Excited states of hadronic constituents with masses about 2.8 and 7–8 GeV

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A model which breaks Bjorken scaling in the deep-inelastic electron-nucleon scattering is presented and compared with the recent data of the MIT-SLAC collaboration and Fermilab. In this model it is assumed that the scaling violation is mainly due to the structure of the hadronic constituents and the transitions of the constituents to their excited states by absorbing a virtual photon. Fitting the formula of the structure function derived from this model to the data, we find the possibility that there exist two excited states with masses about 2.8 and 7–8 GeV.

I. INTRODUCTION

Since the first experimental evidence of Bjorken scaling (found in 1968) was successfully explained by the parton model,[1,2] the subsequent experiments on deep-inelastic lepton scattering have gradually confirmed that the parton model provides a qualitative description of its main features and a consistent picture for both electron-nucleon and neutrino-nucleon scattering.[3] Thus it has become very probable that hadrons are composed of constituents which appear to behave as quasifree and pointlike particles (partons) in the deep-inelastic region.

On the other hand, in 1972 one of the present authors (K.M.) predicted that the finite size of the hadronic constituents would cause the breakdown of scaling in the high-Q^2 (>20 GeV^2) region.[4] Later many other mechanisms of the breakdown were analyzed. A typical example of the structure of the constituents is an anomalous magnetic moment.[5] Other typical analyses were performed in renormalized field theory with and without anomalous dimensions by using the Wilson operator-product expansion.[6,7] Now we believe that the results of the renormalized field theory should be applied in the region of Q^2 higher than the present experimentally available Q^2 ($\lesssim 50$ GeV^2).[8]

Since 1973 further experiments to test the scaling have been continued at SLAC[9,10] and Fermilab.[11–13] In particular, the data of the ratios of muon scattering cross sections at 150 and 56 GeV, reported from Fermilab[12] are still rough but show a rather clear pattern of scaling violation. The main feature is that, as Q^2 increases, the ratios at large fixed ω (>5) decrease rapidly and after that change to grow steeply. Such a feature is just the one expected from our model of scaling violation based on the possible existence of excited states of the hadronic constituents.[14] The notion of constituents' excited states has been based on the analogy of nucleon's resonances.

Examining the recently reported data of the MIT-SLAC collaboration,[10] we find a feature similar to the Fermilab data. That is, the values of the proton structure function νW_2^p at fixed $x=0.1$ ($x=1/\omega$) show a rise of about 15% as Q^2 increases from 1 to 2.5 GeV^2. The same tendency is seen also in the data at $x=0.15$ and 0.2.

The MIT-SLAC data present another evident feature. The values of the proton structure functions $2MW_1^p$ and νW_2^p at fixed x (≥ 0.25) decrease monotonically as Q^2 grows. It is natural to regard this feature as the emergence of the finite-size effect of the constituents.

In this paper we will show that the finite-size effect of the constituents and the inclusion of the contribution from the transitions of the constituents to their excited states by absorbing a virtual photon can explain the main features of MIT-SLAC and Fermilab results. There two excited states can be expected to exist. Their masses are estimated to be about 2.8 and 7–8 GeV. In particular, the existence of the excited state with mass about 2.8 GeV is supported also by the recent SPEAR data[15] on R $[=\sigma(e^+e^-\to \text{hadrons})/(4\pi\alpha^2/3s)]$, which show a slight rise starting at about $\sqrt{s}=5.6$ GeV.

Our paper is organized as follows. In Sec. II we derive the formula for the structure function $F_2(\omega, Q^2)$ ($=\nu W_2$) from our model. In Sec. III the mass of the first excited state is determined from both MIT-SLAC and SPEAR data. At the end of Sec. III we extract the experimental values of the scaling function $\mathfrak{F}_2(\omega)$ included in the formula of Sec. II from MIT-SLAC data. In Sec. IV the necessity of the second excited state is shown from Fermilab data. Next we test the validity of our formula derived in Sec. II by comparing it with both Fermilab and MIT-SLAC data. Concluding remarks are given in Sec. V.

