Photon shadowing in nuclei*

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Photon processes in nuclei are studied using a conventional optical model, interpreted unconventionally. To calibrate the method excellent fits to hadron-nucleus total cross sections are obtained. When applied to photon processes the "self-absorption" feature of the model tends to suppress the shadowing from what has previously been predicted. A further suppression is shown to result from the finite width of vector mesons. Reasonable agreement with recent electroproduction data is obtained.

I. INTRODUCTION

As a photon wave propagates through nuclear matter a regeneration process takes place in which vector-meson waves (mainly ρ) build up coherently. The amplitude for any reaction to take place is the coherent sum of amplitudes from these waves. According to vector-meson dominance (VMD) this interference is largely destructive. As a consequence total photon cross sections are reduced from what they would be in the absence of regeneration: an effect called "shadowing."

In the present paper two effects are described, both of which reduce the predicted shadowing (i.e., increase the ratio of the total photon cross section on, say, lead to that on the proton). The first effect is due to the finite lifetime of the ρ and the second is due to a feature to be called "self-absorption." The latter effect also alters the numerical values of the vector-meson parameters extracted from vector-meson photoproduction experiments.

Before starting it is important to emphasize (as many have done[1]) that the shadowing phenomenon does not depend on the hypothesis of VMD. That hypothesis boils down to the prediction of arithmetic relations among experimentally measurable numbers. Since those relationships seem to be rather well satisfied, it is accurate (though misleading) to incorporate VMD relations among amplitudes as one goes along. We will avoid doing that. At the stage of substituting numerical values into the formulas the connections with VMD will be made clear. Unfortunately, there is no universal agreement about what parameters should be regarded as sacred and used as inputs and what should be calculated. Hence, it will be necessary to specify in boring detail exactly what is being held fixed and what is allowed to vary when the two effects in question are introduced.

No attempt will be made to include theoretical ideas such as "generalized" or "extended" or "off-diagonal" VMD. The ideas in those models are quite compatible with the model presented here and the magnitudes of the corrections would be roughly equal. The evidence supporting such generalizations was not deemed persuasive enough to justify the complication of including them.

Also, this paper does not contain a thorough survey of experimental data. Some data are included for illustrative purposes and to establish the parameters. Data on total photon cross sections and Compton scattering are somewhat ragged. What was felt to be a representative sampling has been included.

II. QUALITATIVE DISCUSSION AND ASSUMPTIONS

In this section we will set down most of the assumptions to be made and give a qualitative explanation of the ideas.

As has been stated, shadowing depends on the near cancellation of a photon and a ρ amplitude. If the ρ wave function is further attenuated owing to spontaneous decay, the cancellation is less perfect and hence there is less shadowing. It can be argued that the two-pion state should be treated as a separate channel which can coherently feed back into (and be fed from) the photon and ρ channels. Our formalism is less general than that and is, in fact, a one-particle formalism: A photon can coherently regenerate single vector mesons but not pion pairs; similarly, pion pairs cannot regenerate photons. The decay of a ρ meson into pion pairs has exactly the same effect (attenuation of the ρ wave) as does nuclear absorption in the nuclear matter.

With this hypothesis the equations become closely analogous to the equations governing K^0 regeneration in matter. In that case, though there are interference effects altering the observed rates into a specific channel, such as the two-pion channel, one neglects the coherent amplitude for "inverse decay", of two pions back into a K^0. For ρ's in nuclear matter such a result is less obvious. If it were flagrantly untrue, however, one would expect the shape of the ρ to depend on the nucleus from which it is produced. No such effect is observed.[2]

A more serious assumption is to neglect the direct coupling of a photon to two pions. Pion pairs of mass remote from the ρ are known[3] to be coherently produced from nuclei. Hence there should be an effect which has been called "inelastic shadowing"[4] in which the photon couples to two pions which couple back to a photon.[5] It is unclear to what extent this effect is included in "ρ dominance." If it is not, then there will be an extra shadowing effect which we will not have included. (It depends on the mass of the pion pairs.) This, however, is a different issue which we do not pretend to attack.

The other effect to be discussed is "self-absorption." All calculations in this field, in the end, boil down to the evaluation of integrals over the nuclear volume. Such integrals can be derived from an optical model in which the nuclear matter is replaced by an equivalent (presumably homogeneous) optical medium, or they can be derived from a more sophisticated particle theory (Glauber theory).[6] The simplest example is the calculation of the total cross section, $\sigma_h(A)$, for a hadron, h, incident on a nucleus containing A nucleons. (We will not distinguish between neutrons and protons.) The single-nucleon cross section $\sigma_h^{(0)}$ is assumed known. $\sigma_h(A)$ is given by

$$\sigma_h(A) = 2\int d^2b[1-\exp(-\tfrac{1}{2}n\sigma_h^{(0)}L_b)], \tag{2.1}$$

where the geometry is illustrated in Fig. 1. L_b is the distance through the nucleus at impact parameter b. n is the nucleon-number density which we assume to be *constant and independent of* A.

Equation (2.1) can be expanded as

$$\sigma_h(A) = 2\int d^2b\,[\tfrac{1}{2}n\sigma_h^{(0)}L_b - \tfrac{1}{2}(\tfrac{1}{2}n\sigma_h^{(0)}L_b)^2 + \cdots]. \tag{2.2}$$

The nuclear volume is related to the nucleon-number density by

$$n = \frac{A}{\int L_b\, d^2b}, \tag{2.3}$$

and, as a result, we obtain

$$\sigma_h(A) = A\sigma_h^{(0)}[1-\Delta(A)], \tag{2.4}$$

where $\Delta(A)$ is positive.

Formulas such as (2.2) are usually applied only to large nuclei. Let us, however, consider applying it to a single nucleon. Putting $A=1$ we obtain

$$\sigma_h(1) = \sigma_h^{(0)}[1-\Delta(1)]. \tag{2.5}$$

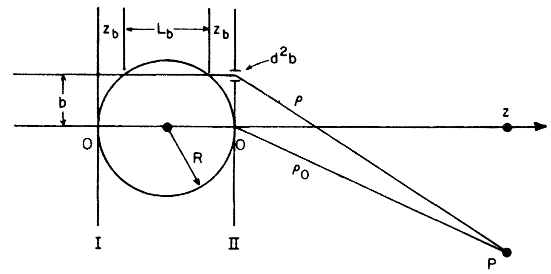

FIG. 1. Scattering geometry. A plane wave is incident from the left.

But $\sigma_h^{(0)}$ has previously been identified as the one-nucleon amplitude, $\sigma_h(1)$, and hence (2.5) is self-contradictory.

We might be willing to tolerate this contradiction if $\Delta(1)$ were small. But, looking at (2.2), one anticipates that $\Delta(A)$ is proportional to $A^{1/3}$. Hence, the fractional reduction in (2.5) is less for $A=1$ than it is for a big nucleus, say, lead, by a factor $(207)^{1/3}\simeq 6$. The shadowing in lead is known to be large. While speaking loosely one says that such cross sections are "geometric," i.e., σ_h is proportional to $A^{2/3}$, which in this case would correspond to $\Delta(A)\simeq\frac{5}{6}$. Hence, the left-hand and right-hand sides of (2.5) appear to differ by perhaps 15%. To avoid this paradox we will now reinterpret $\sigma_h^{(0)}$.

An intuitive way to consider the situation is to think of the nuclear diameter as a target thickness. Lead is a thick target. A single nucleon is a thin target, but not thin enough. A natural procedure is to define a quantity, $\sigma_h^{(0)}$ (to be obtained by extrapolation to zero target thickness), which is the cross section per nucleon for $A \ll 1$. We will employ the terminology that the one-nucleon cross section, $\sigma_h(1)$, is less than $\sigma_h^{(0)}$ owing to self-absorption. With this interpretation, formula (2.4) is self-consistent.

As has been implied by these comments, the single nucleon, as an entity, is to play no special role. A complex nucleus is not to be regarded as an assemblage of nucleons but just as a sphere of nuclear matter. A single nucleon is a smaller sphere of nuclear matter, and it is even meaningful to discuss nuclei with A small compared to 1. If single nucleons are, in fact, built from smaller constituents, then these assumptions may represent a plausible averaging over the true fine-grained structure in the nucleus, but we make no such claim. We are only striving for self-consistency in an outright optical model.

