Possibility of a ninth $J^P = \frac{1}{2}^+$ baryon

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It is shown that (i) the small but definite discrepancy between the mass-squared Gell-Mann–Okubo mass formula of the $\frac{1}{2}^+$ baryons and experiment, (ii) the trouble of the Σ-Λ degeneracy, and (iii) the deviation of the D/F ratio of the axial-vector semileptonic couplings from the SU(6) value as indicated by the value $|G_A/G_V| = 0.435 \pm 0.035$ of the latest high-statistics experiment on the $\Sigma^- \to n + e^- + \bar{\nu}$ decay as compared with the SU(6) value $\simeq 0.25$, can be explained simultaneously if a ninth $I = 0$, $\frac{1}{2}^+$ baryon Λ' exists around 1700 MeV with a broad width of about 1 GeV. The hyperon semileptonic decays in the presence of the Λ' are analyzed in detail. The Cabibbo angles are determined to be $\sin\theta_V = 0.227 \pm 0.008$ and $\sin\theta_A = 0.220 \pm 0.020$. The theoretical frameworks used are asymptotic SU(3), chiral SU(3) $\otimes$ SU(3) charge algebra, and the simple mechanism of symmetry breaking.

I. INTRODUCTION AND SUMMARY

In the real world where SU(3) is broken, the use of *exact* SU(3) and its implied sum rules is not *a priori* justified, since we have to use physical masses in computing actual transition rates and other physical quantities. Still, its use, combined with the phenomenological prescription of using physical masses whenever masses appear, gives a reasonable *overall* description of SU(3) in many instances.

For example, in the application of the Cabibbo theory of weak currents[1] to the semileptonic decays of hyperons, we customarily parametrize the axial-vector amplitudes in terms of three quantities, the D and F couplings and the Cabibbo angle θ, by using exact SU(3) for the axial-vector matrix elements. Armed with the recent experimental data, one indeed finds that the *overall* fit of the exact-SU(3) sum rules to the data is remarkably good. The latest review of Kleinknecht[2] shows that the data can be fitted with (assuming one Cabibbo angle)

$$\frac{D}{D+F} = 0.658 \pm 0.007, \quad \sin\theta = 0.230 \pm 0.003. \tag{1.1}$$

Probably, this is one of the situations where the conventional treatment of SU(3) works particularly well. Actually, there is the following theoretical reason why *exact*-SU(3) sum rules work well in this case.

A promising and theoretically more rigorous way to approach such problems of broken SU(3) is the purely algebraic approach which has been formulated[3] in the following theoretical frameworks: (a) the chiral SU(3) $\otimes$ SU(3) charge algebra of Gell-Mann[4] which is valid in broken SU(3), (b) asymptotic SU(3) symmetry proposed by Matsuda, Oneda, and Umezawa[3,5], and (c) the presence of "exotic" charge commutation relations[3] (C. R.'s) involving the time derivative of the SU(3) vector charges V_i ($i = 4, 5, 6, 7$) which expresses in an algebraic fashion the simple machanism of SU(3) and chiral SU(3) $\otimes$ SU(3) breaking. It was realized[6] that the broken-SU(3) sum rules for the *physical* axial-vector semileptonic decay coupling constants (but defined only at the zero four-momentum transfer limit), obtained in the above theoretical frameworks (a) and (b), take exactly the *same form* [in the absence of the SU(3) mixings] as the usual exact-SU(3) sum rules. That is, the usual Cabibbo analysis of the semileptonic decays using *exact*-SU(3) parametrization and the physical masses of the hyperons is, in fact, justified in this particular case in the theoretical frameworks (a) and (b).

In this paper we further add the theoretical constraint (c) and mainly study the following problems.

(i) SU(3) mass relations for the $J^P = \frac{1}{2}^+$ baryons. The imposition of the exotic C. R.'s $[\dot{V}_{K^0}, V_{K^0}] = 0$ and $[\dot{V}_{K^0}, A_{K^0}] = 0$, etc. ($V_{K^0} = V_6 + iV_7$, $A_{K^0} = A_6 + iA_7$, etc.), upon our theoretical frameworks (a) and (b), implies the existence of the following SU(3) mass formulas[3,7] for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons ($m^2 \equiv \Lambda^2$, etc.):

$$3(\Lambda^0)^2 + (\Sigma^0)^2 = 2[(n)^2 + (\Xi^0)^2], \tag{1.2}$$

and

$$\begin{aligned} &(\Omega^-)^2 + (\Sigma^{*-})^2 = 2(\Xi^{*-})^2, \\ &(\Xi^{*-})^2 + (\Delta^-)^2 = 2(\Sigma^{*-})^2. \end{aligned} \tag{1.3}$$

These mass formulas are exact [except for the possible effect of intermultiplet SU(3) mixing which was not considered] and take the *mass-squared* form. Recent high-statistics experiments indicate[8] that the *mass-squared* equal mass spacings, Eq. (1.3), among the $\frac{3}{2}^+$ decuplet are better satisfied than the usually assumed linear equal mass spacings, and they agree with the experiments well.[9,10] However, experimentally Eq. (1.2) reads $5.156 = 5.222$ in $(\mathrm{GeV})^2$. Therefore, there is a small, but nonvanishing meaningful discrepancy between the predictions, Eq. (1.2), and experiment to which we wish to address (the sign as well as the magnitude).

(ii) The problem of Σ-Λ degeneracy. By sandwiching another type of exotic C. R., $[\dot{V}_{K^0}, A_{\pi^-}] = 0$, etc., between the states $\langle n(\vec{k})|$ and $|\Sigma^+(\vec{k})\rangle$ and also between $\langle\Sigma^-(\vec{k})|$ and $|\Xi^0(\vec{k})\rangle$ with $\vec{k}\to\infty$, Matsuda and Oneda have encountered[3] the problem of the Σ-Λ mass degeneracy in the above theoretical framework. The Σ-Λ degeneracy is, in fact, reminiscent of the troubles of the simple quark-counting model and also of the nonrelativistic SU(6) with a simple mass breaking interaction. We wish to remove simultaneously the Σ-Λ degeneracy as well as the discrepancy between the SU(3) mass formula, Eq. (1.2), and experiment.

(iii) The deviation of the D/F ratio of semileptonic hyperon decays from the SU(6) prediction. SU(6) predicts in contrast to Eq. (1.1) that

$$D/F = \tfrac{3}{2}, \quad D/(D+F) = 0.6. \tag{1.4}$$

Since the other SU(6) prediction, $(G_A/G_V)_\beta = \frac{5}{3}$ for the nuclear β decay, differs significantly from the experimental value $(G_A/G_V)_\beta \simeq 1.25$, the D/F ratio is also expected to deviate to some extent from the SU(6) value, Eq. (1.4). Present experiment [i.e., Eq. (1.1)] seems to demonstrate that this is indeed the case. As a matter of fact, the SU(6) value $D/F = \frac{3}{2}$ predicts, when combined with the experimental value $(G_A/G_V)_\beta \simeq 1.25$, that

$$(G_A/G_V)_{\Sigma^-\to ne^-\bar{\nu}} \simeq -0.25$$

as compared with the value obtained by the recent *high-statistics* experiment[11] of

$$|(G_A/G_V)_{\Sigma^-\to ne^-\bar{\nu}}| \simeq 0.435 \pm 0.035.$$

Since we now work in the presence of the theoretical constraints, $[\dot{V}_{K^0}, A_{\pi^-}] = 0$, etc., which *impose* dynamical constraints upon the axial-vector hyperon semileptonic couplings and the masses of the hyperons, our proposed mechanism of solving the problems (i) and (ii) is also hoped to be consistent with the present data of the hyperon semileptonic decays which exhibit a deviation from the exact-SU(6) prediction, $D/F = \frac{3}{2}$.

