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Properties of supergravity theory*

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The commutator algebra of the recently proposed supergravity theory is elucidated. Several equivalent forms of the spin-3/2 field equations are derived for this purpose. It is also argued that this new theory is unique.

I. INTRODUCTION

Fermi-Bose supersymmetry is known in quantum field theory as a symmetry transformation generated by spinor charges Q_α and constant spinor parameters ϵ_α. Because the commutator of two such charges is a translation,

$$[\bar\epsilon_1 Q, \bar\epsilon_2 Q] = (\bar\epsilon_1\gamma^\alpha\epsilon_2)P_\alpha. \tag{1}$$

Supersymmetry is not purely an internal symmetry, but involves the space-time structure. A close connection with gravitation was therefore suspected at an early stage of the development.[1]

Because of the intimate relation between gravitation and general covariance, a supersymmetric theory of gravitation must possess a *local* spinor invariance with transformation parameters which are space-time-dependent spinor fields $\epsilon_\alpha(x)$. This is necessary because the notion of a constant spinor is coordinate dependent and violates general covariance.

An explicit Lagrangian field theory with local supersymmetry has recently been constructed.[2] It may be related to earlier attempts[3] based on the superspace concept.[4] The theory describes massless interacting spin-2 and spin-$\frac{3}{2}$ fields and the action contains the expected minimally coupled Einstein and Rarita-Schwinger[5] Lagrangians with an added four-fermion coupling of gravitational strength. The requirement of local supersymmetry was used[2] to derive this nonminimal coupling. The uniqueness of this theory will be one of the subjects discussed in the present paper.

Certain four-fermion couplings in the Dirac-Einstein system can be eliminated by casting the theory in first-order form.[6] It was therefore natural to try to reformulate the theory of Ref. 2 in first-order form, and a very useful formulation of this type has recently been given.[7]

The relation between gravitation and local supersymmetry is manifest in the commutator algebra which involves general coordinate transformations.[2] It is extremely interesting that in this gravitational theory the general coordinate transformations are not the primary invariance but are instead obtained by commutation of supersymmetry transformations. Further aspects of the commutator algebra will be clarified below. The discussion requires knowledge of the various different but equivalent forms of the spin-$\frac{3}{2}$ field equations. Since this topic may be of general use, for example for the quantization of spin-$\frac{3}{2}$ systems, it will be treated separately.

II. ACTION, FIELD EQUATIONS, AND SUPERSYMMETRY TRANSFORMATIONS

The action describes massless spin-2 and spin-$\frac{3}{2}$ Majorana fields and is the sum of the Einstein and Rarita-Schwinger[5] actions, coupled in an unusual way:

$$I = \int d^4x[(4\kappa^2)^{-1}VR(V,\omega) - \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar\psi_\mu\gamma_5\gamma_\nu\mathfrak{D}_\rho\psi_\sigma]. \tag{2}$$

In this first-order formulation,[7] the connection $\omega_{\mu ab}$ is an independent variable. We have defined

$$\begin{aligned}
&R_{\mu\nu ab} = (\partial_\mu\omega_{\nu ab} + \omega_{\mu a}{}^c\omega_{\nu cb}) - (\mu \longleftrightarrow \nu),\\
&V = \det V^a{}_\mu, \quad R_{\mu a} = V^{b\nu}R_{\mu\nu ab}, \quad R = V^{a\mu}R_{\mu a},\\
&\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3,\\
&\sigma^{ab} = \tfrac{1}{4}[\gamma^a, \gamma^b], \quad g_{\mu\nu} = (+---), \quad \epsilon^{0123} = 1.
\end{aligned} \tag{3}$$

The Rarita-Schwinger field satisfies the Majorana constraint $\psi = C\bar\psi^T$. The unusual coupling is due to the derivative

$$\mathfrak{D}_\nu\psi_\rho = \partial_\nu\psi_\rho + \tfrac{1}{2}\omega_{\nu ab}\sigma^{ab}\psi_\rho, \tag{4}$$

which is not a proper tensor under coordinate transformations and differs from the fully covariant derivative, consistently denoted by D_ν,

$$D_\nu\psi_\rho = \partial_\nu\psi_\rho - \Gamma^\sigma_{\nu\rho}\psi_\sigma + \tfrac{1}{2}\omega_{\nu ab}\sigma^{ab}\psi_\rho. \tag{5}$$

