A solution to the ρ-π puzzle: Spontaneously broken symmetries of the quark model*

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(Received 24 November 1975)

This article proposes a solution to the long-standing ρ-π puzzle: How can the ρ and π be members of a quark model U(6) 36 and the π be a Nambu-Goldstone boson satisfying partial conservation of the axial-vector current (PCAC)? Our solution to the puzzle requires a revision of conventional concepts regarding the vector mesons ρ, ω, K^*, and ϕ. Just as the π is a Goldstone state, a collective excitation of the Nambu–Jona-Lasinio type, transforming as a member of the $(3,\bar{3})+(\bar{3},3)$ representation of the chiral SU(3) × SU(3) group, so also the ρ transforms like $(3,\bar{3})+(\bar{3},3)$ and is also a collective state, a "dormant" Goldstone boson that is a true Goldstone boson in the static chiral U(6) × U(6) limit. The static chiral U(6) × U(6) is to be spontaneously broken to static U(6) in the vacuum. Relativisitc effects provide for U(6) breaking and a massive ρ. This viewpoint has many consequences. Vector-meson dominance is a consequence of spontaneously broken chiral symmetry—the mechanism that couples the axial-vector current to the π couples the vector current to the ρ. The transition rate is calculated as $\gamma_\rho^{-1} = f_\pi/m_\rho$ in rough agreement with experiment. This picture requires soft ρ's to decouple; but this requirement is not in conflict with any experimental features of the vector mesons. The chiral partner of the ρ is not the A_1 but the $B(1235)$. The experimental absence of the A_1 is no longer a theoretical embarrassment in this scheme. As the analog of PCAC for the pion we establish a tensor-field identity for the ρ meson in which the ρ is interpreted as a dormant Goldstone state. The decays $\delta \to \eta + \pi$, $B \to \omega + \pi$, $\epsilon \to 2\pi$ are estimated and are found to be in agreement with the observed rates. A static U(6) × U(6) generalization of the Σ model is presented with the π, ρ, σ, B in the $(6,\bar{6})+(\bar{6},6)$ representation. The ρ emerges as a dormant Goldstone boson in this model. Symmetry breaking in the model leads to the remarkable relation $m_\rho^2 - m_\pi^2 = m_B^2 - m_\delta^2$, satisfied within 0.5%. Others' efforts towards an integration of PCAC with the quark model, particularly in the context of the Melosh transformation, are discussed.

I. THE ρ-π PUZZLE

A persistent puzzle in the interpretation of hadron symmetries has been with us for almost ten years now: How can one reconcile the success of the approximate static U(6) symmetry of the quark model, in which the π and ρ are classified as members of the same 36, with the apparent role of the π and the K and η as Nambu-Goldstone[1] states of a spontaneously broken SU(3) ×SU(3) chiral symmetry? More simply, how is PCAC (partial conservation of axial-vector current) compatible with the quark model? If the π is a Goldstone state and the π and ρ are members of the same quark-model family what does this imply for the ρ?

We will provide an answer to these questions which requires a revision of our conventional concept of the vector mesons, a revision that has experimental support and unifies in a compact way PCAC with the quark-model symmetry. The essence of our solution to the puzzle is as follows. We accept the π as a Goldstone state so that in a quark model it emerges as a collective excitation as described by Nambu and Jona-Lasinio.[2] So in the chiral-symmetry limit it has strictly zero mass. The ρ meson is also assumed to be a collective state, what we will call a dormant Goldstone boson. A dormant Goldstone boson is defined to be a boson that in the static, nonrelativistic limit becomes a true Goldstone boson associated with the spontaneous vacuum breaking of a static Hamiltonian symmetry. While it has been established that there can be no Goldstone bosons of spin ⩾1 in a relativistic theory,[3] there is no such requirement for a nonrelativistic theory. We propose that the ρ meson and all the ground-state vector mesons are such dormant Goldstone states which, of course, become massive in a relativistic theory for which SU(6) is broken. This viewpoint has powerful consequences and resolves several outstanding difficulties in our understanding of hadron symmetries. Before describing these consequences in more detail we will elaborate on the (apparent) conflict between PCAC and the quark model.

The success of the quark model has been extensively described in the literature.[4] The simple quark model works. But it is not understood why it works. Presumably such an understanding awaits a dynamically consistent model of the hadrons. Nevertheless, we do know that the quark model (i) correctly classifies the hadrons into a Wigner-Weyl-realized static SU(6)×O(3) symmetry, (ii) gives mass formulas, mixing angles, magnetic moments, decay rates, etc. and (iii) more recently, in the guise of the parton model, has been successfully applied to the description of weak and electromagnetic processes at high momentum transfer. The salient dynamical assumption of the model is that the hadrons are ordinary bound states of quasifree quarks and antiquarks. The distinguishing feature of these bound states is that the quarks may be permanently trapped. It is a model in which the quarks are confined in a potential well of infinite height; the problem is then to guess the further details of the potential well.

On the other hand, PCAC works. Chiral SU(2) $\times$SU(2) symmetry with the π as a Nambu-Goldstone state is the best hadronic Hamiltonian symmetry after isospin.[5] It is hard to imagine that a theory of strong interactions could be correct if it ignores PCAC. Like the quark model, PCAC has strong experimental confirmation (which has recently been reviewed by one of us[5]). Included in the successes of PCAC are (i) the Goldberger-Treiman relation, (ii) all the soft-pion results that follow from the Adler-Nambu-Shrauner zero condition, and (iii) current-algebra results supplemented with PCAC, such as the Adler-Weisberger relation and the theorem on K_{l3} decay. However, the dynamical features of hadron interactions implied by PCAC appear to be orthogonal to those implied by the quark model (see Table I). Since the γ_5 symmetry is badly broken in the vacuum and gives rise to the Goldstone phenomenon, interactions must play an important and nonperturbative role. There is no Nambu-Goldstone realization in a free field theory. This is seen in the Σ model[6] which implements PCAC. If we assume that the π, K, and η are bound states of quarks and antiquarks then PCAC is consistent with the requirement that these ground-state mesons are collective excitations of $q\bar{q}$ pairs analogous to a type II superconductor.[7] The quarks are not at all quasifree. Such a bound-state model of the pion was actually constructed by Nambu and Jona-Lasinio[2] in analogy with superconductivity and subsequently developed as a renormalizable field theory.[8]

There is an evident conflict between the quark-model picture of the π and the PCAC picture. In an earlier day this conflict led Brandt and Preparata[9] to abandon (strong) PCAC in favor of weak PCAC, a formulation which was compatible with the quark model but in which the success of PCAC had to be seen as accidental. Our response to this conflict will be in the opposite direction altogether—to look for new manifestations of collective phenomena in the quark model and keep PCAC.

The conflict can be appreciated in another way by asking how one builds a soft pion. There have been at least two attitudes taken in the face of this question. The attitude of most contemporary hadron builders and quark trappers is to build the hadrons out of quarks and gluons and then try to put PCAC in at the end. From their point of view PCAC is a nuisance that has to be accomodated; it has no fundamental role in the construction of hadrons. For example, the sundry potential models[10] and most recently the bag models of hadrons[11] provide a reasonable description of hadron levels. However, they fail when applied to the lowest excitations such as the π. The π mass comes out wrong. Further, there is no reason why such a bound-state pion should satisfy PCAC and the decoupling theorems.

An alternate attitude, which has not been vigorously pursued,[12] is to recognize that the Goldstone nature of the π is a strong clue to how to build the other hadrons. *All* hadrons could be collective excitations, a color-singlet condensate in the context of quantum chromodynamics.[13] The central problem of hadron dynamics is then simi-

TABLE I. Quark-model picture contrasted with PCAC picture.

Quark model	PCAC
Quarks are quasifree	Interactions are essential
Vacuum state is simple and nondegenerate	Vacuum complex and infinitely degenerate chiral sea
Bohr-Sommerfeld model of hadrons, Schrödinger potential models: hadrons are bound states, quarks in bag or potential	Superconductivity model of hadrons, many-body theory: hadrons are collective excitations
Symmetry realized as Wigner-Weyl symmetry	Symmetry realized as Nambu-Goldstone symmetry
Weak PCAC	Strong PCAC

lar to the problem of superconductivity, which is to guess the correct ground state. This is to be contrasted with guessing Schrödinger potentials or building Bohr-Sommerfeld models of the hadrons. The major physical consequences for hadron dynamics are the dynamics of the vacuum. The physical vacuum, rather then being an "empty" state into which one deposits bound states of the quarks, is instead a chiral sea of colored quark-antiquark pairs. The reason that quarks do not get out of hadrons is that they are not in the hadrons—they are everywhere. A hadron is then a collective motion of the sea bearing its distinguishing quantum numbers—a wave on the sea. The difficulty with this approach is that it is hard to implement mathematically; perturbative techniques of field theory fail *ab initio*. New techniques will have to be found to implement this viewpoint.

The difficult problem of actually building a soft pion, or any hadron, is not the subject of this article. Rather we address the simpler question of how to reconcile the quark-model approximate *symmetry* of static $U(6)$ and its classification of the π and the ρ as members of the $\underline{36}$ with the Nambu-Goldstone nature of the π.

It is important to emphasize that we are here concerned with the description of hadron states at low momentum—essentially the static limit. The description of hadron states in terms of their quark content is momentum-dependent. This is because the quark-number operator does not commute with the Hamiltonian. It is this feature, among others, that has led to extensive studies in the literature of the $p_z \to \infty$ limit, where $\vec{p}$ is the hadron momentum.[14] In the $p_z \to \infty$ limit the hadrons may be classified according to the group $SU(6)_W$, and in terms of their constituent-quark content the states may have a simple description.[15] The difficulty with this approach is that in singling out the z direction the content of angular momentum conservation is extremely hard to recover.[16]

Lightlike charges[17] have also been used to study this question[18] and have many formal advantages. However, on the light cone the Wigner-Weyl realization of a symmetry and the Nambu-Goldstone realization merge.[19] So going on the light cone solves the ρ-π problem by avoidance. As one probes the light cone at high momentum transfer the contribution of the wee-quark sea presumably falls away, leaving the contribution primarily from valence quarks, whose quantum numbers and orbital states can be used to classify the hadrons. However, for states *at rest*, whose classification scheme in existing hadron levels we endeavor to understand, the ρ-π problem remains. What we wish to suggest in our remarks here is that for high-momentum-transfer processes, for which the quark-sea contribution to single-particle hadron states is negligible, the problem can disappear since only the Wigner-Weyl realization is possible. However, one state, the vacuum, must have a momentum-independent description, and for this reason the various approaches to this problem can be distinguished by their treatment of the vacuum.

