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Parton model and the Bethe-Salpeter wave function*

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The quark-parton model is discussed from the point of view that the quark partons are the quanta created by the Fourier transforms of the quark fields at constant x^+. We argue using this picture that, for a given behavior of a hadron's deep-inelastic structure function $W_2(x)$ as $x \to 1$, the Drell-Yan-West relation provides a lower bound on the behavior of its form factor $F_1(Q^2)$ as $Q^2 \to \infty$. The connection between the parton and Bethe-Salpeter descriptions of hadron structure is described and used to translate known information about the pion's Bethe-Salpeter wave function into information about the amplitude for a pion to consist of precisely two quark partons. We find that the two-parton contributions to $\nu W_2(x)$ and $F(Q^2)$ behave roughly like $\nu W_2 \propto (1-x)^2$ for small $(1-x)$ and $F(Q^2) \propto (Q^2)^{-1}$ for large Q^2, respectively.

I. INTRODUCTION

The parton model of hadron structure has proved to be a fruitful tool, since its invention by Feynman[1] and Bjorken,[2] to explain scaling in deep-inelastic electron scattering. It is widely applied today in such areas as the interpretation of neutrino scattering experiments. This use continues despite the recognition that pure Bjorken scaling is probably not quite right and therefore that the parton model is not quite right. On the other hand, the parton model seems to be *almost* right, and it has a simplicity and intuitive appeal that account for its continuing usefulness as a theoretical guide.

Another framework for understanding the properties of a bound state of elementary constituents was invented by Bethe and Salpeter in the early days of quantum field theory.[3] If one assumes that the underlying field theory is asymptotically free or that it has a renormalization-group fixed point such that the anomalous dimensions of all the fields are very small, then the Bethe-Salpeter field-theoretic picture will give (almost) the same results as the parton picture.

In principle, all the information that one needs to know about hadron structure is contained in the Bethe-Salpeter (BS) wave functions of baryons as bound states created by a quark field and an antiquark field. Given these wave functions and the appropriate quark Green's functions, hadronic scattering amplitudes can be computed by integration. Of course no one knows the exact quark Green's functions or the exact BS wave functions, but some information about these objects that is relevant for high-energy processes can be obtained by using such tools as the renormalization group, the operator-product expansion, and conformal invariance.

The purpose of this paper is to elucidate the connection between the parton description of hadron structure and the Bethe-Salpeter description, and to use this connection to learn something about the structure of the pion in the parton description. In particular, we translate the information about the pion's BS wave function in a (nongauge) field theory obtained by Goldberger, Guth, and the present author[4] into the equivalent information about the amplitude for a pion to consist of precisely two quark partons. The information obtained concerns the behavior of this wave function as the transverse separation between the partons approaches zero, and its behavior as the momentum fraction x of one of the partons approaches 1.

We find that the two-parton contribution to the pion form factor behaves for large Q^2 roughly like

$$F^{(2)}(Q^2) \propto (Q^2)^{-1} \tag{1.1}$$

(as in Ref. 4 and elsewhere[5,6]), and that the two-parton contribution to the pion structure function $\nu W_2(x)$ behaves for small $(1-x)$ roughly like

$$\nu W_2^{(2)}(x) \propto (1-x)^2 \tag{1.2}$$

[in agreement with the results for $\nu W_2(x)$ of Ezawa[6] and of Farrar and Jackson[7]].

In Sec. II the quark-parton model is developed from the point of view that the quark "partons" are the quanta created by the Fourier transforms of the quark field operators on a surface of fixed $x^+ \equiv (x^0 + x^3)/\sqrt{2}$. This version of the parton model is essentially equivalent to the $P \to \infty$ version[1,2,18] but it is more amenable to field-theoretic interpretation and to the use of Lorentz invariance. The "equal-x^+" version of the parton model is not new.[8] However, it has not received a systematic exposition and analysis, which is the goal of Sec. II.

In this analysis, we describe the deep-inelastic structure function $\nu W_2(x)$ of hadrons, paying particular attention to the way bilocal operators[9] arise from the application of the impulse approximation to current commutators. The hadronic electromagnetic form factor $F_1(Q^2)$ is also discussed. We find that νW_2 for small $(1-x)$ and F_1 for large Q^2 need not be related by the Drell-Yan-West (DYW)[10] relation. [Indeed, Eqs. (1.1) and (1.2) above are not so related.] However, the DYW relation is found to provide a *lower bound* on the behavior of $F_1(Q^2)$ for large Q^2, barring a cancellation between the contributions to F_1 from different quark flavors.

In Sec. III the connection is made between the pion's Bethe-Salpeter wave function and the two-parton part of the parton wave function. This connection is then used to study the two-parton wave function in the short-distance and small-$(1-x)$ limits, leading to the results (1.1) and (1.2).

II. THE QUARK-PARTON MODEL

In the quark-parton model hadrons are composed of constituents called "partons." The partons are charged pointlike spin-$\frac{1}{2}$ quarks and neutral gluons, which are presumably the vector mesons of a non-Abelian gauge theory. (However, the parton wave function discussed in the next section is derived for a simpler class of theories in which the gluons are scalar mesons.)

The collection of partons in a hadron is often described by giving the amplitude to find the partons in a given configuration at time $\bar{t}=0$, in a reference frame in which the hadron is moving in the z direction with nearly the speed of light. As viewed from the rest frame of the hadron, this wave function tells the parton configuration as it would be determined by making local measurements on a space-time surface that is nearly the surface $x^0+x^3=0$. Thus an economical approach, which will be adopted here, is to treat the coordinate $x^+=(x^0+x^3)/\sqrt{2}$ as a "time" coordinate and to describe the hadron by the amplitude for the partons to be in a given configuration at a fixed "time" x^+.

A. The parton distribution function

Let us agree to describe each four-vector a^μ by its components $a^\mu=(a^+,a^1,a^2,a^-)$, where $a^\pm=(a^0\pm a^3)/\sqrt{2}$. We will write $\vec{a}$ for the transverse components (a^1,a^2) of a^μ. The scalar product is $a_\mu b^\mu=a^+b^-+a^-b^+-\vec{a}\cdot\vec{b}$.