The action is a proper scalar owing to the curl structure

$$\mathfrak{D}_\nu\psi_\rho - \mathfrak{D}_\rho\psi_\nu = (D_\nu\psi_\rho - D_\rho\psi_\nu) + 2S^\sigma_{\nu\rho}\psi_\sigma, \tag{6}$$

since the torsion field S is a proper tensor, defined by

$$S^\rho_{\mu\nu} = \tfrac{1}{2}(\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\nu\mu}). \tag{7}$$

The relation between S and ω follows from the metric postulate $D_\nu g_{\rho\sigma}=0$ and its vierbein equivalent $D_\nu V^a{}_\rho=0$. One easily obtains

$$\Gamma^\rho_{\mu\nu}=\left\{{}^{\rho}_{\mu\nu}\right\}-K^\rho_{\mu\nu}\,,$$
$$\omega_{\mu ab}=\omega_{\mu ab}(S=0)+K_{\mu ab}\,, \tag{8}$$

where the contortion tensor K is defined by $K_{\mu\nu\rho}=-S_{\mu\nu\rho}+S_{\nu\rho\mu}-S_{\rho\mu\nu}$ and $\{\,\}$ is the usual Christoffel symbol in terms of the metric $g_{\mu\nu}=V^a{}_\mu V_{a\nu}$. Indices are treated in the usual way ($K_{\mu ab}=V_a{}^\rho V_b{}^\sigma K_{\mu\rho\sigma}$ and $K^\rho_{\mu\nu}=g^{\rho\sigma}K_{\mu\nu\sigma}$) and the symbol $\omega(S=0)$ is the vierbein connection in the absence of torsion, expressed in terms of vierbein fields

$$\omega_{\mu ab}(S=0)=\tfrac{1}{2}[V_a{}^\nu(\partial_\mu V_{b\nu}-\partial_\nu V_{b\mu})+V_a{}^\rho V_b{}^\sigma(\partial_\sigma V_{c\rho})V^c{}_\mu]$$
$$-[a\longleftrightarrow b]\,. \tag{9}$$

Variation of the action with respect to $\omega_{\mu ab}$ yields the field equation

$$\omega_{\mu ab}=\omega_{\mu ab}(S=0)+K_{\mu ab}\,, \tag{10}$$

$$S_{\mu\nu\rho}=\tfrac{1}{2}(i\kappa^2)(\bar\psi_\mu\gamma_\rho\psi_\nu)\,. \tag{11}$$

If one substitutes this result for ω back into (1), the action becomes identical to the second-order action of Ref. 2. Variation of (2) with respect to the vierbein field $V^a{}_\mu$ yields at once the nonsymmetric Einstein equation

$$R_a{}^\mu-\tfrac{1}{2}V_a{}^\mu R=-\kappa^2V^{-1}\epsilon^{\lambda\mu\nu\rho}(\bar\psi_\lambda\gamma_5\gamma_a\mathfrak{D}_\nu\psi_\rho)\,. \tag{12}$$

Variation with respect to the Majorana field ψ_μ yields the spin-$\frac{3}{2}$ equation of motion

$$\epsilon^{\lambda\mu\nu\rho}(\gamma_5\gamma_\mu\mathfrak{D}_\nu\psi_\rho-\tfrac{1}{2}\gamma_5\gamma_\sigma S_{\mu\nu}{}^\sigma\psi_\rho)=0\,. \tag{13}$$

A simple Fierz transformation after substitution of (11) shows that the last term in (13) vanishes, and in this form the wave equation (13) can be obtained directly from the second-order action of Ref. 2.

