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Unification of supergravity and Yang-Mills theory*

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Supersymmetric Yang-Mills theory (including the Abelian case) is combined with supergravity so as to achieve local supersymmetry. The Lagrangian, the supersymmetry transformation rules, and their commutator algebra are presented.

I. INTRODUCTION

A Lagrangian field theory of supergravity has recently been constructed.[1] It may be regarded as a locally supersymmetric extension of Einstein's general relativity which describes interacting massless spin-2 and spin-$\frac{3}{2}$ fields. It is now very natural to attempt to extend previous globally supersymmetric field theories to include local invariance by coupling them to the spin-2–spin-$\frac{3}{2}$ gravitational multiplet. In this paper we give results on the locally supersymmetric extension[2] of the Yang-Mills multiplet[3] with arbitrary internal symmetry group. This multiplet contains vector gauge fields and Majorana spin-$\frac{1}{2}$ fields, transforming in the adjoint representation. We were motivated to consider this multiplet (rather than the chiral multiplet,[4] which might seem simpler) because of the useful restrictions of gauge and conformal invariance[5] and because of a close parallel with the structure of the dual fermion string model.[6] It is known that the zero-slope limit of the open-string sector of this dual model gives globally supersymmetric Yang-Mills theory, and it has been conjectured by J. H. Schwarz that the zero-slope limit of the closed-string sector gives supergravity.

The locally supersymmetric theory is constructed by a two-stage method, using the first-order formalism for gravitation which has already led to a useful reformulation[7] of supergravity. In the first stage, discussed in Sec. II, the Lagrangian is determined by requiring the vanishing of leading derivative terms in the consistency condition[7] for the spin-$\frac{3}{2}$ field equation. After the Lagrangian is under good control it is easier to obtain the supersymmetry transformation laws under which it is invariant, and this is done by a systematic method in Sec. II. The commutator algebra of the supersymmetry variations and additional material are discussed in Sec. III.

II. THE LAGRANGIAN

The action functional of the combined gravitational and matter multiplets is written as

$$I=\int d^4x(\mathcal{L}_{\rm SG}+\mathcal{L}_{\rm M}+\mathcal{L}_1+\mathcal{L}_2)\,, \tag{1}$$

with the supergravity Lagrangian density[7,8]

$$\mathcal{L}_{\rm SG}=-\frac{1}{4\kappa^2}VV^{a\mu}V^{b\nu}R_{\mu\nu ab}-\tfrac{1}{2}\epsilon^{\lambda\rho\mu\nu}\bar\psi_\lambda\gamma_5\gamma_\mu\mathcal{D}_\nu\psi_\rho \tag{2}$$

and matter kinetic term

$$\mathcal{L}_{\rm M}=-\tfrac{1}{4}Vg^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma}+\tfrac{1}{2}iV\bar\chi\not{D}\chi\,, \tag{3}$$

with

$$\begin{aligned}\mathcal{D}_\nu\psi_\rho&=(\partial_\nu+\tfrac{1}{2}\omega_{\nu ab}\sigma^{ab})\psi_\rho\,,\\ F_{\mu\nu}{}^i&=\partial_\mu A_\nu{}^i-\partial_\nu A_\mu{}^i+gf^{ijk}A_\mu{}^jA_\nu{}^k\,,\\ (D_\mu\chi)^i&=(\partial_\mu+\tfrac{1}{2}\omega_{\mu ab}\sigma^{ab})\chi^i+gf^{ijk}A_\mu{}^j\chi^k\,,\end{aligned} \tag{4}$$

where g is the gauge coupling, f^{ijk} are the structure constants of the internal symmetry group, and κ is the gravitational coupling. Except at one point in the analysis below, it is totally unnecessary to distinguish between Abelian and non-Abelian internal symmetry groups, and therefore we adopt the notational convention that all bilinear products among $F_{\mu\nu}$, χ, and $D_\mu\chi$ shall be understood as the group-invariant product, e.g.,

$$F_{\mu\nu}\chi=\sum_i F_{\mu\nu}{}^i\chi^i\,.$$

The linear coupling of the spin-$\frac{3}{2}$ field to the matter fields is of the form