Consider a hadron that contains, for simplicity, a definite number of partons. The nth parton can be described by its + component of momentum, p_n^+, and its transverse position $\vec{x}_n$. Let $P^+=\sum p_n^+$ be the total + momentum of the hadron and let

$$\vec{R}=\frac{1}{P^+}\sum p_n^+\vec{x}_n$$

be its center of P^+.

The center of P^+ plays an important role in parton physics, just as the center of mass is an important variable in nonrelativistic physics. This is because the subgroup of the Poincaré group that leaves the surfaces $x^+=\text{const}$ invariant is isomorphic to the Galilean group in two dimensions. The momentum P^+ plays the role of "mass" and the operator $P^+\vec{R}$ is the generator of Galilean boosts.[11,12]

We let $\vec{r}_n$ be the transverse position of the nth parton relative to the center of P^+:

$$\vec{r}_n=\vec{x}_n-\vec{R}\ . \tag{2.1}$$

It is also convenient to let x_n be the fraction of the total P^+ carried by the nth parton:

$$x_n=p_n^+/P^+\ .$$

With this introduction we can now define the parton distribution function. Let $\mathcal{P}_A(x,\vec{r})=dN_A/dx\,d\vec{r}$ be the average number of partons of type A that carry a fraction x of the hadron's P^+ and are located a distance $\vec{r}$ away from its center of P^+.

As is well known, the deep-inelastic structure function $\nu W_2(x)$ of the hadron is related to the x distribution of partons by

$$\nu W_2(x)=\sum_A \mathcal{Q}_A{}^2 x\int d\vec{r}\,\mathcal{P}_A(x,\vec{r}). \tag{2.2}$$

Here $\mathcal{Q}_A$ is the charge of partons of type A in units of the electron charge. (We will give a field-theoretic derivation of this relation later in this section.)

The hadron's electromagnetic form factor $F_1(Q^2)$ can also be written compactly in terms of the parton distribution function. By analogy with nonrelativistic quantum mechanics, $F_1(Q^2)$ is the Fourier transform of the electric charge distribution:

$$F_1(Q^2)=\sum_A \mathcal{Q}_A\int d\vec{r}\,e^{i\vec{Q}\cdot\vec{r}}\int_0^1 dx\,\mathcal{P}_A(x,\vec{r})\ . \tag{2.3}$$

This expression is derived later in this section, but first we discuss the relation between $F_1(Q^2)$ for large Q^2 and $W_2(x)$ for x near 1.

B. The Drell-Yan-West relation

According to Eq. (2.3), the form factor is the Fourier transform of the charge density as a function of $\vec{r}$, the transverse distance between the struck parton and the center of P^+ of *all* of the partons. Let us define the distance $\vec{y}$ between the struck parton and the center of P^+ of the *rest* of the partons:

$$\vec{r}=(1-x)\vec{y}\ . \tag{2.4}$$

Let us also define

$$f(x,\vec{k}^2)=\sum_A \mathcal{Q}_A\int d\vec{r}\,e^{i\vec{k}\cdot\vec{y}}\mathcal{P}_A(x,\vec{r})\ . \tag{2.5}$$

Since $\vec{y}$ measures the interparton separations, $f(x, \vec{k}^2)$ for large $\vec{k}^2$ probes the behavior of the parton wave function when the struck parton has transverse momentum $\vec{k}$.

In terms of the function $f(x, \vec{k}^2)$, the form factor is

$$F(Q^2) = \int_0^1 dx f(x, (1-x)^2Q^2) \quad . \tag{2.6}$$

By a change of variables $x \to k \equiv (1-x)Q$ we can write

$$F(Q^2) = Q^{-1}\int_0^Q dk f\left(1-\frac{k}{Q}, k^2\right) \quad . \tag{2.7}$$

From Eqs. (2.6) and (2.7) it is apparent that the large-Q^2 behavior of the form factor is sensitive to both the large-k^2 and the small-$(1-x)$ behavior of the Fourier-transformed quark distribution function $f(x, \vec{k}^2)$. Let us suppose that $f(x, \vec{k}^2)$ is power behaved in these limits:

$$\begin{aligned} &f(x, \vec{k}^2) \sim g(x)(\vec{k}^2)^{\alpha}, \quad \vec{k}^2 \to \infty \\ &f(x, \vec{k}^2) \sim h(\vec{k}^2)(1-x)^{-2\beta-1}, \quad (1-x) \to 0 \ . \end{aligned} \tag{2.8}$$

If $\alpha > \beta$, the large-Q^2 behavior of $F(Q^2)$ can be obtained by taking the limit under the integral sign in Eq. (2.6):

$$F(Q^2) \sim A(Q^2)^{\alpha}, \quad \alpha > \beta \tag{2.9}$$

where

$$A = \int_0^1 dx\, g(x)(1-x)^{2\alpha} \quad .$$

(The integral for A converges at its $x=1$ end because $\alpha > \beta$.)

If $\beta > \alpha$, the large-Q^2 behavior of $F(Q^2)$ can be obtained by taking the limit under the integral sign in Eq. (2.7):

$$F(Q^2) \sim B(Q^2)^{\beta}, \quad \beta > \alpha \tag{2.10}$$

where

$$B = \int_0^{\infty} dk\, h(k^2) k^{-2\beta-1} \quad .$$

The integral for B converges at its $k \to \infty$ end because $\beta > \alpha$ and at its $k \to 0$ end provided $\beta < 0$, as one can safely assume in physical applications.

The relation (2.10) between the form factor and the small-$(1-x)$ behavior of the parton distribution function was first obtained by Drell and Yan and by West.[10] We see here that the Drell-Yan-West relation holds in models with sufficiently strong damping of large transverse momenta ($\alpha \approx -\infty$). In general, the DYW relation survives as an inequality; the form factor cannot fall faster than $(Q^2)^{\beta}$. [The contribution B_A to the coefficient B from quark type A is proportional to the Mellin transform of the $(1-x) \to 0$ limit of the positive quantity $\mathcal{P}_A(x, \vec{r})$. Thus $B_A \neq 0$ as long as $\mathcal{P}_A$ does not vanish identically. However, we must *assume* that the contributions from different quark types do not cancel each other, leaving $\sum B_A \equiv B = 0$.]