The action is invariant under the following supersymmetry transformations[7]:

$$\delta V^a{}_\mu=i\kappa(\bar\epsilon\gamma^a\psi_\mu)\,, \tag{14}$$

$$\delta\psi_\mu=\kappa^{-1}D_\mu\epsilon=\kappa^{-1}(\partial_\mu\epsilon+\tfrac{1}{2}\omega_{\mu ab}\sigma^{ab}\epsilon)\,, \tag{15}$$

$$\delta\omega_{\mu ab}=B_{\mu ab}-\tfrac{1}{2}V_{b\mu}B_{ca}{}^c+\tfrac{1}{2}V_{a\mu}B_{cb}{}^c\,, \tag{16}$$

$$B_c{}^{\sigma\tau}=-\kappa V^{-1}(\bar\epsilon\gamma_5\gamma_c\mathfrak{D}_\nu\psi_\rho)\epsilon^{\sigma\tau\nu\rho}\,. \tag{17}$$

The transformations (14) and (15) are the first-order form of the transformation laws given in Ref. 2 and become identical to the previously given expressions upon substitution of (10) and (11).

The unusual feature of the first-order action (2) is the appearance of a not-quite-covariant derivative $\mathfrak{D}$, or, equivalently, of a nonminimal coupling [see (4) and (6)]. A similar situation arises in the first-order Maxwell-Einstein system, where electromagnetic gauge invariance requires a particular nonminimal coupling. The analogy is not complete,[7] however, and the origin of this coupling is not understood. We only remark that a fully minimal coupling[8] in (2) does not reproduce the invariant action of Ref. 2 and is inconsistent with local supersymmetry.

III. EQUIVALENT FORMS OF THE SPIN-$\frac{3}{2}$ WAVE EQUATION

The equation of motion (13) of the spin-$\frac{3}{2}$ field in supergravity theory has the generic form

$$\epsilon^{\lambda\mu\nu\rho}\gamma_5\gamma_\mu\mathfrak{D}_\nu\psi_\rho-M^\lambda=0\,. \tag{18}$$

Similar forms appear in other situations, such as the case[9,10] of a massive charged spin-$\frac{3}{2}$ particle in an electromagnetic field. We now derive several equivalent forms of Eq. (18), using simple properties of the Dirac algebra such as the identity

$$\epsilon^{\lambda\mu\nu\rho}\gamma_5\gamma_\mu=i[g^{\lambda\rho}\gamma^\nu-g^{\lambda\nu}\gamma^\rho-\gamma^\lambda(g^{\nu\rho}-\gamma^\nu\gamma^\rho)]\,. \tag{19}$$

Most of the manipulations are independent of specific properties of $\mathfrak{D}_\nu\psi_\rho$ and M^λ.

After contraction of (18) with γ_λ and use of Eq. (19), we find

$$i(g^{\nu\rho}-\gamma^\nu\gamma^\rho)\mathfrak{D}_\nu\psi_\rho+\tfrac{1}{2}\gamma_\lambda M^\lambda=0\,, \tag{20}$$

or equivalently

$$i\sigma^{\nu\rho}\mathfrak{D}_\nu\psi_\rho-\tfrac{1}{4}\gamma_\lambda M^\lambda=0\,, \tag{21}$$

which is satisfied by any solution of (18). Using (19) we find from (18) and (20) the equation

$$i\gamma^\rho(\mathfrak{D}_\rho\psi_\lambda-\mathfrak{D}_\lambda\psi_\rho)-M_\lambda+\tfrac{1}{2}\gamma_\lambda\gamma\cdot M=0\,, \tag{22}$$

which is generally simpler than (18), especially in cases such as supergravity where $M_\lambda=0$. Since contraction of Eq. (22) with γ^λ leads again to (20) and by trivial manipulations back to (18), it is clear that (22) is equivalent to the original form of the equation of motion. The form (22) of the Rarita-Schwinger equation is so simple that it may well have been discussed in the literature, yet we have not seen such a discussion, perhaps because this form cannot be obtained from any Hermitian Lagrangian. In fact, Eq. (18) seems to be a Hermitianizing of Eq. (22).