Our resolution of the ρ-π problem is as follows. We will adopt the quark model and abstract commutation relations from quantum chromodynamics.[13] Further, we assume that in the static limit the ρ and the π and their U(3) partners transform like members of a $(6,\overline{6})+(\overline{6},6)$ representation of the Feynman–Gell-Mann–Zweig (FGZ) chiral U(6)×U(6) algebra.[20] Then the π and the ρ family transform like the $\underline{36}$ of the U(6) subgroup,

$$\pi^a \sim \overline{q} i\gamma_5 \tfrac{1}{2}\lambda^a q,$$

$$\rho_i^a \sim \overline{q}\,\sigma_{0i}\,\tfrac{1}{2}\lambda^a q.$$

This means that under the chiral SU(3)×SU(3) group of currents the operators π^a and ρ_i^a *both* transform like $(3,\overline{3})+(\overline{3},3)$. While this representation content is what is usual for the pion [for example, in the Gell-Mann–Oakes–Renner (GOR) model[21]] and in agreement with experiment,[5] it is not usual for the vector mesons, which conventionally are thought to transform like (1, 8) + (8, 1) or $\overline{q}\gamma_i\frac{1}{2}\lambda^a q$. *We will assume that the π and the ρ belong to $(3,\overline{3})+(\overline{3},3)$ of chiral SU(3) ×SU(3).* (There can of course be mixing of representations, but we will ignore it to emphasize our viewpoint.) This is the principal requirement of the reconciliation of PCAC with static U(6), and it has several consequences.

The first point is that the chiral SU(3)×SU(3) partners of the π and ρ transform like

$$\sigma^a \sim \overline{q}\tfrac{1}{2}\lambda^a q,$$

$$B_i^a \sim \epsilon_{ijk}\overline{q}\sigma_{jk}\tfrac{1}{2}\lambda^a q.$$

So the chiral partner of the ρ meson is the B(1235), $J^{PC}=1^{+-}$, not the A_1, $J^{PC}=1^{++}$. As is well known, the A_1 has escaped attempts to establish it as a real resonance, while the $B(1235)$ is a well-established state. Our classification does not require an A_1 or strong coupling of the $\pi\rho$ channel to the A_1 if it exists. In the present state of affairs with no A_1 this seems to us to be desirable.

The action of the group generators on these operators is described as follows:

$$\begin{array}{cc} & \longleftarrow \text{static } U(6) \longrightarrow \\ \begin{array}{c}\uparrow \\ \text{chiral} \\ \text{SU(3)}\times\text{SU(3)} \\ \downarrow\end{array} & \begin{bmatrix} \pi^a & \rho_i^a \\ \sigma^a & B_i^a \end{bmatrix} \end{array}.$$

The (π^a, ρ_i^a) and (σ^a, B_i^a) transform as $\underline{36}$ representations under static U(6), while (π^a, σ^a) and (ρ_i^a, B_i^a) transform like $(3,\overline{3})+(\overline{3},3)$ under chiral SU(3)×SU(3). Under chiral U(6)×U(6) all these operators together transform as members of a $(6,\overline{6})+(\overline{6},6)$ representation. This is essentially one of the assignments described in FGZ.[20]

The mode of symmetry breaking we use here is shown in Fig. 1, which is designed to contrast the Wigner-Weyl route of the breaking of U(6) ×U(6) with the Nambu-Goldstone route. The removal of hadron mass degeneracies is shown in Fig.2. What we have done here is to propose that the Nambu-Goldstone route is in fact closer to the existing phenomenology of the hadrons. This is accomplished by extending the picture of chiral SU(3)×SU(3) spontaneous breaking developed by Glashow and Weinberg[22] and Gell-Mann, Oakes, and Renner[21] to spontaneous breaking of static U(6)×U(6). The Wigner-Weyl route shown in Fig. 1 is the alternate route which arises in potential models of the quarks (no spontaneous breaking) or as suggested by the work of Brandt and Preparata.[9]

Our starting point is to imagine a nonrelativistic world with a Hamiltonian symmetry chiral U(6) ×U(6). The vacuum symmetry is spontaneously broken to U(6) or SU(6) and this is the classificatory group for hadrons at rest.[23] The π and ρ along with their U(3) partners are true Goldstone bosons in this nonrelativistic world. In the relativistic world, in which the U(6) vacuum symmetry is necessarily broken, the ρ meson will be massive—however, it remembers its origin as a Goldstone state (more about this later). The pseudoscalars can remain strictly massless true Goldstone states in this relativistic world with a chiral SU(3)×SU(3) Hamiltonian symmetry. The breaking of chiral SU(3)×SU(3) then proceeds as in the GOR model.[21]

Next we turn to the experimental fact of vector-meson dominance (VMD) of current matrix elements.[24] If the ρ field operator transforms like $(3,\overline{3})+(\overline{3},3)$ and the vector current like (1, 8) + (8, 1) under the chiral group, how can the current couple to the ρ? It is well known that the axial-vector current, transforming like (1, 8) + (8, 1), couples to the π transforming like $(3,\overline{3})+(\overline{3},3)$ precisely because of the spontaneous breaking of the chiral symmetry. That is, the coupling proceeds via a σ^0 going into the vacuum and hence the strength of the coupling is specified by $f_\pi \sim \langle\sigma^0\rangle_0$ (see Fig. 3). So too in our picture of vector mesons is the above question answered because *VMD is a consequence of spontaneously broken chiral symmetry*. The same mechanism that couples the axial-vector current to the π couples the vector current to the ρ. Computing the transition matrix element corresponding to Fig. 3(b) we find

$$\frac{1}{\gamma_\rho} = \frac{f_\pi}{m_\rho}\left(\frac{Z_\pi}{Z_\rho}\right)^{1/2},$$

where $(Z_\pi/Z_\rho)^{1/2}$ is a ratio of normalization constants which is equal to unity in the SU(6) limit. With $(Z_\pi/Z_\rho)^{1/2} \simeq 1.5$ this relation agrees with the measured transition rate.

Our picture of the vector mesons implies PCTC (partial conservation of tensor current) relations[25]

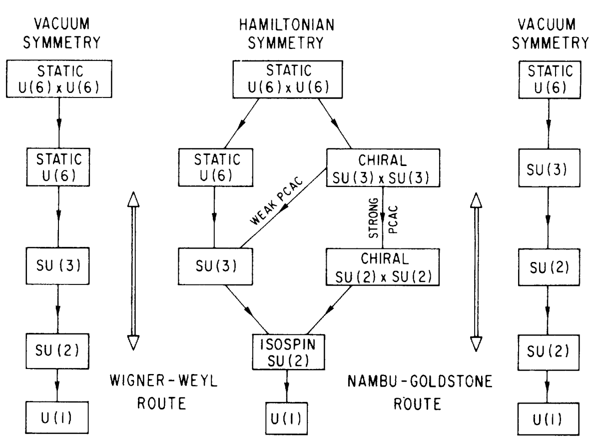

FIG. 1. Group diagram for the breaking of U(6)× U(6) symmetry with the Nambu-Goldstone route (collective states) contrasted to the Wigner-Weyl route (quark model).

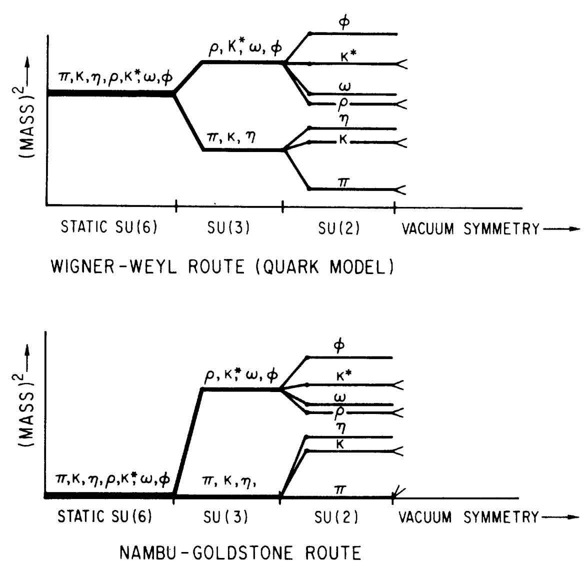

FIG. 2. Level diagrams of the ground-state vector and pseudoscalar mesons in the two routings shown in Fig. 1.

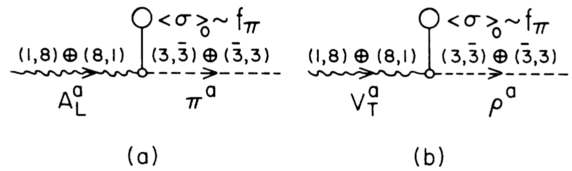

FIG. 3. Coupling of (a) axial-vector current to pion and (b) vector current to the ρ via spontaneous breaking of chiral symmetry.

of the form

$$\partial^\mu t^a_{\mu\nu} = (m_\rho{}^2 Z_\rho)^{1/2}\rho^a_\nu,$$

$$\partial^\mu {}^*t^a_{\mu\nu} = (m_B{}^2 Z_B)^{1/2} B^a_\nu,$$

where the tensor current is $t^a_{\mu\nu} = \bar{q}\frac{1}{2}\lambda^a\sigma_{\mu\nu}q$, the asterisk denotes the dual, and ρ^a_ν and B^a_ν are the $J^{PC} = 1^{--}$ and 1^{+-} field operators, respectively. PCTC is *not* the analog of PCAC. It is simply a way of projecting the ρ and B fields transforming like $(3,\bar{3})+(\bar{3},3)$ out of the skew tensor $t_{\mu\nu}$. PCAC in the form $\partial^\mu A^a_\mu = \mu_\pi{}^2 f_\pi Z_\pi{}^{-1/2}\pi^a$ relates the nonconservation of a generator of the symmetry to a field operator. By contrast in PCTC $t^a_{\mu\nu}$ is *not* related to a generator of the FGZ U(6)×U(6) algebra. We derive the analog of PCAC for the vector current as a tensor-field identity (TFI) from which the dormant-Goldstone-boson character of the vector mesons is evident.

The definition of the ρ field given by PCTC above *implies that soft* ρ*'s decouple*. This is how the ρ meson (and the other ground-state vector mesons) remembers its origin as a dormant Goldstone boson. If $G(q^2)$ is the vector-meson coupling constant for virtual momentum q_μ then PCTC requires the decoupling theorems

$$G_{\rho\pi\pi}(0) = G_{\rho NN}(0) = 0, \text{ etc.}$$

However, as we discuss below, there is no conflict with universality of vector-meson couplings which we obtain in the usual form on the mass shell:

$$\gamma_\rho = G_{\rho\pi\pi}(m_\rho{}^2) = G_{\rho NN}(m_\rho{}^2) \neq 0.$$

Further, PCTC is not in conflict with VMD of electromagnetic form factors or, as far as we have been able to determine, with any other experimental feature of the vector mesons.

We have also examined the decays

$$\delta \to \eta + \pi, \quad B \to \omega + \pi, \quad \epsilon \to \pi + \pi,$$

which test the chiral-representation content of the mesons. Using our representation assignment and hard-meson techniques the calculated decay rates are in good accord with experiment.

It would be desirable to have a field theory model, such as the Σ model,[6] in which these ideas of spontaneous symmetry breaking applied to U(6)×U(6) could be explicitly studied. Unfortunately, as is well known, it is impossible to construct an interacting relativistic field theory with this U(6)×U(6) symmetry.[26] However, one can make a static model with U(6)×U(6) symmetry. We have constructed a static generalized Σ model with elementary fields corresponding to σ, π, ρ, B transforming like $(6,\bar{6})+(\bar{6},6)$ under the chiral U(6)×U(6). By appropriate choice of Lagrangian parameters corresponding to U(6)×U(6) spontaneously broken to U(6) vacuum symmetry, $\langle\sigma^0\rangle_0 \neq 0$, one finds that the π and ρ are massless and the σ and B states are degenerate and massive. So in this static model the ρ shares the massless fate of the π; it is a dormant Goldstone boson. In the real relativistic world, which explicitly breaks the static U(6) to U(3), the ρ acquires a mass while the π remains massless as a consequence of the usual Goldstone theorem.

