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Vector-meson-dominance approach to e^+e^- annihilation and deep-inelastic scattering

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Within the framework of the generalized vector-dominance model a simple solution for the vector-meson–photon couplings, consistent with an asymptotically scaling total cross section for e^+e^- annihilation, is obtained. Using this result, the level spacing for the new vector mesons is discussed. The nucleon structure functions are calculated and an agreement with experimental data is observed.

I. INTRODUCTION

The generalized vector-dominance model (GVDM)[1,2] has been successful, qualitatively, in understanding the electromagnetic properties of the hadrons. According to the GVDM assumptions, the photon at high energies interacts with matter like a series of vector mesons. Several applications of this idea have been useful in deriving scaling results in e^+e^- annihilation into hadrons, deep-inelastic electron-nucleon scattering, and lepton-pair production in hadron collisions.[3]

The aim of this paper is to provide a simple expression exhibiting prominent peaks in the low-energy region and behaving like a power of the total energy in the asymptotic region for the total cross section of e^+e^- annihilation into hadrons.

We first study e^+e^- annihilation into hadrons. With our scheme it is possible to determine the asymptotic value of the ratio $R=\sigma(e^+e^-\to \text{hadrons})/(e^+e^-\to\mu^+\mu^-)$ from the known parameters of the low-lying vector mesons. We find that the value of R obtained from this calculation is the same as the value obtained from the quark-parton models[4] for the well-known ρ, ω, and ϕ mesons.

Assuming that this also applies for the newly discovered vector mesons, we predict a level spacing dramatically different from the level spacing of the ρ, ω, ϕ mesons. We discuss some implications of this result for the new mesons. In the last part, we use our results to discuss the nucleon structure functions in the framework of two-component duality[2,5] (diffractive + resonance). The model accounts for scaling behavior of the transverse parts of the structure functions.

II. ELECTRON-POSITRON ANNIHILATION INTO HADRONS

Electron-positron annihilation into hadrons[6] seems to be dominated by prominent peaks in low-energy region, and behaves as a decreasing power of the total energy in the asymptotic region. Models describing e^+e^- annihilation should be consistent with these observations. We consider e^+e^- annihilation into hadrons in the one-photon-exchange approximation and assume the hypothesis of dominance by an intermediate vector-meson state. Assuming an infinite set of vector states, with a Veneziano-type spectrum, we parametrize the contributions of a vector meson and its recurrences to the total annihilation cross section as

$$\sigma(e^+e^-\to \text{hadrons})=R_0\frac{4\pi\alpha^2 m_0^2}{3s\sqrt{s}}\,\mathrm{Im}\left[\frac{\Gamma(1/a-z)}{\Gamma(\beta+1/a-z)}\right], \tag{1}$$

which satisfies the conditions given above. Here R_0 and β are arbitrary constants to be fixed later, α is the fine-structure constant, and

$$z=\frac{(1+i\gamma)s}{(1+\gamma^2)am_0^2},$$

where we have assumed a Veneziano-type mass spectrum

$$m_n^2=(1+an)m_0^2 \tag{2}$$

for the masses of the recurrences and γ denotes the ratio (Γ_0/m_0), Γ_0 being the width of the vector meson. Our motivation in writing the total cross section as a ratio of two Γ functions is to have an expression where an infinite set of resonance terms can build up a simple scaling behavior in the asymptotic region.

First we determine the parameter β from the asymptotic behavior of the total cross section. From Eq. (1) we see that the total cross section having $1/s$ behavior in the asymptotic region requires[7] that $\beta=-\frac{1}{2}$, which determines β.

Writing the series representation of the Γ functions and calculating the imaginary part, we obtain from Eq. (1)

$$R=\frac{\sigma(e^+e^-\to \text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)}=\frac{24\Gamma(\frac{1}{2})m_\rho^2}{f_\rho^2(s/m_\rho^2)^{1/2}}\sum_{n=0}^{\infty}\frac{\Gamma(n+\frac{3}{2})m_n\Gamma_n}{\Gamma(n+1)[(m_n^2-s)^2+m_n^2\Gamma_n^2]} \tag{3}$$

for the isovector part. Here we have fixed the constant R_0 from the ρ term, and we have defined Γ_n as $\Gamma_n = \gamma m_n$, which corresponds to Greco's formula[2] for the widths of the recurrences. This expressions shows that our first equation is a representation of the diagonal form of the generalized vector-dominance model. Bramon, Etim, and Greco[1,2] have proposed a set of solutions for the parameters in order to sum the infinite series of GVDM. In our formulation the sum is already given by Eq. (1), and this implies some conditions for the parameters involved. We note that z does not take real integer values ($\gamma \neq 0$), and thus we have an analytic expression.