The simple mechanism we wish to propose is the existence of a ninth[12] $J^P = \frac{1}{2}^+$ baryon, Λ', which will weakly mix via the SU(3)-breaking interaction with the Λ resolving the problems (i) and (ii) and at the same time explain the present hyperon semileptonic data [problem (iii)].

Apart from simplicity, one of the motivations for introducing the Λ' comes, of course, from the simple observation that the SU(3) decomposition of the baryon configuration in the popular qqq description of the quark model involves a singlet, i.e., $3\otimes 3\otimes 3 = 1+8+8+10$. For bosons nature seems to realize all the representations of the $q\bar{q}$ state, i.e., the singlet as well as the octet. Therefore, it may not be unrealistic to imagine the existence of the Λ'. Matsuda and Oneda have derived[3] the SU(6) result $D/F = \frac{3}{2}$, etc. [but not encountering the bad result of SU(6), $(G_A/G_V)_\beta = \frac{5}{3}$] in a purely algebraic approach without assuming SU(6), by adding to the theoretical frameworks (a) and (b) a (further) constraint dealing with the C. R., $[A_i, A_j] = if_{ijk}V_k$, and assuming that the ground-state baryons consist of the $\frac{1}{2}^+$ octet and the $\frac{3}{2}^+$ decuplet. In a subsequent paper we make this picture more complete by adding a $\frac{1}{2}^+$ singlet. In Sec. VIII we add a brief remark about the Λ' when viewed from the point of view of the SU(6)$\otimes$O(3) classification.

The role played by the singlet-octet mixing for bosons tempts us to expect a similar role played by the Λ-Λ' mixing. We recall that such mixings as the ω-ϕ and η-η' mixings take care of the deficiencies of the original Gell-Mann-Okubo (GMO) mass formulas for bosons. In our formulation of asymptotic SU(3), these mixings automatically appear[3] in the mass-squared SU(3) mass relations for the 0^{-+} and 1^{--} mesons obtained from the "exotic" C. R.'s, $[\dot{V}_{K^0}, V_{K^0}] = 0$ and $[\dot{V}_{K^0}, A_{K^0}] = 0$. However, in the absence of the ninth bosons, the imposition of the "exotic" C. R.'s $[\dot{V}_{K^0}, A_{\pi^-}] = 0$, etc., indeed lead, for example, to the ρ-ϕ or η-π degeneracies similar to the Σ-Λ mass degeneracy. In the presence of the ninth bosons, these "exotic" C. R.'s in fact play a remarkable role producing the sum rules[3] which exhibit the interesting dynamical interplay between the masses, SU(3) mixing angles, and the axial-vector matrix elements which is consistent with experiment. In Sec. II we briefly describe the derivation of the broken-SU(3) sum rules using our theoretical framework (a), (b), and (c). In Sec. III, we use some of the latest hyperon semileptonic data to derive the vector Cabibbo angle θ_V. We calculate that $\sin\theta_V = 0.227 \pm 0.008$. We also find that $\tau(n\to pe^-\bar{\nu}) = 975 \pm 12$ sec, from which we infer that the radiative correction to G^β in the O^{14} decay is $(3\pm 1)\%$, in good agreement with the radiative corrections predicted by the $SU(2)_L \otimes U(1)$ gauge models. In Sec. IV, we fix the mass of the Λ' and simultaneously determine the axial-vector Cabibbo angle θ_A by using some of the latest experimental data on hyperon semileptonic decays and from the internal consistency of our sum rules involving the axial-vector couplings and the masses of the $\frac{1}{2}^+$ baryons. We find that $\sin\theta_A \cong 0.220 \pm 0.020$ and $\Lambda' = 1.711$ GeV. We also obtain some of the axial-vector matrix elements. Using the calculated values of θ_A, θ_V, and Λ', we derive and present in Sec. V branching ratios for various hyperon semileptonic decays and also their respective G_A/G_V ratios. In Sec. VI we give the simple broken-SU(3) parametrization for the various observable hyperon semileptonic couplings. In Sec. VIII we calculate the branching ratios for the strong decay modes of the Λ' and find a large width $\simeq 1$ GeV for the Λ'.

II. BROKEN-SU(3) SUM RULES FOR THE SEMILEPTONIC HYPERON DECAYS IN THE PRESENCE OF THE Λ'

The semileptonic weak Hamiltonian is given by

$$H_{\text{weak}} = (G_\mu/\sqrt{2})[J^\nu(x) j_\nu^+(x) + j^\nu(x) J_\nu^+(x)],$$

where $G_\mu = 1.43583 \times 10^{-49}$ erg cm^3, $j_\nu(x)$ is the leptonic current, and the hadronic current $J^\nu(x)$ is given by

$$J^\nu(x) = [\cos\theta_V V^\nu_{\pi^+}(x) + \cos\theta_A A^\nu_{\pi^+}(x)] + [\sin\theta_V V^\nu_{K^+}(x) + \sin\theta_A A^\nu_{K^+}(x)].$$

The vector and axial-vector form factors of the hyperon semileptonic decay $B_\alpha \to B_\beta + l + \bar{\nu}$ through the currents $V^\nu_\gamma(x)$ and $A^\nu_\gamma(x)$, respectively, are defined by

$$\langle B_\alpha(\vec{q}\,', s')| V^\nu_\gamma(0)|B_\beta(\vec{q}, s)\rangle = \left(\frac{m_\alpha}{E_\alpha}\right)^{1/2}\left(\frac{m_\beta}{E_\beta}\right)^{1/2} f_{\alpha\beta}(p^2)\bar{u}_\alpha(\vec{q}\,', s')\gamma_\nu u_\beta(\vec{q}, s) + \cdots, \tag{2.1}$$

$$\langle B_\alpha(\vec{q}\,', s')| A^\nu_\gamma(0)|B_\beta(\vec{q}, s)\rangle = \left(\frac{m_\alpha}{E_\alpha}\right)^{1/2}\left(\frac{m_\beta}{E_\beta}\right)^{1/2} g_{\alpha\beta}(p^2)\bar{u}_\alpha(\vec{q}\,', s')\gamma_5\gamma_\nu u_\beta(\vec{q}, s) + \cdots, \tag{2.2}$$

where $E_\alpha = (m_\alpha{}^2 + \vec{q}\,'^2)^{1/2}$, $p^2 = (q-q')^2$, s and s' denote the helicity.

The important fact we utilize below is that the weak currents $V^\nu_i(x)$ and $A^\nu_i(x)$ $(i = 1, 2, \ldots, 8)$ satisfy the chiral SU(3)$\otimes$SU(3) current algebras which are valid in *broken* SU(3). As will be seen, for the discussion of only the vector and axial-vector form factors at *zero* four-momentum transfer, only the use of charge algebras, $[V_i, V_j] = if_{ijk}V_k$ and $[V_i, A_j] = if_{ijk}A_k$, is sufficient.