By consideration of U(6)-symmetry breaking in this static model we obtain the remarkable new mass relation

$$m_\rho{}^2 - m_\pi{}^2 = m_B{}^2 - m_\delta{}^2 .$$

This relation is obeyed within 0.5%.

Our study is concluded with a discussion of others' efforts toward a resolution of PCAC with the quark model,[18] particularly in the context of the Melosh transformation.[27] We also list unsolved problems and suggestions for further research.

II. CHIRAL U(6)×U(6) REVISITED

A. Quantum chromodynamics (QCD)

The model from which we abstract commutation relations is quantum chromodynamics,[13] the gauge theory of strong interactions. The Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\rm YM} + \bar{q}\not{D}q + \mathcal{L}_{\rm SB} , \tag{1}$$

where $\mathcal{L}_{\rm YM}$ is the Yang-Mills Lagrangian for an octet of colored $SU_c(3)$ gauge fields, q is the quark field operator transforming like a triplet under $SU_c(3)$ and like $(1,3)+(3,1)$ under chiral SU(3)×SU(3), and D_μ is the covariant gauge derivative. In the absence of the symmetry-breaking terms $\mathcal{L}_{\rm SB}$ the Lagrangian symmetry is $SU_c(3)\times SU(3)\times SU(3)\times U_A(1)\times U(1)$. The SU(3)×SU(3) chiral group has associated currents

$$V^a_\mu = \bar{q}\gamma_\mu \tfrac{1}{2}\lambda^a q ,$$
$$A^a_\mu = \bar{q}\gamma_\mu\gamma_5 \tfrac{1}{2}\lambda^a q , \tag{2}$$

which if $\mathcal{L}_{\rm SB} = 0$ are conserved.

There are numerous advantages to considering this model of the strong interactions.[13] The principal assumptions (as yet completely unproved) of this model are the following: (i) only $SU_c(3)$ singlets appear as physical states,[28] (ii) if $\mathcal{L}_{SB}=0$ the chiral SU(3)×SU(3) symmetry is spontaneously broken to SU(3) in the vacuum, accompanied by an octet of pseudoscalar Nambu-Goldstone states, (iii) the ninth axial-vector current conservation corresponding to the $U_A(1)$ symmetry is absent,[29] and (iv) the superficial scale invariance of $\mathcal{L}$ is absent and explicitly broken.[30] If any of these assumptions is correct, perturbation theory in the gauge coupling constant is useless.

B. The U(6) × U(6) algebra

Chiral SU(3)×SU(3) is generated by the charges

$$Q^a=\int d^3x\, V_0^a,\quad {}^5Q^a=\int d^3x\, A_0^a, \tag{3}$$

and

$$\begin{aligned}&[Q^a,Q^b]=if^{abc}Q^c,\\&[Q^a,{}^5Q^b]=if^{abc}\,{}^5Q^c,\\&[{}^5Q^a,{}^5Q^b]=if^{abc}Q^c.\end{aligned} \tag{4}$$

According to assumption (ii) the vacuum does not have the chiral symmetry,

$${}^5Q^a|0\rangle\neq 0,\quad Q^a|0\rangle=0,$$

and this symmetry is realized in the manner of Nambu and Goldstone.

We also define the operators

$$Q_i^a=\int d^3x\, V_i^a,\quad {}^5Q_i^a=\int d^3x\, A_i^a, \tag{5}$$

which are not time-independent. Further, there is no reasonable limit in which these charges are time-independent. So they are not properly the generators of a symmetry.[26] If one considers the algebra of these charges (5) with the chiral charges (3) the system closes on the chiral U(6)×U(6) algebra of FGZ,[20]

$$[Q^a,Q_i^b]=if^{abc}Q_i^c, \tag{6a}$$

$$[{}^5Q^a,Q_i^b]=if^{abc}\,{}^5Q_i^c, \tag{6b}$$

$$[Q^a,{}^5Q_i^b]=if^{abc}\,{}^5Q_i^c, \tag{6c}$$

$$[{}^5Q^a,{}^5Q_i^b]=if^{abc}Q_i^c, \tag{6d}$$

$$[Q_i^a,Q_j^b]=i\delta_{ij}f^{abc}Q^c-i\epsilon_{ijk}d^{abc}\,{}^5Q_k^c, \tag{6e}$$

$$[{}^5Q_i^a,{}^5Q_j^b]=i\delta_{ij}f^{abc}Q^c-i\epsilon_{ijk}d^{abc}\,{}^5Q_k^c, \tag{6f}$$

$$[Q_i^a,{}^5Q_j^b]=i\delta_{ij}f^{abc}\,{}^5Q^c-i\epsilon_{ijk}d^{abc}Q_k^c. \tag{6g}$$

There is a U(6) subalgebra of this U(6)×U(6) algebra generated by Q^a and ${}^5Q_i^a$ with commutation rules given by (4a), (6c), and (6f). This U(6) group corresponds to static U(6) and hence may be termed the classificatory group for hadrons at rest. There is an SU(3)×SU(3) subgroup of this U(6) generated by ${}^SQ_\pm^a=Q^a\pm{}^5Q_z^a$ which is the collinear group of the strong interaction and may also be a classificatory group. It is to be distinguished from the chiral SU(3)×SU(3) generated by $Q_\pm^a=Q^a\pm{}^5Q^a$ which is supposed to be realized in the Nambu-Goldstone fashion.

The group that correctly classifies the physical hadrons may in fact be generated by $V{}^SQ_\pm^aV^{-1}$ with V some unitary transformation and ${}^SQ^a$ a generator of the collinear group. ${}^SQ^a$ is explicitly represented in terms of current quarks, $q(x)$, while $V{}^SQ_\pm^aV^{-1}$ is simply represented in terms of constituent quarks.[27] For our present purposes it suffices to suppose $V=1$.

Some remarks are in order about the equal-time algebra of U(6)×U(6) given by (4) and (6). As is well known there are anomalies which appear in perturbation theory in commutation rules involving the space components of currents.[31] Such anomalies, if present, destroy the closure of the commutation rules and the validity of the FGZ algebra. Remarkably, the asymptotic freedom of QCD implies the absence of any such anomalies and the closure of the U(6)×U(6) algebra.[32] So this framework is algebraically consistent.

C. Chiral-representation content of meson states

We assume that single-particle hadron states *at rest* can be classified according to irreducible representations of static U(6), with the π and ρ in the $\underline{36}$.

Chiral SU(3)×SU(3) symmetry breaking is accommodated by the quark mass matrix

$$\mathcal{L}_{SB}=\epsilon_a\sigma^a, \tag{7}$$

$$\sigma^a=\bar{q}\tfrac{1}{2}\lambda^a q, \tag{8}$$

where ϵ_a are parameters related to the quark masses and σ^a belongs to $(3,\bar{3})+(\bar{3},3)$ of the chiral SU(3)×SU(3). Then the divergences of the axial-vector current $\partial^\mu A_\mu^a=d^{abc}\epsilon^b\pi^c$ are also $(3,\bar{3})+(\bar{3},3)$, with the pseudoscalar field operator given by

$$\pi^a=\bar{q}i\gamma_5\tfrac{1}{2}\lambda^a q. \tag{9}$$

The relevant commutation rules are

$$[{}^5Q^a,\sigma^b]=id^{abc}\pi^c, \tag{10a}$$

$$[{}^5Q^a,\pi^b]=-id^{abc}\sigma^c, \tag{10b}$$

$$[Q^a,\sigma^b]=if^{abc}\sigma^c, \tag{10c}$$

$$[Q^a,\pi^b]=if^{abc}\pi^c. \tag{10d}$$

According to PCAC the divergences of the axial-vector currents have matrix elements dominated by the ground-state pseudoscalars π, K, and η. We are led to suppose that π^a strongly connects the vacuum to these physical states,

$$\langle 0|\pi^a(0)|\pi^b(k)\rangle\!\rangle = Z_\pi^{1/2}\delta^{ab}. \tag{11}$$

Similarly

$$\langle 0|\sigma^a(0)|\sigma^b(k)\rangle\!\rangle = Z_\sigma^{1/2}\delta^{ab}, \tag{12}$$

with $|\sigma^b(k)\rangle\!\rangle$ scalar states corresponding to δ, κ, S^*, and ϵ.[33] Introducing the meson decay constant f_π,

$$\langle 0|A_\mu^a|\pi^b(k)\rangle = ik_\mu f_\pi \delta^{ab}, \tag{13}$$

it is straightforward to show that in the chiral-symmetry limit, $\mathcal{L}_{SB}=0$, assuming the vacuum is SU(3)-invariant,

$$f_\pi = Z_\pi^{1/2}(\tfrac{2}{3})^{1/2}\langle\sigma^0\rangle_0, \tag{14}$$

and the pseudoscalars are all massless Goldstone states.

We next consider the action of the U(6) generators Q^a and ${}^5Q_i^a$ upon π^a and σ^a. One obtains (10) and

$$[{}^5Q_i^a, \sigma^b] = -if^{abc}\,{}^*t_{0i}^c, \tag{15a}$$

$$[{}^5Q_i^a, \pi^b] = +if^{abc}t_{0i}^c, \tag{15b}$$

$$[{}^5Q_i^a, t_{0j}^b] = +if^{abc}\delta_{ij}\pi^c - id^{abc}\epsilon_{ijk}t_{0k}^c, \tag{15c}$$

$$[{}^5Q_i^a, t_{jk}^b] = -if^{abc}\epsilon_{ijk}\sigma^c - id^{abc}\epsilon_{jkl}t_{il}^c, \tag{15d}$$

also

$$[{}^5Q^a, t_{\mu\nu}^b] = id^{abc}\,{}^*t_{\mu\nu}^c, \quad [Q^a, t_{\mu\nu}^b] = if^{abc}t_{\mu\nu}^c. \tag{15e}$$

Here

$$t_{\mu\nu}^a = \bar{q}\sigma_{\mu\nu}\tfrac{1}{2}\lambda^a q, \quad \sigma_{\mu\nu} = \tfrac{1}{2}i[\gamma_\mu, \gamma_\nu], \qquad {}^*t_{\mu\nu}^a = \tfrac{1}{2}\epsilon_{\mu\nu\lambda\delta}t^{a\lambda\delta} = \bar{q}i\gamma_5\sigma_{\mu\nu}\tfrac{1}{2}\lambda^a q, \tag{16}$$

are fundamental skew tensors. What one learns from (15) is that (π^a, t_{0i}^a) and $(\sigma^a, {}^*t_{0i}^a)$ each transform like a $\underline{36}$ under U(6). Further, t_{0i}^a and ${}^*t_{0i}^a$ transform like chiral partners, as members of the $(3,\bar{3})+(\bar{3},3)$ representation of chiral SU(3) $\times$SU(3). For completeness we record the commutation rules with the remaining U(6)$\times$U(6) generators

$$[Q_i^a, \sigma^b] = id^{abc}t_{0i}^c, \tag{17a}$$

$$[Q_i^a, \pi^b] = id^{abc}\,{}^*t_{0i}^c, \tag{17b}$$

$$[Q_i^a, t_{0j}^b] = -id^{abc}\delta_{ij}\sigma^c + if^{abc}\epsilon_{ijk}\,{}^*t_{0k}^c, \tag{17c}$$

$$[Q_i^a, t_{jk}^b] = if^{abc}(\delta_{ij}t_{0k}^c - \delta_{ik}t_{0j}^c) - id^{abc}\epsilon_{ijk}\pi^c. \tag{17d}$$

The $\sigma^a, \pi^a, t_{\mu\nu}^a$ transform as members of $(6,\bar{6})+(\bar{6},6)$ under U(6)$\times$U(6).