$$\mathcal{L}_1=-\kappa\bar\psi_\lambda\mathcal{J}^\lambda\,,\quad \mathcal{J}^\lambda=\frac{i}{\sqrt{2}}V\sigma^{\mu\nu}F_{\mu\nu}\gamma^\lambda\chi\,, \tag{5}$$

where, as is to be expected in a theory where ψ_λ is the gauge field of supersymmetry, $\mathcal{J}^\lambda$ is proportional to the covariant generalization of the Noether current of globally supersymmetric Yang-Mills theory.[3,9]

The final term in the Lagrangian is bilinear in ψ_λ and is given by

$$\mathcal{L}_2 = -\tfrac{1}{2}V\kappa^2(\bar{\psi}_\lambda\sigma^{\mu\nu}\gamma^\lambda\chi)(\bar{\psi}_\mu\gamma_\nu\chi)$$
$$= \tfrac{1}{4}V\kappa^2[(\bar{\chi}\chi)\bar{\psi}_\mu(g^{\mu\nu}-\tfrac{1}{4}\gamma^\mu\gamma^\nu)\psi_\nu - (\bar{\chi}\gamma_5\chi)\bar{\psi}_\mu\gamma_5(g^{\mu\nu}-\tfrac{1}{4}\gamma^\mu\gamma^\nu)\psi_\nu - \tfrac{1}{4}(\bar{\chi}\gamma_5\gamma_\lambda\chi)(\bar{\psi}_\mu\gamma_5\gamma^\lambda\psi^\mu - 2\bar{\psi}_\mu\gamma^\mu\gamma_5\psi^\lambda + i\epsilon^{\lambda\mu\nu\rho}\bar{\psi}_\mu\gamma_\nu\psi_\rho)]\,, \tag{6}$$

where the two forms are related by Fierz rearrangement.

The form of this Lagrangian was determined by the following procedure. We started from a general ansatz of the form

$$\mathcal{L} = \mathcal{L}_{SG} + \mathcal{L}_M + a_1\mathcal{L}_1 + \mathcal{L}_2'\,, \tag{7}$$

where the coefficient a_1 of the Noether current term was a free parameter and $\mathcal{L}_2'$ was the most general expression containing products of bilinears in χ and in ψ. This expression was similar to the second form of (6), but involved seven arbitrary parameters instead of the specific values given above. The values of the eight parameters were then determined by an examination of the consistency condition[7] for the spin-$\frac{3}{2}$ wave operator

$$Q^\lambda = -\frac{\delta I}{\delta\bar{\psi}^\lambda}\,. \tag{8}$$

The consistency condition for the coupled system of matter and gravitation is

$$\mathfrak{D}_\lambda Q^\lambda = \tfrac{1}{2}iV(R^{a\mu} - \tfrac{1}{2}V^{a\mu}R)\gamma_a\psi_\mu + \tfrac{1}{2}\epsilon^{\lambda\rho\mu\nu}S_{\lambda\mu}{}^\sigma\gamma_5\gamma_\sigma\mathfrak{D}_\nu\psi_\rho + a_1\kappa\mathfrak{D}_\lambda\mathcal{J}^\lambda - \mathfrak{D}_\lambda\frac{\delta\mathcal{L}_2'}{\delta\bar{\psi}^\lambda}\,, \tag{9}$$

and is required to vanish when the Euler-Lagrange equations associated with (7) are substituted in the right-hand side. It is sufficient to examine the leading derivative terms of the form $FF\psi$, $\bar{\chi}D\chi\psi$, and $\bar{\chi}\chi\mathfrak{D}\psi$ which appear in (9). The first two classes of terms are required to cancel (although terms of the form $\not{D}\chi$ are ignored at this stage because they correspond to the equation of motion for χ), and this determines the values of the parameters given in (3) and (6). As a critical consistency check on these parameters, the $\bar{\chi}\chi\mathfrak{D}\psi$ terms are required to arrange themselves so that they contain as a factor one of the several forms[8] of the spin-$\frac{3}{2}$ equation of motion, and this test is satisfied. The rationale for this requirement is that at the second state of our procedure, such equation-of-motion terms also appear and induce a change in the transformation rule $\Delta\psi_\lambda$, depending on the matter fields.