II. THE FORMULA FOR THE STRUCTURE FUNCTION

In the following discussions we use the quark-parton model in order to make the formula for the structure function easily comparable with the experiments.

We first derive a threshold value of Q^2 for the transition from the quark of a type q to its excited state q_i^* by absorbing a virtual photon with four-momentum squared Q^2 when ω is fixed ($\omega = 2M\nu/Q^2$). Figure 1 shows the vertex of the transition where the longitudinal momentum of the quark is a fraction x_i of the momentum of the nucleon. Using Bjorken-Paschos–type calculation[2] in the parton model, we obtain the relation

$$x_i = x(Q^2 + m_i^2)/Q^2 \,, \tag{2.1}$$

where $x = 1/\omega$. Here the masses of quarks are assumed to be negligibly small as compared with the masses of the excited states m_i. Furthermore, assuming $x_i \leqslant 1$, we get the inequality $Q^2 \geqslant m_i^2/(\omega - 1)$. So the threshold value of Q^2 at fixed ω is given as

$$Q^2 = m_i^2/(\omega - 1)\,. \tag{2.2}$$

We next derive the formula for the structure function $F_2(\omega, Q^2)$ $[= \nu W_2(\omega, Q^2)]$. When the virtual photon interacts with the quarks having an electromagnetic form factor $f_q(Q^2)$, the structure function is given as

$$F_2^{\rm el}(\omega, Q^2) = f_q^{\,2}(Q^2)\mathfrak{F}_2(\omega)\,,$$

where[16]

$$\mathfrak{F}_2(\omega) = \sum_N P(N)\langle \sum_l Q_l^{\,2}\rangle_N x\, f_N(x)\,.$$

Here we ignore the dependence of the form factor $f_q(Q^2)$ on the kinds of the quarks. Furthermore, we note that $P(N)$ depends on x implicitly.

Using the parton-model calculation, we obtain the formula

$$F_{2,i}^{\rm inel}(\omega, Q^2) = g_i(Q^2)\sum_N P(N) n_i(N) x_i f_N(x_i)$$

for the contribution to $F_2(\omega, Q^2)$ from the transitions of the quarks of a type q to its excited state q_i^*. Here $g_i(Q^2)$ is the transition form factor of the vertex shown in Fig. 1 and $n_i(N)$ is the number of the same type of the quarks as q in the configuration of N quarks. In order to simplify the above formula, we will follow the picture that the physical nucleon is composed of three valence quarks, which contribute all of the nucleon's quantum number, plus a "sea" of quark-antiquark pairs and neutral gluons.[17] From various phenomenological analyses it is known that the valence contribution dominates the value of the structure function in the region of $x \gtrsim 0.2$.[17,18] Therefore $\mathfrak{F}_2(\omega_i)$ and $F_{2,i}^{\rm inel}(\omega, Q^2)$ are approximated for $x_i \gtrsim 0.2$ as $\langle \sum_l Q_l^{\,2}\rangle_v x_i f_v(x_i)$ and $g_i(Q^2) n_{iv} x_i f_v(x_i)$, respectively, where $\omega_i = 1/x_i$ and the subscript v refers to the values in the configuration of the three valence quarks. So we obtain a simple formula $F_{2,i}^{\rm inel}(\omega, Q^2) = G_i(Q^2)\mathfrak{F}_2(\omega_i)$ for $x_i \gtrsim 0.2$, where $G_i(Q^2) = n_{iv}\langle \sum_l Q_l^{\,2}\rangle_v^{-1} g_i(Q^2)$.

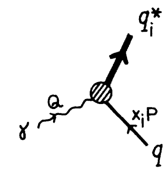

FIG. 1. The vertex of the electromagnetic transition from q to q_i^*.

Thus we get the formula

$$F_2(\omega, Q^2) = f_q^{\,2}(Q^2)\mathfrak{F}_2(\omega) + \sum_i G_i(Q^2)\mathfrak{F}_2(\omega_i) \tag{2.3}$$

in the region of $x_i \gtrsim 0.2$ $[x \gtrsim 0.2Q^2/(Q^2 + m_i^2)]$, where the summation in the second term is carried over various excited states. The ith term in the summation starts to work at Q^2 given by (2.2).