The physical vector mesons are to be classified in the same $\underline{36}$ as the pseudoscalars. Let $|\rho^a(k,\epsilon)\rangle\!\rangle$ be the $J^{PC}=1^{--}$ states identified with ρ, ω, ϕ, K^*. Then, in general,

$$\langle 0|t_{\mu\nu}^a|\rho^b(k,\epsilon)\rangle\!\rangle = \frac{Z_\rho^{1/2}}{m_\rho}\delta^{ab}(k_\mu\epsilon_\nu - k_\nu\epsilon_\mu), \tag{18}$$

where $\epsilon(k)$ is the polarization of the vector-meson states. It follows from (18) that for a state at rest, $\vec{k}=0$,

$$\langle 0|t_{0i}^a|\rho^b(k,\epsilon)\rangle_{\vec{k}=0} = Z_\rho^{1/2}\epsilon_i\delta^{ab},$$

$$\langle 0|t_{ij}^a|\rho^b(k,\epsilon)\rangle_{\vec{k}=0} = 0.$$

By performing a static-U(6) transformation one can relate (18) at $\vec{k}=0$ to (11) and one finds as a consequence of static U(6)

$$Z_\rho^{1/2} = Z_\pi^{1/2}. \tag{19}$$

We will assume that this normalization is approximately valid in the real world.

D. The absence of the A_1 meson

Interestingly, this development implies that the chiral partner of the ρ is not the A_1 axial-vector meson, $J^{PC}=1^{++}$, since $\langle 0|\,{}^*t_{\mu\nu}^a|A_1\rangle=0$ on account of C invariance. Instead we identify the chiral partner of the ρ as the $B(1235)$ with $I=1$, $J^{PC}=1^{+-}$. The B is a well-established resonance although the remaining members of the octet are not well established. Presumably the strange partner of the B lies in the Q region. We will suppose that the ρ and B are members of $(3,\bar{3})+(\bar{3},3)$ of chiral SU(3) $\times$SU(3) and

$$\langle 0|\,{}^*t_{\mu\nu}^a|B^b(k,\epsilon)\rangle = \frac{Z_B^{1/2}}{m_B}\delta^{ab}(k_\mu\epsilon_\nu - k_\nu\epsilon_\mu). \tag{20}$$

The states $|B^b(k,\epsilon)\rangle$ and $|\sigma^b(k)\rangle$ belong to a U(6) $\underline{36}$. The result of static U(6) is the condition $Z_B^{1/2} = Z_\sigma^{1/2}$.

It is well known that the A_1 meson has escaped all experimental attempts to produce it as an unambiguous resonant state in spite of theorists' insistence that it must be there.[34] It could be that the A_1 is hiding and the experimental searches would thus far have failed to find it.[35]

A state with the A_1 quantum numbers is certainly present in the simple quark model. However, the quark model provides no unambiguous information on how strongly this state is required to couple to other hadrons. A *raison d'être* for the A_1 has always been the results obtained from the Weinberg spectral-function sum rules.[36] However, the axial-vector spectral function could well be saturated with $\rho\pi$ continuum states. There seems to be no reason why the A_1, if it exists, does not couple weakly to other hadrons.[37] Usually the arguments that the A_1 couples strongly presuppose the A_1 as the chiral partner to the ρ. As our scheme has avoided this requirement, the experimental absence of the A_1 and the presence of the B is a feature that supports our view.

The usual picture is that the phenomenological ρ field transforms like $\bar{q}\frac{1}{2}\lambda^a\gamma_\mu q$ and its chiral partner in this instance is $\bar{q}\frac{1}{2}\lambda^a\gamma_\mu\gamma_5 q$, which can connect strongly to an A_1 state. This picture is orthogonal to the one presented here. Our ρ and B transform like the components of the skew tensor $\bar{q}\frac{1}{2}\lambda^a\sigma_{\mu\nu}q$—like the electric and magnetic fields in the Maxwell tensor. Under the homogeneous Lorentz group they transform like $(0,1)+(1,0)$ rather than $(\frac{1}{2},\frac{1}{2})$. Our classification also precludes any attempt to describe these vector mesons as gauge bosons of a spontaneously broken gauge theory.

Furthermore, not only may the A_1 be missing but, if our scheme is correct, so is the particle that transforms like the vector current, at least insofar as the ρ is pure $(3,\bar{3})+(\bar{3},3)$. Of course, there may be representation mixing with $(1,8)+(8,1)$. But, modulo such mixing, two possibilities are offered: one is that the physical states whose associated phenomenological fields transform like $\bar{q}\gamma_\mu\frac{1}{2}\lambda^a q$ and $\bar{q}\gamma_\mu\gamma_5\frac{1}{2}\lambda^a q$ do exist but lie very much higher in mass than usually supposed and/or couple weakly to other hadrons. The other possibility (at variance with the usual quark-model assumptions) is that there are no physical states whose associated phenomenological fields transform like $\bar{q}\gamma_\mu\frac{1}{2}\lambda^a q$ and $\bar{q}\gamma_\mu\gamma_5\frac{1}{2}\lambda^a q$, but that these objects only play the role of currents and that they connect to physical states (the ρ^a and π^a, respectively) only via spontaneous symmetry breaking (see Fig. 3). We explain this further in Sec. III.

E. PCTC

We establish a phenomenological form of PCTC[25] (partial conservation of tensor current). The effective vector and axial-vector fields ρ^a_μ and B^a_μ, normalized so that

$$\langle 0|\rho^a_\mu|\rho^b(k,\epsilon)\rangle = -i\delta^{ab}\epsilon_\mu\,,$$
$$\langle 0|B^a_\mu|B^b(k,\epsilon)\rangle = -i\delta^{ab}\epsilon_\mu\,,$$

are given by PCTC

$$\partial^\mu t^a_{\mu\nu} = m_\rho Z_\rho^{1/2}\rho^a_\nu\,,\qquad \partial^\mu\, {}^*t^a_{\mu\nu} = m_B Z_B^{1/2} B^a_\nu\,, \tag{21}$$

consistent with (20) and (18). The fields ρ^a_ν and B^a_ν defined by PCTC, (21), have only three independent components since (21) automatically requires $\partial^\nu\rho^a_\nu = \partial^\nu B^a_\nu = 0$.

Historically, relations of the type (21) were called PCTC[25] in analogy with PCAC. However, we have found this analogy misleading. The divergence of the axial-vector current is a measure of symmetry breaking of the chiral generator ${}^5Q^a$. However, the divergence of the tensor current $t^a_{\mu\nu}$ is not a measure of symmetry breaking. If the U(6)×U(6) generators were all required to be conserved then the appropriate current-conservation law following from $\dot{Q}^a_i = 0$ is for $\Lambda^a_{\lambda\mu\nu} = g_{\lambda\mu}V^a_\nu - g_{\lambda\nu}V^a_\mu$ in the form $\partial^\lambda\Lambda^a_{\lambda\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu = 0$. Our version of PCTC, (21), is simply a covariant means of defining effective ρ^a_ν and B^a_ν fields which transform as $(3,\bar{3})+(\bar{3},3)$ under the chiral group.

The usefulness of PCTC, in contrast to PCAC, is mitigated by the circumstance that the tensor currents $t^a_{\mu\nu}$ are not known to participate in the weak interactions, unlike the axial-vector current A^a_μ. However, the matrix elements of $t^a_{\mu\nu}$ are in principle normalized by equal-time commutation relations $[t^a_{\mu\nu}, t^b_{\lambda\delta}]$ which extend the algebra to U(12).[38]

As emphasized, our interpretation is that PCTC, (21), is simply the consequence of our chiral-representation assignment of the vector mesons. It has, however, an important physical consequence. Since the left-hand side of (21) must vanish between states of zero momentum transfer (in the absence of any zero-mass vector state) it requires *that soft vector mesons decouple*. Physical vector mesons are not soft, so this PCTC analog of the Adler-Nambu-Shrauner[39] zero is difficult to test experimentally. Such tests should be sought to see if this scheme is viable; we discuss this further in the next section.

PCTC in the form given above precludes the possibility of any current-field identities[40] of the form $V^a_\mu = C_\rho\rho^a_\mu$. Such a current-field identity taken along with the observation that PCTC implies vector-meson decoupling would imply that the charge associated with V^a_μ must vanish. Since this charge is an SU(3) generator, the proposition is nonsense.[41]

III. VECTOR-MESON DOMINANCE

A. Spontaneous symmetry breaking and VMD

An immediate objection to putting the vector mesons in $(3,\bar{3})+(\bar{3},3)$ instead of $(1,8)+(8,1)$ is that they then would not couple to the vector current V^a_μ which transforms like $(1,8)+(8,1)$. This would be true were it not for spontaneous symmetry breaking. The same mechanism that couples the axial-vector current directly to the π couples the vector current to the ρ [see Figs. 3(a) and 3(b)]. Vacuum symmetry breaking in these two instances corresponds to the nonvanishing vacuum values of

(10b) and (17c),

$$\langle[{}^5Q^a, \pi^b]\rangle_0 = -i(\tfrac{2}{3})^{1/2}\delta^{ab}\langle\sigma^0\rangle_0 ,$$
$$\langle[Q_i^a, t_{0j}^b]\rangle_0 = -i(\tfrac{2}{3})^{1/2}\delta^{ab}\langle\sigma^0\rangle_0 , \tag{22}$$

which imply that *VMD is a consequence of spontaneous symmetry breaking.*

This feature of VMD permits us to estimate the current–vector-meson transition amplitude defined by

$$\langle 0|V_\mu^a|\rho^b(k,\epsilon)\rangle = -i\epsilon_\mu\delta^{ab}\frac{m_\rho^2}{\gamma_\rho} . \tag{23}$$

Consider

$$\Gamma_{\mu;\lambda\nu}^{ab}(k) = \int d^4x\, e^{ik\cdot x}\langle 0|T(V_\mu^a(0)t_{\lambda\nu}^b(x))|0\rangle , \tag{24}$$

so that

$$k^\lambda\Gamma_{i;\lambda j}^{ab}(k) = i\int d^4x\, e^{ik\cdot x}\langle 0|T(V_i^a(0)\partial^\lambda t_{\lambda j}^b(x))|0\rangle + i\langle 0|[Q_i^a, t_{0j}^b]|0\rangle . \tag{25}$$

Setting $k^\lambda = 0$, using PCTC (21), (22), and (14), one has a sum rule

$$-i\int d^4x\langle 0|T(V_i^a(0)\rho_j^b(x))|0\rangle = \delta^{ab}\delta_{ij}\left(\frac{f_\pi}{m_\rho}\right)\left(\frac{Z_\pi}{Z_\rho}\right)^{1/2} . \tag{26}$$

If just the vector-meson state $\rho(770)$ is inserted in the integral on the left, after expressing it as a sum on states, one obtains

$$\frac{1}{\gamma_\rho} = \left(\frac{f_\pi}{m_\rho}\right)\left(\frac{Z_\pi}{Z_\rho}\right)^{1/2} \tag{27}$$

for the transition amplitude (23). Of course other states with ρ quantum numbers can contribute to the integral. Approximate SU(6) invariance implies $(Z_\pi/Z_\rho)^{1/2} \simeq 1$, Eq. (19). Our result (27) is similar to the KSRF relation,[42] although the derivation bears no resemblance to the KSRF derivation. In our notation the KSRF relation is $\gamma_\rho^{-2} = 2f_\pi^2/m_\rho^2$ or $\gamma_\rho^{-1} = \sqrt{2}\, f_\pi/m_\rho$ and this result is numerically in good agreement with experiment. However, as is well known,[43] the KSRF relation does not follow, even in an approximate fashion, from general principles such as current algebra or PCAC. As has been adequately described in the literature,[43] using current algebra one can obtain any relation one pleases for γ_ρ. Only with additional assumptions, whose ultimate justification rests on the fact that the KSRF relation works, does one obtain the desired result.