We now briefly explain our dynamical assumption, asymptotic SU(3).[3,5] Consider the creation or annihilation operator $a_\alpha(\vec{q}, s)$ of the physical particle with physical SU(3) index α, helicity s, and momentum $\vec{k}$. The transformation of $a_\alpha(\vec{q}, s)$ under SU(3) can be expressed (suppressing the time-dependent factor in the Schrödinger representation) as

$$[V_i, a_\alpha(\vec{q}, s)] = i\Sigma_\beta u_{i\alpha\beta}(\vec{q}, s)a_\beta(\vec{q}, s) + \delta u_{i\alpha}(\vec{q}, s).$$

In exact SU(3), $\delta u_{i\alpha}(\vec{q}, s) = 0$, particles α and β belong to the same SU(3) multiplet, $u_{i\alpha\beta}$ is a linear combination of structure constants, and the above expression is valid for *any* $\vec{q}$. In broken SU(3), this is no longer true. However, asymptotic SU(3) assumes[3,5] that $a_\alpha(\vec{q}, s)$ still transforms *linearly* under SU(3) but only in the limit $\vec{q} \to \infty$, i.e.,

$$\delta u_{i\alpha}(\vec{q}, s) \to \frac{1}{|\vec{q}|^{1+\epsilon}} \quad (\epsilon > 0) \quad \text{as } \vec{q} \to \infty. \tag{2.3}$$

In the sum Σ_β, β can no longer be restricted to the members of the same SU(3) multiplet involving the α and β should, therefore, be extended, in principle, to all particles which have the same J^{PC} or J^P as the α in anticipation of the occurence of SU(3) mixing. However, with Eq. (2.3) the $a_\beta(\vec{q}, s)$'s can be related (only in the limit $\vec{q} \to \infty$) *linearly* to the hypothetical SU(3) representation operators $a_i(\vec{q}, s)$'s. These linear relations define the SU(3) mixing parameters. Since we only consider in this paper the mixing between the octet hyperon and the SU(3)-singlet ninth $\frac{1}{2}^+$ baryon Λ', the annihilation operators $a_\Lambda(\vec{q}, s)$ and $a_{\Lambda'}(\vec{q}, s)$ will be related linearly to the SU(3) representation operators $a_8(\vec{q}, s)$ and $a_0(\vec{q}, s)$ in the limit $\vec{q} \to \infty$ by

$$\begin{pmatrix} a_\Lambda(\vec{q}, s) \\ a_{\Lambda'}(\vec{q}, s) \end{pmatrix} = \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} a_8(\vec{q}, s) \\ a_0(\vec{q}, s) \end{pmatrix}, \quad \vec{q} \to \infty \tag{2.4}$$

where ω is the SU(3) Λ-Λ' mixing angle.

Let us rewrite the charge algebras in terms of physical SU(3) indices as $[V_\alpha, V_\beta] = F_{\alpha\beta\gamma}V_\gamma$ and $[V_\alpha, A_\beta] = F_{\alpha\beta\gamma}A_\gamma$, where α, β, and γ stand for π^+, K^+, etc. $F_{\alpha\beta\gamma}$ is a linear combination of f_{ijk}. Inserting the vector algebras between the $\frac{1}{2}^+$ baryon states $\langle B_\lambda(\vec{q}, s)|$ and $|B_\sigma(\vec{q}, s)\rangle$ with $\vec{q} \to \infty$, we obtain a set of algebraic equations,

$$\Sigma_\mu \langle B_\lambda(\vec{q}, s)| V_\alpha |B_\mu\rangle\langle B_\mu| V_\beta |B_\sigma(\vec{q}, s)\rangle - \Sigma_\nu\langle B_\lambda(\vec{q}, s)| V_\beta|B_\nu\rangle\langle B_\nu| V_\alpha |B_\sigma(\vec{q}, s)\rangle = F_{\alpha\beta\gamma}\langle B_\lambda(\vec{q}, s)| V_\gamma |B_\sigma(\vec{q}, s)\rangle\,,$$

in which the intermediate states are restricted to the $\frac{1}{2}^+$ baryon states (with $\vec{q}$ and s) according to asymptotic SU(3). [In exact SU(3) the above set of equations is valid for any $\vec{q}$.] We thus see that, in the *absence* of the Λ', $\langle B_\lambda(\vec{q},s)|V_\alpha|B_\sigma(\vec{q},s)\rangle$ with $\vec{q}\to\infty$ takes the exact-SU(3) value even in broken SU(3) thanks to asymptotic SU(3). With the normalization

$$\langle p(\vec{q},s)|V_{\pi^0}|p(\vec{q},s)\rangle = \tfrac{1}{2}$$

(note that $V_{\pi^0}\equiv I_3$) we obtain, for example,

$$\langle n(\vec{q},s)|V_{K^0}|\Lambda(\vec{q},s)\rangle = (\tfrac{3}{2})^{1/2}$$

for $\vec{q}\to\infty$. In the presence of the Λ', the Λ' must be included in the above set of algebraic equations. Upon solving these equations we find that the matrix elements involving the state Λ' are now modified and we find, for example,

$$\langle n(\vec{q},s)|V_{K^0}|\Lambda(\vec{q},s)\rangle = (\tfrac{3}{2})^{1/2}\cos\omega$$

for $\vec{q}\to\infty$. We may also conjecture this result via the observation (in the limit $\vec{q}\to\infty$)

$$\begin{aligned}\langle n(\vec{q},s)|V_{K^0}|\Lambda(\vec{q},s)\rangle &= \langle n(\vec{q},s)|V_{K^0}|\cos\omega\Lambda_8(\vec{q},s) + \sin\omega\Lambda_0(\vec{q},s)\rangle\\ &= \cos\omega\langle n(\vec{q},s)|V_{K^0}|\Lambda_8(\vec{q},s)\rangle\\ &= (\tfrac{3}{2})^{1/2}\cos\omega.\end{aligned}$$

Therefore, we have shown that in the framework of asymptotic SU(3) $\langle B_\lambda(\vec{q},s)|V_\alpha|B_\sigma(\vec{q},s)\rangle$ with $\vec{q}\to\infty$ can be parametrized in broken SU(3) by using exact SU(3) with a simple modification due to mixing. Proceeding in a similar way for the algebra $[V_\alpha,A_\beta]=f_{\alpha\beta\gamma}A_\gamma$, we obtain for $\vec{q}\to\infty$

$$\Sigma_\mu\langle B_\lambda(\vec{q},s)|V_\alpha|B_\mu\rangle\langle B_\mu|A_\beta|B_\sigma(\vec{q},s)\rangle - \Sigma_\nu\langle B_\lambda(\vec{q},s)|A_\beta|B_\nu|\rangle\langle B_\nu|V_\alpha|B_\sigma(\vec{q},s)\rangle = F_{\alpha\beta\gamma}\langle B_\lambda(\vec{q},s)|V_\gamma|B_\sigma(\vec{q},s)\rangle.$$

Again the intermediate states are restricted to the $\frac{1}{2}^+$ baryons thanks to asymptotic SU(3). It is then clear that in the *absence* of the Λ', the axial-vector matrix elements $\langle B_\lambda(\vec{q},s)|A_\alpha|B_\sigma(\vec{q},s)\rangle$ can still be parametrized (but only in the limit $\vec{q}\to\infty$) by exact SU(3) in terms of two independent matrix elements usually called D and F couplings. In the presence of the Λ', we again find that a simple modification due to mixing takes place.

