem to agree with ours.