The procedure at this stage is straightforward, although moderately complicated Dirac algebra is necessary. Two perhaps unfamiliar results which are required are the identity[2]

$$\tilde{F}^{\mu\rho}F^\nu{}_\rho = \tfrac{1}{4}g^{\mu\nu}\tilde{F}^{\lambda\rho}F_{\lambda\rho} \tag{10}$$

for products of any antisymmetric tensor and its dual, and the result

$$\sigma^{\mu\nu}\gamma\cdot\psi(\bar{\chi}\gamma_\mu D_\nu\chi) = -\gamma_5\sigma^{\mu\nu}\gamma\cdot\psi(\bar{\chi}\gamma_5\gamma_\mu D_\nu\chi) + \sigma^{\mu\nu}\gamma\cdot\psi(\bar{\chi}\sigma_{\mu\nu}\not{D}\chi) \tag{11}$$

which is required to relate terms of a vector character in the consistency condition to terms of an axial-vector character which cancel against similar terms from $\mathcal{L}_2'$.

The examination of the consistency equation for cancellation of leading derivative terms would not reveal the presence in the Lagrangian of terms independent of ψ_λ such as terms of higher order in the matter fields. The second stage of our procedure is sensitive to such terms, and it turns out that none are required.

III. LOCAL SUPERSYMMETRY

Consistency of the spin-$\frac{3}{2}$ wave equation is closely related to but not sufficient for local supersymmetry. There is still work to do to determine the transformation laws $\delta\phi_i$ for the fields ψ_λ, $V_{a\mu}$, $\omega_{\mu ab}$, A_μ, and χ—each linear in the supersymmetry transformation parameter which is an arbitrary Majorana spinor field $\epsilon(x)$—such that the action is invariant. We assume that the variation of the spin-$\frac{3}{2}$ field is of the form $\delta\psi_\lambda = \kappa^{-1}D_\lambda\epsilon + \Delta\psi_\lambda$, where $\Delta\psi_\lambda$ does not involve derivatives of $\epsilon(x)$ and is of higher order in κ. Local supersymmetry then requires that

$$\delta I = \int d^4x\left[\kappa^{-1}\bar{\epsilon}(x)\mathfrak{D}_\lambda Q^\lambda - \Delta\bar{\psi}_\lambda Q^\lambda + \sum_{i\neq\psi_\lambda}\frac{\delta\mathcal{L}}{\delta\varphi_i}\,\delta\varphi_i\right] \tag{12}$$

vanish for arbitrary $\epsilon(x)$ and arbitrary field configuration.

We first give the field variations under which the action of the previous section is invariant and then discuss the procedure used to find them. The supersymmetry transformation laws of the matter fields are

$$\delta A_\mu = \frac{i}{\sqrt{2}}\bar{\epsilon}\gamma_\mu\chi\,, \tag{13}$$

$$\delta\chi = \left(\frac{1}{\sqrt{2}}F_{\mu\nu} - i\kappa\bar{\psi}_\mu\gamma_\nu\chi\right)\sigma^{\mu\nu}\epsilon\,, \tag{14}$$

and consist, as expected, of a covariant form of the variations associated with the global supersymmetric theory together with an order-κ correction involving $\bar{\psi}_\mu$. The transformation rules of the gravitational fields consist of the expected terms of the first-order form[7] of pure supergravity plus corrections of higher order in κ involving matter fields, and are given by

$$\delta V_{a\mu} = i\kappa\bar{\psi}_\mu\gamma_a\epsilon\,, \tag{15}$$

$$\delta\psi_\lambda = \kappa^{-1}D_\lambda\epsilon + \tfrac{1}{4}i\kappa(\bar{\chi}\gamma_5\gamma^\nu\chi)\gamma_\nu\gamma_\lambda\gamma_5\epsilon\,, \tag{16}$$

