ω-ϕ-ψ current-mixing angles and mass formulas from SU(4) spectral-function sum rules

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Spectral-function sum rules for broken SU(4) symmetry are used to obtain the current-mixing angles for the vector mesons ω, ϕ, and $\psi(3095)$ and the mass relations among the 16 vector mesons, from which the masses of the F^* and D^* mesons are determined. In addition, the electron-pair partial widths of the neutral vector mesons are given.

I. INTRODUCTION

It is popularly believed that the new narrow resonances[1] discovered recently are bound states of the "charmed" quark and its antimatter partner. Several authors[2] have already studied the $1\oplus 15$ representation of SU(4) to obtain the hadron mass spectrum by assuming the SU(4)-symmetry-breaking interaction

$$H_{\text{int}} = \epsilon\lambda_8 + \epsilon'\lambda_{15}\,.$$

In the case of SU(3), the nonet vector-meson mass spectrum, the ω-ϕ mixing, and the leptonic decay widths of ρ^0, ω, ϕ can be successfully explained[3] by the two spectral sum rules[4] of Weinberg for which SU(3)-symmetry breaking (particularly for the second sum rule) is assumed to be of the form $(\lambda_8)_{ij}$. Within the context of the gauge-field algebra,[5] one can show that[6] the first sum rule must take a diagonal form so long as a current-mixing model is employed, because the "bare" mass term of the gauge-field Lagrangian density is diagonal in current-mixing models.[7] Also, the algebra of the gauge fields gives equal c-number Schwinger terms both for vector and axial-vector currents, but it yields the current-algebra relations only in current-mixing models.[6]

In this paper, we extend the discussions of Ref. 3 to the SU(4) spectral sum rules with symmetry breaking $\epsilon(\lambda_8)_{ij}+\epsilon'(\lambda_{15})_{ij}$ and with current mixing in mind. We then obtain mass relations among the 16 vector mesons, the ω-ϕ-ψ mixing angles, and the leptonic decay widths of $\rho^0, \omega, \phi, \psi$ mesons.

II. SPECTRAL-FUNCTION SUM RULES

We postulate the ij dependence of the two sum rules as[3]

$$J_{ij} \equiv \int_0^\infty dm^2[m^{-2}\rho_{ij}^{(1)}(m^2)+\rho_{ij}^{(0)}(m^2)] = A\,\delta_{ij}\,, \quad (1)$$

$$K_{ij} \equiv \int_0^\infty dm^2\rho_{ij}^{(1)}(m^2) = B\delta_{ij}+Cd_{8ij}+Dd_{15ij}\,, \quad (2)$$

where $\rho_{ij}^{(1)}$ and $\rho_{ij}^{(0)}$ are the spin-1 and spin-0 spectral functions of the SU(4) currents $\mathcal{J}_\mu^i$ ($i=0,1,\ldots,15$) defined as

$$\int d^4x\, e^{-iq\cdot x}\langle 0|\,T(\mathcal{J}_\mu^i(x)\mathcal{J}_\nu^j(0))\,|0\rangle = i\int_0^\infty \frac{dm^2}{m^2+q^2-i\epsilon}\left[\rho_{ij}^{(1)}(m^2)\left(g_{\mu\nu}+\frac{q_\mu q_\nu}{m^2}\right)+\rho_{ij}^{(0)}(m^2)q_\mu q_\nu\right] + \text{Schwinger terms}\,.$$

Eliminating the unknown coefficients A,B,C,D in (1) and (2), we then obtain

$$J_I = J_S = J_C = J_{CS} = J_{88} = J_{15\,15} = J_{00}\,, \quad (3a)$$

$$J_{08} = J_{0\,15} = J_{8\,15} = 0\,, \quad (3b)$$

$$K_I - K_S = K_C - K_{CS} = \tfrac{3}{4}(K_I - K_{88}) = (\tfrac{3}{2})^{1/2}K_{08}\,, \quad (4a)$$

$$K_I - K_{CS} = 2[(\tfrac{2}{3})^{1/2}K_{08} + (\tfrac{1}{3})^{1/2}K_{0\,15}]\,, \quad (4b)$$

$$K_{00} - K_{15\,15} = \frac{2}{\sqrt{3}}K_{0\,15}\,, \quad (4c)$$

$$K_{08} = \sqrt{3}\,K_{8\,15}\,, \quad (4d)$$

$$K_I + K_{CS} = K_S + K_C = 2K_{00}\,, \quad (4e)$$

where the subscripts I, S, C, CS denote $i=j=1,2,3$, $i=j=4,5,6,7$, $i=j=9,10,11,12$, and $i=j=13,14$, respectively.