If one insists on treating nucleons as entities, then the effect we want to get at may be the following one. In an independent-particle approximation, the presence of a single nucleon in a given region in a nucleus does not inhibit any other nucleon from occupying the same region. This seems wrong. Presumably, there is a correlation effect causing nucleons not to occupy the same space. This means that for a short distance after a vector meson is born it should be protected from all nucleons except its parent. On the other hand, any "absorption" in the parent is present even for production on a single nucleon. It may well be possible to account for this effect by a correlation correction to the independent-particle model. It has been said[6] that the main effect of correlations is to eliminate voids between nucleons, thereby effectively shortening the mean free path of hadrons in nuclear matter. This would increase the cross section for nuclei without changing the single-nucleon cross section, i.e., less shadowing would be predicted. At least qualitatively that agrees with the present work.

It seems to me that treating the nucleus as a homogeneous and incompressible droplet is a closer first approximation to the nucleon correlations than is the independent-particle model. The absence of voids and overlap is guaranteed. It is not, however, necessary to insist on this point. Our purpose is to obtain photon cross sections. Since the nuclear parameters are empirically determined by fitting hadron scattering, it does not much matter how credible is the reasoning by which the formulas are derived. This argument will be repeated below in a more concrete context.

Let it then be granted that even single-nucleon cross sections are reduced by shadowing. How does that affect experimental observations? As regards total photon cross sections, what one measures is the ratio of the cross section on a large nucleus to that on a single nucleon. In this ratio it has always been assumed that only the numerator has been influenced by shadowing. The present claim is that shadowing has also reduced the denominator. That is the main effect. To be self-consistent, however, it is necessary to check whether the result depends on other results which should have been corrected for self-absorption. Unfortunately, there are two such effects. The nuclear radius (the single most important parameter) must be obtained from some source. The sound way[7] of obtaining it is from high-energy neutron-nucleus total cross sections. One fits the ratio of the cross section on the nucleus to that on a single nucleon by varying the nuclear radius. Obviously self-absorption alters this procedure. This will be explained below. Similar modifications must be made in the analysis of vector-meson photoproduction from which vector-meson parameters are obtained.

Again we call attention to our assumption that the nuclear radius is proportional to $A^{1/3}$,

$$R = R_0 A^{1/3}. \tag{2.6}$$

While such a trend is well established for complex nuclei, it is optimistic to apply it for $A = 1$.[8] Since the single-nucleon cross section is obtained as an integral over the single-nucleon density, the result is sensitive to this assumption. Furthermore, there is little reason to expect a uniform distribution to represent the true situation accurately. This latter comment applies also to complex nuclei. Two points can be made to justify our assumption (2.6). We will find that hadron cross sections are, to excellent accuracy, consistent with the assumption of constant nuclear densities satisfying (2.6). Short of incorporating results from elastic electron scattering, one can ask for nothing more. Furthermore, we are evaluating corrections to previous models[1,9] which have assumed uniform nuclei.

We have now made all the introductory, qualitative, and subjective comments we intend to. It remains to derive formulas for the various cross sections. The new effects we are discussing result in minor modifications of well-known formulas. There is, however, a confusing proliferation of such formulas resulting from the diverse processes under consideration (total and differential cross sections for vector-meson photoproduction and for hadron and Compton scattering). To reduce confusion and to make clear the role of the two effects in question, we will sketch the derivation of all results obtained, even though most of the arguments are elementary and well known.

In the final sections numerical results are given and compared, in a limited way, with experiment. Care is needed in doing this to make clear exactly how the various parameters are determined.

III. PHOTON-VECTOR-MESON PROPAGATION IN A UNIFORM NUCLEUS

Referring to Fig. 1, consider a photon wave incident from the left on a uniform spherical nucleus. Let $\psi_\gamma(\mathrm{I})$ be its amplitude on a plane I at the entrance. Inside the nucleus a vector-meson wave, ψ_V, is generated. V will later stand for ρ, ω, ϕ, ρ', etc. To obtain the amplitudes ψ_γ and ψ_V at a distant point P we first obtain them on a plane II at the nuclear exit and then employ Huygens's principle. The eikonal approximation is employed in obtaining the amplitudes on plane II. Rays are assumed to pass, undeflected, through the nucleus. Phase shifts (and attenuation) due to interaction with the nuclear matter are accounted for, but those due to geometrical deflection are not. This requires that the energy be high enough that the scattering is highly peaked forward.

These assumptions allow us to concentrate on an area, d^2b, at impact parameter b, for which we can deal with one-dimensional propagation through a slab of thickness L_b beginning at $z=z_b$.

To obtain simpler equations one factors out from ψ_γ and ψ_V, the main oscillatory factor, $\exp(i|\vec{k}|z)$, where $\vec{k}$ is the photon three-momentum,

$$\begin{aligned}\psi_\gamma &= e^{i|\vec{k}|z}\phi_\gamma,\\ \psi_V &= e^{i|\vec{k}|z}\phi_V .\end{aligned} \tag{3.1}$$

If the photon is virtual, its momentum and its energy ν are related by

$$|\vec{k}| \simeq \nu + \frac{Q^2}{2\nu} , \tag{3.2}$$

where Q^2 is assumed small compared to ν^2. Similarly, the vector mesons, though real, have different momentum and energy

$$|\vec{k}_V| \cong \nu - \frac{M_V{}^2}{2\nu} \quad (M_V << \nu). \tag{3.3}$$

Here it is implicitly assumed that the V energy is given by ν.

The choice of the factor in (3.1) was biased to make the photon propagation in free space simple.

$$\phi_\gamma(z \leq z_b) = 1. \tag{3.4a}$$

Since there is no incident V wave, we also have

$$\phi_V(z \leq z_b) = 0. \tag{3.4b}$$

Let us concentrate initially on vector-meson propagation since, to a first approximation, the photon wave passes unattenuated through the nucleus. Propagation is governed, as in optics, by a complex index of refraction,

$$m_V = 1 + \frac{2\pi}{\nu^2} n f_{VV}^{(0)}(0) , \tag{3.5}$$

where the second term is assumed small compared to the first. $f_{VV}^{(0)}(0)$ is the forward V-nucleon scattering amplitude. It can be separated into real and imaginary parts with the latter given by the optical theorem,

$$f_{VV}^{(0)}(0) = i\frac{\nu}{4\pi}\sigma_V^{(0)}(1 - i\alpha_V). \tag{3.6}$$

$\sigma_V^{(0)}$ is subject to the interpretation described in the previous section. Subsequent formulas will be simplified by introducing a quantity a_{VV} by

$$a_{VV} = \frac{n\sigma_V^{(0)}}{2}(1 - i\alpha_V). \tag{3.7}$$

It is the so-called optical potential multiplied by a factor $i\nu/2$. The quantities m_V and a_{VV} should for consistency have superscript (0)'s on them but, since they are not directly comparable to experimental quantities, we can leave them off with small risk of confusion.

The effects discussed so far cause a vector-meson wave, ϕ_V, to have the form

$$e^{-z a'_{VV}} , \tag{3.8}$$

where

$$a'_{VV} = a_{VV} + i q_{11,V} \tag{3.9}$$

and

$$q_{11,V} = \frac{Q^2 + M_V{}^2}{2\nu} . \tag{3.10}$$

In nuclear matter ϕ_V will satisfy a differential equation

$$\frac{d\phi_V}{dz} = - a'_{VV}\phi_V + \text{other terms to be discussed.} \tag{3.11}$$

At this point we can incorporate the effect of the instability of the vector mesons. As shown be Gottfried and Julius[10] this instability has only a small effect, at high energy, on ρ photoproduction. Such experiments detect the decay products and it does not much matter whether the decay takes place inside or outside the nucleus. But the total photon cross section will be altered owing to the extra attenuation of the vector-meson wave. Define

$$\Delta_V = \frac{g_V M_V}{2\nu} , \tag{3.12}$$

where g_V is the vector-meson decay width. The effect of decay is to replace a'_{VV} by $a'_{VV} + \Delta_V$ in (3.11). In evaluating photoproduction formulas, Δ_V will be set to zero (thereby giving the result of Gottfried and Julius).