$$\delta\omega_{\mu ab} = V_a{}^\nu V_b{}^\rho(C_{\mu\nu\rho} + \tfrac{1}{2}g_{\mu\nu}D_\rho - \tfrac{1}{2}g_{\mu\rho}D_\nu)\,, \tag{17}$$

with

$$\begin{aligned} C_{\mu\nu\rho} = {} & \kappa V^{-1}\epsilon_{\nu\rho}{}^{\sigma\tau}\bar{\epsilon}\gamma_5\gamma_\mu\mathfrak{D}_\sigma\psi_\tau + \sqrt{2}\,i\kappa^2\bar{\epsilon}\{\sigma_{\nu\rho}, \sigma^{\alpha\beta}F_{\alpha\beta}\}\gamma_\mu\chi \\ & + 2\kappa^3(\bar{\epsilon}\gamma_\mu\chi)(\bar{\chi}\gamma^\lambda\sigma_{\nu\rho}\psi_\lambda) \\ & + \tfrac{1}{4}i\kappa^3V^{-1}\epsilon_{\nu\rho}{}^{\sigma\tau}(\bar{\chi}\gamma_5\gamma^\lambda\chi)\bar{\epsilon}\gamma_\sigma\gamma_\lambda(\gamma_\mu\psi_\tau - \gamma_\tau\psi_\mu) \end{aligned} \tag{18}$$

and

$$C^\lambda{}_{\lambda\mu} + D_\mu = -\frac{i}{\sqrt{2}}\kappa^2\bar{\epsilon}\sigma^{\rho\lambda}F_{\rho\lambda}\gamma_\mu\chi + \kappa^3(\bar{\epsilon}\gamma^\rho\chi)(\bar{\chi}\gamma^\lambda\sigma_{\rho\mu}\psi_\lambda)\,. \tag{19}$$

To determine (13)–(17) and establish local supersymmetry we start from expressions for the variations which are the expected terms from the global matter supersymmetry and pure supergravity only, and then study carefully the various independent terms in the variation (12). Some terms, such as those involving purely ψ and ψ^3, are already known to cancel from the pure-supergravity calculation,[7] but we must study explicitly terms of the forms

(a) $DF\chi$ and $FD\chi$,

(b) $SF\chi$,

(c) $\kappa F^2\psi$,

(d) $\kappa\chi D\chi\psi$,

(e) $\kappa\chi\chi\mathfrak{D}\psi$,

(f) $\kappa S\chi\chi\psi$,

(g) $\kappa^2F\chi^3$,

(h) $\kappa^2F\chi\psi^2$,

(i) $\kappa^3\chi^4\psi$,

and

(j) $\kappa^3\chi^2\psi^3$.

Each type of term must either vanish or determine the corrections to the starting expression for the variations.

We now discuss the salient features of the various stages of calculation. The cancellation of type-(a) terms essentially reflects the global supersymmetry of the matter kinetic Lagrangian, although the Noether-current term comes in to cancel expressions which would be total derivatives in the global limit. Luckily, it is only at this early stage that the non-Abelian internal symmetry has any distinct effect. Namely, a term of the form

$$f^{ijk}\bar{\epsilon}\gamma^\mu\chi^i\bar{\chi}^j\gamma_\mu\chi^k \tag{20}$$

appears from the variation of $\bar{\chi}\not{D}\chi$ with respect to $A_\mu{}^i$. This same term appears in the variation of the globally supersymmetric Yang-Mills Lagrangian[9] and is known to vanish because of the antisymmetry of f^{ijk}.