C. The parton distribution function in operator form

How can the partons be described in quantum field theory? We shall identify the quark partons with the quanta that are created and destroyed by the Fourier transforms of the quark field operators at a fixed x^+. Thus, for instance, the amplitude for a pion to consist of two quark-antiquark pairs is given by

$$\langle 0|\Psi(0)\Psi(x_1)\overline{\Psi}(x_2)\overline{\Psi}(x_3)|P\rangle$$

on the surface $x_1^+ = x_2^+ = x_3^+ = 0$. In particular, the amplitude for a pion to consist of exactly one quark and one antiquark can be determined by examining the pion Bethe-Salpeter wave function

$$\langle 0|T(\Psi(0)\overline{\Psi}(x))|P\rangle$$

on the surface $x^+ = 0$. This is what we shall do in the next section.

In order to make these ideas more precise, let us review some pertinent facts about spinor field theories canonically quantized on equal x^+ surfaces.[12] Of the four components of a Dirac field $\Psi_\alpha(x)$, only two are independent dynamical variables. These are the components projected out by the matrix $\frac{1}{2}\gamma^-\gamma^+$; the other two components, $\frac{1}{2}\gamma^+\gamma^-\Psi$, are determined by a constraint equation. Choosing the representation of the γ matrices in which

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^j = \begin{pmatrix} 0 & -\sigma^j \\ \sigma^j & 0 \end{pmatrix} ,$$

the independent components of Ψ_α are Ψ_1 and Ψ_4.

It is convenient[8] to define a two-component field ψ_s, $s = \pm\frac{1}{2}$, with $\psi_{+1/2} = 2^{1/4}\Psi_1$ and $\psi_{-1/2} = i2^{1/4}\Psi_4$ to describe the independent quark variables. The two-component field ψ obeys the canonical equal-x^+ anticommutation relations

$$\{\psi_s(x), \psi_{s'}^{\dagger}(0)\}\delta(x^+) = \delta^4(x)\delta_{ss'} \quad . \tag{2.11}$$

Consider the Fourier transform of $\psi_s(x)$ with respect to x^- at $x^+ = 0$:

$$\tilde{\psi}_s(0, \vec{x}; P^+) = \int dx^- e^{iP^+x^-}\psi_s(0, \vec{x}, x^-) \quad .$$

When $P^+ > 0$, this operator destroys a quark at transverse position $\vec{x}$ with + component of momentum P^+ and helicity[13] s. When $P^+ < 0$, $\psi_s(0, \vec{x}; P^+)$ creates an antiquark at transverse position $\vec{x}$, with + component of momentum $-P^+$ and helicity $-s$.

Using the definition of $\mathcal{P}_A(x, \vec{r})$ and the interpretation of the quark fields as quark-parton creation and annihilation operators, we have

$$\mathcal{P}_A(x,\vec{r}) = \frac{1}{4\pi}\int dx^- e^{ixP^+x^-}\langle P^+,\vec{0},s|\delta(\vec{R}_{op})\psi^{(A)}(0,\vec{r},0)^\dagger\psi^{(A)}(0,\vec{r},x^-)|P^+,\vec{0},s\rangle \quad . \tag{2.12}$$

Here $|P^+,\vec{0},s\rangle$ is a hadron state with + momentum P^+, transverse momentum $\vec{P}=\vec{0}$, and helicity[13] s. The factor $\delta(\vec{R}_{op})$ sets the hadron center of P^+ at the origin; then the Fourier-transformed $\psi^\dagger\psi$ is the appropriate number operator for the quarks. The factor $(4\pi)^{-1}$ results from our covariant normalization of states:

$$\langle P^+,\vec{P}|K^+,\vec{K}\rangle = (2\pi)^3 2P^+\delta(P^+-K^+)\,\delta(\vec{P}-\vec{K}).$$

For antiquarks of type $\bar{A}$ one has

$$\mathcal{P}_{\bar{A}}(x,\vec{r}) = -\frac{1}{4\pi}\int dx^- e^{-ixP^+x^-}\langle P^+,\vec{0},s|\delta(\vec{R}_{op}):\psi^{(A)}(0,\vec{r},0)^\dagger\,\psi^{(A)}(0,\vec{r},x^-):|P^+,\vec{0},s\rangle \quad . \tag{2.13}$$

The $:\psi^\dagger\psi:$ denotes the normal-ordered product, equivalent to $-\psi\psi^\dagger$ here.

The factor $\delta(R_{op})$ in these equations can be eliminated by using the Poincaré-group commutation relation

$$[P^j_{op}, R^k_{op}] = -i\delta^{jk} \quad .$$

For $\mathcal{P}_A(x,\vec{r})$ we have

$$\begin{aligned}\mathcal{P}_A(x,\vec{r}) &= \frac{1}{4\pi}\int dx^- e^{ixP^+x^-}\int\frac{d\vec{P}}{(2\pi)^2}\langle P^+,0,s|e^{-i\vec{P}\cdot(\vec{R}_{op}+\vec{r})}\psi^{(A)}(0,\vec{0},0)^\dagger\psi^{(A)}(0,\vec{0},x^-)|P^+,0,s\rangle \\ &= \int\frac{d\vec{P}}{(2\pi)^2}e^{-i\vec{P}\cdot\vec{r}}\,\frac{1}{4\pi}\int dx^- e^{ixP^+x^-}\langle P^+,\vec{P},s|\psi^{(A)}(0,\vec{0},0)^\dagger\psi^{(A)}(0,\vec{0},x^-)|P^+,\vec{0},s\rangle .\end{aligned} \tag{2.14}$$

A similar relation holds for $\mathcal{P}_{\bar{A}}(x,\vec{r})$.