Other forms of the spin-$\frac{3}{2}$ wave equation which occurred in the course of our work are the form

$$\gamma_\alpha(\mathfrak{D}_\beta\psi_\gamma-\mathfrak{D}_\gamma\psi_\beta)+\gamma_\beta(\mathfrak{D}_\gamma\psi_\alpha-\mathfrak{D}_\alpha\psi_\gamma)+\gamma_\gamma(\mathfrak{D}_\alpha\psi_\beta-\mathfrak{D}_\beta\psi_\alpha)$$
$$=\epsilon_{\alpha\beta\gamma\delta}\gamma_5M^\delta\,, \tag{23}$$

which follows from contraction of (18) with $\epsilon_{\alpha\beta\gamma\lambda}$, and the form

$$\gamma_\lambda(\mathfrak{D}_\alpha\psi_\beta-\mathfrak{D}_\beta\psi_\alpha)+i\epsilon_{\alpha\beta}{}^{\rho\sigma}\gamma_5\gamma_\sigma(\mathfrak{D}_\rho\psi_\lambda-\mathfrak{D}_\lambda\psi_\rho)=0\,, \tag{24}$$

valid when $M_\lambda=0$, which follows after multiplication of (22) by $\sigma_{\alpha\beta}$.

It should be noted that all previous results can be extended to a one-parameter family of equivalent spin-$\frac{3}{2}$ actions and wave equations by the substitution[9] $\psi_\rho \to \psi_\rho + a\gamma_\rho\gamma\cdot\psi$.

IV. THE COMMUTATOR ALGEBRA

The structure of the algebra generated by commutators of local supersymmetry transformations was studied in Ref. 2, and we now clarify some of the results which were tentatively stated there.

From (14) and (15) we easily find

$$[\delta_1, \delta_2] V_{a\mu} = i(\bar{\epsilon}_2\gamma_a D_\mu \epsilon_1 - \bar{\epsilon}_1\gamma_a D_\mu\epsilon_2) = D_\mu \xi_a , \tag{25}$$

with

$$\xi_a(x) = i\bar{\epsilon}_2(x)\gamma_a\epsilon_1(x) . \tag{26}$$

A precise interpretation of (25) will be given shortly, but we note that $\xi_\mu = V^a{}_\mu \xi_a$ will be interpreted as the parameter of an infinitesimal general coordinate transformation.

For ψ_μ, we have

$$[\delta_1, \delta_2]\psi_\mu = (2\kappa)^{-1}[(\delta_1\omega_{\mu ab})\sigma^{ab}\epsilon_2 - (\delta_2\omega_{\mu ab})\sigma^{ab}\epsilon_1] . \tag{27}$$

Further calculation is facilitated by making the "flat-space ansatz" $V_{a\mu} = \eta_{a\mu}$ and invoking general covariance to restore the correct tensor structure in the final result. We therefore write

$$\delta\omega_{\mu ab} = (-\kappa)[\epsilon_{\nu\rho ab}(\bar{\epsilon}\gamma_5\gamma_\mu\mathfrak{D}^\nu\psi^\rho) - \tfrac{1}{2}\eta_{\mu b}\epsilon_{\nu\rho a\tau}(\bar{\epsilon}\gamma_5\gamma^\tau\mathfrak{D}^\nu\psi^\rho) + \tfrac{1}{2}\eta_{\mu a}\epsilon_{\nu\rho b\tau}(\bar{\epsilon}\gamma_5\gamma^\tau\mathfrak{D}^\nu\psi^\rho)] . \tag{28}$$

A more convenient form for $\delta\omega_{\mu bc}$ is obtained by using the identity

$$\eta_{\mu\tau}\epsilon_{\nu\rho ab} = \eta_{\mu\nu}\epsilon_{\tau\rho ab} + \eta_{\mu\rho}\epsilon_{\nu\tau ab} + \eta_{\mu a}\epsilon_{\nu\rho\tau b} + \eta_{\mu b}\epsilon_{\nu\rho a\tau}$$

contracted with γ^τ; it is given by

$$\delta\omega_{\mu ab} = (-\kappa)[\epsilon_{\tau\rho ab}\bar{\epsilon}_1\gamma_5\gamma^\tau(\mathfrak{D}_\mu\psi_\rho - \mathfrak{D}_\rho\psi_\mu) - \tfrac{1}{2}\eta_{\mu a}\epsilon_{\nu\rho b\tau}(\bar{\epsilon}_1\gamma_5\gamma^\tau\mathfrak{D}^\nu\psi^\rho) + \tfrac{1}{2}\eta_{\mu b}\epsilon_{\nu\rho a\tau}(\bar{\epsilon}_1\gamma_5\gamma^\tau\mathfrak{D}^\nu\psi^\rho)] . \tag{29}$$

The last two terms vanish as a consequence of the equations of motion (11) and (13), and will be dropped here, because such terms do not affect the algebra of physical states and the Ward identities of observable amplitudes.