We do not have an independent estimate of the ratio $(Z_\pi/Z_\rho)^{1/2}$, although on symmetry grounds it should not be very different from unity. With $(Z_\pi/Z_\rho)^{1/2} \simeq 1.5$ one obtains the observed rate[44] for $\rho \to e^+ + e^-$. One can generalize this discussion to the vector mesons ω, ϕ, and ϕ_c assuming the ϕ_c to be the state at 3.1 GeV. These vector mesons also have their coupling to the photon given by spontaneous vacuum breaking. To understand the ϕ_c leptonic decay rate one must assume there is large SU(4) breaking, and it is not at all clear how to incorporate this properly.

B. Form factors and PCTC

1. Pion form factor

One might think that PCTC and the vector-meson decoupling theorems would be in conflict with the observed behavior of electromagnetic form factors. This is not so.

We simplify the discussion by considering the internal symmetry to be SU(2). Then the matrix element of the tensor current between pion states is

$$\langle\pi^b(p)|t_{\mu\nu}^a|\pi^c(k)\rangle = iT(q^2)\epsilon^{abc}(q_\mu P_\nu - q_\nu P_\mu) , \tag{28}$$
$$q = p - k ,\quad P = p + k$$

while that of the vector current is

$$\langle\pi^b(p)|V_\mu^a|\pi^c(k)\rangle = F_\pi(q^2)\epsilon^{abc}P_\mu , \tag{29}$$
$$F_\pi(0) = 1 .$$

With the effective ρ field defined by PCTC, Eq. (21), and the ρ source defined by $(\Box + m_\rho^2)\rho_\mu^a = {}^\rho J_\mu^a$, one obtains a relation between the $\rho\pi\pi$ matrix element,

$$\langle\pi^b(p)|\,{}^\rho J_\mu^a|\pi^c(k)\rangle = G_{\rho\pi\pi}(q^2)\epsilon^{abc}P_\mu , \tag{30}$$

and the tensor current $T(q^2)$. It is

$$G_{\rho\pi\pi}(q^2) = q^2\frac{(-q^2 + m_\rho^2)}{m_\rho Z_\rho^{1/2}}\, T(q^2) . \tag{31}$$

This implies, since $G_{\rho\pi\pi}(m_\rho^2) = g_{\rho\pi\pi}$ is nonvanishing, that $T(q^2)$ has a ρ pole. Since $T(q^2)$ has no pole at $q^2 = 0$, we have $G_{\rho\pi\pi}(0) = 0$, the decoupling theorem.

If we assume that $F_\pi(q^2)$ obeys an unsubtracted dispersion relation, $F_\pi(\infty) = 0$, and saturate this relation with just the ρ pole according to $\mathrm{Im}F_\pi(q^2) = (m_\rho^2/\gamma_\rho)G_{\rho\pi\pi}(m_\rho^2)\delta(q^2 - m_\rho^2)$, we obtain

$$F_\pi(q^2) = \frac{1}{\gamma_\rho}\,\frac{G_{\rho\pi\pi}(m_\rho^2)m_\rho^2}{-q^2 + m_\rho^2} , \tag{32}$$

the usual vector-meson dominance. $F_\pi(0) = 1$ implies

$$\gamma_\rho = G_{\rho\pi\pi}(m_\rho^2) = g_{\rho\pi\pi} , \tag{33}$$

the universality relation.[24] The point is that $G_{\rho\pi\pi}(q^2)$ is not a smooth function; it vanishes at $q^2 = 0$, and at $q^2 = m_\rho^2$ it is the $\rho\pi\pi$ coupling.

An alternate approach consistent with the above is to assume that after removing the ρ pole from the tensor current what remains is smooth. Then $(-q^2+m_\rho^2)T(q^2)=$constant is smooth and $G_{\rho\pi\pi}(q^2) = (q^2/m_\rho^2)g_{\rho\pi\pi}$ from (31). If we now consider the contributions to the pion form factor $F_\pi(q^2)$ they are given by a direct coupling of the current to the pionic isospin plus a ρ pole term (see Fig. 4). This direct coupling is required since in our approach the ρ does not couple to the isotopic charge. This is further elaborated on in Sec. III C. Denoting the direct coupling by 1 we have for the two processes in Fig. 4

$$F_\pi(q^2)=1+\frac{m_\rho^2}{\gamma_\rho}\,\frac{G_{\rho\pi\pi}(q^2)}{-q^2+m_\rho^2}$$
$$=1+\frac{g_{\rho\pi\pi}}{\gamma_\rho}\,\frac{q^2}{-q^2+m_\rho^2}\,. \tag{34}$$

This is identical to (32). The no-subtraction hypothesis, $F_\pi(\infty)=0$, yields the usual vector universality relation (33) from (34).

2. *Nucleon form factor*

The same development applies to nucleonic form factors. The general matrix element of the tensor current is

$$\langle N(p)|t^a_{\mu\nu}|N(k)\rangle$$
$$=\bar{u}(p)\tfrac{1}{2}\tau^a\left[G(q^2)\sigma_{\mu\nu}+iH(q^2)(\gamma_\mu q_\nu-\gamma_\nu q_\mu)\right.$$
$$\left.+\,iR(q^2)(P_\mu q_\nu-P_\nu q_\mu)\right].$$

The form factors $G(q^2)$, $H(q^2)$, $R(q^2)$ have poles corresponding to the ρ and B mesons. Defining the ρ^a_μ and B^a_μ sources as ${}^\rho J^a_\mu=(\Box+m_\rho^2)\rho^a_\mu$ and ${}^B J^a_\mu = (\Box+m_B^2)B^a_\mu$ one has

$$\langle N(p)|\,{}^\rho J^a_\mu|N(k)\rangle=\bar{u}(p)\tfrac{1}{2}\tau^a\left[F_1^\rho(q^2)\gamma_\mu\right.$$
$$\left.+\,i\sigma_{\mu\nu}q^\nu F_2^\rho(q^2)\right]u(k)\,, \tag{36}$$
$$\langle N(p)|\,{}^B J^a_\mu|N(k)\rangle=\bar{u}(p)F^B(q^2)\tfrac{1}{2}\tau^a P_\mu i\gamma_5 u(k)\,.$$

Using PCTC and the useful identity

$$\bar{u}(p)\gamma_5\epsilon_{\alpha\beta\gamma\delta}q^\gamma p^\delta u(k)$$
$$=\bar{u}(p)(-q^2\sigma_{\alpha\beta}+\sigma_{\alpha\nu}q^\nu q_\beta-\sigma_{\beta\nu}q^\nu q_\alpha)u(k)\,,$$

these form factors are related by

$$\frac{Z_\rho^{1/2}m_\rho F_1^\rho(q^2)}{-q^2+m_\rho^2}=-q^2[H(q^2)+2MR(q^2)]\,, \tag{37a}$$

$$\frac{Z_\rho^{1/2}m_\rho F_2^\rho(q^2)}{-q^2+m_\rho^2}=+[G(q^2)+q^2R(q^2)]\,, \tag{37b}$$

$$\frac{Z_B^{1/2}m_B F^B(q^2)}{-q^2+m_B^2}=-G(q^2)\,. \tag{37c}$$

From (37a) follows the ρ-decoupling theorem

$$F_1^\rho(0)=0\,. \tag{38}$$

Notice that there is no similar requirement for the BNN coupling to vanish, $F^B(0)\neq 0$. So PCTC requires only vector-meson decoupling, not axial-vector decoupling.[45] We also learn that the tensor form factor $G(q^2)$ has only the B pole, while the form factors $H(q^2)$ and $R(q^2)$ have both B and ρ poles; the combinations of G, H, and R in Eqs. (37a) and (37b) are free of the B pole contribution.

The vector form factor is specified by

$$\langle N(p)|V^a_\mu|N(k)\rangle=\bar{u}(p)\tfrac{1}{2}\tau^a\left[F_1^V(q^2)\gamma_\mu\right.$$
$$\left.+\,F_2^V(q^2)i\sigma_{\mu\nu}q^\nu\right]u(k)\,, \tag{39}$$

with

$$F_1^V(0)=1\,,\quad F_2^V(0)=\tfrac{1}{2}(\kappa_p-\kappa_n)\,. \tag{40}$$

We assume that there is both a direct coupling to the nucleons and a contribution mediated by the ρ. For the direct coupling we assume for simplicity no anomalous magnetic moment, and so we specify this coupling as $\bar{u}(p)\frac{1}{2}\tau^a\gamma_\mu u(k)$. Then one has

$$F_1^V(q^2)=1+\frac{m_\rho^2F_1^\rho(q^2)}{\gamma_\rho(m_\rho^2-q^2)}\,, \tag{41a}$$

$$F_2^V(q^2)=\frac{m_\rho^2}{\gamma_\rho}\,\frac{F_2^\rho(q^2)}{(m_\rho^2-q^2)}\,. \tag{41b}$$

Assuming that $F_1^V(q^2)$ satisfies an unsubtracted dispersion relation and saturating this relation with just the ρ state one has the universality relation

$$\gamma_\rho=F_1^\rho(m_\rho^2)=F_\pi(m_\rho^2)$$

for the on-shell ρNN coupling. Again this is only possible if $F_1^\rho(q^2)\simeq g_{\rho NN}(q^2/m_\rho^2)$ is not a smooth function but $F_1^\rho(q^2)/q^2$ *is* a smooth function. We have found no experimental evidence from photoproduction or electroproduction of vector mesons that is inconsistent with this proposition.