Let us now look at the photon–vector-meson couplings. On the basis of the available data[5,8] we take $a = 2$ for the level spacing of the recurrences of ρ. From Eq. (3) we have

$$\frac{1}{f_n^2} = \frac{\Gamma(n+\frac{1}{2})}{f_\rho^2\,\Gamma(n+1)\,[\pi(1+2n)]^{1/2}} \tag{4}$$

for the coupling of the photon to the vector mesons. Labeling the first even daughter of ρ as ρ'', Eq. (4) gives $f_{\rho''}^2/f_\rho^2 = 5.9$. The properties of these vector states have been studied extensively, and the recent data[6] give $f_{\rho''}^2/f_\rho^2 \approx 5$, suggesting that our parametrization is reasonable for the low-lying vector mesons.

We would like, now to study the asymptotic value of the ratio R. Using Stirling's formula for the Γ functions we obtain from Eq. (1)

$$R = \frac{6\sqrt{\pi}}{(f_0^2/4\pi)}\,\frac{1}{(a)^{3/2}} \tag{5}$$

for large values of s. We note that, in this formulation, the coupling of the low-lying vector meson and the level spacing determine the contribution of that vector meson and its recurrences to R in the asymptotic region. In this framework the total value of R is given by the sum $\sum_V R_V$. Including only ρ, ω, ϕ terms in this sum with the $9:1:2$ ratio of the photon couplings, and using the experimental value[9] $f_\rho^2/(4\pi) = 2.54$ we obtain

$$\begin{aligned} R &= R_\rho + R_\omega + R_\phi \\ &= \frac{4}{3}\,\frac{6\sqrt{\pi}}{(f_\rho^2/4\pi)2\sqrt{2}} = 2. \end{aligned} \tag{6}$$

This value agrees surprisingly well with the values determined from the sum of the squared quark charges in quark-parton models.[4]

This result encourages us to investigate the consistency between the values of R obtained in these two ways for the newly discovered vector mesons. First we note that for these vector mesons the difference between hadronic decay width and total width is not negligible, and this introduces a factor $\Gamma(\text{hadron})/\Gamma(\text{total})$ into the right-hand side of Eq. (5). Calculation of R from the quark-parton model with the interpretation of $J(3.1)$ as a bound state of charm-anticharm quarks (charge $= \frac{2}{3}$) gives

$$3(\tfrac{2}{3})^2 = \frac{6\sqrt{\pi}}{f_J^2/4\pi}\,\frac{1}{a_c^{3/2}}\,\frac{\Gamma(\text{hadron})}{\Gamma(\text{total})} \tag{7}$$

where a_c determines the level spacing of the charm-anticharm vector-meson bound states and the factor 3 takes into account the color degree of freedom. Using the experimental values[9] $(4\pi/f_J^2) \sim 0.09$, $m_J \sim 3.1$ GeV, $\Gamma(\text{hadron})/\Gamma(\text{total}) \sim 0.86$ we obtain $a_c = 0.73$. If this value is taken literally as the level spacing of a spectrum of daughters of the $J(3.1)$ one finds

$$m_J(1+a_c)^{1/2} = 4.1 \text{ GeV} \tag{8}$$

for the first recurrence. Thus we observe a regularity between 3.1- and 4.1-GeV mesons, which makes it tempting to identify the 4.1-GeV meson as a recurrence of the 3.1-GeV one. Such level spacing for the new particles has been estimated from the "new duality"[10] where one assumes that the low-energy resonances in e^+e^- annihilation simulate, globally, in the sense of finite-energy sum rules, a pointlike coupling of currents to partons. Several authors[8,11] have used this conjecture to calculate the level spacing for the new mesons. We note that our observation is in agreement with their results.

III. INELASTIC ELECTRON-NUCLEON SCATTERING

Let us now consider electron-proton scattering in the deep inelastic region. According to the diagonal[2] GVDM the transverse virtual-photon–proton total cross section is given by

$$\begin{aligned} \sigma_T^D &= (4\pi\alpha)\left(1-\frac{1}{\omega}\right)\sum_n \frac{m_n^2}{f_n^2}\,\frac{m_n^2}{(m_n^2+q^2)^2}\,\sigma_n^D \\ &\quad + \text{isoscalars}, \\ \sigma_T^R &= (4\pi\alpha)\left(1-\frac{1}{\omega}\right)\sum_n \frac{m_n^2}{f_n^2}\,\frac{m_n^2}{(m_n^2+q^2)^2}\,\sigma_n^R \\ &\quad + \text{isoscalars}, \end{aligned} \tag{9}$$

where α is the fine-structure constant, σ_n^D and σ_n^R are the diffractive and the nondiffractive vector-meson–nucleon total cross sections. The Bjorken variable is defined as $\omega = (2M\nu)/q^2$.