In what follows, we consider only the case of vector currents, and we shall use the convention that the electromagnetic current is given as in the fractional-charge assignment of the quarks:

$$\mathcal{J}_\mu^{em} = \mathcal{J}_\mu^3 + \frac{1}{\sqrt{3}}\mathcal{J}_\mu^8 + \frac{\sqrt{2}}{3}\mathcal{J}_\mu^0 - (\tfrac{2}{3})^{1/2}\mathcal{J}_\mu^{15}. \quad (5)$$

Also, we introduce the hypercharge, baryon, and charm currents by

$$Y_\mu = \frac{2}{\sqrt{3}}\mathcal{J}_\mu^8, \quad B_\mu = \frac{2\sqrt{2}}{3}\mathcal{J}_\mu^0, \quad C_\mu = -2(\tfrac{2}{3})^{1/2}\mathcal{J}_\mu^{15}. \quad (6)$$

It should be noted that the symmetry is broken, in the current-mixing model, by certain current-operator terms, so that there are in general different mixing angles associated with different current operators.[7] Within the context of the field-current identity, we must then write the vacuum-to-single-vector-meson matrix elements of Y_μ, B_μ, and C_μ as

$$\langle 0|Y_\mu|\phi\rangle = \frac{m_\phi^2}{f_Y}T_{11}\epsilon_\mu(\phi),$$
$$\langle 0|Y_\mu|\omega\rangle = \frac{m_\omega^2}{f_Y}T_{21}\epsilon_\mu(\omega), \quad (7)$$
$$\langle 0|Y_\mu|\psi\rangle = \frac{m_\psi^2}{f_Y}T_{31}\epsilon_\mu(\psi),$$

$$\langle 0|B_\mu|\phi\rangle = \frac{m_\phi^2}{f_B}T_{12}\epsilon_\mu(\phi),$$
$$\langle 0|B_\mu|\omega\rangle = \frac{m_\omega^2}{f_B}T_{22}\epsilon_\mu(\omega), \quad (8)$$
$$\langle 0|B_\mu|\psi\rangle = \frac{m_\psi^2}{f_B}T_{32}\epsilon_\mu(\psi),$$

$$\langle 0|C_\mu|\phi\rangle = \frac{m_\phi^2}{f_C}T_{13}\epsilon_\mu(\phi),$$
$$\langle 0|C_\mu|\omega\rangle = \frac{m_\omega^2}{f_C}T_{23}\epsilon_\mu(\omega), \quad (9)$$
$$\langle 0|C_\mu|\psi\rangle = \frac{m_\psi^2}{f_C}T_{33}\epsilon_\mu(\psi),$$

where the matrix elements T_{ij} are given by

$$(T_{ij}) = \begin{pmatrix} \cos\theta_Y & \sin\theta_B\cos\varphi_B & \sin\theta_C\sin\varphi_C \\ -\sin\theta_Y\cos\psi_Y & (\cos\theta_B\cos\varphi_B\cos\psi_B - \sin\varphi_B\sin\psi_B) & (\cos\theta_C\sin\varphi_C\cos\psi_C + \cos\varphi_C\sin\psi_C) \\ \sin\theta_Y\sin\psi_Y & (-\cos\theta_B\cos\varphi_B\sin\psi_B - \sin\varphi_B\cos\psi_B) & (-\cos\theta_C\sin\varphi_C\sin\psi_C + \cos\varphi_C\cos\psi_C) \end{pmatrix}. \quad (10)$$

By saturating the relation (3.2) with single vector mesons, we obtain

$$\tan\theta \equiv \frac{m_0}{m_\phi}\tan\theta_Y = \frac{m_\phi}{m_0}\tan\theta_B, \quad \theta_C = \theta_B \quad (11)$$

$$\tan\psi \equiv a\tan\psi_Y = a^{-1}\tan\psi_B, \quad \psi_C = \psi_B \quad (12)$$

$$\cot\varphi_C = \frac{\alpha + \beta\tan\varphi_B}{\beta + \gamma\tan\varphi_B}, \quad (13)$$

where

$$m_0 = m_\omega\left(\frac{1 + a^2\tan^2\psi}{1 + a^{-2}\tan^2\psi}\right)^{1/4},$$

$$\alpha = 1 + a^4\tan^2\psi + \frac{(1 + a^2\tan^2\psi)^{3/2}}{(1 + a^{-2}\tan^2\psi)^{1/2}}\tan^2\theta,$$

$$\beta = ab(a^2 - 1)\tan\psi,$$

$$\gamma = a^2b^2/\cos^2\psi,$$

$$b^2 = 1 + \frac{m_0^2}{m_\phi^2}\tan^2\theta,$$

and

$$a = \frac{m_\psi}{m_\omega}.$$

Note that, in the limit where the angle $\psi \to 0$, the $\psi(3.095)$ meson gets decoupled from the ω and ϕ mesons, i.e., $m_0 = m_\omega$, and (11) becomes the usual mixing angle[3,7] of the SU(3) symmetry. By saturating the rest of the relations in (3) and (4) again by vector dominance, i.e., by using (7)–(9) and[8]

$$\langle 0|\mathcal{J}_\mu^3|\rho^0\rangle = \frac{m_\rho^2}{f_\rho}\epsilon_\mu(\rho),$$