The final term to be added is an amplitude, $-a_{\gamma V}\phi_\gamma$, for the vector-meson wave to be regenerated by the photon wave. We get

$$\frac{d\phi_V}{dz} = -(a'_{VV} + \Delta_V)\phi_V - a_{\gamma V}\phi_\gamma . \tag{3.13}$$

There is such an equation for each vector meson. It is assumed that there are no couplings of the various vector mesons among themselves.

The photon wave satisfies a similar equation

$$\frac{d\phi_\gamma}{dz} = - a_{\gamma\gamma}\phi_\gamma - \sum_V a_{\gamma V}\phi_V , \tag{3.14}$$

which involves summing over all vector mesons. We assume $a_{\gamma V} = a_{V\gamma}$.

The problem is to solve (3.13) and (3.14) subject to the initial conditions (3.4). It is convenient to use operational methods, with the Laplace transform symbolized by an overbar. Transforming the differential equations and incorporating the initial conditions, we get (s is the transform variable)

$$s\overline{\phi}_\gamma - 1 = -a_{\gamma\gamma}\overline{\phi}_\gamma - \sum_V a_{\gamma V}\overline{\phi}_V\ , \tag{3.15}$$

$$s\overline{\phi}_V = -a_{\gamma V}\overline{\phi}_\gamma - (a'_{VV} + \Delta_V)\overline{\phi}_V\ . \tag{3.16}$$

Solving for $\overline{\phi}_\gamma$, we get

$$\overline{\phi}_\gamma = \frac{1}{s}\left[1 + \frac{a_{\gamma\gamma}}{s} - \sum_V \frac{a_{\gamma V}{}^2}{s(s + a'_{VV} + \Delta_V)}\right]^{-1}\ . \tag{3.17}$$

It is possible to invert this in closed form, but for several vector mesons the algebra is complicated. Fortunately, we can assume

$$a_{\gamma\gamma} \ll a_{\gamma V} \ll a_{VV}\ . \tag{3.18}$$

(Actual numerical values will be given below to justify this.) Hence, (3.17) can be written

$$\overline{\phi}_\gamma \simeq \frac{1}{s} - \frac{a_{\gamma\gamma}}{s^2} + \sum_V \frac{a_{\gamma V}{}^2}{s^2(s + a'_{VV} + \Delta_V)} = \frac{1}{s} - \frac{1}{s^2}\left(a_{\gamma\gamma} - \sum_V \frac{a_{\gamma V}{}^2}{a'_{VV} + \Delta_V}\right) - \sum_V \frac{a_{\gamma V}{}^2}{(a'_{VV} + \Delta_V)^2}\left(\frac{1}{s} - \frac{1}{s + a'_{VV} + \Delta_V}\right)\ , \tag{3.19}$$

where we have followed the standard procedure of decomposing the transform into partial fractions. Before inverting this, and introducing similar expressions for ϕ_V, let us, following convention, introduce profile functions Γ_γ and Γ_V by

$$\Gamma_\gamma(b) = 1 - \phi_\gamma(\mathrm{II}, b), \tag{3.20a}$$

$$\Gamma_V(b) = -\phi_V(\mathrm{II}, b)\ , \tag{3.20b}$$

where ϕ_γ and ϕ_V are evaluated on the exit plane II shown in Fig. 1. In subsequent formulas the argument b will frequently be dropped.

Putting all these things together and inverting we get

$$\Gamma_\gamma = \left(a_{\gamma\gamma} - \sum_V \frac{a_{\gamma V}{}^2}{a'_{VV} + \Delta_V}\right) L_b + \sum_V \frac{a_{\gamma V}{}^2}{(a'_{VV} + \Delta_V)^2}\{1 - \exp[-(a'_{VV} + \Delta_V)\, L_b]\}. \tag{3.21}$$

To obtain Γ_V to the same approximation we substitute $\phi_\gamma = s^{-1}$ into (3.16) and invert to get

$$\Gamma_V = \frac{a_{\gamma V}}{a'_{VV}}[1 - \exp(-a'_{VV}\, L_b)]\exp(-iq_{11,V}\, z_b)\ , \tag{3.22}$$

where the final factor accounts for the phase lag of the vector-meson wave in the interval z_b from the nuclear exit boundary to the plane II. As justified previously, Δ_V has been set to zero in (3.22).

Particularly for small values of L_b, there tend to be large cancellations in (3.21) and (3.22). Because of this, and to abbreviate the formulas we introduce two new functions $E_1(x)$ and $E_2(x)$ according to

$$e(x) = 1 + x\, E_1(x) \tag{3.23a}$$

$$= 1 + x + \tfrac{1}{2}x^2\, E_2(x)\ . \tag{3.23b}$$

They are dimensionless numerical factors in the range from 0 to 1 with 0.5 being a typical value. In words, $E_1(x)$ [and $E_2(x)$] are related to the exponential function by normalizing to 1 at $x = 0$ after having dropped the first (and second) terms of the power-series expansion. Hence we get

$$\Gamma_\gamma = a_{\gamma\gamma} L_b - \tfrac{1}{2}L_b{}^2 \sum_V a_{\gamma V}{}^2 E_2(-(a'_{VV} + \Delta_V)L_b)\ , \tag{3.24}$$

$$\Gamma_V = a_{\gamma V} L_b E_1(-a'_{VV}\, L_b)\exp(-iq_{11,V}\, z_b)\ . \tag{3.25}$$

The second term of (3.24) is the shadowing contribution. In general, the arguments of $E_1(x)$ and $E_2(x)$ are complex, but as an intuitive aid they are graphed for real arguments in Fig. 2. They resemble the exponential function and will be called modified exponentials.

In passing we comment that the arguments of E_1 and E_2 depend only on the vector-meson factor a'_{VV} and not on $a_{\gamma V}$.

IV. APPLICATION OF HUYGENS'S PRINCIPLE TO OBTAIN MEASURABLE QUANTITIES

To obtain the propagation from plane II to the distant point P, we apply Huygens's principle.[11] For example, for the photon

$$\phi_\gamma(P) = -\int_{\mathrm{II}} d^2b\, i\, \frac{\nu}{2\pi}\, \frac{e^{i\nu\rho}}{\rho}\, \phi_\gamma(\mathrm{II})\ , \tag{4.1}$$

where, for convenience, the origin has been shifted to O' in Fig. 1. If $\vec{k}'$ is the scattered momentum vector, one gets

$$\psi_\gamma(z) = e^{i\nu z} + \frac{e^{i\nu\rho_0}}{\rho_0}\int_{\mathrm{II}} i\, \frac{\nu}{2\pi}\, d^2b\,[1 - \phi_\gamma(\mathrm{II})]\, e^{-i\vec{k}'\cdot\vec{b}}, \tag{4.2}$$

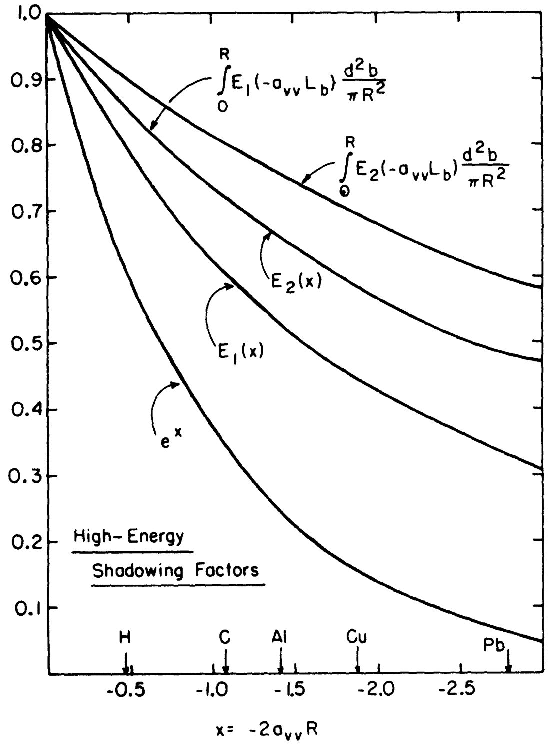

FIG. 2. Some shadowing functions described in the text. The identification of x values with specific nuclei is valid only for a specific (but typical) choice of paramters.