The first term of (29) is inserted in (27) and we use the relation $\epsilon_{\tau\rho ab}\sigma^{ab} = -2i\gamma_5\sigma_{\tau\rho}$. A Fierz rearrangement is made leading to the expression

$$[\delta_1, \delta_2]\psi_\mu = -\tfrac{1}{2}i\sum_F C_F(\bar{\epsilon}_1\Gamma^F\epsilon_2)\gamma_5\sigma^{\tau\rho} \times \Gamma_F\gamma_5\gamma_\tau(\mathfrak{D}_\mu\psi_\rho - \mathfrak{D}_\rho\psi_\mu) ,$$

to which only the vector (with $C_V = 1$, $\Gamma_V = \gamma_\sigma$) and tensor (with $C_T = -4$, $\Gamma_T = \sigma^{ab}$) invariants contribute because of the Majorana condition. After straightforward Dirac algebra we find, in correct tensor form,

$$[\delta_1, \delta_2]\psi_\mu = \xi^\nu(\mathfrak{D}_\nu\psi_\mu - \mathfrak{D}_\mu\psi_\nu) + (\tfrac{1}{4}\gamma\cdot\xi - \xi^{ab}\sigma^{ab})\gamma^\rho(\mathfrak{D}_\mu\psi_\rho - \mathfrak{D}_\rho\psi_\mu) , \tag{30}$$

with $\xi^{ab} = i\bar{\epsilon}_2\sigma^{ab}\epsilon_1$. The last term is the spin-$\frac{3}{2}$ equation of motion (22) and it may be dropped. Note that if the last two terms of (29) had been kept and Fierz rearrangement had then been made, we would have found some of the more exotic forms of the spin-$\frac{3}{2}$ wave equation noted in Sec. III.

To interpret (25) and (30), we note that the textbook formulas

$$V'_{a\mu}(x') = \frac{\partial x^\nu}{\partial x'^\mu} V_{a\nu}(x) , \qquad \psi'_\mu(x') = \frac{\partial x^\nu}{\partial x'^\mu}\psi_\nu(x) \tag{31}$$

for general coordinate transformations can be rewritten in infinitesimal form ($x'^\mu = x^\mu - \xi^\mu$) as

$$\delta_G V_{a\mu} = D_\mu\xi_a - \xi^\nu\omega_{\nu a}{}^b V_{b\mu} + 2\xi^\nu S_{\nu\mu a} , \tag{32}$$

$$\delta_G\psi_\mu = \xi^\nu D_\nu\psi_\mu + (D_\mu\xi^\nu)\psi_\nu - \tfrac{1}{2}\xi^\nu\omega_{\nu ab}\sigma^{ab}\psi_\mu + 2\xi^\nu S_{\nu\mu}{}^a\psi_a . \tag{33}$$

The terms involving $\xi^\nu\omega_{\nu ab}$ are local Lorentz transformations. They do not appear in the transformation laws of strict world tensors such as $g_{\mu\nu}(x)$.