As an illustration, we consider a special case, i.e., the algebra $[V_{K^0},A_{\pi^+}]=-A_{K^+}$ inserted between the states $\langle p(\vec{q},s)|$ and $|\Lambda(\vec{q},s)\rangle$ with $\vec{q}\to\infty$, i.e.,

$$\langle p(\vec{q},s)|V_{K^0}|\Sigma^+\rangle\langle\Sigma^+|A_{\pi^+}|\Lambda(\vec{q},s)\rangle - \langle p(\vec{q},s)|A_{\pi^+}|n\rangle\langle n|V_{K^0}|\Lambda(\vec{q},s)\rangle = -\langle p(\vec{q},s)|A_{K^+}|\Lambda(\vec{q},s)\rangle.$$

By noting that $V_\gamma=\int V_\gamma^0(x)d^3x$ and using our normalization of the spinor wave function $u_\alpha(\vec{q},s=\frac{1}{2}) =(E_\alpha+m_\alpha)^{1/2}(2m_\alpha)^{-1}(1,0,0,|\vec{q}|(E_\alpha+m_\alpha)^{-1})$, we see from Eqs. (2.1) and (2.2) that

$$\lim_{|\vec{q}|\to\infty}\langle B_\alpha(\vec{q},s|V_\gamma|B_\beta(\vec{q},s)\rangle = f_{\alpha\beta}(0)$$

and

$$\lim_{|\vec{q}|\to\infty}\langle B_\alpha(\vec{q},s)|A_\gamma|B_\beta(\vec{q},s)\rangle = g_{\alpha\beta}(0)\ ,$$

i.e., the matrix elements of the vector and axial-vector charges at $\vec{q}\to\infty$ are directly related to the vector and axial-vector couplings at the *zero* four-momentum transfer limit. The contributions of *other* form factors in Eqs. (2.1) and (2.2) to the matrix elements of the charges V_γ and A_γ vanish in the limit $\vec{q}\to\infty$. $(G_\mu/\sqrt{2})g_{p\Lambda}(0)\sin\theta_A$, for example, is the observed axial-vector coupling constant (at zero four-momentum transfer) for the $\Lambda\to p+e^-+\nu$ decay. The values of $f_{\alpha\beta}(0)$ relevant for the hyperon semileptonic decays are listed below:

$$f_{pn}(0)=1,\ \ f_{\Lambda\Sigma^\pm}(0)=0,\ \ f_{pn}(0)=(\tfrac{3}{2})^{1/2}\cos\omega,\ \ f_{n\Sigma^-}(0)=1\ ,$$

$$f_{\Lambda\Xi^-}(0)=(\tfrac{3}{2})^{1/2}\cos\omega\ ,\ \ f_{\Sigma^0\Xi^-}(0)=(\tfrac{1}{2})^{1/2}\ ,\ \ f_{\Sigma^+\Xi^0}(0)=1\ .$$

Noting that

$$\langle p(\vec{q},s)|V_{K^0}|\Sigma^+\rangle = f_{p\Sigma^+}(0) = -1$$

and

$$\langle n(\vec{q},s)|V_{K^0}|\Lambda\rangle = f_{n\Lambda}(0) = (\tfrac{3}{2})^{1/2}\cos\omega\ ,$$

we obtain from Eq. (2.5) the sum rule

$$g_{\Lambda p}(0) = g_{\Sigma^+\Lambda}(0) + (\tfrac{3}{2})^{1/2}\cos\omega g_{np}(0)\ . \tag{2.6}$$

[Hereafter we write $g_{\Lambda p}(0)=g_{\Lambda p}$, etc.] We emphasize that Eq. (2.6) is a broken-SU(3) sum rule derived without recourse to *exact* SU(3).

In addition to Eq. (2.6) we obtain the following set of sum rules which allows us to determine all the couplings in terms of *three* (the inclusion of the Λ' brings in one more coupling) independent couplings and the mixing angle ω:

$$-g_{\Sigma^-n} = -\frac{1}{\sqrt{2}}g_{\Sigma^-\Sigma^0} + (\tfrac{3}{2})^{1/2}\cos\omega g_{\Sigma^-\Lambda} - (\tfrac{3}{2})^{1/2}\sin\omega g_{\Sigma^-\Lambda'}\ , \tag{2.7}$$

$$g_{\Sigma^0\Sigma^+} = \frac{1}{\sqrt{2}}(g_{np}+g_{\Sigma^-n})\ , \tag{2.8}$$

$$g_{\Sigma^+\Lambda'} = -g_{\Sigma^-\Lambda'}\ , \tag{2.9}$$

$$g_{\Sigma^+\Lambda} = -g_{\Sigma^-\Lambda}\,, \tag{2.10}$$

$$g_{\Sigma^0\Sigma^+} = g_{\Sigma^-\Sigma^0}\,, \tag{2.11}$$

$$g_{\Xi^-\Xi^0} = g_{\Sigma^- n}\,, \tag{2.12}$$

$$g_{\Xi^-\Lambda} = (\tfrac{3}{2})^{1/2}\cos\omega\, g_{\Xi^-\Xi^0} + g_{\Sigma^-\Lambda}\,, \tag{2.13}$$

$$-g_{np} = -(\tfrac{1}{2})^{1/2} g_{\Sigma^0\Sigma^+} + (\tfrac{3}{2})^{1/2}\cos\omega\, g_{\Sigma^+\Lambda} - (\tfrac{3}{2})^{1/2}\sin\omega\, g_{\Lambda'\Sigma^+}\,. \tag{2.14}$$

We now specify the mechanism of SU(3) and chiral SU(3) ⊗ SU(3) breaking according to (c). We first use the exotic C. R.'s $[\dot{V}_{K^0}, V_{K^0}] = 0$ and $[\dot{V}_{K^0}, A_{K^0}] = 0$, etc., and then the exotic C. R. $[\dot{V}_{K^0}, A_{\pi^-}] = 0$.

Consider $[\dot{V}_{K^0}, V_{K^0}] = 0$. By inserting the commutator between the states $\langle n(\vec{q})|$ and $|\Xi^0(\vec{q}\,')\rangle$, we obtain via the asymptotic SU(3) assumption the *quadratic* GMO mass formula with mixing in the limit $\vec{q} \to \infty$:

$$\sin^2\omega[(\Lambda')^2 - (\Lambda)^2] = \tfrac{1}{3}\{2[(n)^2 + (\Xi^0)^2] - [(\Sigma^0)^2 + 3(\Lambda)^2]\}\,. \tag{2.15}$$

The use of the exotic C. R. $[\dot{V}_{K^0}, A_{K^0}] = 0$, etc. also produces[13] the same mass constraint Eq. (2.15) after eliminating the axial-vector matrix elements through Eqs. (2.6)–(2.14). Thus if the Λ' mass is larger than the Λ mass, the small Λ-Λ' mixing angle ω can fix the small deviation of the GMO mass formula (quadratic) from experiment.