The proceed further we need the vector-meson-photon coupling constants and the purely strong-interaction cross sections σ_n. We have already discussed the coupling constants in the previous section, and we found that they can be represented as in Eq. (4). For the total cross sections σ_n we will follow Ref. 1. In this paper it is argued that the diffractive part of the cross section behaves like

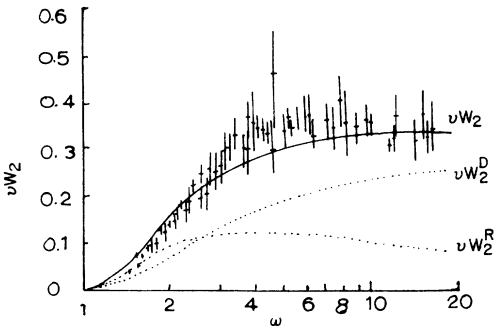

FIG. 1. Diffractive (D) and resonant (R) contributions to the structure function $\nu W_2(\omega)$.

$\sigma_n^D \sim 1/m_n{}^2$, and the nondiffractive part behaves like $\sigma_n^R \sim 1/[m_n(2M\nu)^{1/2}]$. Making use of these arguments in Eq. (9), and expressing the sum as an integral over the variable $y = m_n{}^2/q^2$, we find

$$\begin{aligned}\sigma_T^D &= \left(\frac{4\pi\alpha}{f_\rho{}^2}\right)\left(1-\frac{1}{\omega}\right)\frac{\sigma_\rho^D}{2\pi}\left(\frac{2m_\rho^2}{q^2}\right)^{1/2} \\ &\quad\times\int_0^\infty dy\, B\left(\frac{yq^2}{2m_\rho{}^2},\frac{1}{2}\right)\frac{\sqrt{y}}{(1+y)^2} \\ &\quad+\text{isoscalars}, \\ \sigma_T^R &= \left(\frac{4\pi\alpha}{f_\rho{}^2}\right)\left(1-\frac{1}{\omega}\right)\frac{\sigma_\rho^R}{2\pi}\int_0^\infty dy\, B\left(\frac{yq^2}{2m_\rho{}^2},\frac{1}{2}\right)\frac{y}{(1+y)^2} \\ &\quad+\text{isoscalars},\end{aligned} \tag{10}$$

where B is the usual beta function.

Recalling the definition of the structure functions in terms of the transverse and longitudinal virtual-photon cross sections, we can now calculate the structure functions. We have

$$\begin{aligned}W_1 &= \frac{1}{4\pi^2\alpha}\left(\nu-\frac{q^2}{2M}\right)\sigma_T, \\ W_2 &= \frac{1}{4\pi^2\alpha}\left(\nu-\frac{q^2}{2M}\right)\frac{q^2}{q^2+\nu^2}(\sigma_T+\sigma_L),\end{aligned} \tag{11}$$

where M is the proton mass, and σ_L is the longitudinal virtual-photon cross section. In the following, the longitudinal contribution will be taken into account by means of the experimental relation $\sigma_L/\sigma_T \sim 0.2$. Let us now calculate these expressions in the Bjorken limit. In this limit the integrals of Eq. (10) behave like

$$\left(\frac{2\pi m_\rho{}^2}{q^2}\right)^{1/2}\int_0^\infty \frac{dy}{(1+y)^2} = \left(\frac{2\pi m_\rho{}^2}{q^2}\right)^{1/2}$$

and

$$\left(\frac{2\pi m_\rho{}^2}{q^2}\right)^{1/2}\int_0^\infty \frac{\sqrt{y}}{(1+y)^2}dy = \left(\frac{2\pi m_\rho{}^2}{q^2}\right)^{1/2}\frac{\pi}{2} \tag{12}$$

for the diffractive and the nondiffractive parts. From Eqs. (8), (12) we find that in the Bjorken limit the transverse part of the structure function W_2 scales as

$$\begin{aligned}\nu W_{2T}^D &= \text{const}\times\left(1-\frac{1}{\omega}\right)^2, \\ \nu W_{2T}^R &= \text{const}\times\left(1-\frac{1}{\omega}\right)^2\frac{1}{\sqrt{\omega}}.\end{aligned} \tag{13}$$

Comparison of these expressions with data,[12] as a function of ω, is given in Fig. 1. We note that we can take into account the off-diagonal contributions by means of the off-diagonal GVDM proposed by Fraas, Read, and Schildknecht.[13] This can easily be done by incorporating our solution for the photon–vector-meson couplings into this model.

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