In the following analyses we will use the formula (2.3) almost in the region of $x_i \gtrsim 0.2$, but try to apply it even in the region of $0.1 \lesssim x_i < 0.2$. Though up to now it is not so definite to what extent the sea contributes, the sea contribution is known phenomenologically to be less than about 20% in the region of $x \gtrsim 0.1$[17,18] So we can consider the formula almost precise in our following analyses. Then, if the sea contribution increases as $x_i \to 0$, the formula (2.3) should be mostly corrected at $x_i \sim 0$.

III. THE FIRST EXCITED STATE AND VALUES OF $\mathfrak{F}_2(\omega)$

The MIT-SLAC collaboration recently presented fairly precise values of the structure functions $2MW_1$ and νW_2 for a proton and a deuteron target.[10] These data are extracted from the three experiments which cover the kinematical range $2M < W < 4.84$ GeV, $2.1 < \nu < 13.4$ GeV, $1.0 < Q^2 < 16.0$ GeV^2, and $0.1 < x < 0.8$. In the following we will discuss only $F_2(\omega, Q^2)$ for a proton target, the data of which are shown in Figs. 2(a)–2(k).

As seen in Fig. 2(a) the values of $F_2(\omega, Q^2)$ at $x = 0.1$ show a rise which starts at $Q^2 = 1$ GeV^2. In our model we regard this rise as the appearance of the contribution from the transitions of quarks to the excited state q_1^*. From the relation (2.2) we