We can then write, using (6),

$$[\delta_1, \delta_2]V_{a\mu} = \delta_G V_{a\mu} + \xi^\nu\omega_{\nu a}{}^b V_{b\mu} - 2\xi^\nu S_{\nu\mu a} , \tag{34}$$

$$[\delta_1,\delta_2]\psi_\mu = \delta_G\psi_\mu + \tfrac{1}{2}\xi^\nu\omega_{\nu ab}\sigma^{ab}\psi_\mu - D_\mu(\xi\cdot\psi) , \tag{35}$$

ignoring all equation-of-motion terms. The first two terms in each equation describe uniform general coordinate transformations and (field-dependent) local Lorentz rotations. The term $-D_\mu(\xi\cdot\psi)$ is exactly a local supersymmetry transformation [Eq. (15)], but with field-dependent parameter[2] $\epsilon'(x) = -\kappa\xi\cdot\psi$. The term $-2\xi^\nu S_{\nu\mu a}$ can be interpreted similarly after insertion of the torsion equation of motion (11); specifically,

$$-2\xi^\nu S_{\nu\mu a} = -i\kappa^2\xi^\nu(\bar{\psi}_\nu\gamma_a\psi_\mu) = i\kappa(\bar{\epsilon}'\gamma_a\psi_\mu) . \tag{36}$$

It is not necessary to give a direct calculation of $[\delta_1, \delta_2]\omega_{\mu ab}$, which appears to be very complicated, because we have restricted ourselves to the algebra of field transformations subject to the equation of motion, and the connection field $\omega_{\mu ab}$ is no longer an independent variable when this restriction is made.

We therefore find that the algebra of local supersymmetry transformations, when restricted to fields satisfying the equations of motion, consists of general coordinate transformations plus field-dependent local Lorentz rotations[11] and further field-dependent supersymmetry transformations. We note that terms corresponding to local symmetries of the action with field-dependent parameters have occurred previously in the algebra of commutators in global supersymmetry models with electromagnetic and Yang-Mills gauge invariance.[12] Similarly, terms vanishing as a consequence of the field equations also appear in previous models if auxiliary fields are eliminated before commutators are calculated. This suggests that there may be a formulation of the present supergravity theory with additional auxiliary fields where the equation-of-motion terms do not appear. Such a formulation may be equivalent to one of the earlier superspace approaches[3] to supergravity.

The remaining question is whether the supersymmetry algebra of field transformations reduces to the standard algebra [Eq. (1)] in the Hilbert space of particle states. We think that it does because local gauge transformations should not affect the algebra as applied to physical states and because there is axiomatic work[13] which indicates that the algebra (1) is essentially unique. Equation (1) has been verified[14] in the linearized limit of the present theory which describes free massless spin-2 and spin-$\frac{3}{2}$ particles.

V. UNIQUENESS OF THE ACTION AND TRANSFORMATION LAWS[15]

The starting point in the derivation of the theory in Ref. 2 was the trial transformation law $\delta\psi_\mu \sim D_\mu\epsilon = \partial_\mu\epsilon + \frac{1}{2}\omega_{\mu ab}(S=0)\sigma^{ab}\epsilon$ and $\delta V^a{}_\mu \sim \bar\epsilon\gamma_a\psi_\mu$ and the minimally coupled Einstein and Rarita-Schwinger actions. The trial rule for $\delta\psi_\mu$ seems to be unique because in the flat-space limit the only available local symmetry is the Rarita-Schwinger[5] transformation $\delta\psi_\mu = \partial_\mu\epsilon(x)$. Similarly, the symmetric part of $\delta V_{a\mu}$ is determined from the requirement that terms linear in ψ cancel in the variation of the action.[2] The antisymmetric part of $\delta V^a{}_\mu$ is completely free at this level because $(\delta\mathfrak{L}_2/\delta V^a{}_\mu)\delta V^a{}_\mu$ depends on the metric only and we therefore entertain the most general ansatz consistent with Lorentz invariance and dimensional requirements:

$$\delta V^a{}_\mu = i\kappa[(\bar\epsilon\gamma^a\psi_\mu) + \alpha(\bar\epsilon\gamma^a\psi_\mu - \bar\epsilon\gamma_\mu\psi^a) + i\beta(\bar\epsilon\gamma_5\gamma^c\psi^d)\epsilon^a{}_{bcd}V^b{}_\mu]. \tag{37}$$

We now investigate whether there are nonzero values of α and β which lead to a locally supersymmetric action.

