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Verification of the equivalence of gravitational and inertial mass for the neutron

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A comparison of neutron scattering lengths measured dependent on and independent of gravity leads to a value γ for the ratio of gravitational to inertial mass for the neutron. We obtained $\gamma = 1.000\,16 \pm 0.000\,25$. This means the first verification of the equivalence for the neutron with an uncertainty of only 1/4000.

I. INTRODUCTION

Very precise measurements of the gravitational force have been performed only on bulk matter[1,2] and most recently on very massive bodies[3,4]. By these experiments the principle of equivalence of inertial and gravitational mass could be verified within stringent limits of 2 parts in 10^{12}. Furthermore, it has been shown that the gravity acceleration is independent of the composition of matter within one part in 10^{11}.[2] Thus it may be concluded that neutrons and protons bound in nuclei experience the same gravitational acceleration within about $10^{-10}\Delta g/g$. On the other hand the behavior of free particles, atoms,[5] neutrons,[6,7,8] electrons,[9] and photons[10] in the earth's gravitational field has been studied with lower accuracy of about only one part in 10^2 or 10^3. Among the particles, the neutron is best suited for a study of the gravitational force in the "quantum limit"[11] since it may experience the gravity simultaneously while reacting as a matter wave.

Thus, experiments with freely falling matter waves may provide a direct proof of the statement that the action of gravity does not affect the validity of the quantum physical laws for matter waves. A verification of this fact includes also the verification of the Einstein equivalence principle. On the other hand, experiments with freely falling neutrons in the "classical" limit, without neutron reactions other than simple detection, are not suitable for verifying the equivalence.

In this note I will report on evaluations of exact measurements of scattering length for slow neutrons which led to a direct verification of the equivalence for the neutron in the quantum limit.

II. METHOD AND RESULT

From the experimentally (with high accuracy) confirmed principle of the universality of free fall it follows that the gravitational accelerations of the free neutron (g_f) and of bulk matter, the local value (g_0), are equal. Thus, neutron experiments which result in a value for the equivalence factor $\gamma = (m_i/m)(g_f/g_0)$ provide a test for the equality of inertial (m_i) and passive gravitational mass (m) for the neutron. A verification of this equality with $\gamma = 1$ would confirm that the universality of free fall implies the Einstein equivalence principle since $g_f/g_0 = m/m_i$. Suitable for this purpose are experiments which have been performed to measure neutron scattering lengths dependent on and independent of gravity. The gravity-dependent measurements were made in the neutron gravity refractometer.[8,12,13] In this device (see Fig. 1) very slow neutrons are reflected from liquid mirrors after having fallen a distance h. By the free fall they gain an energy mg_fh in the direction of gravity. If this energy equals the potential energy of neutrons inside the mirror substance (scattering length b, N atoms per cm^3) the relation

$$mg_fh_0 = 2\pi\hbar^2 m_i^{-1} Nb$$

is valid. h_0 denotes the critical height for total reflection. Measurements of h_0 for liquid lead[14] and previous experiments[15] on liquid bismuth yielded values for the scattering length b_f listed in line 12 of Table I. These quantities were calculated with effective values h_0^* for the critical height of fall[13] according to

$$Nb_f = m_i m g_f h_0^*/2\pi\hbar^2 = \gamma m^2 g_0 h_0^*/2\pi\hbar^2 .$$

The equivalence factor $\gamma = (m_i/m)(g_f/g_0)$ was taken to be unity. If this assumption is not fulfilled,

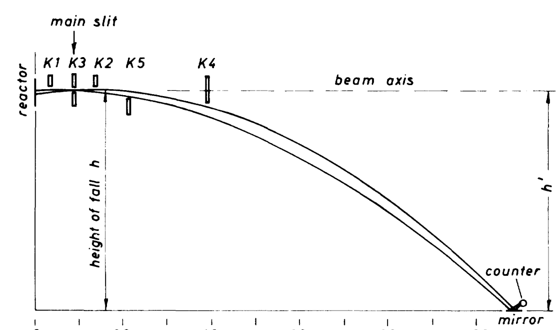

FIG. 1. Principle of the neutron gravity refractometer. $K1, \ldots, K5$: slits and stopper for the neutron beam (Ref. 8).