Terms involving the torsion tensor appear at stage (b) because relations such as

$$\partial_\mu(VF^{\mu\nu i}) + Vgf^{ijk}B_\mu{}^jF^{\mu\nu k} = V(D_\mu F^{\mu\nu} + 2S_{\mu\tau}{}^\tau F^{\mu\nu} - S_{\lambda\rho}{}^\mu F^{\lambda\rho})\,, \tag{21}$$

where D_μ is the combined relativistic and gauge-covariant derivative, and

$$\epsilon^{\sigma\lambda\mu\nu}D_\lambda F_{\mu\nu} = 2\epsilon^{\sigma\lambda\mu\nu}S_{\lambda\mu}{}^\rho F_{\nu\rho}\,, \tag{22}$$

which is the gauge-field Bianchi identity in first-order gravitation, are used at the earlier stage. These terms are canceled by adding the order-κ^2 correction to $\delta\omega_{\mu ab}$ in (17). Similar remarks apply to the torsion terms of type (f).

The calculations at stages (c), (d), and (e) are, in large part, identical to those described in Sec. II, where the Lagrangian was determined. However, terms involving $\not{D}\chi$, which were previously ignored, must be studied here, and to cancel them we make the order-κ modification of $\delta\chi$ in (14).

As a more detailed example, we describe the determination of the order-κ correction to the transformation law of ψ_λ. We note that the terms of type (e) turn out after some algebra to be of the form

$$\delta I\big|_{(e)} = \tfrac{1}{4}i\kappa\int d^4x\,(\bar{\chi}\gamma_5\gamma_\sigma\chi)\bar{\epsilon}\gamma_5\gamma_\lambda\gamma^\sigma \times \epsilon^{\lambda\rho\mu\nu}\gamma_5\gamma_\mu\mathfrak{D}_\nu\psi_\rho\,. \tag{23}$$

Because this term involves as a factor the leading part of the spin-$\frac{3}{2}$ wave operator, it can be canceled by the modification of $\delta\psi_\lambda$ given in (16). If a combination of derivatives of ψ_ρ occurred in (23) which were not of the form of the spin-$\frac{3}{2}$ wave operator, the Lagrangian could not be locally supersymmetric. The correction term just determined also contributes to terms (f), (g), and (i) because it multiplies subsidiary terms in the spin-$\frac{3}{2}$ Euler variation Q^λ.

The terms (g) receive contributions due to the order-κ corrections to $\delta\psi_\lambda$ (as just described) and to the κ^2 corrections to $\delta\omega_{\mu ab}$, and these contributions cancel exactly. If the $\kappa^2 F\chi^3$ term did not cancel at this point, one could introduce an order-χ^4 term into the Lagrangian, whose variation with respect to χ would cause cancellation. It is fortunate that this complication does not occur.

Terms of the types (h), (i), and (j) must vanish because there is no further freedom to modify the previously determined Lagrangian and transformation laws. The calculations require Fierz rearrangement and a steady hand at Dirac algebra. We have verified that the $\kappa^2 F\chi\psi^2$ and $\kappa^3\chi^4\psi$ terms vanish. We have not studied the $\kappa^3\chi^2\psi^3$ terms, which seem to require a particularly tedious calculation, because we are confident that the previously determined structure is correct.

IV. THE COMMUTATOR ALGEBRA AND SUPERCOVARIANT DERIVATIVES

Before discussing commutators of the supersymmetry variations (13)–(17), we note that it is possible to simplify some of the previous expressions by introducing a supercovariant derivative[10] $\overline{D}_\mu$ defined by

$$\overline{D}_\mu A_\nu \equiv \partial_\mu A_\nu - \frac{i}{\sqrt{2}}\kappa\overline{\psi}_\mu\gamma_\nu\chi\,,$$
$$\overline{D}_\mu\chi \equiv D_\mu\chi - \frac{\kappa}{\sqrt{2}}\sigma^{\lambda\rho}\overline{F}_{\lambda\rho}\psi_\mu\,, \tag{24}$$

with

$$\overline{F}_{\mu\nu}{}^i \equiv \overline{D}_\mu A^i{}_\nu - \overline{D}_\nu A_\mu{}^i + gf^{ijk}A_\mu{}^j A_\nu{}^k\,. \tag{25}$$

Using (24) and (25), we can write the supersymmetry variation (14) of the field χ as

$$\delta\chi = \frac{1}{\sqrt{2}}\sigma^{\mu\nu}\overline{F}_{\mu\nu}\epsilon\,. \tag{26}$$