We now study *all* the constraints obtained by imposing the exotic C. R. $[\dot{V}_{K^0}, A_{\pi^-}] = 0$. For example, we insert this C. R. between the states $\langle n(\vec{q})|$ and $|\Sigma^+(\vec{q}\,')\rangle$ with $|\vec{q}| \to \infty$, and we obtain (actually this is the only independent information from the C. R.'s of the type $[\dot{V}_{K^0}, A_{\pi^-}] = 0$, if we use Eq. (2.15))

$$[(\Sigma^0)^2 - (\Lambda)^2](\tfrac{3}{2})^{1/2}\cos\omega\, g_{\Sigma^+\Lambda} - [(\Sigma^0)^2 - (\Lambda')^2](\tfrac{3}{2})^{1/2}\sin\omega\, g_{\Lambda'\Sigma^+} = [(n)^2 - (\Sigma^0)^2]\left\{\frac{1}{\sqrt{2}} g_{\Sigma^0\Sigma^+} - (\tfrac{3}{2})^{1/2}\cos\omega\, g_{\Sigma^+\Lambda} + (\tfrac{3}{2})^{1/2}\sin\omega\, g_{\Lambda'\Sigma^+} - \left[\frac{(p)^2 - (\Sigma^+)^2}{(n)^2 - (\Sigma^0)^2}\right] g_{np}\right\}. \tag{2.16}$$

When Eq. (2.16) is combined with Eq. (2.14), we find that

$$[(\Sigma^0)^2 - (\Lambda)^2]\cos\omega\, g_{\Sigma^+\Lambda} - [(\Sigma^0)^2 - (\Lambda')^2]\sin\omega\, g_{\Lambda'\Sigma^+} = -\delta'[(\Sigma^0)^2 - (n)^2] g_{np}\,, \tag{2.17}$$

where

$$\delta' \equiv (\tfrac{2}{3})^{1/2}\{1 - [(p)^2 - (\Sigma^+)^2]/[(n)^2 - (\Sigma^0)^2]\} \simeq 0\,.$$

$\delta' = 0$ if we neglect the p-n mass difference. [If we consider SU(2) breaking, then Λ-Λ-Σ^0 mixing takes place. In this paper we only consider SU(3) Λ-Λ' mixing and neglect the SU(2) mixing. However, we keep SU(2) breaking in the masses like δ' in order to study a partial effect of SU(2) breaking.] In the absence of the Λ', i.e., $\sin\omega = 0$ and $\cos\omega = 1$, the Σ^0-Λ mass degeneracy[3] is apparent ($g_{\Sigma^+\Lambda} \neq 0$) from the sum rule Eq. (2.17). Equation (2.17) can be written in the form

$$g_{\Lambda'\Sigma^+} = \beta\cot\omega\, g_{\Sigma^+\Lambda} + \frac{\delta'\beta'}{\sin\omega} g_{np}\,, \tag{2.18}$$

where

$$\beta = \frac{(\Sigma^0)^2 - (\Lambda)^2}{(\Sigma^0)^2 - (\Lambda')^2}\,, \quad \beta' \equiv \frac{(\Sigma^0)^2 - (n)^2}{(\Sigma^0)^2 - (\Lambda')^2}\,.$$

We can also eliminate $g_{\Lambda'\Sigma^+}$ from Eq. (2.14) using Eq. (2.16); then we obtain

$$g_{np} = \frac{1}{\sqrt{2}}\epsilon' g_{\Sigma^0\Sigma^+} + (\tfrac{3}{2})^{1/2}\epsilon'(\beta - 1)\cos\omega\, g_{\Sigma^+\Lambda} \equiv G\sqrt{2}(D + F)\,, \tag{2.19}$$

where

$$\epsilon' \equiv [1 - (\tfrac{3}{2})^{1/2}\delta'\beta']^{-1} \simeq 1\,.$$

GD and GF are defined by

$$G\sqrt{2}F \equiv \frac{1}{\sqrt{2}}\epsilon' g_{\Sigma^0\Sigma^+}\,, \qquad G\sqrt{2}D \equiv (\tfrac{3}{2})^{1/2}\epsilon'(\beta - 1)\cos\omega\, g_{\Sigma^+\Lambda} \tag{2.20}$$

with $D + F = 1$. G is simply a scale factor. In the absence of the Λ', our D and F couplings can be shown to coincide with the familiar D and F couplings of exact SU(3).

III. EVALUATION OF THE VECTOR CABIBBO ANGLE FROM THE HYPERON SEMILEPTONIC DECAYS

For the hypercharge-conserving and the hypercharge-changing semileptonic decay $B_i \to B_f + l + \nu$, define

$$(G_A)_{if} \equiv g_{if}(0)\cos\theta_A\,, \quad (G_V)_{if} \equiv f_{if}(0)\cos\theta_V\,, \qquad (G_A)_{if} \equiv g_{if}(0)\sin\theta_A\,, \quad (G)_{if} \equiv f_{if}(0)\sin\theta_V\,. \tag{3.1}$$

We also define $X_{if}^{-1} \equiv (G_A)_{if}/(G_V)_{if}$.

In order to evaluate θ_V, it is necessary to know X_{if}^{-1} and $\Gamma(B_i \to B_f + l + \nu)$ and it is convenient to have the superb numerical tables of Nieto[14] (corrected for the more recent value of G_μ) at one's disposal. Nieto's tables relate directly the partial decay rates to the various form factors. We will consider only the vector and axial-vector form factors in this paper, as corrections due to other form factors are expected to be relatively minor. We also neglect the q^2 dependence of the $f_{if}(q^2)$ and $g_{if}(q^2)$. These effects tend to decrease the Cabibbo angles θ_V and θ_A by a few percent.

We choose the decay $\Sigma^- \to ne^-\bar{\nu}$ to evaluate θ_V, as its statistics are much better than any other decay. In addition, the problems of radiative and nuclear corrections for the $n \to pe^-\bar{\nu}$ decay are avoided. Using the experiment, i.e., $X_{\Sigma^- n}{}^{-1}$ from Table I and $\Gamma(\Sigma^- \to ne^-\bar{\nu})$ from Table II, and Nieto's tables,[14] we find that

$$\begin{aligned}(G_V)_{\Sigma^- n} &= \{\Gamma(\Sigma^- \to ne^-\bar{\nu})/[9.059 + 26.73(X_{\Sigma^- n}{}^{-1})^2] \times 10^7\}^{1/2} \\ &= 0.227 \pm 0.008 = f_{\Sigma^- n} \sin\theta_V \ .\end{aligned}$$

Thus

$$\sin\theta_V = 0.227 \pm 0.008 \ [f_{\Sigma^- n} = 1 \text{ from asymptotic SU(3)}] . \tag{3.2}$$

From the definition of $X_{if}{}^{-1}$ (we take $X_{\Sigma^- n}{}^{-1}$ negative), we find that

$$\begin{aligned}(G_A)_{\Sigma^- n} &= (-0.435)(G_V)_{\Sigma^- n} \\ &= -0.0989 \pm 0.0078 \ .\end{aligned} \tag{3.3}$$

Similarly, using $\Gamma(\Lambda \to pe^-\bar{\nu})$ and $X_{\Lambda p}{}^{-1}$ (see Tables I and II), we obtain

$$\begin{aligned}(G_V)_{\Lambda p} &= 0.298 \pm 0.027 \ , \\ (G_A)_{\Lambda p} &= 0.196 \pm 0.008 \ .\end{aligned} \tag{3.4}$$

For the decays $\Sigma^\pm \to \Lambda e^\pm \nu$, we find in a similar way

$$\begin{aligned}(G_A)_{\Sigma^+\Lambda} &= -0.615 \pm 0.072 \ , \\ (G_V)_{\Sigma^+\Lambda} &= 0 \text{ by assumption (no } \Sigma^0\text{-}\Lambda\text{-}\Lambda' \text{ mixing)} ,\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}(G_A)_{\Sigma^-\Lambda} &= 0.608 \pm 0.030 \ , \\ (G_V)_{\Sigma^-\Lambda} &= 0 \text{ by assumption (no } \Sigma^0\text{-}\Lambda\text{-}\Lambda' \text{ mixing)} .\end{aligned} \tag{3.6}$$

For the β decay $n \to pe^-\bar{\nu}$, we obtain,[15] using $X_{np}{}^{-1}$ from Table I and Eq. (3.2),

$$\begin{aligned}(G_V)_{np} &= f_{np} \cos\theta_V = \cos\theta_V = 0.974 \pm 0.002 \ , \\ (G_A)_{np} &= 1.217 \pm 0.009 .\end{aligned} \tag{3.7}$$

Thus we find that $\tau(n \to pe^-\bar{\nu}) = 975 \pm 12$ sec, which implies[16] that the radiative correction to G^β in O^{14} decay is $(3 \pm 1)\%$, in good agreement with the ra-

TABLE I. Axial-vector coupling constant to vector coupling constant ratios from experiment [a] for three hyperon semileptonic decays.

