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Simple relation between the fine-structure and gravitational constants

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A simple relation between the fine-structure constant α and the gravitational constant G is derived in a "truly" unified model of the Nambu–Jona-Lasinio type for all elementary-particle forces including gravity.

A unified model of the Nambu–Jona-Lasinio type[1] for "all" elementary-particle forces has recently been proposed by three of the present authors (H.T., K.A., and Y.C.).[2] Starting with a nonlinear fermion Lagrangian of the Heisenberg type and imposing the massless conditions of Bjorken[1] on vector fields, we have constructed an effective Lagrangian which combines the unified gauge theory of Weinberg and Salam[3] for the weak and electromagnetic interactions of leptons and quarks and the asymptotically free gauge theory of Gross, Wilczek, and Politzer[4] for the strong interaction of quarks. The photon, weak vector bosons, and Higgs scalars appear as composites of lepton-antilepton or quark-antiquark pairs, while the color-octet gluons appear as those of quark-antiquark pairs. In analyzing our model, we have made full use of the recent remarkable progress in the field-theoretical treatment of nonlinear fermion Lagrangians.[5] As a result, the Weinberg angle is determined[6] to be $\sin^2\theta_W = \frac{3}{8}$ for fractionally charged quarks, which coincides with the prediction of Georgi and Glashow in their unified SU(5) gauge model.[7] The gluon coupling constant is also determined to be $\frac{8}{3}$ times the fine-structure constant. The masses of the weak vector bosons and physical Higgs scalars are related to those of leptons and quarks. In the previous paper,[2] we have also proposed a unified spinor-subquark model[8] in which the gauge bosons and Higgs scalars as well as leptons and quarks are all composites of subquarks of spin $\frac{1}{2}$. In such a model, we have predicted, among other things, the mass of the charged weak vector bosons to be approximately $\sqrt{3}$ times the subquark mass. From these results, we have strongly suggested either that there exist much heavier leptons and/or quarks whose masses reach or go beyond the weak-vector-boson masses, or that there exist heavy subquarks whose pair-production threshold lies very close to the weak-vector-boson masses.

What are left undetermined in our unified model are the Cabibbo angle, the masses of leptons and quarks, and a single arbitrary coupling constant, the fine-structure constant. What is excluded by the "all" elementary-particle forces is the gravitational force. In this note, we show how to include gravity in our original unified model so that a "truly" unified model of the Nambu–Jona-Lasinio type for *all* elementary-particle forces may be achieved. In the following, we shall assume that a graviton is a collective excitation of a fermion-antifermion pair. A similar idea was first proposed by Phillips[9] in 1966. Recently, Adler *et al.*[10] have analyzed in detail an alternative picture that the gravitational fields be identified with photon pairing amplitudes of a superconductor type. A main result of our truly unified model for all elementary-particle forces is a simple relation (call it the G-α relation) between the fine-structure constant α and the gravitational constant G, whose derivation will be sketched below.

Let us start with the following nonlinear Lagrangian for a massless fermion field ψ:

$$L = \bar{\psi} i \partial\!\!\!/ \psi + f_0[\bar{\psi} i \tfrac{1}{2}(\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu)\psi]^2 , \tag{1}$$

where f_0 is the fundamental coupling constant. Introducing the auxiliary tensor field $H_{\mu\nu}$,[11] we construct the Lagrangian

$$L' = \bar{\psi} i \partial\!\!\!/ \psi + \bar{\psi} i \tfrac{1}{2}(\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu)\psi H^{\mu\nu} + c_0 (H_{\mu\nu})^2 , \tag{2}$$

where c_0 is a constant. Variation with respect to $H_{\mu\nu}$ gives the "equation of motion"

$$H_{\mu\nu} = -(1/2c_0)\bar{\psi} i \tfrac{1}{2}(\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu)\psi , \tag{3}$$

which shows that $H_{\mu\nu}$ is symmetric, traceless, and divergenceless up to the order of f_0. Substitution of this equation indicates that the Lagrangian L' is effectively equivalent to the original L if

$$c_0 = -1/4f_0 \,. \tag{4}$$

Define the effective Lagrangian $L_{\rm eff}$ for the auxiliary field by the path integrals over the fermion fields:

$$\exp\left(i\int d^4x\, L_{\rm eff}\right) = \int [d\psi][d\bar{\psi}]\exp\left(i\int d^4x\, L'\right). \tag{5}$$

Performing the path integration formally, we then obtain

$$\int d^4x\, L_{\rm eff} = \int d^4x [c_0(H_{\mu\nu})^2] - i\,{\rm Tr}\ln[1 + (1/i\partial\!\!\!/)i\tfrac{1}{2}(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu)H^{\mu\nu}], \tag{6}$$

where Tr denotes the trace operation with respect to the space-time points as well as the γ matrices. The second term in (6) corresponds to a series of fermion-loop diagrams if it is expanded into a Taylor series in the auxiliary field. All actual calculations of such diagrams involve quartically divergent integrals. We must, therefore, introduce the cutoff momentum Λ. Then, the divergent part of $L_{\rm eff}$, which we call $L_{\rm div}$, can be calculated to be

$$L_{\rm div} = \tfrac{1}{6}I_0(H_{\mu\nu})^2 + \tfrac{1}{4}\kappa_0 I_1(\partial_\lambda H_{\mu\nu})^2 + \cdots, \tag{7}$$

with

$$I_0 = \Lambda^4/2(4\pi)^2 \quad\text{and}\quad I_1 = \Lambda^2/(4\pi)^2, \tag{8}$$

where the constant $\kappa_0 = \frac{5}{9}$ and $\frac{2}{3}$, depending on the invariant and straight cutoffs, respectively. All the remaining terms are irrelevant for the purpose of this note. We define $L'_{\rm div}$ by the first two terms in $L_{\rm div}$ given in (7).