where we have approximated ρ by ρ_0 in the denominator, but not in the rapidly varying phase factor. We have also exploited the remarkable way in which Huygens's principle applied to a plane wave at II generates a plane wave of just the right phase and amplitude at P. Written in the form (4.2) the integral can be recognized as the photon scattering amplitude, $f_{\gamma\gamma}(\vec{k}')$:

$$f_{\gamma\gamma}(\vec{k}')=\int_{\text{II}} i\,\frac{\nu}{2\pi}\,d^2b\;\Gamma_\gamma(b)e^{-i\vec{k}'\cdot\vec{b}}\;. \tag{4.3}$$

A similar expression holds for the vector-meson photoproduction amplitude $f_{\gamma V}(\vec{k}')$. We will be concerned only with forward amplitudes, which are given by

$$f_{\gamma\gamma}(0)=\frac{i\nu}{2\pi}\int\Gamma_\gamma\,d^2b, \tag{4.3$'$}$$

$$f_{\gamma V}(0)=\frac{i\nu}{2\pi}\int\Gamma_V\,d^2b. \tag{4.3$''$}$$

One must distinguish between these amplitudes, which are measurable, and amplitudes $f^{(0)}_{VV}$ in formula (3.5), which are to be obtained by extrapolation to $A=0$.

We can extract the following measurable quantities:

σ_γ = total cross section ($\gamma\to$anything)

$$=2\,\mathrm{Re}\int\Gamma_\gamma\,d^2b, \tag{4.4}$$

$\sigma_{\gamma V}$ = total cross section ($\gamma\to V$)

$$=\int|\Gamma_V|^2\,d^2b, \tag{4.5}$$

$\sigma_{\gamma\gamma}$ = total cross section ($\gamma\to\gamma$)

$$=\int|\Gamma_\gamma|^2\,d^2b\;, \tag{4.6}$$

$\frac{d\sigma_{\gamma V}}{dt}(0)$ = forward V photoproduction cross section

$$=\frac{1}{4\pi}\left|\int\Gamma_V\,d^2b\right|^2, \tag{4.7}$$

$\frac{d\sigma_{\gamma\gamma}}{dt}(0)$ = forward Compton differential cross section

$$=\frac{1}{4\pi}\left|\int\Gamma_\gamma\,d^2b\right|^2. \tag{4.8}$$

V. ELASTIC HADRON SCATTERING

Hadron scattering was discussed earlier. We can now write out the formulas in more detail. A hadron wave, ϕ_h, inside the nucleus satisfies the equation

$$\frac{d\phi_h}{dz}=-a_{hh}\phi_h\;, \tag{5.1}$$

for which the appropriate solution is

$$\phi_h(\text{II})=e^{-a_{hh}L_b}\;. \tag{5.2}$$

This yields for the profile function

$$\Gamma_h=1-e^{-a_{hh}L_b} \tag{5.3}$$

$$=a_{hh}L_b\,E_1(-a_{hh}L_b). \tag{5.4}$$

The following measurable quantities can be derived:

σ_h = total cross section ($h\to$anything)

$$=2\,\mathrm{Re}\int\Gamma_h\,d^2b\;, \tag{5.5}$$

σ_{hh} = total cross section ($h\to h$)

$$=\int|\Gamma_h|^2\,d^2b\;, \tag{5.6}$$

$\frac{d\sigma_{hh}}{dt}(0^\circ)$ = forward elastic cross section

$$=\frac{1}{4\pi}\left|\int\Gamma_h\,d^2b\right|^2. \tag{5.7}$$

VI. SMALL-R LIMITS

It is normally assumed that the formulas given in the previous two sections are applicable only to large nuclei. But there is nothing to prevent our evaluating them for very small values of R; small even compared to the proton radius. On taking these limits we get

$$\sigma_\gamma^{(0)} = (\tfrac{4}{3}\pi R^3) 2\,\mathrm{Re}\, a_{\gamma\gamma}\ , \tag{6.1}$$

$$\sigma_{\gamma V}^{(0)} = 2\pi R^4 |a_{\gamma V}|^2\ , \tag{6.2}$$

$$\sigma_{\gamma\gamma}^{(0)} = 2\pi R^4 |a_{\gamma\gamma}|^2\ , \tag{6.3}$$

$$\frac{d\sigma_{\gamma V}^{(0)}}{dt}(0) = \frac{1}{4\pi}(\tfrac{4}{3}\pi R^3)^2\, |a_{\gamma V}|^2\ , \tag{6.4}$$

$$\frac{d\sigma_{\gamma\gamma}^{(0)}}{dt}(0) = \frac{1}{4\pi}(\tfrac{4}{3}\pi R^3)^2\, |a_{\gamma\gamma}|^2\ , \tag{6.5}$$

$$\sigma_h^{(0)} = (\tfrac{4}{3}\pi R^3) 2\,\mathrm{Re}\, a_{hh}, \tag{6.6}$$

$$\sigma_{hh}^{(0)} = 2\pi R^4 |a_{hh}|^2\ , \tag{6.7}$$

$$\frac{d\sigma_{hh}^{(0)}}{dt}(0) = \frac{1}{4\pi}(\tfrac{4}{3}\pi R^3)^2\, |a_{hh}|^2\ . \tag{6.8}$$

The conventional interpretation of these formulas would be that they are single-nucleon cross sections when R is set to R_0. In our new interpretation they are limits as $R \to 0$.

VII. EVALUATION OF THE INTEGRALS FOR UNIFORM NUCLEI

For uniform nuclei all the integrals can be written in terms of a single integral. Formulas for this are given in Appendix I. But, for purposes of discussion, in this section we will give simpler approximate expressions obtained when one sets $q_{11,V}$ to zero in the vector-meson profile function (3.25) and ignores real parts. We rewrite the profile functions

$$\Gamma_\gamma = a_{\gamma\gamma} L_b - \tfrac{1}{2} L_b^2 \sum_V a_{\gamma V}^2 E_2(-(a_{VV}+\Delta_V)L_b), \tag{7.1}$$

$$\Gamma_V = a_{\gamma V} L_b E_1(-a_{VV} L_b), \tag{7.2}$$

$$\Gamma_h = a_{hh} L_b E_1(-a_{hh} L_b)\ . \tag{7.3}$$

From the latter two it is clear that vector-meson photoproduction is, except for a constant factor, indistinguishable from elastic scattering of the same vector meson ($a_{hh} = a_{VV}$). (From this it follows that VMD in the small implies VMD in the large; that is, the ratio of photoproduction to elastic scattering is independent of A.)

We are primarily interested in integrals of Γ_γ, Γ_V, and T_h over a sphere. For these we need the results

$$\int_0^R E_1(2\mu(R^2-b^2)^{1/2})\frac{d^2b}{\pi R^2} = E_2(2\mu R), \tag{7.4}$$

$$\int_0^R E_2(2\mu(R^2-b^2)^{1/2})\frac{d^2b}{\pi R^2} = I_2(2\mu R), \tag{7.5}$$

where all these functions are plotted in Fig. 2. μ is to be replaced by $-a_{VV}$ or $-a_{hh}$ and hence the argument can be interpreted as the nuclear diameter measured in attenuation lengths (amplitude, not intensity). The arguments are defined as negative numbers to emphasize the similarity of the functions E_1, E_2, and I_2 to exponentials. The integrals can also be interpreted as attenuations averaged over impact parameters. Since the average distance through the nucleus is less than $2R$, the effect of averaging over impact parameters is to give less attenuation than would apply to a flat slab of thickness $2R$. That is why, in Fig. 2, E_2 is less steep than E_1 and I_2 is less steep than E_2.