The purpose of this exercise was to show that the usual lore about form factors is in no way inconsistent with PCTC and the decoupling theorems it implies. It would be worthwhile to search for experimental evidence or new experiments that could serve to verify or disqualify the hypothesis of PCTC. It is important to emphasize that although our discussion of PCTC was motivated by

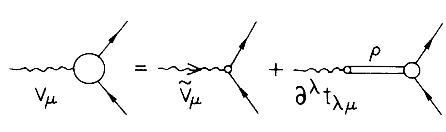

FIG. 4. Separation of the vector form factor into a direct plus a ρ pole contribution. The ρ coupling to hadrons vanishes at zero momentum transfer.

the resolution of the ρ-π puzzle, our discussion is essentially independent of this motivation and is an interesting question in its own right.

C. Tensor-field identities (TFI)

As remarked before, PCTC is not the analog of PCAC. However, there is a tensor-field identity (TFI) which is the analog of PCAC.

We begin our discussion with the vector current $V^a_\beta(x) = \bar{q}(x)\frac{1}{2}\lambda^a\gamma_\beta q(x)$, for which we write

$$V^a_\beta(x) = \tilde{V}^a_\beta(x) + \frac{m_\rho{}^2}{\gamma_\rho}\rho^a_\beta(x). \tag{42}$$

Here ρ^a_β is specified by PCTC (21),

$$\partial^\lambda t^a_{\lambda\alpha} = (m_\rho{}^2 Z_\rho)^{1/2}\rho^a_\alpha, \quad t^a_{\alpha\beta} = \bar{q}\sigma_{\alpha\beta}\tfrac{1}{2}\lambda^a q,$$

and so (42) is just a definition of $\tilde{V}^a_\beta(x)$. The separation of the vector current into two pieces given by (42) just corresponds to the two diagrams in Fig. 4. There $\tilde{V}^a_\beta$ corresponds to the direct coupling of the current V^a_β to hadrons and has no ρ pole term, and ρ^a_β corresponds to the ρ pole piece. From (42) and (21) we have

$$\tilde{V}^a_\beta(x) = \bar{q}\gamma_\beta\tfrac{1}{2}\lambda^a q - (m_\rho/\gamma_\rho Z_\rho{}^{1/2})\partial^\lambda(\bar{q}\sigma_{\lambda\beta}\tfrac{1}{2}\lambda^a q), \tag{43}$$

so we see that the charge of $\tilde{V}^a_\beta$ is identical to the charge associated with V^a_β, the group generator, providing there are no massless states.

This procedure is familiar in the separation of the axial-vector current $A^a_\mu = \bar{q}\gamma_\mu\gamma_5\frac{1}{2}\lambda^a q$ as

$$A^a_\mu = \tilde{A}^a_\mu - f_\pi Z_\pi{}^{-1/2}\partial_\mu\pi^a, \tag{44}$$

where $\pi^a = \bar{q}i\gamma_5\frac{1}{2}\lambda^a q$. $\tilde{A}^a_\mu$ thus defined has no pion pole. The utility of the separation (44) has been elaborated on by Dashen and Weinstein[46] to understand the $m_\pi{}^2 \to 0$ limit for chiral dynamics. Using (27), $\gamma_\rho{}^{-1} = (f_\pi/m_\rho)(Z_\pi/Z_\rho)^{1/2}$, and (43) one has

$$V^a_\mu = \tilde{V}^a_\mu + \frac{f_\pi Z_\pi{}^{1/2}}{Z_\rho}\partial^\lambda t^a_{\lambda\mu}, \tag{45}$$

where $\tilde{V}^a_\mu$ has no ρ pole terms. This is the analog of (44).

Note that (44) and (45) imply that the curls of A_μ and $\tilde{A}_\mu$ are equal and the divergences of V_μ and $\tilde{V}_\mu$ are equal. Conversely, it is the divergence of A_μ and the curl of V_μ that are measures of U(6) $\times$ U(6)-symmetry breaking, since they imply ${}^5\dot{Q}^a \neq 0$ and $\dot{Q}^a_i \neq 0$, respectively.

Next we make use of the identity, valid in the free quark model,

$$\Box t^a_{\alpha\beta} = \epsilon_{\alpha\beta\gamma\delta}\partial^\gamma C^{a\delta} + \partial_\alpha\partial^\lambda t^a_{\lambda\beta} - \partial_\beta\partial^\lambda t^a_{\lambda\alpha}, \tag{46}$$

where $t^a_{\alpha\beta}$ is the usual tensor current and $C^a_\delta = (\partial_\delta\bar{q})\gamma_5\frac{1}{2}\lambda^a q - \bar{q}\gamma_5\frac{1}{2}\lambda^a\partial_\delta q$ is a second-class axial-vector current. We can assume that (46) is valid in an interacting theory; in this case it is a definition of C^a_δ.[47] Using (45) in (46) we obtain the desired form of TFI, the analog of PCAC:

$$\partial_\alpha V^a_\beta - \partial_\beta V^a_\alpha = \partial_\alpha\tilde{V}^a_\beta - \partial_\beta\tilde{V}^a_\alpha + \frac{f_\pi Z_\pi{}^{1/2}}{Z_\rho}(\Box t^a_{\alpha\beta} - \epsilon_{\alpha\beta\gamma\delta}\partial^\gamma C^{a\delta}). \tag{47}$$

This TFI can be used to study the $m_\rho{}^2 \to 0$ limit of chiral U(6) $\times$ U(6). The utility of TFI lies in the fact that the curl of the vector current is on the left and this is a measure of U(6) $\times$ U(6) breaking.

It is illuminating at this stage to suppose that we are in the U(6) $\times$ U(6) limit. It is impossible, in fact, to accomplish this limit, in contrast with the chiral SU(3) $\times$ SU(3) limit, in a relativistic field theory such as quantum chromodynamics. Let us suppose anyway that we have $\dot{Q}^a_i = 0$ and the corresponding conservation law $\partial_\mu V^a_\nu - \partial_\nu V^a_\mu = 0$, so the left side of (47) vanishes. The consequences of this may be seen by taking nucleonic matrix elements of TFI, (47). Denote [for SU(2) symmetry]

$$\langle N(p)|\tilde{V}^a_\alpha|N(k)\rangle = \bar{u}(p)\tfrac{1}{2}\tau^a[\tilde{F}_1(q^2)\gamma_\alpha + \tilde{F}_2(q^2)i\sigma_{\alpha\beta}q^\beta]u(k),$$

$$\langle N(p)|C^a_\alpha|N(k)\rangle = \bar{u}(p)\tfrac{1}{2}\tau^a[C(q^2)i\gamma_5 P_\alpha]u(k),$$

and the matrix elements of $t^a_{\alpha\beta}$ given by (35). Then $\partial_\alpha V^a_\beta - \partial_\beta V^a_\alpha = 0$ implies from TFI

$$\tilde{F}_1(q^2) = q^2[2MR(q^2) + H(q^2)]f_\pi\left(\frac{Z_\pi{}^{1/2}}{Z_\rho}\right),$$

$$\tilde{F}_2(q^2) = -[C(q^2) + q^2R(q^2)]f_\pi\left(\frac{Z_\pi{}^{1/2}}{Z_\rho}\right), \tag{48}$$

$$C(q^2) = G(q^2).$$

Since $C(q^2)$ can have no ρ pole and $\tilde{F}_{1,2}(q^2)$ has no ρ poles — by construction — we learn from (48) that as $q^2 \to 0$, since $\tilde{F}_1(0) = 1$ and $C(0) \neq 0$, $2MR(q^2) + H(q^2)$ has a pole at $q^2 = 0$ corresponding to a massless particle. This is the Goldstone theorem. The ρ mass must vanish — it is dormant Goldstone boson.

Since there does not exist a quark field theory for which $\dot{Q}^a_i = 0$ and $\partial_\mu V^a_\nu - \partial_\nu V^a_\mu = 0$ the above exercise is pedagogic. It is intended to emphasize the analogy of PCAC and TFI and the dormant Goldstone character of the ρ. However, the impossibility of a relativistic SU(6) is the only feature that prevents a strict one-to-one analogy between our treatment of the π with PCAC and the ρ with TFI.

IV. DECAYS OF SCALAR AND VECTOR MESONS

We consider the decays

$$\delta \to \eta + \pi,$$

$$B \to \omega + \pi,$$

$$\epsilon \to \pi + \pi,$$

each of which has a pion in the final state. These decays are chosen to test the chiral representation content of the mesons. The experimental widths are

$$\Gamma_{\delta\to\eta+\pi} = (50 \pm 20)\ \text{MeV},$$

$$\Gamma_{B\to\omega+\pi} = (120 \pm 20)\ \text{MeV}, \tag{49}$$

$$\Gamma_{\epsilon\to 2\pi} \approx (600\text{–}700)\ \text{MeV (large errors).}^{33}$$

Elementary application of current algebra does not suffice to determine these decay amplitudes. Only if one supplements current algebra with some rather questionable extrapolation procedures does one obtain definite answers. We present our results without detailed justification for these extrapolations, which are typical for hard-meson techniques.[48]

For the $\delta^b(p) \to \pi^a(q) + \eta(k)$ decay, with the $\delta(970)$ identified as the $I = 1$ chiral partner of the pion, we write the amplitude

$$\langle \eta(k) | j_\pi^a | \delta^b(p) \rangle = \delta^{ab} A_\delta(k^2, p^2, q^2), \quad p = k + q. \tag{50}$$

We will work in the chiral SU(2) × SU(2) limit for which $m_\pi^2 = 0$ and always keep the final pion on the mass shell. Extrapolations of $O(m_\pi^2)$ we consider to be harmless. We examine the amplitude, with the initial state δ off-shell,

$$\langle \eta(k) | A_\mu^a(0) | \delta^b(p) \rangle = \frac{i(p^2 - m_\delta^2)}{Z_\delta^{1/2}} \times \int d^4x\, e^{ip\cdot x} \langle \eta(k) T(\delta^b(x) A_\mu^a(0)) | 0 \rangle, \tag{51}$$

where $\delta^a(x) = \bar{q}\, \tfrac{1}{2}\lambda^a q$ $(a = 1, 2, 3)$ is identified as the δ field [see Eq. (8)]. On the mass shell $p^2 = m_\delta^2$ (51) is an identity. The relevant commutator is

$$[{}^5Q^a, \delta^b] = \frac{i}{\sqrt{3}} \eta, \quad a, b = 1, 2, 3 \tag{52}$$

with $\eta = \bar{q}\, \tfrac{1}{2}\lambda^8 i\gamma_5 q$ the η field operator [see Eq. (9)]. Multiplying (51) by $-iq_\mu$ and taking $q_\mu \to 0$, using

$$\langle \eta(k) | A_\mu^a | \delta^b(p) \rangle = \frac{iq_\mu}{q^2} f_\pi \delta^{ab} A(k^2, p^2, 0) + \text{terms regular in } q_\mu,$$

the commutator (52), and the normalizations (11) and (12), one obtains

$$|A(m_\eta^2, m_\eta^2, 0)| = \frac{|m_\eta^2 - m_\delta^2|}{\sqrt{3} f_\pi} \left(\frac{Z_\eta}{Z_\delta} \right)^{1/2}. \tag{53}$$

The major assumption is to assume that (53) approximates the physical amplitude

$$A_\delta = A_\delta(m_\eta^2, m_\delta^2, 0) \simeq A_\delta(m_\eta^2, m_\eta^2, 0). \tag{54}$$