D. The form factor

The expression (2.14) for $\mathcal{P}_A(x,\vec{r})$ can be used to give a simple derivation of the relation between $\mathcal{P}_A(x,\vec{r})$ and the form factor. First integrate $\mathcal{P}_A(x,\vec{r})$ over the physical range $0<x<1$. The integral from $x=1$ to $x=\infty$ can be added at no cost since the integrand vanishes in this range (because there are no physical states with negative P^+). Next, subtract the integral of $\mathcal{P}_{\bar{A}}(x,\vec{r})$ from $x=0$ to $x=1$, which is the same as the integral from $x=0$ to $x=\infty$. In the $\mathcal{P}_{\bar{A}}$ integral, change variables from x to $-x$. This gives

$$\begin{aligned}\int_0^1 dx[\mathcal{P}_A(x,\vec{r})-\mathcal{P}_{\bar{A}}(x,\vec{r})] &= \int\frac{d\vec{P}}{(2\pi)^2}e^{-i\vec{P}\cdot\vec{r}}\frac{1}{4\pi}\int dx^-\int_{-\infty}^{\infty}dx\,e^{ixP^+x^-}\langle P^+,\vec{P},s|:\psi^{(A)}(0,\vec{0},0)^\dagger\psi^{(A)}(0,\vec{0},x^-):|P^+,\vec{0},s\rangle \\ &= \int\frac{d\vec{P}}{(2\pi)^2}e^{-i\vec{P}\cdot\vec{r}}\,\frac{1}{2P^+}\langle P^+,\vec{P},s|:\psi^{(A)}(0)^\dagger\psi^{(A)}(0):|P^+,\vec{0},s\rangle \quad .\end{aligned}$$

Finally, Fourier-transform, multiply by the charge $\mathcal{Q}_A$ of the quark of type A, and note that the charge $\mathcal{Q}_{\bar{A}}$ of the corresponding antiquark is $-\mathcal{Q}_A$:

$$\int d\vec{r}\,e^{i\vec{Q}\cdot\vec{r}}\int dx[\mathcal{Q}_A\mathcal{P}_A(x,\vec{r})+\mathcal{Q}_{\bar{A}}\mathcal{P}_{\bar{A}}(x,\vec{r})] = \frac{1}{2P^+}\langle P^+,\vec{Q},s|\mathcal{Q}_A:\psi^{(A)}(0)^\dagger\psi^{(A)}(0):|P^+,\vec{0},s\rangle \quad .$$

The operator $e\mathcal{Q}_A:\psi^{(A)}(0)^\dagger\psi^{(A)}(0):$ is the contribution from quarks and antiquarks of type A to the + component of the electromagnetic-current operator, $J^+(0)$.[12] Thus, if we sum over quark types we have

$$\sum_A\int d\vec{r}\,e^{i\vec{Q}\cdot\vec{r}}\int_0^1 dx\;\mathcal{Q}_A\mathcal{P}_A(x,\vec{r}) = \frac{1}{2P^+}\,\frac{1}{e}\,\langle P^+,\vec{Q},s|J^+(0)|P^+,\vec{0},s\rangle \quad . \tag{2.15}$$

The right-hand side of (2.15) is precisely the form factor F_1 of the hadron. Thus, we obtain the result (2.3) that one might have guessed based on the similar result in nonrelativistic quantum mechanics.

E. The structure function

We turn now to the scaling behavior of the deep-inelastic structure function $W_2(\nu,Q^2)$ of a hadron. Let us adopt a coordinate system in which the hadron has momentum components

$$P^\mu = \left(P^+,\vec{0},\frac{M^2}{2P^+}\right) \;,$$

and the virtual photon that strikes the hadron has momentum components

$$q^\mu = \left(0, \vec{Q}, \frac{M}{P^+}\nu\right) .$$

Then $P_\mu P^\mu = M^2$, $-q_\mu q^\mu = \vec{Q}^2 \equiv Q^2$, and $P^\mu q_\mu = M\nu$. As is well known, W_2 can be written as a matrix element of the commutator of two electromagnetic currents. In the present coordinate system

$$W_2(\nu, Q^2) = \frac{M}{4\pi e^2 (P^+)^2} \int d^4x\, e^{-iq\cdot x} \langle P^\nu | [J^+(0), J^+(x)] | P^\nu \rangle . \tag{2.16}$$

(We consider a spinless hadron here. Otherwise one must sum over spins.)

We wish to investigate the behavior of W_2 in the scaling limit, $\nu \to \infty$, $Q^2 \to \infty$, with $x \equiv Q^2/2M$ held fixed. Begin with the Fourier-transformed current that appears in W_2:

$$\tilde{J}^+(q^\nu) \equiv e\mathcal{Q} \int d^4x\, e^{-iq\cdot x} : \psi^\dagger(x)\psi(x):$$

$$= e\mathcal{Q} \int dx^+ e^{-i(M/P^+)\nu x^+} (2\pi)^{-3} \int dk^+ d\vec{k} : \tilde{\psi}^\dagger(k^+, \vec{k} + \vec{Q}; x^+) \tilde{\psi}(k^+, \vec{k}; x^+) : . \tag{2.17}$$

Here $\tilde{\psi}(k^+, \vec{k}; x^+)$ is the Fourier transform of the quark field operator at constant x^+, $e\mathcal{Q}$ is the quark charge, and the sum over quark types A has been suppressed.

Now we apply the essential approximation of the parton model, the impulse approximation. This approximation is known to be not quite right in field theory, but it is almost right if the dimensions of the various fields are nearly canonical. Notice that the current $\tilde{J}(q^\nu)$ transfers a large "energy" $q^- = M\nu/P^+$ to one of the quarks or antiquarks in the hadron. (Since $q^+ = 0$ the current cannot create or destroy a quark-antiquark pair, which must have $P^+ > 0$.) One assumes that this energy is so large compared to quark-quark interaction energies that the struck quark can be treated as if it were free. Thus the x^+ dependence of the quark field operator can be approximated by that of a free field:

$$\tilde{\psi}(k^+, \vec{k}; x^+) \approx \exp\left(-i\, \frac{\vec{k}^2 + M^2}{2k^+}\, x^+\right) \tilde{\psi}(k^+, \vec{k}; 0) .$$

With this approximation, the x^+ dependence of the integrand in Eq. (2.17) is known and the x^+ integration can be done, producing a δ function:

$$\int dx^+ \to \frac{|k^+|}{M\nu}\, 2\pi\delta\left(\frac{k^+}{P^+} - \frac{(\vec{k} + \vec{Q})^2 - \vec{k}^2}{2M\nu}\right) .$$

The factor $|k^+|$ here can be replaced by $Q^2/2M\nu \equiv x$ for the following reason. The dominant contribution to W_2 comes from the integration region $|\vec{k}| \ll Q$ (and the region $|\vec{k} + \vec{Q}| \ll Q$), since the probability to find a quark (or an antiquark) in the hadron falls off with increasing transverse momentum. Thus the δ function requires that k^+/P^+ equals approximately $Q^2/2M\nu$ (or $-Q^2/2M\nu$) within the dominant integration regions.