Decay process	$X^{-1} \equiv (G_A/G_V)$ ratio
$\Sigma^- \to ne^-\bar{\nu}$	$\pm(0.435 \pm 0.035)$
$n \to pe^-\bar{\nu}$	1.250 ± 0.009
$\Lambda \to pe^-\bar{\nu}$	0.658 ± 0.054

[a] The data are taken from Ref. 2.

TABLE II. Hyperon semileptonic branching ratios: (i) from experiment, (ii) from a one-angle Cabibbo fit, and (iii) from broken-SU(3) sum rules in the presence of $\Lambda'(1711)$.

Decay process	Branching ratio (i) [a]	(ii) [a]	(iii)
$\Sigma^- \to ne^-\bar{\nu}$	$(1.082 \pm 0.038) \times 10^{-3}$	1.07×10^{-3}	1.082×10^{-3}
$\Sigma^- \to n\mu^-\bar{\nu}$	$(4.47 \pm 0.43) \times 10^{-4}$	4.95×10^{-4}	4.80×10^{-4}
$\Lambda \to pe^-\bar{\nu}$	$(8.13 \pm 0.29) \times 10^{-4}$	8.13×10^{-4}	7.64×10^{-4}
$\Lambda \to p\mu^-\bar{\nu}$	$(1.57 \pm 0.35) \times 10^{-4}$	1.34×10^{-4}	1.23×10^{-4}
$\Sigma^- \to \Lambda e^-\bar{\nu}$	$(6.04 \pm 0.60) \times 10^{-5}$	6.98×10^{-5}	6.11×10^{-5}
$\Sigma^+ \to \Lambda e^+\nu$	$(2.02 \pm 0.47) \times 10^{-5}$	2.28×10^{-5}	2.00×10^{-5}
$\Xi^- \to \Lambda e^-\bar{\nu}$	$(1.15^{+0.90}_{-0.55}) \times 10^{-3}$	0.46×10^{-3}	0.41×10^{-3}
$\Xi^- \to \Sigma^0 e^-\bar{\nu}$	$<0.5 \times 10^{-3}$	...	0.08×10^{-3}
$\left.\begin{matrix}\Xi^- \to \Lambda e^-\bar{\nu} \\ \Xi^- \to \Sigma^0 e^-\bar{\nu}\end{matrix}\right\}$	$(0.68 \pm 0.22) \times 10^{-3}$	0.55×10^{-3}	0.49×10^{-3}
$\Xi^- \to \Lambda\mu^-\bar{\nu}$	$<1.3 \times 10^{-3}$	...	0.11×10^{-3}
$\Xi^- \to \Sigma^0\mu^-\bar{\nu}$	<0.005	...	9.77×10^{-7}
$\Xi^0 \to \Sigma^+ e^-\bar{\nu}$	$<1.5 \times 10^{-3}$	...	0.24×10^{-3}

$\tau_\Lambda = 2.624 \times 10^{-10}$ sec $\tau_{\Sigma^+} = 0.80 \times 10^{-10}$ sec $\tau_{\Sigma^-} = 1.482 \times 10^{-10}$ sec $\tau_{\Xi^-} = 1.652 \times 10^{-10}$ sec
$\tau_{\Xi^0} = 2.96 \times 10^{-10}$ sec

[a] See Ref. 2.

diative corrections predicted by $SU(2)_L \otimes U(1)$ gauge models.[17]

IV. EVALUATION OF THE AXIAL-VECTOR CABIBBO ANGLE AND THE MASS OF Λ'

With the definition of $(G_A)_{if}$ and Eq. (2.6) we obtain

$$\tan\theta_A = \frac{(G_A)_{\Lambda p}}{f_{n\Lambda}(G_A)_{pn} + (G_A)_{\Sigma^+\Lambda}} \quad . \tag{4.1}$$

On the other hand, Eqs. (2.8) and (2.19) tell us that

$$\tan\theta_A = \frac{(\epsilon'/2)(G_A)_{n\Sigma^-}}{(1-\epsilon'/2)(G_A)_{pn} - \epsilon'(\beta-1)f_{n\Lambda}(G_A)_{\Sigma^+\Lambda}} \,. \tag{4.2}$$

From Eqs. (4.1) and (4.2) we find that

$$\frac{(G_A)_{\Lambda p}}{f_{n\Lambda}(G_A)_{pn} + (G_A)_{\Sigma^+\Lambda}} = \frac{(\epsilon'/2)(G_A)_{n\Sigma^-}}{(1-\epsilon'/2)(G_A)_{pn} - \epsilon'(\beta-1)f_{n\Lambda}(G_A)_{\Sigma^+\Lambda}} \,. \tag{4.3}$$

Now Eq. (4.3) is a function only of the mass of Λ', $(G_A)_{n\Sigma^-}$, $(G_A)_{\Sigma^+\Lambda}$, $(G_A)_{\Lambda p}$, and $(G_A)_{pn}$. Therefore, Λ' can be determined from the data. Use of Eq. (2.15), i.e., the SU(3) mass formula for the $\frac{1}{2}^+$ baryons, then gives ω and thus $f_{n\Lambda} = (\frac{3}{2})^{1/2}\cos\omega$; Eq. (4.1) then yields the value of θ_A. What is the value to be assigned to $(G_A)_{\Sigma^+\Lambda}$? Under our hypothesis of no SU(2) mixing, $|(G_A)_{\Sigma^+\Lambda}| = |(G_A)_{\Sigma^-\Lambda}|$.

Even with SU(2) mixing $|(G_A)_{\Sigma^+\Lambda}| \simeq |(G_A)_{\Sigma^-\Lambda}|$ is expected to hold. Therefore, in the absence of an estimate of the SU(2) mixing, the average of the magnitudes of $(G_A)_{\Sigma^+\Lambda}$ and $(G_A)_{\Sigma^-\Lambda}$ may be the more meaningful quantity to use in Eq. (4.3). We then have

$$(\overline{G}_A)_{\Sigma^+\Lambda} = -0.612 \pm 0.039 = -(\overline{G}_A)_{\Sigma^-\Lambda} \,. \tag{4.4}$$

With the values of $(\overline{G}_A)_{\Sigma^+\Lambda}$, $(G_A)_{np}$, $(G_A)_{\Lambda p}$, $(G_A)_{\Sigma^- n}$, and Eq. (4.3), we obtain, using a computer, the following results:

$$\Lambda' = 1.711 \text{ GeV}, \quad \omega = 6.594^\circ \,, \quad \text{and} \quad \sin\theta_A = 0.220 \pm 0.020 \,. \tag{4.5}$$

We note that most of the error in $\sin\theta_A$ is due to the error in the value of $(\overline{G}_A)_{\Sigma^+\Lambda}$.