For typical values of the parameters, values of $2a_{VV}R$ for the various elements are indicated at the bottom of Fig. 2. $|E_1(-2a_{VV}R)|^2$ gives the shadowing factor for vector-meson photoproduction and for vector-meson elastic scattering. Γ_γ is more complicated. It will turn out though that if a single vector meson, say the ρ, were dominant, and $a_{\gamma\gamma}$, $a_{\gamma V}$, and a_{VV} are related by VMD then (7.1) would also be proportional to $E_1(-2a_{VV}R)$.

VIII. MODIFICATION FOR NONUNIFORM NUCLEI

Though we shall not exploit the results, we will give, for reference, the modifications required to handle nonuniform nuclear-density distributions.

The main dependence can be taken into account by defining a new "optical thickness" variable

$$\tilde z = \frac{1}{n}\int_{-\infty}^{z} \tilde n\, dz'\ , \tag{8.1}$$

where $\tilde n$ is the variable density and n is an assumed central density which will be identified with the value n used previously. In terms of $\tilde n$, Eqs. (3.13) and (3.14) become

$$\frac{d\phi_\gamma}{d\tilde z} = \frac{n}{\tilde n}\left[-\tilde a_{\gamma\gamma}\phi_\gamma - \sum_V \tilde a_{\gamma V}\phi_V\right]\ , \tag{8.2}$$

$$\frac{d\phi_V}{d\tilde z} = \frac{n}{\tilde n}\left[-\tilde a_{\gamma V}\phi_\gamma - (\tilde a_{VV} + \Delta_V + iq_{11,V})\phi_V\right]. \tag{8.3}$$

Since amplitudes $\tilde a_{\gamma\gamma}$, $\tilde a_V$, and $\tilde a_{VV}$ are proportional to $\tilde n$, these equations simplify to

$$\frac{d\phi_\gamma}{d\tilde z} = -a_{\gamma\gamma}\phi_\gamma - \sum_V a_{\gamma V}\phi_V\ , \tag{8.4}$$

$$\frac{d\phi_V}{d\tilde z} = -a_{\gamma V}\phi_\gamma - \left[a_{VV} + (\Delta_V + iq_{11,V})\frac{n}{\tilde n}\right]\phi_V\ . \tag{8.5}$$

TABLE I. Vector-meson parameters.

	m_V (GeV)	γ_V (GeV)	σ_V (mb)	C_V	$f_V^2/4\pi$
ρ	0.77	0.14	$\begin{pmatrix}26.0\\28.1\end{pmatrix}^{a}$	$\begin{pmatrix}0.053\\0.050\end{pmatrix}^{a}$	$\begin{pmatrix}2.56\\2.88\end{pmatrix}^{a}$
ω	0.78	0.009	28.2	0.0199	18.4
ϕ	1.02	0.003	13.5	0.0254	11.3
ρ'	1.60	0.45	28.2	0.0208	16.8

[a]As determined in the text.

Except for the term with $\Delta_V + q_{11,V}$ these are the equations we have solved already. We will set the factor $n/\bar{n}$ to 1, thereby making a surface-dependent error of importance only at low energy (since $\Delta_V + q_{11,V}$ vanishes at high energy).

All formulas given in early sections are to be modified by replacing L_b by

$$\tilde{L}_b = \frac{1}{n}\int_{-\infty}^{\infty} \bar{n}\, dz\ . \tag{8.6}$$

The factor $\exp(-iq_{11,V}z_b)$ in the expression (3.25) for Γ_V loses definition. To a good approximation one can presumably replace z_b by a quantity $\tilde{z}_b$ which is the distance from the exit of an "equivalent" uniform sphere to plane II.

IX. SECONDARY NUMERICAL VALUES

We have now exhibited, more or less explicitly, formulas for all relevant photon and hadron processes. In this section we will specify the numerical values to be used for the various parameters which enter. Most quantities are reasonably independent of energy. Values will be specified for ν near 8 GeV, along with formulas for the ν dependence.

(a) Vector mesons. We take account only of ρ, ω, ϕ, and ρ'. Since the ρ is dominant, it will be treated separately. Parameters for ω, ϕ, and ρ' are taken from the best available values[12] and held fixed throughout. The values assumed are given in Table I. The amplitudes a_{VV} are obtained from σ_V using (3.7). Then the regeneration amplitudes are given by

$$a_{\gamma V} = C_V a_{VV}\ , \tag{9.1}$$

where

$$C_V = \left(\alpha \frac{4\pi}{f_V^2}\right)^{1/2}\ . \tag{9.2}$$

$a_{\gamma\gamma}$ can be obtained from Compton scattering. According to VMD it should satisfy the so-called Compton sum rule

$$a_{\gamma\gamma} = \sum_V C_V^2 a_{VV}(1+h)\ , \tag{9.3}$$

where h is an empirical parameter which should vanish, but has been added to turn (9.3) into an equality. Its value is in the range 0–0.2.

(b) Real parts. All real parts, α_V, are assumed to be given by

$$\alpha_V = -0.19\left(\frac{8.0\ \text{GeV}}{\nu}\right)^{1/2}\ . \tag{9.4}$$

This is known to be reasonable for the ρ, and for the other vector mesons it has little effect.

(c) Energy dependence. We assume

$$\sigma_V = \sigma_V(8.0\ \text{GeV})\left(\frac{1+0.2(3.3\ \text{GeV}/\nu)^{1/2}}{1.128}\right)\ . \tag{9.5}$$

This is a fit to the average of π^+ and π^- single-nucleon cross sections, that is, the quark-model prediction for σ_V. σ_V(8 GeV) is determined below.

(d) Q^2 dependence. Most results are quoted for $Q^2=0$, but for small Q^2 (0.1 GeV^2) we use

$$a_{\gamma V}(Q^2) = a_{\gamma V}(0)(1+Q^2/m_V^2)^{-1} \tag{9.6}$$

and

$$a_{\gamma\gamma}(Q^2) = a_{\gamma\gamma}(0)\,(1+Q^2/0.6\ \text{GeV}^2)^{-1.2}, \tag{9.7}$$

which give reasonable fits to low-Q^2 electroproduction.[13]

X. PRIMARY NUMERICAL VALUES

Our main purpose is to study the consequences of introducing ρ instability and self-absorption. Since ρ instability alters the shadowing but does nothing else, it needs no special discussion. But consistent treatment of self-absorption necessitates a reconsideration of the determination of the ρ-meson parameters.

The value of $f_\rho^2/4\pi$ can be obtained from colliding-beam results or from ρ photoproduction. σ_ρ can be obtained from the quark model (theory) or ρ photoproduction (experiment). In both cases we will use photoproduction. After the fact we can take note of the agreement with the other determinations (Table II).

Fits were obtained with the two widths ($g_\rho = 0$ and $g_\rho = 0.14$ GeV) and with and without self-absorption (labeled new and old in Table II).

Five experimental cross sections come in:

$$\sigma(n+p\to\text{all}) = 39.3\ \text{mb (at 10.0 GeV)}$$

$$\text{(Ref. 14)}, \tag{10.1}$$

$$\frac{1}{A}\,\frac{\sigma(n+\text{Pb}\to\text{all})}{\sigma(n+p\to\text{all})} = 0.396\ \text{(at 10.0 GeV)}$$

$$\text{(Ref. 14)}, \tag{10.2}$$

TABLE II. Parameters determined. "New" and "old" refer to the assumption of self-absorption or no self-absorption. Values obtained from other sources are given in parentheses on the left. The Roman-numeral symbols are used in labeling the graphs.