The supercovariant derivative is defined (in analogy to covariant derivatives for an ordinary internal symmetry) by subtracting from an ordinary derivative the supersymmetry variation of the field with $\epsilon(x)$ replaced by $\kappa\psi_\mu(x)$. This derivative enjoys some, but not all, of the properties of covariant derivatives for internal symmetry and general covariance. For example, the matter-field interaction terms are not given by the "minimal replacement" $\partial_\mu A_\nu \to \overline{D}_\mu A_\nu$ and $D_\mu\chi \to \overline{D}_\mu\chi$. However, one tantalizing fact which could lead to a simplification ot the theory is that the form (but not the precise coefficients) of the interaction terms $\mathcal{L}_1$ and $\mathcal{L}_2$ is suggested by the replacement $D_\mu\chi \to \overline{D}_\mu\chi$ in the spin-$\frac{1}{2}$ kinetic Lagrangian.

Another useful property of the supercovariant derivative is the simple transformation rule (for local Lorentz indices)

$$\delta\overline{F}_{ab} = \frac{-i}{\sqrt{2}}\overline{\epsilon}(\gamma_a\overline{D}_b - \gamma_b\overline{D}_a)\chi - [S_{ab}{}^\rho + \tfrac{1}{2}i\,\kappa^2\overline{\psi}_a\gamma^\rho\psi_b + \tfrac{1}{4}\kappa^2 V^{-1}\epsilon_{ab}{}^{\sigma\rho}(\overline{\chi}\gamma_5\gamma^\sigma\chi)](\overline{\epsilon}\gamma_\rho\chi)\,. \tag{27}$$

The second term vanishes when one substitutes the value of the torsion for the Lagrangian of our system (obtained by solution of the equation $\delta\mathcal{L}/\delta\omega_{\mu ab} = 0$), which is

$$S_{\mu\nu\rho} = -\tfrac{1}{2}i\kappa^2\overline{\psi}_\mu\gamma_\rho\psi_\nu + \tfrac{1}{4}\kappa^2 V^{-1}\epsilon_{\mu\nu\rho\sigma}(\overline{\chi}\gamma_5\gamma^\sigma\chi)\,. \tag{28}$$

It is also noteworthy that the Euler variation of $\mathcal{L}$ with respect to χ can be written as

$$-iV^{-1}\frac{\delta\mathcal{L}}{\delta\overline{\chi}} = \gamma^\mu\overline{D}_\mu\chi + (S_{\mu\tau}{}^\tau - \tfrac{1}{2}i\kappa^2\overline{\psi}\cdot\gamma\psi_\mu)\gamma^\mu\chi \tag{29}$$

and that the second term again vanishes if (28) is used. The Euler-Lagrange equation for χ is therefore essentially given by the free Dirac equation with minimal substitution of the supercovariant derivative.

The commutator of two supersymmetry transformations of the matter fields is given by [where δ_1 and δ_2 denote variations with spinor parameters $\epsilon_1(x)$ and $\epsilon_2(x)$, respectively].

$$[\delta_1, \delta_2]A_\mu = \xi^\nu\left(F_{\nu\mu} - \frac{i\kappa}{\sqrt{2}}\overline{\psi}_\nu\gamma_\mu\chi\right), \tag{30}$$