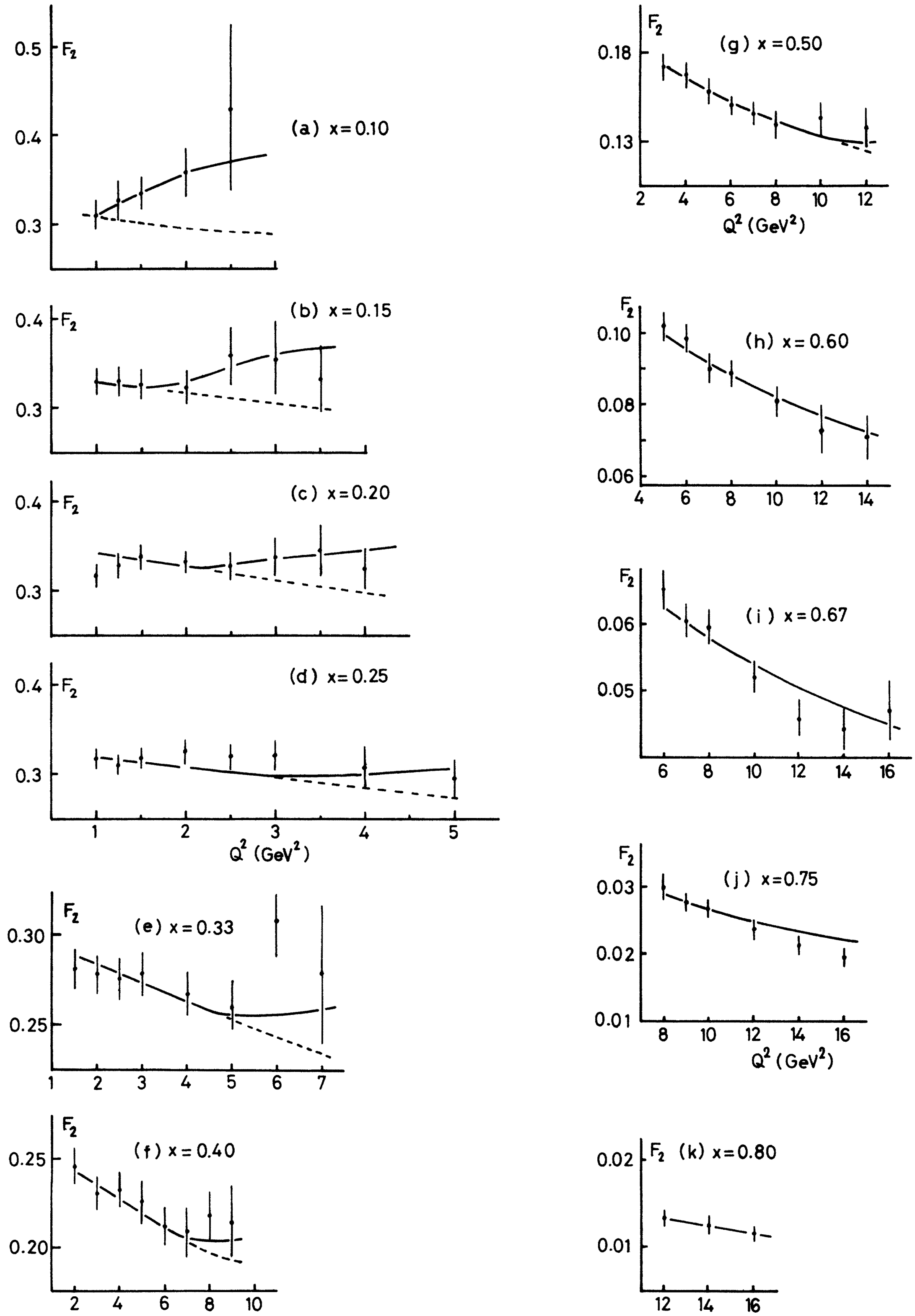

FIG. 2. The experimental values of $F_2(\omega, Q^2)$ for a proton target versus Q^2 for fixed values of x. The solid lines are resulting plots of the formula (2.3), which is discussed in Sec. IV. The dashed lines represent plots of the values of $F_2^{el}(\omega, Q^2)$, where $F_2^{el}(\omega, Q^2) = (1 + Q^2/\Lambda^2)^{-2}\mathfrak{F}_2(\omega)$ with $\Lambda^2 = 50$ GeV^2.

find that the mass m_1 of q_1^* should be less than 3 GeV. On the other hand, the values of $F_2(\omega, Q^2)$ at fixed $x \geq 0.6$ decrease monotonically as Q^2 increases. This means that the second term of (2.3) does not almost contribute so that $\mathfrak{F}_2(\omega_1) \simeq 0$, that is, ω_1 $[=\omega Q^2/(Q^2+m_1^2)]$ is very near to or less than unity in $x \geq 0.6$. Then we obtain $m_1 \gtrsim 3$ GeV at $Q^2 = 14$ GeV^2 of $x = 0.6$. As a result of the above discussions m_1 is supposed to be near to 3 GeV.

If the first excited state q_1^* with $m_1 \sim 3$ GeV really exists, we can expect that in the $e^+e^- \to$ hadrons experiment a virtual production of a $q_1^*\bar{q}_1^*$ pair from a virtual photon occurs from $\sqrt{s} \sim 6$ GeV, where this pair state decays into ordinary hadrons after the production. Then R $[=\sigma(e^+e^- \to \text{hadrons})/(4\pi\alpha^2/3s)]$ is considered to rise from $\sqrt{s} \sim 6$ GeV. As we expect, we can really find a slight rise which starts at $\sqrt{s} \sim 5.6$ GeV in the data of R, which is given in Fig. 3, reported recently from SPEAR.[15] Therefore we can assume the mass m_1 to be 2.8 GeV.

Finally, we will extract values of the scaling function $\mathfrak{F}_2(\omega)$ from MIT-SLAC data. At each fixed x, we choose the values of Q^2 where $\omega_1 \lesssim 1$, that is, the second term of (2.3) does not contribute at all. So we can extract the values of $\mathfrak{F}_2(\omega)$ from the data of $F_2(\omega, Q^2)$ at these values of Q^2 in each fixed x through the formula $F_2(\omega, Q^2) = f_q^2(Q^2)\mathfrak{F}_2(\omega)$. We will employ the form $(1+Q^2/\Lambda^2)^{-2}$ for $f_q^2(Q^2)$. The MIT-SLAC collaboration[10] obtains $\Lambda^2 \sim 50$ GeV^2 in both cases of νW_2 and $2MW_1$. So we use this value for Λ^2. We show in Fig. 4 the values of $\mathfrak{F}_2(\omega)$ obtained by taking an average of the values extracted from the data in each fixed x.[19] Precise data of $\mathfrak{F}_2(\omega)$ for $\omega > 10$ are now lacking.