The required impulse approximation on $\tilde{J}(q^\mu)$ has now been made. If we express the Fourier-transformed fields $\tilde{\psi}(k^+, \vec{k}; 0)$ in terms of the $x^+ = 0$ fields $\psi(0, \vec{x}, x^-)$ and do the k^+ and $\vec{k}$ integrations, we obtain

$$\tilde{J}(q^\nu) \approx \frac{x(P^+)^2}{M\nu}\, e\mathcal{Q} \int d\vec{x}\, e^{i\vec{Q}\cdot\vec{x}} \int dz' dz : \psi^\dagger(0, \vec{x} + \vec{\Delta}, z') \psi(0, \vec{x} - \vec{\Delta}, z) : , \tag{2.18}$$

where

$$\vec{\Delta} = (z' - z)\, \frac{P^+}{2M\nu}\, \vec{Q} .$$

Notice that the two field operators in this approximation both act at the same "time" x^+, but at different points in the $x^+ = 0$ plane. Thus $\tilde{J}(q^\nu)$ is a certain integral transform of a "bilocal" operator.[9] The transverse separation $\vec{\Delta}$ between the points at which the fields act is small in the scaling limit, but it is important not to set $\vec{\Delta} = 0$ here because of the rapidly oscillating phase factor $\exp(i\vec{Q}\cdot\vec{x})$.

When we insert the approximate expression (2.18) for $\tilde{J}(q^\nu)$ into the expression (2.16) for W_2 we obtain

$$\nu W_2(\nu, Q^2) \approx x\mathcal{Q}^2 \int d\vec{x}\, e^{i\vec{Q}\cdot\vec{x}} \int dz' dz \langle P | [:\psi^\dagger(0)\psi(0):, :\psi^\dagger(0, \vec{x} + \vec{\Delta}, z') \psi(0, \vec{x} - \vec{\Delta}, z):] | P \rangle .$$

It is now a simple matter to calculate the current commutator, using the canonical commutation relations (2.11). In computing the commutator one obtains a factor $\delta(\vec{x}+\vec{\Delta})$ or a factor $\delta(\vec{x}-\vec{\Delta})$. Thus the rapidly oscillating phase factor becomes

$$e^{i\vec{Q}\cdot\vec{x}} \to e^{\pm i\vec{Q}\cdot\vec{\Delta}} = e^{\pm ixP^+[z'-z]} \quad .$$

One of the fields then acts at transverse position $\pm 2\vec{\Delta}$, but since $\vec{\Delta}\to 0$ in the scaling limit we can replace $\psi(0,\pm 2\vec{\Delta},x^-)$ by $\psi(0,\vec{0},x^-)$. After a little algebra we obtain in the scaling limit

$$\nu W_2(\nu,Q^2)\approx x\mathcal{Q}^2\int dz\,\frac{i}{2\pi}\,\sin(xP^+z) \times\langle P|:\psi^\dagger(0)\psi(0,\vec{0},z):|P\rangle\,. \quad (2.19)$$

Comparison with the definition (2.14) of $\mathcal{P}(x,\vec{r})$ gives

$$\nu W_2(\nu,Q^2)\approx x\mathcal{Q}_A{}^2\int d\vec{r}[\mathcal{P}_A(x,\vec{r})-\mathcal{P}_{\overline{A}}(x,\vec{r})] \quad .$$

If we restore the sum over quark types that has been suppressed above, this is precisely the familiar relation (2.2) between νW_2 and the parton distribution function.

III. THE QUARK-ANTIQUARK CONTRIBUTION TO THE PION'S PARTON DISTRIBUTION FUNCTION

In this section we examine the two-parton contribution to the parton distribution function $\mathcal{P}_A(x,\vec{r})$ of a pion (or some other pseudoscalar meson).

The pion can be considered to be a bound state of quark-type A and antiquark of type $\overline{B}$, plus a "sea" consisting of an indefinite number of quark-antiquark pairs. Such a bound state is described in field theory by the Bethe-Salpeter wave function

$$\langle 0|\Psi^{(A)}(0)\overline{\Psi}^{(B)}(-y^\nu)|P\rangle \quad .$$

If one sets $y^+=0$ in this BS wave function and takes an appropriate trace to project out the dynamically independent components of the Dirac fields, one obtains the amplitude for the pion to consist of precisely the A-$\overline{B}$ "valence" quarks with no "sea."

Specifically, consider the amplitude for a pion at rest,

$$\phi(x,\vec{y})=(8\pi)^{-1/2}\times\int dy^- e^{-i(1-x)P^+y^-}\times\mathrm{Tr}\{\gamma^+\gamma_5\langle 0|\Psi^{(A)}(0)\overline{\Psi}^{(B)}(0,-\vec{y},-y^-)|P\rangle\}\,. \quad (3.1)$$

In terms of the independent fields $\psi(x)$ defined above Eq. (2.11) this amplitude is

$$\phi(x,\vec{y})=(8\pi)^{-1/2}\int dy^- e^{-i(1-x)P^+y^-}\times[\langle 0|\psi^{(A)}_{+1/2}(0)\psi^{(B)}_{+1/2}(0,-\vec{y},-y^-)^\dagger|P\rangle -\langle 0|\psi^{(A)}_{-1/2}(0)\psi^{(B)}_{-1/2}(0,-\vec{y},-y^-)^\dagger|P\rangle]\,.$$

Thus $\phi(x,\vec{y})$ is the amplitude for the pion bound state to consist of one type-A quark and one type-$\overline{B}$ antiquark, separated by a transverse distance $\vec{y}$, with the quark carrying a fraction x of the + component of the momentum of the bound state. The spin state projected in ϕ is the singlet state

$$|+\tfrac{1}{2}\rangle_A|-\tfrac{1}{2}\rangle_{\overline{B}}-|-\tfrac{1}{2}\rangle_A|+\tfrac{1}{2}\rangle_{\overline{B}} \;;$$

the triplet state $(|+\rangle\,|-\rangle+|-\rangle|+\rangle)$ amplitudes can be obtained by replacing γ_5 by 1 in Eq. (3.1); parity invariance thus requires that this triplet amplitude be zero.[17] The normalization has been chosen so that

$$\int_0^1 dx\int d\vec{y}|\phi(x,\vec{y})|^2$$

is the probability that the pion consists of just the A-$\overline{B}$ quark pair.

