Comments and Addenda

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Supergravity with axial-gauge invariance*

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An extension of the locally supersymmetric theory of the spin-(2,3/2) gravitational multiplet and the spin-(1,1/2) Abelian gauge multiplet is obtained in which the spin-1 field becomes an axial-gauge field coupled to both the spin-3/2 and spin-1/2 fields.

The first stage of the development of explicit locally supersymmetric field theories in four-dimensional spacetime was marked by the formulation of the Lagrangian and transformation rules for the spin-2–spin-$\frac{3}{2}$ gravitational multiplet.[1-3] In the second stage the local extensions[4-6] of the kinetic actions of known global supersymmetry multiplets[7] were constructed.

In this note we describe the locally supersymmetric extension of the Fayet-Iliopoulos term[8] of the Abelian vector multiplet. We find the curious feature that the gauge field $A_\mu(x)$ which was coupled only to the vierbein field $V_{a\mu}(x)$ in the previous construction[4,5] now couples to both the spin-$\frac{1}{2}$ field $\chi(x)$ and the spin-$\frac{3}{2}$ field $\psi_\rho(x)$ with an axial-gauge coupling. The result is a closely knit field theory which has general covariance, supersymmetry, and chiral symmetry all as local invariances. Since local supersymmetry implies consistency[2] and causal propagation,[9] this appears to be the first field theory in which these properties hold in the presence of an "electromagnetic-like" interaction for the spin-$\frac{3}{2}$ field.[10] However, there is a cosmological term which leads to difficulties of interpretation as we discuss below.

In the notation of Ref. 5 the Lagrangian density of this field theory is (using a first-order description of gravitation)

$$\mathcal{L}=\mathcal{L}_{SG}+\mathcal{L}_M+\mathcal{L}_1+\mathcal{L}_2\,, \tag{1}$$

with $[D(x)$ is an auxiliary spinless field and e is the axial charge of the spinor fields$]$

$$\begin{aligned}\mathcal{L}_{SG}&=-\frac{1}{4\kappa^2}VV^{a\mu}V^{b\nu}R_{\mu\nu ab}\\&\quad-\tfrac{1}{2}\epsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu\hat{\mathfrak{D}}_\nu\psi_\rho\,,\\ \mathcal{L}_M&=-\tfrac{1}{4}Vg^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma}+\tfrac{1}{2}V\bar{\chi}\gamma^\mu\hat{D}_\mu\chi\\&\quad+\tfrac{1}{2}VD^2-e\kappa^{-2}VD\end{aligned} \tag{2}$$

and with the Noether current and contact terms

$$\begin{aligned}\mathcal{L}_1&=-\frac{i\kappa}{\sqrt{2}}V\bar{\psi}_\lambda\sigma^{\mu\nu}F_{\mu\nu}\gamma^\lambda\chi+\frac{e}{\sqrt{2}\,\kappa}V\bar{\psi}\cdot\gamma\gamma_5\chi\,,\\ \mathcal{L}_2&=-\tfrac{1}{2}V\kappa^2(\bar{\psi}_\lambda\sigma^{\mu\nu}\gamma^\lambda\chi)(\bar{\psi}_\mu\gamma_\nu\chi)\,.\end{aligned} \tag{3}$$

This Lagrangian is invariant under the local chiral transformation

$$\begin{aligned}\delta\psi_\rho&=i\,\alpha(x)\gamma_5\psi_\rho\,,\\ \delta\chi&=i\,\alpha(x)\gamma_5\chi\,,\\ \delta A_\mu&=e^{-1}\partial_\mu\alpha(x)\,,\end{aligned} \tag{4}$$

so that A_μ plays the role of an axial-gauge field. For this reason the combined first-order gravitational and chiral covariant derivatives

$$\begin{aligned}\hat{\mathfrak{D}}_\nu\psi_\rho&=(\partial_\nu+\tfrac{1}{2}\omega_{\nu ab}\sigma^{ab}-ieA_\nu\gamma_5)\psi_\rho\,,\\ \hat{D}_\mu\chi&=(\partial_\mu+\tfrac{1}{2}\omega_{\mu ab}\sigma^{ab}-ieA_\mu\gamma_5)\chi\end{aligned} \tag{5}$$

appear in $\mathcal{L}$. The Lagrangian changes by a total derivative under the supersymmetry transformation

$$\delta A_\mu = \frac{i}{\sqrt{2}} \bar{\epsilon} \gamma_\mu \chi ,$$

$$\delta \chi = \frac{1}{\sqrt{2}} (\sigma^{\mu\nu} \bar{F}_{\mu\nu} + i\gamma_5 D) \epsilon ,$$