IV. THE SECOND EXCITED STATE AND TESTS OF THE FORMULA

The data of the ratios of muon-scattering cross sections on an iron target at 150 and 56 GeV, reported from Fermilab[12] are shown in Fig. 5, where the experimentally measured ratio $r(\omega, Q^2)$ is de-

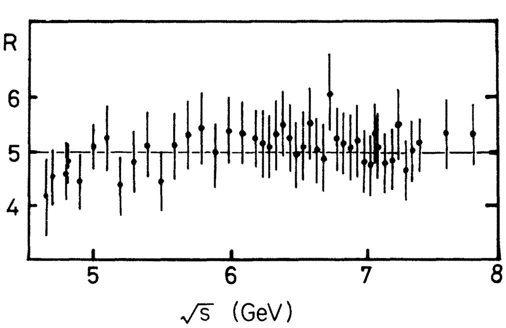

FIG. 3. The experimental data of $R = \sigma(e^+e^- \to \text{hadrons})/(4\pi\alpha^2/3s)$.

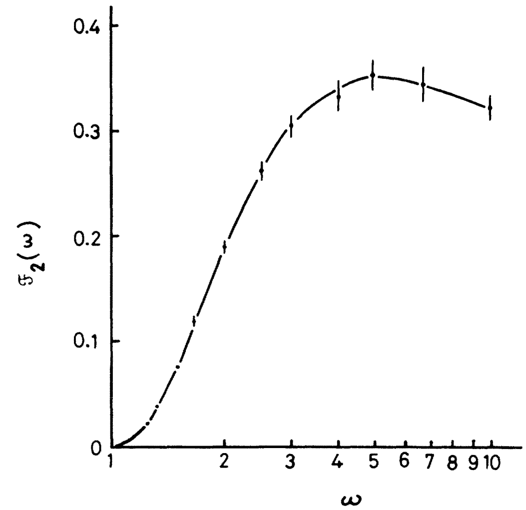

FIG. 4. Plots of the values of $\mathfrak{F}_2(\omega)$. The error bars shown are typical ones due to the data of $F_2(\omega, Q^2)$. The solid line is an eyeball fit of the plots.

fined as

$$r(\omega, Q^2) = \frac{E\dfrac{d^2\sigma}{dxdy}\ (E=150)}{E\dfrac{d^2\sigma}{dxdy}\ (E=56)} \,. \tag{4.1}$$

Here for small scattering angles the cross section is related to the structure functions as

$$E\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2}{2M}\frac{F_2(\omega, Q^2)}{x^2y^2}[1-y+y^2/2(1+R)] \,, \tag{4.2}$$

where $\nu = E - E'$ (E' is final muon energy), $y = \nu/E$, and $R = \sigma_L/\sigma_T$. The experimental apparatus was scaled to change experimental quantities by $\lambda = \frac{8}{3}$

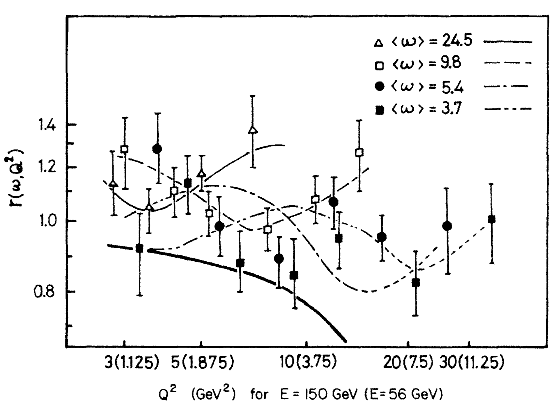

FIG. 5. The experimental data of $r(\omega, Q^2)$ defined by (4.1). Narrow lines show the result of the formula (2.3). A bold line represents the values for the formula $r^{\text{el}}(\omega, Q^2) = F_2^{\text{el}}(\omega, Q^2)/F_2^{\text{el}}(\omega, Q_0^2)$, where $F_2^{\text{el}}(\omega, Q^2) = (1+Q^2/\Lambda^2)^{-2}\mathfrak{F}_2(\omega)$ and $Q_0^2 = 3Q^2/8$.