The parton distribution function defined in Eq. (2.12) is

$$\mathcal{P}_A(x,\vec{r})dxd\vec{r}=|\phi(x,\vec{y})|^2dxdy+\cdots \quad , \quad (3.2)$$

where the transverse distance $\vec{r}$ between the quark and the center of P^+ of the pair is $\vec{r}=(1-x)\vec{y}$. The dots here indicate the contribution to $\mathcal{P}_A$ from the amplitudes for the pion to contain two or more "sea" partons in addition to the $A\overline{B}$ valence pair. We will be unable to say anything about these more complicated amplitudes. What we can do is to translate the information contained in Ref. 4 into the present parton language in order to say how $\phi(x,\vec{y})$ behaves for small $\vec{y}$ and also for x near 1.

A. O(4) Amplitudes and small-$(1-x)$ behavior

In this subsection we write an O(4) expansion of the pion Bethe-Salpeter wave function and then make a Sommerfeld-Watson transform of the sum over J. This enables us to relate the small-$(1-x)$ behavior of the parton wave function $\phi(x,\vec{y})$ to the location of poles of the Bethe-Salpeter O(4) partial-wave amplitudes in the complex J plane. At the same time, we relate the small-$\vec{y}$ behavior of $\phi(x,\vec{y})$ to the small-$\vec{y}$ behavior of the partial-wave amplitudes. Since the mathematical techniques used here are similar to those presented in some detail in Ref. 4, we will omit some of the details.

We begin by writing an O(4) expansion of the Bethe-Salpeter wave function

$$\langle 0|\Psi(0)\bar{\Psi}(-y^\nu)|P\rangle = \not{P}\gamma_5\sum_{J=0}^{\infty} g_{J+1}(-y^2)\left(\frac{2i}{M}\right)^J \{y^{\mu_1}\cdots y^{\mu_J}\}\{P_{\mu_1}\cdots P_{\mu_J}\} + \cdots \quad (3.3)$$

The braces here indicate that the traceless symmetric part of the tensor enclosed is to be taken. The dots indicate omitted terms that are proportional to γ_5, $\gamma_5\sigma_{\mu\nu}$, and $x\cdot\gamma\gamma_5$. We next set $y^+=0$ and take the trace with $\gamma^+\gamma_5$ (the omitted terms do not contribute to this trace):

$$\mathrm{Tr}(\gamma^+\gamma_5\langle 0|\bar{\Psi}(0)\Psi(0,-\vec{y},-y^-)|P\rangle) = -4P^+\sum_{J=0}^{\infty} g_{J+1}(\vec{y}^2)|\vec{y}|^J(-1)^J U_J(\cos\theta) \ , \quad (3.4)$$

where

$$\cos\theta = -\frac{iy^0}{(-y^\nu y_\nu)^{1/2}} = -\frac{i}{\sqrt{2}}\frac{y^-}{|\vec{y}|}$$

and

$$U_J(\cos\theta) = \frac{\sin(J+1)\theta}{\sin\theta}$$

is a Chebyshev polynomial of the second kind.[14]

The sum in Eq. (3.4) is not a useful representation for large values of $|\cos\theta|$. Therefore we rewrite it as a Sommerfeld-Watson integral

$$\mathrm{Tr}(\gamma^+\gamma_5\langle 0|\bar{\Psi}(0)\Psi(0,-y,-y^-)|P\rangle) = -2iP^+\int_{-\epsilon-i\infty}^{-\epsilon+i\infty} dJ\, g_{J+1}(\vec{y}^2)|\vec{y}|^J \frac{U_J(\cos\theta)}{\sin\pi J} \ . \quad (3.5)$$

The contour here initially circles the positive J axis and is then opened up so that it runs just to the left of the imaginary J axis. We assume, as argued in Ref. 4, that $g_J(\vec{y}^2)$ is an analytic function of J except for possible singularities in the left half J plane.

We can now do the Fourier transform with respect to z that is required to form the wave function $\phi(x,\vec{y})$, Eq. (3.1). We use[15]

$$\int dy^- e^{-i(1-x)P^+y^-} U_J\left(-\frac{i}{\sqrt{2}}\frac{y^-}{|\vec{y}|}\right) = i\pi|\vec{y}|\Theta(1-x)[I_{-J-1}((1-x)|\vec{y}|P^+) - I_{J+1}((1-x)|\vec{y}|P^+)] \ ,$$

where $I_J(Z)$ is the usual modified Bessel function.[14] Thus the wave function $\phi(x,\vec{y})$ is

$$\phi(x,\vec{y}) = (\pi/2)^{1/2}P^+\Theta(1-x) \times\int_{-\epsilon-i\infty}^{-\epsilon+i\infty} dJ\, g_{J+1}(\vec{y}^2)|\vec{y}|^{J+1} \times\frac{1}{\sin\pi J}[I_{-J-1}((1-x)|\vec{y}|P^+) - I_{J+1}((1-x)|\vec{y}|P^+)] \ . \quad (3.6)$$

Consider first the term in ϕ containing the factor I_{-J-1}. Since

$$I_\nu(Z)\propto Z^\nu \quad \text{for } Z\to 0$$

the small-$(1-x)$ behavior of this term can be determined by moving the integration contour to the left past the rightmost singularities of $g_{J+1}(\vec{y}^2)$. If $g_J(\vec{y}^2)$ has poles at $J=J_n$, one must include terms ϕ_n in ϕ arising from the integrals around the poles:

$$\phi_n(x,\vec{y}) = (2\pi)^{1/2}iP^+\Theta(1-x)\frac{1}{\sin\pi(J_n-1)} \times\left[\operatorname*{Res}_{J=J_n} g_J(\vec{y}^2)\right]|\vec{y}|^{J_n} I_{-J_n}((x-1)|\vec{y}|P^+) \ . \quad (3.7)$$

The remaining background integral then falls off faster than the $\phi_n(x,\vec{y})$ as $(1-x)\to 0$.