TABLE I. Evaluation of the equivalence factor. b_N denotes the nuclear scattering length and $\bar{f}$ is the averaged x-ray atomic form factor.

	Neutron energy E	Pb 1.26 eV	Pb 5.19 eV	Bi 1.26 eV	Bi 5.19 eV
	in barns:				
1	$\sigma_{\rm tot}$	11.2357(45)	11.2554(44)	9.2566(42)	9.2830(40)
2	$\sigma_{\rm abs}$	− 0.0241(1)	− 0.0119(1)	− 0.0047(6)	− 0.0023(3)
3	$\sigma_{\rm corr}^{\rm calc}$	+ 0.0040(2)	− 0.0033(2)	+ 0.0038(2)	+ 0.0014(2)
4	$\sigma_{\rm sc,\,free}$	11.2156(46)	11.2402(45)	9.2557(43)	9.2821(41)
5	$\sigma_{\rm bound}$	11.3251(46)	11.3499(45)	9.3452(43)	9.3719(41)
6	$\sigma_{\rm inc}$	− 0.0013(5)	− 0.0013(5)	− 0.0071(6)	− 0.0071(6)
7	$4\pi b^2(E)$	11.3237(47)	11.3486(46)	9.3381(44)	9.3648(42)
	in fm:				
8	$(b_N+\bar{f}Zb_{ne})$	9.4927(20)	9.5031(20)	8.6203(20)	8.6326(20)
9	$(1-\bar{f})Z$	68.0(2)	75.12(10)	68.8(2)	76.0(1)
10	$(1-\bar{f})Zb_{ne}$	− 0.0952(34)	− 0.1051(38)	− 0.0963(35)	− 0.1064(38)
11	b_0	9.3975(40)	9.3980(43)	8.5240(40)	8.5262(43)
12	b_f	9.4003(14)	9.4003(14)	8.5256(15)	8.5256(15)
13	b_f/b_0	1.000 29(45)	1.000 24(48)	1.000 19(50)	0.999 93(54)

the b_f calculated above must deviate from the "true" value b_0, which can be measured independently of gravity, such as by means of transmission measurements on liquid probes. Such experiments were carried out most recently by Waschkowski and Koester[16] on Pb and Bi with neutrons of energies $E=1.26$ eV and 5.19 eV. They yielded for molten probes (d cm thick) the total cross section according to

$$Nd\sigma_{\rm tot}(E)=-\ln T(E),$$

where $T(E)$ denotes the measured transmission ratio. For the calculation of b_0 from $\sigma_{\rm tot}(E)$ some correction terms, as given in Table I, are necessary. The cross sections for absorption,[17] $\sigma_{\rm abs}$, and incoherent scattering,[18] $\sigma_{\rm inc}$ are available as measured values.

The small correction term in line 3 of Table I is due to energy-dependent solid-state effects (interference and Doppler effect) and to a resonance contribution and spin-orbit coupling (Schwinger scattering) in the neutron-nucleus interaction. Furthermore, the energy-dependent interaction of the neutron with the shell electrons must be taken into account as given in line 10 of Table I. As the value for the neutron-electron scattering length we used the average $b_{n,e}=-1.40(5)\times10^{-3}$ fm obtained from the published but corrected data from Krohn and Ringo[19] [$b_{n,e}=-1.33(3)\times10^{-3}$ fm corrected for Schwinger scattering], data from Melkonian, Rustad, and Havens[20] [$b_{n,e}=-1.49(5)\times10^{-3}$ fm corrected for Schwinger scattering and a resonance contribution], and uncorrected data from Hughes *et al.*[21] [$b_{n,e}=-1.39(13)\times10^{-3}$ fm].

The results of our investigation are given in line 13 of Table I. The single values for the equivalence factor $\gamma=b_f/b_0$ are consistent within the limit of error $\pm5\times10^{-4}$. The mean value is

$$\gamma=(m_i/m)(g_f/g_0)=1.00016\pm0.00025.$$

If we take $g_f=g_0$ as reality, we consequently verify for the first time the equivalence of gravitational and inertial mass for the neutron with an uncertainty of 1/4000.

This accuracy is far inferior compared with that of macroscopic experiments, but it represents increases by factors of two[13] and ten[7] in comparison with the accuracies of previous experiments in the quantum limit. The interpretation of these experiments, however, has been of concern only in a determination of g_f.

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