Procedure		I Old	II Old	III New	IV New
R_0 (F)		1.30	1.30	1.26	1.26
g_ρ (GeV)		0	0.14	0	0.14
$\sigma_\rho^{(0)}$ (mb)		26.0	26.0	33.7	33.7
σ_ρ (mb)	(28.2?)	26.0	26.0	28.1	28.1
C_ρ	(0.053?)	0.053	0.053	0.050	0.050
h	(0?)	0.15	0.15	0.21	0.21
$\sigma_\gamma(12)/\sigma_\gamma(1)$		0.794	0.802	0.867	0.873
$\sigma_\gamma(207)/\sigma_\gamma(1)$		0.664	0.681	0.705	0.726

$$\frac{d\sigma}{dt}(\gamma+p\to\rho+p)\big|_{0^\circ}=101\ \mu\text{b/GeV}^2\ \text{(at 8.8 GeV)}\qquad \text{(Ref. 2, 15)},\quad (10.3)$$

$$\frac{1}{A^2}\left.\frac{d\sigma/dt(\gamma+\text{Pb}\to\rho+\text{Pb})}{d\sigma/dt(\gamma+p\to\rho+p)}\right|_{0^\circ}=0.1756\ \text{(at 8.8 GeV)}\qquad \text{(Ref. 2)},\quad (10.4)$$

$$\sigma(\gamma+p\to\text{all})=121\ \mu\text{b (at 8.8 GeV)}\qquad \text{(Ref. 15)}.\quad (10.5)$$

The first two values, which are neutron cross sections, determine the nuclear-radius parameter R_0. σ_h is evaluated using (5.5). The real part, α, for neutrons is taken as -0.30.[16] If one ignores self-absorption, then the value a_{hh} is obtained directly from (10.1) using (3.7) [or (6.6) which is equivalent]. R_0 is varied to fit (10.2). To account for self-absorption one makes a two-parameter fit to (10.1) and (10.2) to determine the parameters R_0 and a_{hh}.

The hope in such a procedure for determining R_0 is that errors in the optical model will "cancel out." That is, a value of R_0 known to give correct neutron cross sections from a theory (wrong at some level) should give correct values for the cross section of some other particle using the same theory (presumably wrong only in much the same way).

The next parameters to be determined are σ_ρ and $f_\rho^2/4\pi$. They are obtained by fitting to (10.3) and (10.4). In the absence of self-absorption, (10.4) determines σ_ρ by the comparison with the ratio of (4.7) to (6.4). Then one determines $f_\rho^2/4\pi$ using (10.3). Self-absorption is included by evaluating (4.7) for *both* the proton and for lead to fit the two parameters.

Finally, the parameter h is determined using (4.6) with $a_{\gamma\gamma}$ given by (9.3). h is varied to fit (10.5).

There is no need to haggle over the exact values taken in (10.1)–(10.5). They are experimental numbers known only to lie within certain errors, but for present purposes it is necessary only that all fits are constrained to give those values.

XI. COMPARISON WITH EXPERIMENTS

The result of fitting 10-GeV neutron-nucleus elastic-scattering cross sections of Engler *et al.*[14] is shown in Fig. 3. In this and all subsequent figures the experimental points should be compared with the heavy curve. The experimental points are always ratios of measured cross sections. The curve labeled I (for parameters see Table II) gives the fit obtained ignoring self-absorption. The value obtained for the nuclear radius is $R_0=1.302$ F. Curve III (renormalized to 1 at $A=1$) is the fit given by following the self-absorption philosophy. The value for the nuclear radius is determined to be $R_0=1.26$ F. Both curves fit lead since that was the criterion determining R_0. It can be seen that III gives a better fit to other nuclei. That could be fortuitous as we have taken an over-simplified density distribution. However, as we have argued twice already, once we have a good fit to hadron data we need not worry too much where it came from.

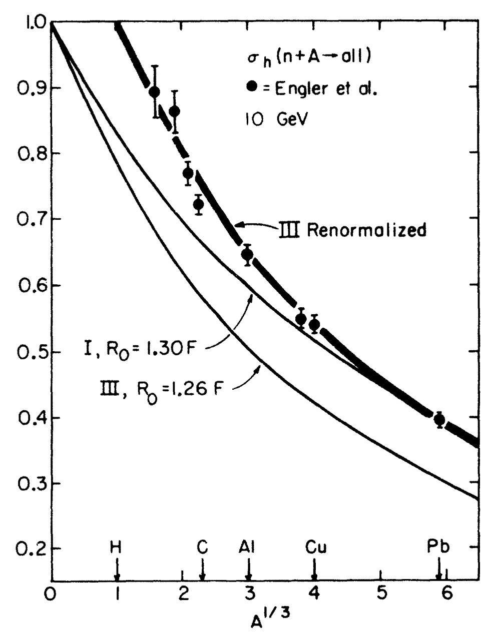

FIG. 3. Total cross sections for 10-GeV neutrons incident on nuclei. Roman numerals in this and following figures refer to parameter values listed in Table II. In this and following curves the extra heavy curve is to be compared with the data (Ref. 14). These data are used to determine nuclear radii.

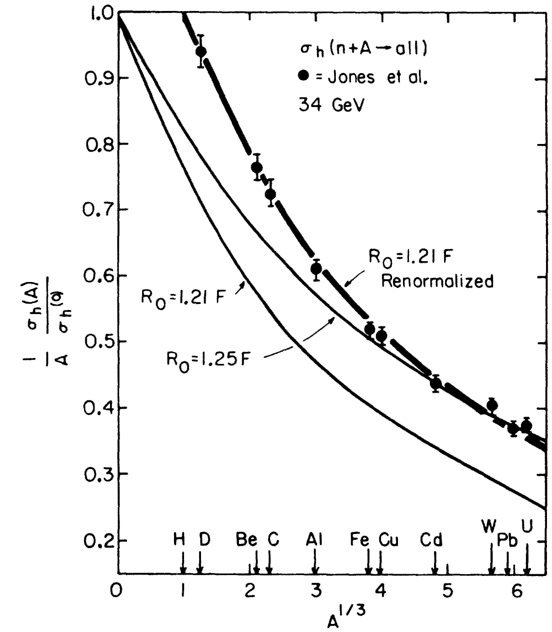

FIG. 4. Total cross sections (Ref. 17) for 34-GeV neutrons incident on nuclei. From this energy up to a few hundred GeV the cross sections are nearly constant.

It is often stated heuristically that hadron cross sections on nuclei are proportional to $A^{2/3}$. Unfortunately, they are not. ($A^{0.8}$ is more typical.) It is gratifying that, with our interpretation, the deviation from $A^{2/3}$ is entirely accounted for, even though the nuclear radius is exactly proportional to $A^{1/3}$.

We can inquire to what extent the data of Engler *et al.* (10 GeV) agree with other measured neutron cross sections. Among low-energy measurements, the data of Engler *et al.* agree well with the 6-GeV data of Jones *et al.*[17] and with the 19.3-GeV proton data of Bellettini *et al.*[18] The 27-GeV data of Longo *et al.*[19] and the 8.3-GeV data of Pantuev *et al.*[20] seem to be in serious disagreement and have been ignored. There are high-energy neutron data (30–270 GeV) of Jones *et al.*[21] for which 34-GeV data are plotted in Fig. 4. Data of Biel *et al.*[22] are in reasonable agreement at these energies. The same fitting procedure yields an excellent fit to all nuclei (even including the deuteron, which is undeserved). The radius determination gives $R_0 = 1.21$ F.

Jones *et al.*[21] have stated that the energy dependence of their neutron cross sections is in rough accord with the expected inelastic screening effect. Since this effect is small at low energy, we will take the nuclear radius determined at 10 GeV in what follows. The smaller radius which we extract at 34 GeV is consistent with the analysis of Jones *et al.*, at least to the extent that it implies an extra screening effect at high energy.

We turn now to photon-initiated processes. For ρ photoproduction the curves obtained by fitting at lead, with and without self-absorption, are shown in Fig. 5(a). The curve labeled IV must be renormalized to 1 at $A=1$ as shown. The values of C_ρ and σ_ρ obtained in this way are given in Table II. They can be seen to be in adequate agreement with the colliding-beam and the quark-model determinations, respectively. The values of h needed to satisfy the Compton sum rule (9.3), are also listed in Table II. With the values for ρ, ω, φ, and ρ' we have used, the sum rule fails by 21%.