Now consider the second term in $\phi(x,y)$, which contains the factor I_{J+1}. In this term one moves the contour to the *right*, picking up contributions from the poles of $1/\sin\pi J$. These contributions cancel the $1/\sin\pi J$ pole terms from the I_{-J-1} part of ϕ, provided

$$g_J(y^2)|\vec{y}|^J + g_{-J}(y^2)|\vec{y}|^{-J} = 0 \quad (3.8)$$

for integer values $J=0,1,2,\ldots$. We will assume, based on the argument given in Ref. 4, that this "Lorentz symmetry" condition does in fact hold. After a suitable change of integration variables, the two parts of the background integral can be combined into

$$I(x,\vec{y}) = -\left(\frac{\pi}{2}\right)^{1/2}P^+\Theta(1-x) \times\int_{\mathrm{Re}J=J_B} dJ[g_J(\vec{y}^2)|\vec{y}|^J + g_{-J}(\vec{y}^2)|\vec{y}|^{-J}] \times\frac{I_{-J}((1-x)|\vec{y}|P^+)}{\sin\pi J} \ , \quad (3.9)$$

where J_B lies to the left of all the pole positions J_n.

Thus we obtain

$$\phi(x,\vec{y}) = \sum_n \phi_n(x,\vec{y}) + I(x,\vec{y}) \ , \quad (3.10)$$

where

$$\phi_n(x,\vec{y}) \propto \left[\operatorname*{Res}_{J=J_n} g_J(\vec{y}^2)\right]\Theta(1-x)(1-x)^{-J_n} \quad (3.11)$$

for small $(1-x)$. The background integral I, Eq. (3.9), falls off at least as fast as $(1-x)^{-J_B}$ as $(1-x)\to 0$, where $J_B < \mathrm{Re}\,J_n$.

B. Behavior of the two-parton wave function

We are now prepared to combine the analysis of the preceding subsection with the properties of the partial-wave amplitudes $g_J(\vec{y}^2)$ found in Ref. 4. The results of Ref. 4 are based on the use of scale invariance and conformal invariance at short distances (with the presumed fixed point of the renormalization group at nonzero coupling constant). Thus they are presumably applicable to nongauge field theories. We must also presume that the anomalous dimension of operators that appear are all small, otherwise scaling and the parton model would be seriously wrong. The results that we need from Ref. 4 are as follows. The small-$\vec{y}^2$ behavior of the partial-wave amplitude $g_J(\vec{y}^2)$, which is determined by counting powers in the operator product expansion for $\Psi(0)\overline{\Psi}(-y^\nu)$, is

$$g_J(\vec{y}^2) \sim \sum_n C(J,n)\,|\vec{y}\,|^{\gamma_n - 2\gamma_\psi} + \text{higher-order terms.} \tag{3.12}$$

Here γ_ψ is the anomalous dimension of the quark field $\Psi(x)$ and γ_n is the anomalous dimension of the twist-two operator

$$\overline{\Psi}(x)\{\gamma_{\mu_1}\overleftrightarrow{\partial}_{\mu_2}\cdots\overleftrightarrow{\partial}_{\mu_{2n+1}}\}\Psi(x).$$

The coefficients $c(J,n)$ in (3.12) have poles at locations in the left-hand J plane that are determined by applying conformal invariance at short distances to the operator-product expansion. One finds that the rightmost pole of $C(J,n)$ occurs at a location

$$J_n = -1 - \tfrac{1}{2}\gamma_n \tag{3.13}$$

near $J=-1$.

We also need to know the J-plane singularities of $g_J(\vec{y}^2)$ when $\vec{y}^2$ is not small. We will simply assume, following Ref. 4, that the rightmost singularities are poles at the same locations, $J=J_n$, that were found in the short-distance limit. This assumption is somewhat *ad hoc*, but it is simple and, more importantly, it is true in the $g\overline{\Psi}\Psi\phi$ ladder model.[16]

We can now deduce how the two-parton wave function $\phi(x,\vec{y})$ behaves for small $(1-x)$ and for small $\vec{y}^2$.

Small $(1-x)$. The leading terms in $\phi(x,\vec{y})$ as $(1-x)\to 0$ are the pole terms $\phi_n(x,\vec{y})$, with

$$\phi_n(x,\vec{y}) \propto (1-x)^{1+\gamma_n/2}\operatorname*{Res}_{J=J_n} g_J(\vec{y}^2)$$

[see Eqs. (3.10), (3.11), (3.13)]. When $y^2\to 0$ also, we have from Eq. (3.12)

$$\operatorname*{Res}_{J=J_n} g_J(\vec{y}^2) \propto |y\,|^{\gamma_n-2\gamma_\psi}.$$

Small $\vec{y}^2$. The pole terms in $\phi(x,\vec{y})$, Eq. (3.10), behave like

$$\phi_n(x,y) \propto |y\,|^{\gamma_n-2\gamma_\psi}(1-x)^{1+\gamma_n/2}.$$

The background integral contains the same powers of $|\vec{y}|$ as $|\vec{y}|\to 0$:

$$I(x,\vec{y}) \sim \sum_n |\vec{y}\,|^{\gamma_n-2\gamma_\psi} I_n(x),$$

where

$$I_n(x) \propto \int_{\operatorname{Re}J=J_B} dJ\,\frac{C(J,n)}{\sin J}(1-x)^J$$

[see Eqs. (3.9) and (3.12)]. When $(1-x)\to 0$ also, the pole terms dominate.