as follows: $E \to \lambda E$ (increase muon energy from 56 to 150 GeV), $E' \to \lambda E'$, and $\theta \to \theta/\sqrt{\lambda}$ (θ is a scattering angle). Then $\nu \to \lambda\nu$ and $Q^2 \to \lambda Q^2$, so that ω and y remain unchanged. Thus the ratio $r(\omega, Q^2)$ is almost equal to the ratio $F_2(\omega, Q^2)/F_2(\omega, Q_0^2)$, where $Q_0^2 = 3Q^2/8$, because R is known to be small (~0.16) experimentally. If the scaling were to hold, the ratio $r(\omega, Q^2)$ should be unity at all values of ω and Q^2.

In order to show our interpretation of the feature of scaling violation of the Fermilab data we first notice the data of $\omega = 9.8$ in Fig. 5. We will show that the rapid decrease of the ratios from $Q^2 = 3$ to 8 GeV^2 can be explained by the existence of q_1^* with the mass 2.8 GeV, which is speculated on in Sec. III from MIT-SLAC data. At $Q^2 = 3$ GeV^2 ($Q_0^2 = 1.125$ GeV^2) at $E = 150$ GeV ($E = 56$ GeV), $\omega_1 = 2.7$ ($\omega_1 = 1.2$) and then $\mathfrak{F}_2(\omega_1 = 2.7) = 0.27$ [$\mathfrak{F}_2(\omega_1 = 1.2) \sim 0.01$], so that the ratio $r(\omega, Q^2 = 3)$ is enhanced because $F_2(\omega, Q^2 = 3)$ is supposed to obtain a large contribution from the second term of (2.3) but $F_2(\omega, Q_0^2 = 1.125)$ is not. As Q^2 increases, the value of $\mathfrak{F}_2(\omega_1(Q_0^2))$ grows rapidly, so that $F_2(\omega, Q^2)$ comes to receive a large contribution from the second term. As a result of this $r(\omega, Q^2)$ decreases rapidly as Q^2 increases.

After the point $Q^2 = 8$ GeV^2 at $E = 150$ GeV the ratios at $\omega = 9.8$ again rise steeply. Looking upon the data of $\omega = 24.5$, we find again a similar rising which starts at about $Q^2 = 3.5$ GeV^2. Both of the starting points, $Q^2 = 8$ GeV^2 of $\omega = 9.8$ and $Q^2 = 3.5$ GeV^2 of $\omega = 24.5$, give roughly the same value for m_i^2 defined in (2.2), that is, 70–80 GeV^2. So it is reasonable to consider that these risings are generated by the transitions of the quarks to one more excited state q_2^* with mass m_2 of about 8–9 GeV. But we cannot here determine the mass m_2 more precisely.

Let us test our formula (2.3) numerically. There remain the unknown form factors $G_i(Q^2)$ ($i = 1, 2$) in the formula. There is not any other knowledge than the restriction $G_i(Q^2) = 0$ at $Q^2 = 0$. Now we will make an attempt to use the following simple form for $G_i(Q^2)$:

$$\begin{aligned} G_i(Q^2) &= a_i Q^2/Q_{ti}^2 \quad \text{for } Q^2 \lesssim Q_{ti}^2 \\ &= a_i \quad \text{for } Q^2 \gtrsim Q_{ti}^2, \end{aligned} \tag{4.3}$$

where a_i and Q_{ti}^2 ($i = 1, 2$) are some parameters. Then, as shown in Fig. 5, we can obtain a good fit of the formula to the data of the ratios at both $\omega = 9.8$ and 24.5 simultaneously. Here we use $m_2 = 7.2$ GeV, $\mathfrak{F}_2(\omega = 24.5) = 0.3$ (see Ref. 20), and, for the parameters of $G_i(Q^2)$, $a_1 = 0.35$, $Q_{t1}^2 = 1.5$ GeV^2, $a_2 = 0.55$, and $Q_{t2}^2 = 7$ GeV^2. From our numerical analyses we could not get a good fit in $m_2 < 7$ GeV or $m_2 > 8$ GeV.