Since terms linear in ψ are already absent in the varied action, we proceed, as in Ref. 2, to the ψ^3 terms, given by

$$\frac{\delta\mathfrak{L}_{3/2}}{\delta V^a{}_\mu}\delta V^a{}_\mu = +\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\bar\psi_\mu\gamma_5\gamma^a\mathfrak{D}_\rho\psi_\sigma)(\delta V_{a\nu}) - \tfrac{1}{8}i(\bar\psi_\mu\gamma_d\psi_\sigma)\epsilon^{\mu\nu\rho\sigma}\epsilon_\nu{}^{abd}[D_\rho(\delta V_{ba}) + D_b(\delta V_{\rho a}) + D_b(\delta V_{a\rho})], \tag{38}$$

where we arrived at this expression by varying the lowest-order form,

$$\omega_{\rho ab}\sigma^{ab} = (\partial_\rho V_{ba} + \partial_b V_{\rho a} + \partial_b V_{a\rho})\sigma^{ab}, \tag{39}$$

and covariantizing afterwards. For antisymmetric $\delta V^a{}_\mu$ the last two terms in (38) cancel.

At the analogous point in Ref. 2, it proved advantageous to divide the ψ^3 terms into terms involving a covariant curl $\epsilon^{\mu\nu\rho\sigma}\mathfrak{D}_\rho\psi_\sigma$ and gradient terms $D_\rho\epsilon$. Inspection of (38) suggests partial integration of the $D_\rho(\delta V_{ba})$ term. This yields an expression with covariant curls only:

$$\tfrac{1}{4}(\delta V_{a\nu} - \delta V_{\nu a})[\epsilon^{\mu\nu\rho\sigma}(\bar\psi_\mu\gamma_5\gamma_a\mathfrak{D}_\rho\psi_\sigma) + \tfrac{1}{2}i\epsilon^{\mu\tau\rho\sigma}\epsilon_\tau{}^{\nu ad}(\bar\psi_\mu\gamma_d\mathfrak{D}_\rho\psi_\sigma)]. \tag{40}$$

According to the method of Ref. 2, one can therefore hope to eliminate the curl terms by adding terms linear in α and β to $\delta\psi_\mu$. This strategy succeeds if one chooses for the extra α,β terms

$$\delta\bar\psi_\mu = (-i\alpha\bar\epsilon\gamma_a\psi_b + \tfrac{1}{2}\beta\epsilon_{abcd}\bar\epsilon\gamma_5\gamma^c\psi^d)(\bar\psi_\mu\sigma^{ab}). \tag{41}$$

No extra four-fermion interaction involving α and β is needed, since no $D_\mu\epsilon$ terms are present.

Again we are in a similar position as in Ref. 2. For any α and β the action is locally supersymmetric up to and including ψ^3 terms, while new quintic terms $(\delta\mathfrak{L}_4/\delta\psi_\mu)\delta\psi_\mu$ with $\delta\psi_\mu$ given in Eq. (41) appear. Again the crucial question is whether these cancel. The same computer program which was previously used to treat the quintic terms in Ref. 2 now showed that a cancellation occurred only for the trivial values $\alpha=\beta=0$, so that Eq. (37)

reduced to the original vierbein field transformation law.

The only possible remaining nonuniqueness resides in the method used here and in Ref. 2 to cancel the ψ^3 terms. There do not seem to be other solutions, although a strict mathematical proof is lacking. We therefore believe that the present supergravity theory is the unique locally supersymmetric theory involving only spin-2 and spin-$\frac{3}{2}$ fields with the general structure of minimal coupling plus quartic contact terms. It may be possible to find additional supersymmetric Lagrangians involving higher derivatives[16] and contact terms of order ψ^6 or greater.

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[14]We thank A. Das for performing calculations on this point.

[15]In this section $\mathcal{L}_2$, $\mathcal{L}_{3/2}$, and $\mathcal{L}_4$ denote respectively the Einstein, Rarita-Schwinger, and quartic contact terms in the Lagrangian of Ref. 2, and the derivative D is the full covariant derivative in second-order formalism.

[16]A supersymmetric expression involving higher derivatives of the spin-2 and spin-$\frac{3}{2}$ fields would seem to be necessary for regularization of Feynman diagrams.