From the experimental width, 50 ± 20 MeV, one obtains from

$$\Gamma_{\delta\to\eta+\pi} = \frac{|\vec{k}|}{8\pi m_\delta^2} |A_\delta|^2, \quad |\vec{k}| = \text{final momentum}$$

and (54) the crude estimate

$$\frac{Z_\eta}{Z_\delta} \approx \frac{1}{4}. \tag{55}$$

Next consider the decay $B^b(p) \to \pi^a(q) + \omega(k)$ with the amplitude

$$\langle \omega(k) | j_\pi^a | B^b(p) \rangle = A_B(k^2, p^2, q^2) \delta^{ab} \left(\epsilon_\mu^\omega - \frac{\epsilon^\omega \cdot k k_\mu}{k^2} \right) \left(\epsilon_B^\mu - \frac{\epsilon^B \cdot p p^\mu}{p^2} \right) + D\text{-wave term}. \tag{56}$$

This decay proceeds by both S and D waves. Experimentally the D-to-S-wave amplitude ratio is 0.24 ± 0.06, and so we will ignore the D wave and try to estimate the S wave. We consider the amplitude

$$\langle \omega(\epsilon^\omega, k) | A_\mu^b(0) | B^b(\epsilon^B, p) \rangle = \frac{i(p^2 - m_B^2)}{Z_B^{1/2}} \int d^4x\, e^{ip\cdot x} \langle \omega(\epsilon^\omega, k) | T(B_\nu^b(x) A_\mu^a) | 0 \rangle \epsilon_B^\nu, \tag{57}$$

with the B_ν^b field operator $m_B B_\nu^b = \partial^\mu\, {}^*t_{\mu\nu}^b$ defined by PCTC. On the mass shell, $p^2 = m_B^2$, (57) is an identity. The commutation rule we need is

$$[{}^5Q^a, {}^*t_{\mu\nu}^b] = i\delta^{ab} t_{\mu\nu}, \tag{58}$$

where $\partial^\mu t_{\mu\nu}$ corresponds to the ω field operator ($\vec{B}$ and ω transform like a chiral quadruplet in our representation assignment). One finds from (58) and (57)

$$|A_B(m_\omega^2, m_\omega^2, 0)| = \frac{|m_\omega^2 - m_B^2|}{f_\pi} \frac{m_\omega Z_\omega^{1/2}}{m_B Z_B^{1/2}}. \tag{59}$$

Again using our assumption of extrapolation to the physical point

$$A_B = A_B(m_\omega^2, m_B^2, 0) \approx A_B(m_\omega^2, m_\omega^2, 0),$$

one obtains for the rate

$$\Gamma_{B\to\omega+\pi}=\frac{m_\omega^2}{8\pi f_\pi^2}\left(1-\frac{m_\omega^2}{m_B^2}\right)^2|\vec{k}|\left(\frac{Z_\omega}{Z_B}\right), \tag{60}$$

with $|\vec{k}|$ the final momentum of the decay. To get a definite answer one makes the assumption of static SU(6) for the normalization constants

$$\frac{Z_\omega}{Z_B}=\frac{Z_\eta}{Z_\delta}\approx\frac{1}{4}. \tag{61}$$

This implies

$$\Gamma_{B\to\omega+\pi}\simeq 90 \text{ MeV},$$

in excellent agreement with the observed rate (49) of 120 ± 20 MeV and $D/S=0.24\pm0.06$.

We now consider the $\epsilon\to2\pi$ decay and identify the ϵ with the $\pi\pi$ resonance in the region 1100–1300 MeV.[33] The decay amplitude for $\epsilon(p)\to\pi^a(q)+\pi^b(k)$ is

$$\langle\pi^b(k)|j_\pi^a(0)|\epsilon(p)\rangle=\delta^{ab}A_\epsilon(k^2,p^2,q^2), \tag{62}$$

$$p=k+q.$$

As before we consider the amplitude

$$\langle\pi^b(k)|A_\mu^a|\epsilon(p)\rangle=\frac{i}{Z_\epsilon^{1/2}}(p^2-m_\epsilon^2)\times\int d^4x\, e^{ip\cdot x}\langle\pi^b(k)|T(\epsilon(x)A_\mu^a(0))|0\rangle,$$

with the initial state off-shell. Using the commutator $[{}^5Q^a,\epsilon]=i\pi^a$ $(a=1,2,3)$ for the ϵ field operator $\epsilon(x)=(1/\sqrt{6})\bar{q}[\lambda^0+(1/\sqrt{2})\lambda^8]q$, one finds

$$|A(0,0,0)|=\frac{m_\epsilon^2}{f_\pi}\left(\frac{Z_\pi}{Z_\epsilon}\right)^{1/2}. \tag{63}$$

Making use of the result valid in the SU(3) limit

$$\frac{Z_\pi}{Z_\epsilon}=\frac{Z_\eta}{Z_\delta}$$

and (55) we obtain $Z_\pi/Z_\epsilon\approx\frac{1}{4}$. Then the rate is determined using the drastic extrapolation assumption

$$A_\epsilon=A_\epsilon(0,m_\epsilon^2,0)\approx A_\epsilon(0,0,0) \tag{64}$$

as

$$\Gamma_{\epsilon\to2\pi}=\frac{3}{16\pi m_\epsilon^2}|A_\epsilon|^2|\vec{k}| \approx 1100\text{–}1800 \text{ MeV}, \tag{65}$$

in fair agreement with the rather uncertain experimental range of 600–700 MeV.

We caution the reader that the use of these extrapolations and symmetry results such as (61) is potentially dangerous. Remarkably, if we make these naive assumptions we can test our representation content of the vector mesons and everything works our far better than one should expect.

V. THE GENERALIZED Σ MODEL

If one wishes to extend the SU(3)×SU(3) Σ model to include the approximate U(6) symmetry observed in the hadron spectrum, one finds that the smallest group which contains these groups and which closes is U(6)×U(6). Hence we consider the generalization of the Σ model to chiral U(6)×U(6) symmetry. Owing to the well-known and apparently insurmountable difficulties in effecting a relativistic generalization of SU(6) symmetry,[26] we are able to study only a static ($\vec{p}=0$) model, but we still of course demand Galilean invariance. Furthermore, as fermions are an unenlightening complication here, we look at a model restricted to mesons.[49]

It is convenient to employ the following notation. The charges which generate chiral U(6)×U(6) obey the commutation relations

$$\begin{aligned}[{}_\alpha Q^A,{}_\alpha Q^B]&=iF^{ABC}{}_\alpha Q^C,\\ [{}_\beta Q^A,{}_\alpha Q^B]&=iF^{ABC}{}_\beta Q^C,\\ [{}_\beta Q^A,{}_\beta Q^B]&=iF^{ABC}{}_\alpha Q^C,\end{aligned} \tag{66}$$

where $A=a$ or ai, and $a=0,\ldots8$, $i=1,\ldots,3$. Note that the generators Λ_A of static U(6) obey

$$\mathrm{Tr}\Lambda_A\Lambda_B=2\delta_{AB},$$

and

$$\begin{aligned}&[\Lambda_A,\Lambda_B]=2iF_{ABC}\Lambda_C,\\ &\{\Lambda_A,\Lambda_B\}=2D_{ABC}\Lambda_C,\\ &F_{abc}=(\tfrac{1}{2})^{1/2}f_{abc},\quad D_{abc}=(\tfrac{1}{2})^{1/2}d_{abc}\\ &F_{ai,bj,ck}=(\tfrac{1}{2})^{1/2}\epsilon_{ijk}d_{abc}+(\tfrac{1}{2})^{1/2}\delta_{ij}f_{abc}\\ &D_{ai,bj,ck}=(\tfrac{1}{2})^{1/2}\epsilon_{ijk}f_{abc}+(\tfrac{1}{2})^{1/2}\delta_{ij}d_{abc},\end{aligned}$$

where f_{abc} and d_{abc} are the usual SU(3) coefficients, and $d_{0aa}=(\frac{2}{3})^{1/2}$.

The mesons are classified into two 36-plets of static U(6). Using Cartesian coordinates we identify an odd-parity 36-plet,

$$M_A:\ M_a=\pi^a,\quad M_{ai}=\rho_i^a,$$

and an even-parity 36-plet,

$$N_A:\ N_a=\sigma^a,\quad N_{ai}=B_i^a.$$

Together these multiplets transform as a $(6,\bar{6})+(\bar{6},6)$ representation of chiral U(6)×U(6).

$$\begin{aligned}[{}_\alpha Q^A,M^B]&=iF^{ABC}M^C,\\ [{}_\alpha Q^A,N^B]&=iF^{ABC}N^C,\\ [{}_\beta Q^A,M^B]&=-iD^{ABC}N^C,\\ [{}_\beta Q^A,N^B]&=iD^{ABC}M^C.\end{aligned} \tag{67}$$

Note that the $(6,\bar{6})+(\bar{6},6)$ representation is forced upon us once we have the pseudoscalar mesons and vector mesons in the same multiplet and then demand that the pseudoscalars be in $(3,\overline{3})+(\overline{3},3)$ under chiral $SU(3)\times SU(3)$, as they are in the usual formulation of the Σ model. The requirement of the $(6,\overline{6})+(\overline{6},6)$ then fixes the charge conjugation property of the axial-vector mesons to be the same as the vectors, i.e. odd, while σ^a and π^a are even as usual.

The potential part of the Lagrangian is

$$\mathcal{L}_{\text{pot}} = -\tfrac{1}{2}\mu^2(M_AM_A+N_BN_B) - \lambda(M_AM_A+N_BN_B)^2 - \gamma[D_{ABC}D_{A'B'C}(M_AM_BM_{A'}M_{B'}+N_AN_BN_{A'}N_{B'}+2M_AM_BN_{A'}N_{B'})+4F_{ABC}F_{A'B'C}M_AN_BM_{A'}N_{B'}]. \tag{68}$$

This is the most general $U(6)\times U(6)$-invariant Lagrangian (restricted to polynomials of degree $\leqslant 4$). It should be pointed out that there is no trilinear term, in contrast with the case for $SU(3)\times SU(3)$. Hence, the maximal group that leaves the Lagrangian invariant is $U(6)\times U(6)$ and not just $SU(6)\times SU(6)$.