With all parameters determined we can finally consider total photon cross sections. The nature of the self-absorption correction is shown in Fig. 5(b). The ordinate is

$$\frac{1}{A}\,\frac{\sigma_\gamma(A)}{\sigma_\gamma^{(0)}}\,, \tag{11.1}$$

which is plotted against $A^{1/3}$ for the two values of $\sigma_\rho^{(0)}$ specified previously. If one ignores self-absorption, this ratio should equal the experimental ratio

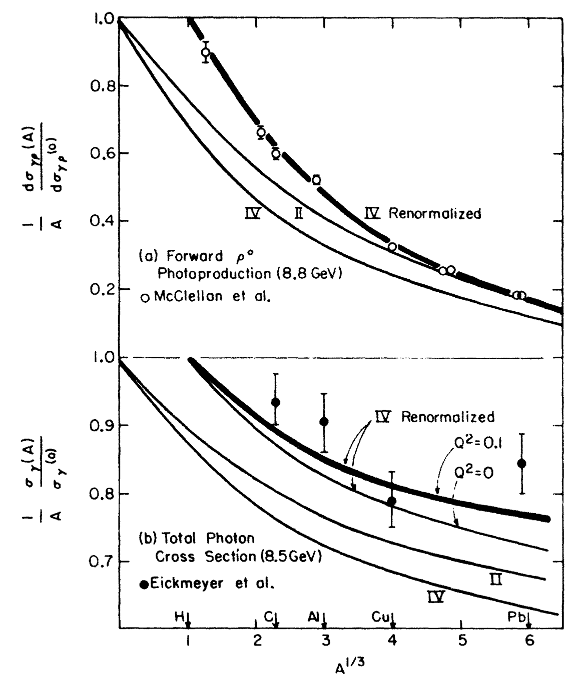

FIG. 5. (a) Forward ρ photoproduction cross sections (Ref. 2) at 8.8 GeV on nuclei. These data are used to determine σ_ρ and C_ρ. (b) Total cross sections (Ref. 23) for 8.5-GeV photons incident on nuclei. The main new claim made in the paper is that the data should be compared with the curves IV, corrected for self-absorption, rather than II, evaluated in the conventional way.

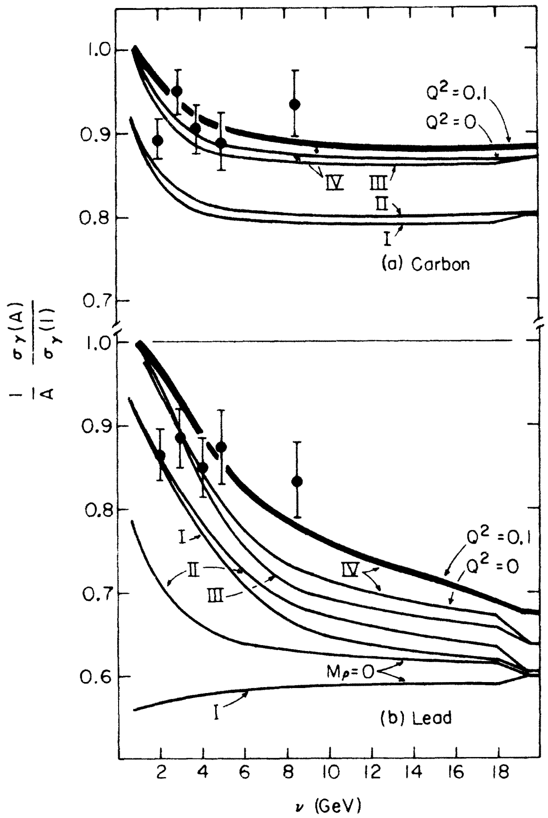

FIG. 6. Total cross sections (Ref. 23) for photons incident on carbon (a) and on lead (b) as a function of ν. The contributions of various effects described in the text can be inferred from the various curves by comparing the parameter values given in Table II.

$$\frac{1}{A}\frac{\sigma_\gamma(A)}{\sigma_\gamma(1)} . \tag{11.2}$$

But when evaluated at $A=1$ it takes the value 0.89. This is another manifestation of the paradox described in the Introduction. According to our philosophy the curve should be normalized by dividing by 0.89 to give the heavy curve. It is this curve which should be compared with the experimental ratios (11.2) of Eickmeyer *et al.*[23]

The ν dependence is illustrated in Fig. 6 for carbon and lead. The data are to be compared with the solid curve which has all effects included. In particular, it applies to $Q^2=0.1$ GeV^2. All the other curves apply to $Q^2=0$. The curve labels refer to the parameters of Table II. The effects of the finite width of the ρ and of the self-absorption correction can be separately inferred by comparing the various curves. We can enumerate a few features.

(i) The maximum amount by which shadowing is suppressed owing to the finite ρ width is about

$$\frac{0.025}{1-0.66}=7\%$$

at 8 GeV from lead.

(ii) Surprisingly, the width effect becomes small at low values of ν. This is an accidental consequence of the longitudinal loss of coherence due to the ρ mass. This can be seen from the curves evaluated with

$$M_\rho=0 .$$

With the width also zero (parameters I) the shadowing can be seen to be roughly independent of ν. With $g_\rho=0.14$ GeV (parameters II) the shadowing is strongly suppressed at low ν. But in the presence of the longitudinal loss of coherence the sensitivity to g_ρ is much reduced.

(iii) The width effect is small for small nuclei.

(iv) The self-absorption correction reduces the shadowing at 8 GeV by 15% at lead and by 35% at carbon.

A comparison with other data,[23-26] from elements near lead, as a function of Q^2, is shown in Fig. 7.

Finally, in Fig. 8 is shown the prediction, obtained from (4.8), for the forward differential Compton cross section. To compare with data of Criegee *et al.*[27] the cross sections are evaluated

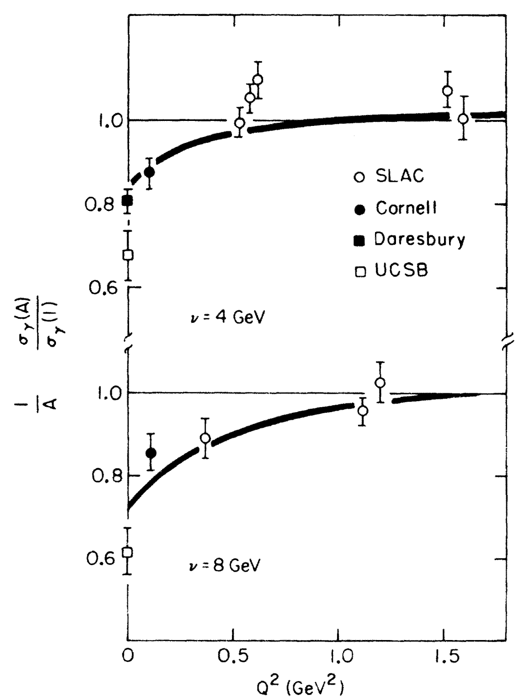

FIG. 7. Comparison of shadowing factors for elements near lead measured near 4 GeV and 8 GeV from UCSB (Ref. 24), Cornell (Ref. 23), SLAC (Ref. 25), and Daresbury (Ref. 26).

at 3 GeV and 5 GeV.

It is not our intention to review all experimental results in this area. The Compton-scattering result is shown since it is a different (though closely related) process. The data of Criegee *et al.* do not support the theory; though, given the 7% systematic error, they may be compatible with it.

Unfortunately, comparable discrepancies exist among total-cross-section experiments. Hence we have chosen mainly to show the Cornell results of Eickmeyer *et al.* The present author's objectivity may well be suspect on this, but the agreement between experiment and theory appears adequate. No experiment, in itself, has ever persuasively observed the ν dependence characteristic of the shadowing effect. The experiment of Eickmeyer *et al.* is no exception, but the average shadowing level is about right and the absence of ν dependence can (plausibly?) be blamed on statistics.