$$\delta D = \frac{1}{\sqrt{2}} [\bar{\epsilon}\gamma_5 \gamma^\mu \hat{\bar{D}}_\mu \chi + 2(S_{\mu\tau}{}^\tau - \tfrac{1}{2} i\kappa^2 \bar{\psi}\cdot\gamma\psi_\mu)(\bar{\epsilon}\gamma_5\gamma^\mu \chi)] .$$

$$\delta V_{a\mu} = -i\bar{\epsilon}\gamma_a \psi_\mu , \tag{6}$$

$$\delta\psi_\rho = \kappa^{-1}\hat{D}_\rho \epsilon + \tfrac{1}{4} i\kappa (\bar{\chi}\gamma_5\gamma^\nu \chi)\gamma_\nu \gamma_\rho \gamma_5 \epsilon ,$$

$$\delta\omega_{\mu ab} = \Delta\omega_{\mu ab} + V_{a\mu}E_b - V_{b\mu}E_a ,$$

$$E_a = \frac{1}{2\sqrt{2}} \kappa^2 D(\bar{\epsilon}\gamma_5\gamma_a \chi) ,$$

where $\hat{D}_\rho \epsilon$ is defined just as $\hat{D}_\rho \chi$ and where $\Delta\omega_{\mu ab}$ is the complicated variation given previously [see Eq. (17) of Ref. 5] with $\mathfrak{D}_\sigma \psi_\tau$ replaced by $\hat{\mathfrak{D}}_\sigma \psi_\tau$. Our notation uses the supercovariant derivatives[4-6]

$$\bar{D}_\mu A_\nu = \partial_\mu A_\nu - \frac{i}{\sqrt{2}} \kappa \bar{\psi}_\mu \gamma_\nu \chi ,$$

$$\bar{F}_{\mu\nu} = \bar{D}_\mu A_\nu - \bar{D}_\nu A_\mu , \tag{7}$$

$$\hat{\bar{D}}_\mu \chi = \hat{D}_\mu \chi - \frac{1}{\sqrt{2}} \kappa \sigma^{\lambda\rho} \bar{F}_{\lambda\rho} \psi_\mu - \frac{i\kappa}{\sqrt{2}} \gamma_5 D \psi_\mu .$$

To discuss the proof of local supersymmetry we shall use the notation $\mathfrak{L}_0$ to refer to all terms in $\mathfrak{L}$ except the terms

$$\mathfrak{L}' = -e\kappa^{-2} V \left(D - \frac{\kappa}{\sqrt{2}} \bar{\psi}\cdot\gamma\gamma_5 \chi \right) . \tag{8}$$

Since local supersymmetry has been established[4,5] for $e=0$, we need only consider terms in $\delta\mathfrak{L}_0$ and $\delta\mathfrak{L}'$ which involve e. Most such terms in $\delta\mathfrak{L}_0$ are merely the chiral-covariant completions of previously studied[5] derivative terms and cancel accordingly. One must give separate study to the terms

$$\delta\mathfrak{L}_0 \sim -\tfrac{1}{2} ie\kappa^{-1} \epsilon^{\lambda\rho\mu\nu} F_{\lambda\rho} \bar{\epsilon}\gamma_\mu \psi_\nu - \tfrac{1}{2} e (i\epsilon^{\lambda\rho\mu\nu} \bar{\psi}_\lambda \gamma_\mu \psi_\rho + V\bar{\chi}\gamma_5\gamma^\nu \chi)\, \delta A_\nu \tag{9}$$

which come from partial integration after the variation $\delta\psi_\rho \sim \hat{\mathfrak{D}}_\rho \epsilon$ and from the explicit variation $\delta A_\nu \sim \bar{\epsilon}\gamma_\nu \chi$. These terms are easily shown to cancel against similar terms in $\delta\mathfrak{L}'$, and all other terms cancel within $\delta\mathfrak{L}'$.

After elimination of the auxiliary field D one finds the cosmological[11] term $-\frac{1}{2} e^2 \kappa^{-4} V$ whose value cannot be changed without violation of local supersymmetry. The experimental limit[12] on the cosmological constant implies the terribly stringent bound $e^2 < 10^{-120}$ on the axial charge. For this reason we will not attempt to give a physical interpretation of the present model. In global supersymmetry the Fayet-Iliopoulos term leads to spontaneous breakdown[8] in theories with coupled vector and scalar multiplets. The axial-gauge coupling found here should also be present in the local extension of such theories, and there may also be a spin-$\frac{3}{2}$ Higgs mechanism. Further, the effective cosmological term is likely to depend on several parameters and may not lead to direct limitation on e^2.

It is possible that the present construction is related to the work of Ferrara and Zumino[13] in global supersymmetry models, where it was shown that energy-momentum tensors, supersymmetry currents, and axial-vector currents transform in a single supermultiplet.

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