As in the usual Σ model, if $\mu^2<0$ then in the classical limit the potential defined by $\mathcal{L}_{\text{pot}}$ has a minimum when σ^0 has a nonvanishing vacuum expectation value,

$$\langle\sigma^0\rangle_0 = a.$$

This condition restricts the vacuum to be just $U(6)$-invariant, and the $U(6)\times U(6)$ symmetry is spontaneously broken. We introduce a displaced field

$$\sigma^{0\prime} = \sigma^0 - a,$$

whose vacuum expectation value is zero. Rewriting the Lagrangian in terms of this well-defined $\sigma^{0\prime}$, we isolate the term linear in $\sigma^{0\prime}$,

$$\sigma^{0\prime}a(\mu^2+4\lambda a^2+\tfrac{4}{3}\gamma a^2).$$

If $\langle\sigma^{0\prime}\rangle$ is to be zero at least at the tree-graph level, we must demand that this term linear in $\sigma^{0\prime}$ vanishes. This is just the condition that the classical potential be at an extremum. Hence we require

$$a^2 = \frac{-\mu^2}{4(\lambda+\gamma/3)}. \tag{69}$$

The masses of the various mesons are found to be

$$\begin{aligned} m_{\sigma^{0\prime}}^2 &= \mu^2+4a^2(3\lambda+\gamma),\\ m_B^2 = m_{\hat\sigma}^2 &= \mu^2+4a^2(\lambda+\gamma),\\ m_\pi^2 = m_\rho^2 &= \mu^2+4a^2(\lambda+\gamma/3)=0, \end{aligned} \tag{70}$$

where $\hat\sigma$ denotes the scalar octet. That is, the masses of the axial-vector nonet and the scalar octet are equal, while the whole odd-parity 36-plet has become massless, corresponding to the Goldstone mode of the broken $U(6)\times U(6)$ symmetry to $U(6)$.[50] The vector mesons cannot of course be true Goldstone bosons in a realistic, relativistic theory. But they are Goldstone states in this static model and hence can be termed dormant Goldstone bosons. As we shall now see, relativity will explicitly break the symmetry and rouse the vector mesons to massive states.

So let us consider explicit symmetry breaking. Unfortunately one cannot be as precise here as in the case of $SU(3)\times SU(3)$-symmetry breaking. However, it has been pointed out that once one demands relativistic invariance, the kinetic energy terms in the free part of a Hamiltonian must give rise to spin-dependent mass terms in the presence of interactions.[23] We will assume that this spin-dependent effect is SU(3)-independent.

It is helpful to use the tensor notation

$$\begin{aligned} \hat M^\alpha_\beta &= i\delta^i_jP^A_B+(\vec\sigma\cdot\vec\epsilon)^i_jV^A_B,\\ \hat N^\alpha_\beta &= i\delta^i_jS^A_B+(\vec\sigma\cdot\vec\epsilon)^i_jB^A_B, \end{aligned} \tag{71}$$

where $P^A_B=(1/\sqrt2)\sum_{a=0}^8\lambda^A_B\pi^a$, $V^A_B=(1/\sqrt2)\sum_{a=0}^8\lambda^A_{Ba}\rho^a$, etc., and $\vec\epsilon$ is the polarization vector. Combining these tensors into

$$\begin{aligned} T &= \hat N + i\hat M,\\ T^\dagger &= \hat N^\dagger - i\hat M^\dagger, \end{aligned} \tag{72}$$

we may then write the mass operator as

$$-\tfrac12\mu^2\,\mathrm{Tr}\,TT^\dagger - \tfrac12\beta^2\,\mathrm{Tr}[\vec\sigma T][\vec\sigma T^\dagger] = -\tfrac12\mu^2(\pi^2+\rho^2+\sigma^2+B^2)-\tfrac12\beta^2(\rho^2+B^2), \tag{73}$$

where [ ] indicates the spin trace. The μ^2 term is as above, Eq. (68), and the β^2 term gives the spin-dependent U(6) breaking, and hence explicitly breaks $U(6)\times U(6)$. The spin-one mesons are now split from their spin-zero multiplet partners, and the mass formulas are

$$\begin{aligned} m_B^2 &= \mu^2+\beta^2+4a^2(\lambda+\gamma),\\ m_{\hat\sigma}^2 &= \mu^2+4a^2(\lambda+\gamma),\\ m_\rho^2 &= \beta^2,\\ m_\pi^2 &= 0. \end{aligned} \tag{74}$$

The vector mesons have acquired a mass, which we expect to be large compared to the actual pion mass since we know that U(6) is a badly broken symmetry compared to SU(3). From these mass formulas we have the relation

$$m_\rho^2 - m_\pi^2 = m_B^2 - m_\delta^2. \quad (75)$$

This relation is in remarkably good agreement with the experimentally determined masses. For example, putting in the masses of the $I=1$ members of the octets, i.e., the $\vec{\pi}$, $\vec{\rho}$, $\vec{B}$, and $\vec{\delta}$, one obtains

$$m_{\vec{\rho}}^2 - m_{\vec{\pi}}^2 = m_{\vec{B}}^2 - m_{\vec{\delta}}^2,$$

$$(0.593-0.019)\ \text{GeV}^2 \text{ vs } (1.53-0.953)\ \text{GeV}^2,$$

$$0.574\ \text{GeV}^2 \text{ vs } 0.577\ \text{GeV}^2.$$

Similarly, for the $K^*(892)$-K mass^2 difference, the result is 0.55 GeV^2. Unfortunately the strange members of the even-parity 36-plet are not well enough identified to make comparison worthwhile. At any rate, from these results it appears that U(6)-symmetry breaking due to spin-dependent effects independent of SU(3) breaking is rather well borne out experimentally, with the parameter $\beta^2 \approx 0.6\ \text{GeV}^2$.

VI. DISCUSSION AND CONCLUSIONS

A. PCAC and the Melosh transformation

We have addressed ourselves to the ρ-π puzzle and thus to the problem of how to coherently incorporate PCAC in the quark model. Our solution is based on a static symmetry, U(6)×U(6). There has been another attempt to incorporate PCAC and the quark model, but one that does not discuss the ρ-π puzzle. This is the work of Carlitz and Tung and their collaborators.[18] More specifically, these authors were concerned with the problem of installing PCAC within the framework of the Melosh transformation in a more integral way than the phenomenological approach of putting it in by hand. They propose some intriguing concepts, especially that the Nambu-Goldstone realization of chiral symmetry is the origin of chiral representation mixing. However, we believe that the problem of incorporating PCAC with the Melosh transformation still exists for the reasons we shall now explain.

The approach of Carlitz and Tung is in the formalism of null-plane or lightlike charges. But since the Wigner-Weyl and Nambu-Goldstone realizations of a symmetry merge on the light cone, the Nambu-Goldstone nature of the pion cannot be distinguished. To do so, Carlitz and Tung go outside their formalism to the usual static or timelike charges. This leads to two pions in their scheme. One is the Nambu-Goldstone pion which transforms like $(3,\overline{3})+(\overline{3},3)$ under the usual chiral SU(3) ×SU(3). This pion they subtract away in defining their lightlike axial charge. The other pion, the quark-model pion, is in a Wigner-Weyl representation along with the ρ meson. It is not subtracted out in defining their lightlike axial charge. This can be seen explicitly in their discussion using the Σ model where the elementary pion field [transforming like $(3,\overline{3})+(\overline{3},3)$] is subtracted but there still remains the pion composed of quarks. In the representations of the algebra of lightlike charges there exists a representation with the ρ and the π as members of the same representation. They are built out of quarks and antiquarks. But the PCAC pion, while removed from the lightlike axial charge, nevertheless exists and plays a very important role. So there are two pions in this approach which are distinct, and the ρ-π problem comes back to haunt one.

Furthermore, having two pions is essentially the same situation that existed before the work of Carlitz and Tung. This can be seen, for example, in the phenomenological applications[51] of the Melosh transformation to the calculation of the transition rate for $\alpha \to \beta + \pi$.

The π that is contracted from the state $\beta+\pi$ is a soft pion, $p_z \to 0$, while β and α are states at $p_z \to \infty$. For this soft pion one uses PCAC, $\partial^\mu A_\mu = \mu_\pi^2 f_\pi \pi$, which is a representation of the π in terms of current quarks. However, a π that is contained in α or β is taken to have $p_z \to \infty$ and this pion has a simple representation content under $\text{SU}(6)_W$ in terms of constituent quarks. So in these calculations the π is treated in two different ways, as a bound-state quark model π and as a PCAC π. As long as one uses PCAC for soft π's and the $\text{SU}(6)_W$ (strong) representation for π's with $p_z \to \infty$, which is done in these phenomenological applications, the approach is consistent. However, the connection between these two treatments of the pion is still obscure. The problem remains to devise a description of the pion that interpolates between a pion at rest and a pion moving with $p_z = \infty$.

The discussions of the Melosh transformation are given in the context of $\text{SU}(6)_W$ symmetry. This symmetry is invoked if one is concerned with saturating sum rules since it involves only "good-good" commutators in the $p_z \to \infty$ limit.[52] Instead we have used the FGZ U(6)×U(6) algebra which involves "good-bad" and "bad-bad" cummutators but for which rotational invariance is evident. Furtherthermore, this symmetry incorporates the static SU(6) symmetry approximately observed in the hadron spectrum. It and its subgroups seem to us to be the symmetries of choice when talking about low-energy phenomena such as PCAC. What we have emphasized in this paper is that it is through PCAC, (more precisely, through the Nambu-Goldstone mode) that the quark model tells us in a fundamental way how hadrons are made in their rest frame.

B. Unsolved problems

It would be desirable to have a real relativistic quantum field theory in which one could study these problems associated with dormant Goldstone bosons. Can one build a Nambu-Jona-Lasinio model of the ρ as a dormant Goldstone boson? Such a model should satisfy the requirement that it has a reasonable static limit with $U(6)\times U(6)$ invariance realized by vector and pseudoscalar Goldstone states which are collective excitations. Some efforts in this direction have been made by the authors but will not be reported here. Eventually one hopes to understand the $m_\rho^2 \to 0$ limit as well as the $m_\pi^2 \to 0$ limit. Conceivably there are other dormant Goldstone bosons such as 2^+ tensor mesons associated with spontaneous breaking of higher symmetries.

If we look at the group diagram, Fig. 1, we see that the relative merits of weak PCAC vs strong PCAC can be parametrized in terms of the GOR "c" parameter.[21] For strong PCAC $c \simeq -\sqrt{2}$, as seems to be observed. This raises the question of how one gives a similar parametrization for the choices describing the breaking of static $U(6)\times(6)$. Such a parametrization is desired to formulate these questions with greater clarity.

We have pointed out that the reconciliation of quark-model symmetries with PCAC requires the ρ to be in the $(3,\overline{3})+(\overline{3},3)$ representation and so it satisfies decoupling theorems. Can one extend the development of phenomological Lagrangians to include such vector mesons? What are the further experimental consequences of the decoupling theorems?

If anything, this work shows that the ρ and the other ground-state vector mesons are collective excitations. So a simple potential model for the quarks in the vector mesons must fail. Recently such potential models for the $\phi_c(3.1)$ and $\phi_c(3.7)$ have had qualitative success.[53] Our picture suggests that these new states are also collective excitations. Does this feature shed any light on the details of charmonium levels?

A final remark: The present work indicates that the ground-state vector mesons, like the ground-state pseudoscalars, are manifestations of collective phenomena of the type familiar in superconductivity. There seems to us to be nothing in principle that prevents understanding of all hadrons as collective phenomena. Contrary to conventional interpretations of the hadron spectrum, perhaps more is to be learned from the paradigm of the theory of superconductivity than from atomic models.

ACKNOWLEDGMENTS

We would like to thank M. A. B. Bég, A. Pais, and V. Rittenberg for discussions on these problems. One of us (H.P.) found a summer visit to the Aspen Center for Physics useful for the completion of this work. He also thanks R. P. Feynman for a pleasant afternoon discussion.

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*Work supported in part by the U. S. Energy Research and Development Administration under Contract No. E(11-1)-2232B.

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