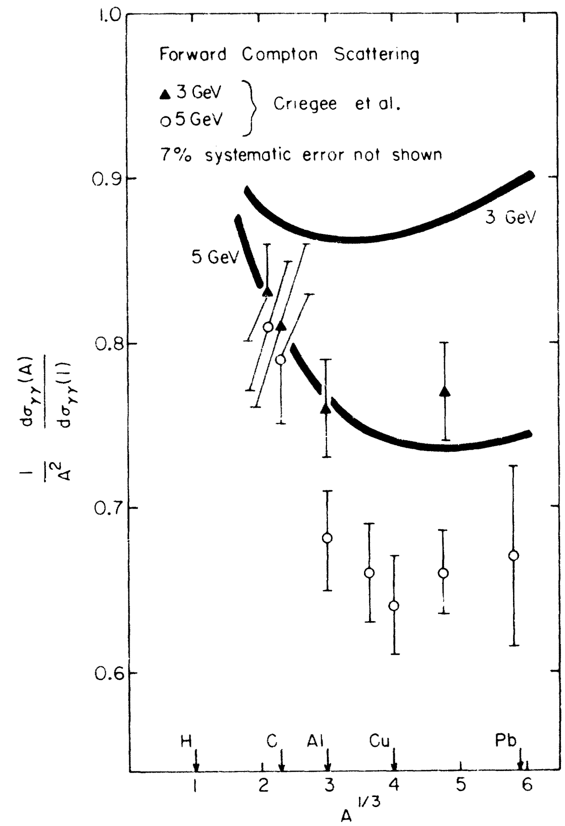

FIG. 8. Forward Compton scattering differential cross section at 3 GeV and 5 GeV. The curves are evaluated using the parameters values favored in the text (labeled IV on other figures).

XII. CONCLUSIONS

A self-consistent solution to a well-defined (essentially classical) problem has been given. Since it may be argued that it is the wrong problem, let us consider objectively what has been accomplished.

Certainly the fit to neutron cross sections (e.g., Fig. 4) is excellent (and analytically very simple). It is a two-parameter fit with R_0 and $\sigma_n^{(0)}$ taking the values required to force the fit at $A=1$ and $A=207$. Such a fit need not have fitted intermediate nuclei. The fact that it does implies that the free proton is qualitatively much the same as any other nucleus. Had it been pointlike, for example, the proton would not fit on a smooth curve through all other nuclei.

Essentially the same theory gives a good fit to ρ photoproduction. Again the fit is forced at $A=1$ and $A=207$, this time by the choice of C_ρ and σ_ρ. The good fit for intermediate nuclei seems like a strong confirmation of the VMD ideas. The *same curve* fits ρ photoproduction and neutron scattering. The credibility of the theory from which the curve was derived is logically not even relevant.

The parameters R_0, C_ρ, and σ_ρ obtained in the above fits do not come out just anything, but rather, agree, at least at the 10–20% level, with independent determinations.

Encouraged by such success one anticipates that photon total cross sections calculated in the same way will be accurate. That is our claim.

The approach has been unabashedly phenomenological. For small nuclei a more sophisticated treatment would certainly be preferable (though the data do not demand it). For the deuteron a standard Glauber treatment could be used and for all nuclei a more realistic surface shape could be used. The $Q^2>0$ treatment given here is also superficial. Longitudinal terms have been ignored. But the main effects, the disappearance of shadowing at large Q^2 and the behavior at low Q^2 have probably been described adequately.

ACKNOWLEDGMENT

I would like to thank John Eickmeyer, Nari Mistry, and Don Yennie, for helpful conversations.

APPENDIX: EVALUATION OF THE INTEGRALS FOR UNIFORM NUCLEI

For a uniform nucleus all the integrals in the text can be written in terms of a single integral

$$J(\mu R)=\int_0^R e^{\mu 2(R^2-b^2)^{1/2}}(d^2b/\pi R^2) \tag{A1}$$

$$=\frac{1}{2(\mu R)^2}(1-e^{2\mu R}+2\mu Re^{2\mu R}) \tag{A2}$$

$$=\sum_{n=0}^{\infty}(2\mu R)^n\frac{2(n+1)}{(n+2)!} \tag{A3}$$

$$=1+\tfrac{2}{3}(2\mu R)+\tfrac{1}{4}(2\mu R)^2+\cdots . \tag{A4}$$

While formula (A2) looks particularly simple, it is numerically treacherous for small R, as can be seen from (A4). The first two powers of R cancel. But the situation is actually worse since, on substituting into (4.4) to obtain σ_γ, the leading term again cancels, as can be seen from (6.1), which is proportional to R^3. For $\sigma_{\gamma V}$ the first two terms cancel leaving a leading R^4.

Fortunately, however, (A3) is rapidly convergent for all relevant values of $2\mu R$. Hence, introduce new functions J_1 and J_2 according to

$$J(\mu R) = 1 + J_1 \tag{A5}$$

$$= 1 + \tfrac{2}{3}(2\mu R) + J_2 \ . \tag{A6}$$

If the leading term is known to cancel, J can be replaced by J_1 (or J_2 if the first two terms are known to cancel). Having followed this route, there is, of course, no escape from numerically summing J_1 and J_2.

Following this procedure we can evaluate the integrals in Secs. IV and V after substituting from (3.21) or (3.22). The main cross sections we need have Γ_γ or Γ_V appearing linearly in the integrand. They are

$$\sigma_\gamma = 2\,\mathrm{Re}\left(a_{\gamma\gamma} - \sum_V \frac{a_{\gamma V}{}^2}{a'_{VV} + \Delta_V}\right)(\tfrac{4}{3}\pi R^3) - 2\,\mathrm{Re}\sum_V \frac{a_{\gamma V}{}^2}{(a'_{VV} + \Delta_V)^2}(\pi R^2) J_1((-a'_{VV} - \Delta_V)R) \ , \tag{A7}$$

$$\frac{d\sigma_{\gamma V}}{dt}(0) = \frac{1}{4\pi}\left|\frac{a_{\gamma V}}{a'_{VV}}\right|^2 (\pi R^2)^2 \left| J_1(\tfrac{1}{2} i q_{11,V} R) - J_1((-a'_{VV} - \Delta_V)R)\right|^2 \ , \tag{A8}$$

$$\frac{d\sigma_{\gamma\gamma}}{dt}(0) = \frac{1}{4\pi}\left|\left(a_{\gamma\gamma} - \sum_V \frac{a_{\gamma V}{}^2}{a'_{VV} + \Delta_V}\right)(\tfrac{4}{3}\pi R^3) - \sum_V \frac{a_{\gamma V}{}^2}{(a'_{VV} + \Delta_V)^2}(\pi R^2) J_1((-a'_{VV} - \Delta_V)R)\right|^2 \ , \tag{A9}$$

$$\sigma_h = -2\,\mathrm{Re}(\pi R^2) J_1(-a_{hh} R) \ , \tag{A10}$$

$$\frac{d\sigma_{hh}}{dt}(0) = \frac{(\pi R^2)^2}{4\pi}\left| -J_1(-a_{hh} R)\right|^2 \ . \tag{A11}$$

The other measurable quantities can also be expressed in terms of the function $J(\mu R)$.

$$\sigma_{\gamma V} = \left|\frac{a_{\gamma V}}{a'_{VV}}\right|^2 (\pi R^2)\left[-J_2(-a'_{VV} R) - J_2(-a'^{*}_{VV} R) + J_2((-a'_{VV} - a'^{*}_{VV})R)\right] \ , \tag{A12}$$

$$\sigma_{hh} = (\pi R^2)\left[-J_1(-a_{hh} R) - J_2(-a^{*}_{hh} R) + J_2((-a_{hh} - a^{*}_{hh})R)\right] \ . \tag{A13}$$

The expression for $\sigma_{\gamma\gamma}$ is not given. It is an extremely small cross section involving yet more cancellations.

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[7]One could start with charge densities obtained from electron scattering, but there are subtle effects at, say, the 10% level which make this less than straightforward. Examples are correlations, differences between neutron and proton distributions, projectile-size effects, and surface effects. Among hadron scattering processes, neutron cross sections escape the complications of the interference with Coulomb scattering the hence are experimentally the easiest. The attitude in this paper is that, having a fit to the neutron total cross section, there is no need to seek further refinements.

[8]Qualitatively, though, the proton size inferred from hadronic diffractive scattering and from electron scattering is consistent with (2.6).

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