Now we can predict the values of the ratios at $\omega = 3.7$ and 5.4. The resultant plots are given in Fig. 5. But we cannot conclude whether they are consistent with the data or not.[21]

Finally, let us make one more test of the validity of the formula (2.3) by evaluating the values of $F_2(\omega, Q^2)$ at the points of x used in the MIT-SLAC data. In Fig. 2 we compare the plots of our result with the experiment. The curves are in fair agreement with the data.

V. CONCLUDING REMARKS

We have shown that the features of the recent data of the scaling violation can be explained by the modified parton model in which the quarks have structure and excitation degrees of freedom. In this section we will give some predictions and remarks related to the further tests of our explanation of the scaling violation. It should be remembered that the structure function should necessarily show a definite rise, which starts at $Q^2 = m_i^2/(\omega - 1)$, at *all* ω if the photoexcited state with mass m_i exists.

The formula (2.3) was shown to describe the data well except the Fermilab data at $\omega = 3.7$ and 5.4. As shown in Fig. 5, the predicted curves of $r(\omega, Q^2)$ at $\omega = 3.7$ and 5.4 have swellings, whose tops are situated near $Q^2 = 10$ and 6 GeV^2, respectively, due to the excitation to q_1^*. It is therefore a serious test of our model whether the future experimental data have these swellings or not.

The threshold values of Q^2 for the transitions of the quarks to q_2^* with mass 7–8 GeV are about 20 and 13 GeV^2 for $\omega = 3.7$ and 5.4, respectively. Therefore, new rises of the ratio $r(\omega, Q^2)$ at $\omega = 3.7$ and 5.4 are expected to occur in the region of $Q^2 \gtrsim 20$ GeV^2 and $Q^2 \gtrsim 13$ GeV^2, respectively. We describe in Fig. 5 typical examples, which are given as the dashed lines continued after the predicted curves of $\omega = 3.7$ and 5.4.

In the same way we can expect the risings of $F_2(\omega, Q^2)$ due to the transitions of the quarks to q_1^* and q_2^*. For examples, $F_2(\omega, Q^2)$ is predicted to show a new rise, which starts at about 5.5–7 GeV^2, due to the transitions to q_2^* at $x = 0.1$, and a new rise, which starts at about 3 GeV^2, due to the transitions to q_1^* at $x = 0.25$.[22]

Finally, we would like to comment that the features of the scaling violation of our model are obviously distinguishable from those of the renormalized field theories with and without anomalous dimensions. These theories predict the features that the structure function decreases at small ω but increases at large ω, monotonically as Q^2 grows. In our model the structure function should be expected to show rises due to the excitations of the quarks even when ω is fixed small. In particular, one can easily find a prominent difference in high-Q^2 behavior of the structure function at small ω between the field-theoretical arguments and our model. We therefore expect future experiments to give precise data at small ω.

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[7]See also Ref. 12 for the recent analyses based on Fermilab data.

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[19]Here we do not use the data point at $Q^2=1.00$ GeV^2 of $x=0.2$ because the value of $F_2(\omega=5, Q^2)$ at this point in the MIT-SLAC data is too small to satisfy the formula (2.3).

[20]This value is taken from MIT-SLAC and Fermilab (see Ref. 13), both of which show the value near 0.3 at $\omega=24.5$.

[21]Here we note that the data point at $Q^2=4.5$ GeV^2 of $\omega=3.7$ is inconsistent with SLAC data. That is, the values of $F_2(\omega, Q^2)$ at $\omega=4$ ($\omega=3$), which are shown in Figs. 2(d) and 2(e), are almost constant between $Q^2=1$ and 5 GeV^2 ($Q^2=1.5$ and 7.5 GeV^2).

[22]The values of $F_2(\omega, Q^2)$ at $Q^2=2-5$ GeV^2 of $x=0.25$ in MIT-SLAC data (Ref. 10), which are shown in Fig. 2(d), are not definite enough for us to discriminate the rising.