Let us now construct the new Lagrangian

$$L'' = L' + L'_{\rm div}, \tag{9}$$

where the auxiliary field $H_{\mu\nu}$ has been promoted to become a "genuine" Bose field. Then, the relation

$$L' = L'' - L'_{\rm div}, \tag{10}$$

which looks trivial, indicates that the original Lagrangian L is effectively equivalent to the new L'' if $L'_{\rm div}$ is subtracted as a counterterm. Furthermore, in order to make the graviton massless we require the massless condition

$$\tfrac{1}{6}I_0 - \tfrac{1}{4}f_0^{-1} = 0\,. \tag{11}$$

Rescaling the gravitational field by

$$H_{\mu\nu} = -(\kappa_0 I_1)^{-1/2}h_{\mu\nu}, \tag{12}$$

we obtain the following form of L'':

$$L'' = \bar{\psi}i\partial\!\!\!/\psi + \tfrac{1}{4}(\partial_\lambda h_{\mu\nu})^2 - g_0\bar{\psi}i\tfrac{1}{2}(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)\psi h_{\mu\nu} \tag{13}$$

with

$$g_0 = (\kappa_0 I_1)^{-1/2}\,. \tag{14}$$

To the lowest order in the coupling constant g_0, this form of the Lagrangian reproduces the familiar Newtonian gravitational potential if

$$G = g_0^2/4\pi = 4\pi/\kappa_0\Lambda^2, \tag{15}$$

where G is the Newtonian gravitational constant ($=6.67\times10^{-8}$ cm$^3g^{-1}$sec^{-2}).

On the other hand, in our original unified model for all elementary-particle forces the fine-structure constant is calculated from our previous paper[2] to be

$$\alpha = 3\pi/(\textstyle\sum Q^2)\ln(\Lambda^2/m^2), \tag{16}$$

where Q and m are the charge and the averaged mass of leptons and quarks, respectively, and the summation $\sum$ runs over all the members of existing Weinberg-Salam multiplets of leptons and quarks $(l_L, l_R, q_L, u_R, d_R)_i$ $(i = 1, 2, \ldots, N)$. An extension of our present model of gravity for including more than one fundamental fermion is almost trivial. All that is needed is to replace κ_0 in the above results by $\kappa_0 N_0$, where N_0 is the number of relevant fermions. Combining Eqs. (15) and (16), we finally derive the G-α relation:

$$\alpha = 3\pi/(\textstyle\sum Q^2)\ln(4\pi/\kappa_0 N_0 G m^2)\,. \tag{17}$$

Historically, a relation of this type was first conjectured by Landau[12] in 1955, based on the idea that the effects of gravitational interaction may exceed the electromagnetic effects at the cutoff energies Λ. Much later, in 1971, such infinity suppression in gravity-modified quantum electrodynamics was formulated and discussed in detail by Isham, Salam, and Strathdee.[13] Together with the weak-interaction cutoff of quantum electrodynamics by the weak-vector-boson mass m_W ($\sim10^2$ GeV),[14] this gravitational cutoff by the Planck mass $G^{-1/2}$ ($\sim10^{19}$ GeV) would eliminate all existing infinities in quantum field theories.

In order to derive some interesting results from the G-α relation (17), we first notice that the right-hand side of the relation does not strongly depend on the ambiguous quantities κ_0 or m. Next, notice that $\sum Q^2 = \frac{8}{3}N$ and $N_0 = \frac{15}{2}N$ for N Weinberg-Salam multiplets, since a neutrino accounts for only $\frac{1}{2}\kappa_0$. Take, for example, $\kappa_0 = \frac{5}{9}$ (or $\frac{2}{3}$) and $m \sim 1$ GeV; then the G-α relation together with the experimental data $G^{-1/2} = 1.221\times10^{19}$ GeV leads to $\alpha \cong 1/25N$. Since there are at least two charged leptons, the electron and the muon, the integer N

is equal to or larger than 2. We, therefore, find a theoretical upper bound on the fine-structure constant $\alpha \lesssim \frac{1}{50}$. Furthermore, the relation $\alpha \cong 1/25N$ with the familiar experimental value of $\alpha^{-1} \cong 137$ leads to an exciting expectation, $N=5$ or 6. From this, we strongly suggest that there exist additional three or four heavy charged leptons, neutrinos, up-quarks, and down-quarks.

In conclusion, an important question as to whether our model of gravity is equivalent to Einstein's general relativity has not been answered. Nor has been answered an even more important question whether our model is consistent with the various presently existing data on gravity. These problems are subjects for future investigations.

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