Actually, the results for Λ', ω, θ_A are not qualitatively much different even when we use $(G_A)_{\Sigma^+\Lambda} = -0.608$ or -0.615. For $(G_A)_{\Sigma^+\Lambda} = -0.608$ we obtain

$$\Lambda' = 1.681 \text{ GeV} \,, \quad \omega = 6.806^\circ \,, \quad \sin\theta_A = 0.219 \pm 0.020 \,. \tag{4.6}$$

For $(G_A)_{\Sigma^+\Lambda} = -0.615$ we get

$$\Lambda' = 1.745 \text{ GeV} \,, \quad \omega = 6.389^\circ \,, \quad \sin\theta_A = 0.221 \pm 0.020 \,. \tag{4.7}$$

It is interesting to note that the average of the last two results yields $\Lambda' = 1.713$ GeV. Using the values of θ_A and θ_V obtained from the data and the broken-SU(3) sum rules, we can calculate all of the axial-vector matrix elements. The results are given in Table III.

V. PREDICTION OF BRANCHING RATIOS AND G_A/G_V RATIOS FOR HYPERON SEMILEPTONIC DECAY

Using the calculated axial-vector matrix elements listed in Table III and the values of θ_A and θ_V ($\sin\theta_V = 0.227 \pm 0.008$ and $\sin\theta_A = 0.220 \pm 0.02$) we can calculate G_A and G_V (the axial-vector and vector form factors, respectively, at zero four-momentum transfer) for each semileptonic decay. With the help of Nieto's tables, we can then easily determine the partial rates or branching ratios. The results are presented in Table II. The G_A/G_V ratios are determined as well and are presented in Table IV. We emphasize that these branching ratios and G_A/G_V ratios [including τ_Σ-$\Gamma(\Sigma^- \to ne^-\overline{\nu})$ and $X_{\Sigma^- n}{}^{-1}$] in column (iii) of Tables II and IV are not input, because they are *recalculated* using the value of θ_A which depends on the experimental data τ_Σ-$\Gamma(\Sigma^- \to ne^-\overline{\nu})$, $X_{\Sigma^- n}{}^{-1}$, τ_Λ $\Gamma(\Lambda \to pe^-\overline{\nu})$, $X_{\Lambda p}{}^{-1}$, $X_{np}{}^{-1}$, and $\tau_\Sigma + \Gamma(\Sigma^+ \to \Lambda e^+\nu)$ as given in column (i) of these tables. We find that the experimental values are well reproduced except for the small discrepancy in the $\Lambda \to pe^-\overline{\nu}$ decay. We believe that this is due

TABLE III. Axial-vector matrix elements calculated using three different values for the mass of Λ': (i) $\Lambda' = 1.681$ GeV, (ii) $\Lambda' = 1.711$ GeV, and (iii) $\Lambda' = 1.745$ GeV.

Axial-vector matrix element	(i)	(ii)	(iii)
$g_{\Sigma^+\Sigma^0}$	+0.5634	+0.5645	+0.5657
$g_{\Sigma^- n}$	−0.4509	−0.4495	−0.4480
$g_{p\Lambda}$	+0.8941	+0.8912	+0.8883
$g_{p\Sigma^0}$	−0.3189	−0.3178	−0.3168
$g_{p\Lambda'}$	+0.4490	+0.4347	+0.4199
$g_{\Lambda'\Sigma^+}$	+0.6300	+0.6102	+0.5896
$g_{\Xi^0\Xi^-}$	−0.4509	−0.4495	−0.4480
$g_{\Sigma^0\Xi^-}$	+0.8822	+0.8824	+0.8825
$g_{\Lambda\Xi^-}$	+0.0748	+0.0802	+0.0856
$g_{\Lambda'\Xi^-}$	−0.5646	−0.5470	−0.5287
g_{pn}	1.2476	1.2479	1.2481
$g_{\Sigma^+\Lambda}$	−0.6232	−0.6270	−0.6308

to the slight inconsistency between the nominal experimental values of $\tau_\Lambda\, \Gamma(\Lambda \to p e^- \bar{\nu})$ and $X_{\Lambda p}^{-1}$.

VI. BROKEN-SU(3) PARAMETRIZATION OF THE SEMILEPTONIC HYPERON DECAY COUPLINGS

Thanks to the constraint from the C. R. $[\dot{V}_{K^0}, A_{\pi^-}] = 0$, even in the presence of the SU(3)-breaking interaction and the singlet $\frac{1}{2}^+$ baryon Λ', we still find it possible to parametrize all the axial-vector couplings with just two parameters D and F *defined* by Eq. (2.20) (which reduce to their canonical Cabibbo values as ω [the SU(3) mixing angle] vanishes) given the mass of Λ'. In the following we list the parametrization for the observable axial-vector couplings. The parametrization of the vector parts is, of course, obtained by setting $D=0$, changing θ_A to θ_V, and scaling the overall coupling G to an appropriate value:

TABLE IV. Axial-vector coupling constant to vector coupling constant ratios: (i) from experiment, (ii) from a one-angle Cabibbo fit, and (iii) from broken-SU(3) sum rules in the presence of $\Lambda'(1711)$.

Decay process	(i) [a]	(ii) [a]	(iii)
$\Sigma^- \to n e^- \bar{\nu}$	$\pm(0.435 \pm 0.035)$	-0.394	-0.435
$\Lambda \to p e^- \bar{\nu}$	0.658 ± 0.054	0.702	0.709
$n \to p e^- \bar{\nu}$	1.250 ± 0.009	1.250	1.250
$\Xi^0 \to \Sigma^+ e^- \bar{\nu}$	···	···	1.208
$\Xi^- \to \Lambda e^- \bar{\nu}$	···	···	0.064
$\Xi^- \to \Sigma^0 e^- \bar{\nu}$	···	···	1.208
$\Xi^- \to \Xi^0 e^- \bar{\nu}$	···	···	-0.450
$\Sigma^- \to \Sigma^0 e^- \bar{\nu}$	···	···	0.340

[a] See Ref. 2.

$$n \to p e^- \nu:\quad (G\sqrt{2})\cos\theta_A (F+D)\ , \tag{6.1}$$

$$\Sigma^- \to \Lambda e^- \nu:\quad (G\sqrt{2})\cos\theta_A \left[\frac{(\frac{2}{3})^{1/2}}{\gamma}\right] D\ , \tag{6.2}$$

$$\Lambda \to p e^- \nu:\quad (G\sqrt{2})\sin\theta_A \left\{ F + \left[1 - \frac{\frac{2}{3}}{\gamma\cos\omega}\right] D \right\} [(\tfrac{3}{2})^{1/2}\cos\omega]\ , \tag{6.3}$$

$$\Sigma^- \to n e^- \nu:\quad (G\sqrt{2})\sin\theta_A \left[F\left(\frac{1}{\epsilon'} - \frac{\{\sin^2\omega[(\Sigma^0)^2 - (\Sigma^+)^2 + (p)^2 - (n)^2]\}(\beta-1)}{\alpha}\right) + D\left(\frac{1}{\epsilon'(\beta-1)} - \frac{(\beta-1)\sin^2\omega[(\Sigma^0)^2 - (\Sigma^+)^2 + (p)^2 - (n)^2]}{\alpha} - \frac{[(\Sigma^0)^2 - (\Lambda)^2]\sin^2\omega}{\epsilon'}\right)\right]\ , \tag{6.4}$$

$$\Xi^- \to \Xi^0 e^- \nu:\quad (\text{same as } \Sigma^- \to n e^- \nu \text{ with } \sin\theta_A \to \cos\theta_A)\ , \tag{6.5}$$

$$\Xi^- \to \Lambda e^- \nu:\quad (\tfrac{3}{2})^{1/2}\cos\omega\, g_{\Sigma^- n} + g_{\Sigma^+\Lambda}\ , \tag{6.6}$$

$$\Lambda' \to \Sigma^+ e^- \nu:\quad (G\sqrt{2})\cos\theta_A (\tfrac{2}{3})^{1/2} \frac{[(\Sigma^0)^2 - (\Sigma^+)^2 + (p)^2 - (n)^2](\beta-1)\sin\omega}{\alpha} \times \left[F + \left(1 + \frac{[(\Sigma^0)^2 - (\Lambda)^2]}{\epsilon'[(\Sigma^0)^2 - (\Sigma^+)^2 + (p)^2 - (n)^2](\beta-1)}\right) D\right]\ . \tag{6.7}$$