In summary, the small-$(1-x)$ and small-$\vec{y}^2$ behavior of $\phi(x,\vec{y})$ is

$$\begin{aligned}
&\phi(x,\vec{y}) \sim \sum_n A_n(y^2)(1-x)^{1+\gamma_n/2}+\cdots, \quad (1-x)\to 0\\
&\phi(x,\vec{y}) \sim \sum_n B_n(x)\,|\vec{y}\,|^{\gamma_n-2\gamma_\psi}+\cdots, \quad \vec{y}^2\to 0\\
&\phi(x,\vec{y}) \sim \sum_n C_n(1-x)^{1+\gamma_n/2}|\vec{y}\,|^{\gamma_n-2\gamma_\psi}+\cdots,\\
&\qquad \vec{y}^2\to 0 \text{ and } (1-x)\to 0.
\end{aligned} \tag{3.14}$$

C. The structure function and form factor

The contribution from the two-parton state to the parton distribution function $\mathcal{P}(x,\vec{r})$ is

$$\mathcal{P}^{(2)}(x,\vec{r})\,dx\,d\vec{r} = |\phi(x,\vec{y})|^2\,dx\,d\vec{y},$$

where $\vec{r}=(1-x)\vec{y}$ [see Eq. (3.2)]. Thus, using Eq. (3.14), the two-parton contribution to $\nu W_2(x)$ behaves for small $(1-x)$ like

$$\nu W_2^{(2)}(x) = x\int d\vec{y}\,|\phi(x,\vec{y})|^2 \sim \sum_{n,m} a(n,m)(1-x)^{2+\gamma_n/2+\gamma_m/2}$$

or, neglecting anomalous dimensions,

$$\nu W_2^{(2)}(x) \propto (1-x)^2. \tag{3.15}$$

To compute the two-parton contribution to the form factor, we take the Fourier transform

$$f^{(2)}(x,\vec{k}^2) = \int d\vec{y}\,e^{i\vec{k}\cdot\vec{y}}|\phi(x,\vec{y})|^2.$$

Using the information (3.14) about $\phi(x,\vec{y})$, we see that

$$f^{(2)}(x,\vec{k}^2) \approx a(k^2)(1-x)^2$$

for small $(1-x)$ and

$$f^{(2)}(x,\vec{k}^2) \approx b(x)(\vec{k}^2)^{-1}$$

for large $\vec{k}^2$, where we have neglected the anomalous dimensions after taking the Fourier transform. According to the analysis of Sec. II, Eqs. (2.8), (2.9), and (2.10), the form factor at large Q^2 receives its leading contribution from the large-$\vec{k}^2$

behavior of $f^{(2)}(x, \vec{k}^2)$:

$$F^{(2)}(Q^2) \propto (Q^2)^{-1}. \tag{3.16}$$

There is also a possible nonleading contribution that behaves like $(Q^2)^{-3/2}$, corresponding to the small-$(1-x)$ behavior of $f^{(2)}(x, \vec{k}^2)$.

D. Comparison with other calculations

The result (3.16) for the form factor is consistent, up to anomalous dimensions, with the result of Ref. 4 obtained in a Bethe-Salpeter calculation including all numbers of partons in intermediate states (but only the "triangle" class of Feynman diagrams). If one keeps track of the anomalous dimensions one finds instead of (3.16) the more precise result

$$F^{(2)}(Q^2) \sim \sum_{n,m} c(n,m)(Q^2)^{-1+2\gamma_\psi-\gamma_n/2-\gamma_m/2}.$$

In the Bethe-Salpeter calculation, before corrections for dressed quarks are applied, the contributions to $F(Q^2)$ that arise from the short-distance behavior of the BS wave function have precisely this form. There are also contributions that behave like $(Q^2)^{-1-\gamma_n/2}$ and correspond to the J-plane poles of the BS wave function. [These poles determine the behavior of the BS wave function when one leg is far off shell and, as we have just seen, determine the behavior of the two-parton wave function for $(1-x) \to 0$.] These $(Q^2)^{-1-\gamma_n/2}$ wave-function pole contributions are absent from the two-parton part of the form factor calculated here. The question of how these contributions arise in the parton model thus remains open.

The present results (3.15) and (3.16), if they are taken to apply to the whole form factor and structure function, disagree with the Drell-Yan-West relation but agree with the results of Ezawa[6] and of Farrar and Jackson.[7]

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[1]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972).

[2]J. D. Bjorken, in *Proceedings of the Third International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, Stanford, Calif., 1967* (Clearing House of Federal Scientific and Technical Information, Washington, D.C., 1968); J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969); see also S. D. Drell, D. J. Levy, and T.-M. Yan, Phys. Rev. 187, 2159 (1969).

[3]E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1232 (1951); M. Gell-Mann and F. E. Low, *ibid.* 84, 350 (1951).

[4]M. L. Goldberger, D. E. Soper, and A. H. Guth, Phys. Rev. D 14, 1117 (1976).

[5]S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. 31, 1153 (1973); V. Matveev, R. Muradyan, and A. Tavkhelidze, Lett. Nuovo Cimento 7, 719 (1973); R. Blankenbecler, S. Brodsky, and J. Gunion, Phys. Rev. D 8, 187 (1973).

[6]Z. Ezawa, Nuovo Cimento 23A, 271 (1974).

[7]G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. 35, 1416 (1975).

[8]J. D. Bjorken, J. B. Kogut, and D. E. Soper, Phys. Rev. D 3, 1382 (1971).

[9]D. J. Gross and S. B. Treiman, Phys. Rev. D 4, 1059 (1971).

[10]S. D. Drell and T.-M. Yan, Phys. Rev. Lett. 24, 181 (1970); G. B. West, *ibid.* 24, 1206 (1970).

[11]L. Susskind, Phys. Rev. 165, 1535 (1968); L. Susskind, in *Lectures in Theoretical Physics*, edited by K. Mahanthappa and W. Brittin (Gordon and Breach, New York, 1969).

[12]J. B. Kogut and D. E. Soper, Phys. Rev. D 1, 2901 (1970).

[13]By "helicity" we mean the null-plane helicity that is unchanged by Galilean boosts. See D. E. Soper, Phys. Rev. D 5, 1956 (1972).

[14]See, for example, *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun, National Bureau of Standards Applied Mathematics Series, No. 55 (U.S. G.P.O., Washington, D.C., 1964).

[15]This integral is computed for $x<1$ by closing the integration contour in the lower half y^- plane and evaluating the integral along the cut of $U_J(\cos\theta)$ from $\cos\theta = -1$ to $\cos\theta = -\infty$. For $x>1$ one closes the contour in the upper half y^- plane, where U_J is analytic, and gets zero.

[16]M. L. Goldberger, D. E. Soper, and A. H. Guth, Phys. Rev. D 14, 2633 (1976).

[17]The amplitudes for the $s_z = \pm 1$ states $|+\rangle|+\rangle$ and $|-\rangle|-\rangle$ have not been included in the present analysis. They can be included, using the same methods, and do not affect the results.

[18]Y. S. Kim and M. E. Noz, Phys. Rev. D 15, 335 (1977).