$$[\delta_1, \delta_2]\chi = \xi^\nu\overline{D}_\nu\chi\,, \tag{31}$$

where $\xi^\nu = i\overline{\epsilon}_1\gamma^\nu\epsilon_2$, and where terms vanishing as a consequence of the equations of motion (28) and (29) are neglected in (31). The calculation of (31) is facilitated by use of (27). Both commutators have a uniform interpretation as the sum of (i) a general coordinate transformation with parameter ξ^ν, (ii) a local Lorentz transformation with field-dependent parameter $\lambda_{ab}(x) = \xi^\nu\omega_{\nu ab}$, (iii) a supersymmetry transformation with field-dependent parameter $\epsilon'(x) = -\kappa\xi\cdot\psi$, and (iv) a local gauge transformation with parameter $\theta(x) = -g\xi\cdot A$. This is completely consistent with the interpretation of the commutators in pure supergravity[1,8] and in globally supersymmetric Yang-Mills theory.[3,9] It is therefore to be expected that the commutators $[\delta_1, \delta_2]V_{a\mu}$ and $[\delta_1, \delta_2]\psi_\mu$ will continue to have that same interpretation when the effect of the matter-field correction terms in (16) and (17) is included in the previous[8] calculation. If this is to be true, then the equation-of-motion terms of pure supergravity which appeared in the previous calculation must now become equations of motion of the coupled gravity-matter theory. This has been verified exactly for $[\delta_1, \delta_2]V_{a\mu}$ and verified to one nontrivial order in the matter-field terms for $[\delta_1, \delta_2]\psi_\mu$.

The presence of matter fields in the transformations (16) and (17) of the gravitational variables is a signal that it may be possible to linearize the transformation rules and simplify the Lagrangian by introducing auxiliary fields which would be regarded as part of the gravitational multiplet. The superspace formulation[11,12] implicitly contains such auxiliary variables, but it appears difficult to make contact[13] with explicit forms of locally supersymmetric theories.

One auxiliary field which can be discussed quite easily is the dimension-2 pseudoscalar field $D^i(x)$, which has a natural place in the globally supersymmetric theory. This field vanishes as a consequence of its equation of motion in both global and local theories, but it can be restored in the Lagrangian and transformation rules as follows. The term

$$\mathcal{L}_D = \tfrac{1}{2}VD^2 \tag{32}$$

is added to the Lagrangian (1), and local supersymmetry is maintained if one takes

$$\delta\chi = \frac{1}{\sqrt{2}}(\sigma^{\mu\nu}\overline{F}_{\mu\nu}\epsilon + iD\gamma_5\epsilon)\,, \tag{33}$$

$$\delta D = \frac{-i}{\sqrt{2}}\bar{\epsilon}\gamma_5 V^{-1}\frac{\delta\mathcal{L}}{\delta\overline{\chi}} + \frac{i}{2}\kappa\bar{\epsilon}\gamma\cdot\psi D \tag{34}$$

with no other change in the transformation laws given previously. This choice, which may not be unique, has the property that the equation of motion for χ no longer appears in the calculation (31) of $[\delta_1, \delta_2]\chi$. The introduction of the field D should be useful for the extension of field theories of interacting gauge and chiral multiplets to local supersymmetry.

As in the initial construction[1] of pure supergravity, the present extensions (we include here Ref. 2) to include matter fields involve very complicated calculations. A set of procedural rules to enable straightforward constructions of locally supersymmetric theories would be very useful. Superspace methods[11,12] may be able to provide the necessary systematic procedures, but it may also be useful to attempt to simplify the present theory. It is possible that some simplification may be achieved by a combination of techniques, such as the introduction of first-order variables for the Yang-Mills field, further development of the concept of supercovariant derivative, or an attempt to find convenient auxiliary fields.

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Added note. The conjecture in the Introduction concerning supergravity and the dual fermion model has also been made and then verified by F. Gliozzi, J. Scherk, and D. Olive [Phys. Lett. 65B, 282 (1976)].

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[2]We have very recently received a report by S. Ferrara, J. Scherk, and P. van Nieuwenhuizen [now published in Phys. Rev. Lett. 37, 1035 (1976)] in which a locally supersymmetric extension of the Abelian- or electromagnetic-gauge multiplet is constructed, using an approach based on the second-order description of gravitation. This paper arrived after the Lagrangian and all transformation rules were obtained within the approach discussed here. After completion of this manuscript, we received a second paper in which the extension to the non-Abelian case is obtained in the second-order formalism [S. Ferrara, F. Gliozzi, J. Scherk, and P. van Nieuwenhuizen, Nucl. Phys. B117, 333 (1976).

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[10]This useful concept is due to Dr. P. Breitenlohner [to

whom one of us (D.Z.F.) is indebted for numerous useful conversations] and to Professor M. Gell-Mann and Professor Y. Ne'eman.

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