Here

$$\gamma \equiv -\epsilon'(\beta-1)\cos\omega\ ,$$

$$\alpha \equiv \tfrac{1}{3}\{2[(n)^2 + (\Xi^0)^2] - [(\Sigma^0)^2 + 3(\Lambda)^2]\}\ , \tag{6.8}$$

and

$$G \equiv g_{pn}(1/\sqrt{2})(D+F)^{-1} \text{ with } D+F=1\ .$$

With $\Lambda' = 1.711$ we find that $F = 0.319$, $D = 0.681$, and $D/(D+F) = 0.681$. These may be compared with the latest fit[2] to the experimental data with *exact* SU(3) parametrization, i.e., $D/D+F = 0.65 \pm 0.02$, $D = 0.65 \pm 0.02$, and $F = 0.35 \pm 0.02$. We, therefore, have shown that even in the presence of the Λ', we can parametrize the data in terms of the newly defined D and F couplings [Eq. (2.20)]. Since we found that Λ' weakly mixes with Λ, our D and F couplings will not be significantly different from original exact-SU(3) D and F couplings. The value of the D/F ratio of the new couplings is found to be close to the best fit of the exact-SU(3) D/F ratio to the present experimental data which deviate rather significantly from the SU(6) D/F value. Thus, in our formulation, all deviations from SU(6) are mainly due to Λ-Λ' mixing.

VII. STRONG DECAYS AND BRANCHING RATIOS OF THE Λ'

In order to calculate the partial rates of the main strong decays of the Λ', we use the broken-SU(3) decay-rate formula given by[7]

$$\Gamma(\Lambda' \to B + P_\gamma) = \frac{g_{\Lambda' B}{}^2}{8\pi f_{p_\gamma}{}^2}\left(\frac{\Lambda' + B}{\Lambda'}\right)^2 [(\Lambda' - B)^2 - P_\gamma{}^2]p \ , \tag{7.1}$$

where P_γ is a $J^P = 0^-$ meson, f_{p_γ} is the appropriate meson decay constant (we take $f_\pi \simeq 0.132$ GeV, $f_K \simeq 0.157$ GeV), p is the center-of-mass momentum of the final-state baryon, and $g_{\Lambda' B}$ is the appropriate axial-vector matrix element. Our results are presented in Table V. Since the mass of the Λ' is large, the total width of Λ' turns out to be of the order of 1 GeV. This will make the observation of the Λ' difficult. However, there is some indication for the existence of a $J^P = \frac{1}{2}^+$ resonance at 1.75 GeV.[18]

TABLE V. Branching ratios for the strong decay modes of $\Lambda'(1711)$ calculated using a broken-SU(3) decay-rate formula.

Decay mode	Branching ratio
$\bar{K}^0 n$	0.11
$K^- p$	0.12
$\Sigma^+\pi^-$	0.26
$\Sigma^0\pi^0$	0.26
$\Sigma^-\pi^+$	0.25
$\Gamma(\Lambda' \to \text{all}) \cong 1005$ MeV	

VIII. FINAL REMARKS: MIXING FROM THE POINT OF VIEW OF SU(6) $\otimes$ O(3) CLASSIFICATION

If it is insisted that SU(6) $\otimes$ O(3) is the underlying classification group of hadrons, our Λ' will probably belong to the SU(3) singlet of the 70 $L^P = 0^+$ which comprises $(1, \frac{1}{2}^+)$, $(8, \frac{1}{2}^+)$, $(10, \frac{1}{2}^+)$, and $(8, \frac{3}{2}^+)$. The predicted mass of our Λ' lies, in fact, in the range of theoretical expectation.[19] The ground-state baryons belong to the 56 $L^P = 0^+$ which comprises $(8, \frac{1}{2}^+)$ and $(10, \frac{3}{2}^+)$.

The seemingly natural point of view toward the inter-SU(6) $\otimes$ O(3)-multiplet SU(3) mixing will be that the mixings between the multiplets belonging to the *same* L^P dominate over others. Then, for the ground state $\frac{1}{2}^+$ octet, the mixings with the $L^P = 0^+$ $(1, \frac{1}{2}^+)_{70}$, $(8, \frac{1}{2}^+)_{70}$, and $(10, \frac{1}{2}^+)_{70}$ will be relatively important. The simplest argument which favors the mixing with the $(1, \frac{1}{2}^+)_{70}$ treated in this paper will then be that the Λ' lies closest to the ground-state $\frac{1}{2}^+$ octet. It may also be noted that the SU(3) mixings between the two SU(3) multiplets of the *same* multiplicity, such as the 8-8 or 10-10 mixings, will be less important *in the SU(3) sum rules* because of the symmetric appearance of the mixing angles. For example, consider the 8-8 baryon mixings. There are four mixing angles θ_n, θ_Λ, θ_Σ, and θ_Ξ. (θ_n denotes, for example, the n-n' mixing). In the SU(3) sum rules, these mixing angles appear *only* in the combination[20] $\theta_i - \theta_j$ $(i, j = n, \Lambda, \Sigma, \Xi)$. However, in the zeroth approximation we expect[20] that θ_i's are the same. Therefore, even if θ_i itself were large, the *net* effect of this type of mixings in the SU(3) sum rules would be expected to be small. On the contrary, in the 8-1 mixing only θ_Λ appears and its effect will be keenly felt. In this connection, 8-10 mixing could also be important. In this case only θ_Σ and θ_Ξ appear (and we expect in the zeroth approximation $\theta_\Sigma = \theta_\Xi$).

In fact, Lipkin[21] has tried to use the 8-10 mixing to patch up the GMO mass formula for the $\frac{1}{2}^+$ octet and also the deviations observed in Σ/Λ transition ratios by considering the mixing between the ground-state $\frac{1}{2}^+$ octet and the $(10, \frac{1}{2}^+)_{70}$ belonging to the $L^P = 2^+$ SU(6) multiplet. The nonstrange member of this $\frac{1}{2}^+$ decuplet is taken to be $\Delta(1910)$.

However, Matsuda, Oneda, and Takasugi have found[22] that the 8-10 mixing [for the decuplet, one may take the 70 $L^P = 0^+$ $(10, \frac{1}{2}^+)$] does not fix the GMO mass formula for the ground state $\frac{1}{2}^+$ (the correction gives the wrong sign) in our theoretical frameworks (a), (b), and (c), whereas the Λ' can easily do the job as shown in this paper. This suggests that the 8-1 mixing should be more important for the ground-state $\frac{1}{2}^+$ baryons. For the ground-state $\frac{3}{2}^+$ decuplet, the effect of the mixing with 70 $L^P = 0^+$ $(8, \frac{3}{2}^+)$ remains to be studied.

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