$$\left\langle 0\left|\mathcal{J}_\mu^6\right|\frac{K^{*0} + \bar{K}^{*0}}{\sqrt{2}}\right\rangle = \frac{m_{K^*}^2}{f_{K^*}}\epsilon_\mu(K^*),$$

$$\left\langle 0\left|\mathcal{J}_\mu^9\right|\frac{D^{*0} + \bar{D}^{*0}}{\sqrt{2}}\right\rangle = \frac{m_{D^*}^2}{f_{D^*}}\epsilon_\mu(D^*), \quad (14)$$

$$\left\langle 0\left|\mathcal{J}_\mu^{13}\right|\frac{F^{*+} + F^{*-}}{\sqrt{2}}\right\rangle = \frac{m_{F^*}^2}{f_{F^*}}\epsilon_\mu(F^*),$$

we obtain

$$4m_{K^*}^2 - m_\rho^2 = 3m_0^2 \frac{m_\phi^2 + m_0^2\tan^2\theta}{m_0^2 + m_1^2\tan^2\theta}, \tag{15}$$

$$m_{K^*}^2 - m_\rho^2 = m_{F^*}^2 - m_{D^*}^2 = (\tfrac{3}{2})^{1/2} \frac{m_\omega X_B \tan\theta}{(m_0^2 + m_1^2\tan^2\theta)^{1/2}(1 + a^{-2}\tan^2\psi)^{1/2}(\alpha + 2\beta\tan\varphi_B + \gamma\tan^2\varphi_B)^{1/2}}, \tag{16}$$

$$m_{K^*}^2 + m_{D^*}^2 = m_\rho^2 + m_{F^*}^2 = \frac{2b^2 Z_B}{\alpha + 2\beta\tan\varphi_B + \gamma\tan^2\varphi_B}, \tag{17}$$

$$\left(\frac{X_C}{X_B}\right)^2 = \frac{1}{3}\,\frac{\gamma\cot\varphi_C - \beta}{\gamma\tan\varphi_B + \beta}, \tag{18}$$

$$3X_C Z_B / X_B - X_B Z_C / X_C = 2U_{BC}, \tag{19}$$

where

$$X_B = (m_\phi^2 - m_\omega^2) - (m_\psi^2 - m_\omega^2)ab\tan\psi\tan\varphi_B - (m_\psi^2 - m_\phi^2)a^2\tan^2\psi,$$

$$X_C = X_B(\tan\varphi_B \to -\cot\varphi_C),$$

$$Z_B = m_2^2 + 2m_\omega^2\frac{a}{b}(a^4 - 1)\tan\psi\tan\varphi_B + m_3^2\tan^2\varphi_B,$$

$$Z_C = Z_B(\tan\varphi_B \to -\cot\varphi_C),$$

$$U_{BC} = m_4^2 + m_\omega^2\frac{a}{b}(a^2 - 1)\tan\psi(\cot\varphi_C - \tan\varphi_B) - m_5^2\cot\varphi_C\tan\varphi_B,$$

$$m_1^2 = m_\omega^2\,\frac{1 + \tan^2\psi}{1 + a^{-2}\tan^2\psi}, \quad m_2^2 = \frac{1}{b^2}\left[m_\omega^2(1 + a^6\tan^2\psi) + m_\phi^2(\alpha - 1 - a^4\tan^2\psi)\right],$$

$$m_3^2 = m_\psi^2(a^2 + \tan^2\psi), \quad m_4^2 = \frac{1}{b^2}\left[m_\omega^2(a^2 - 1) + (m_\psi^2 - m_\phi^2)(\alpha - 1 - a^4\tan^2\psi)\right],$$

and

$$m_5^2 = m_\psi^2(a^2 - 1)\tan^2\psi.$$

Note that in the limit where the angle ψ goes to zero, both m_0 and m_1 approach m_ω, and (15) reduces to the usual Gell-Mann–Okubo mass relation in SU(3) with the "mass-mixing" angle θ. Also, (16) and (17) show the familiar spacing of the vector mesons in SU(4).

On the other hand, the photon-V couplings (V denotes vector mesons $\rho^0, \omega, \phi, \psi$) as defined by em_V^2/f_V can be related through Eqs. (3a), (5), (7), (8), (9), (11), (12), and (14) by

$$\left(\frac{f_\rho}{f_\phi}\right)^2 = \frac{1}{3}\,\frac{m_\rho^2}{m_\phi^2}\left\{\left(\frac{m_\omega^2\tan^2\theta}{m_0^2 + m_1^2\tan^2\theta}\right)^{1/2} + \left[\frac{2(b^2-1)}{3}\right]^{1/2}\frac{m_\phi}{m_\omega}\left(\frac{1 + a^2\tan^2\psi}{\alpha + 2\beta\tan\varphi_B + \gamma\tan^2\varphi_B}\right)^{1/2} - [2(b^2-1)]^{1/2}\frac{m_\phi}{m_\omega}\left(\frac{1 + a^2\tan^2\psi}{\alpha - 2\beta\cot\varphi_C + \gamma\cot^2\varphi_C}\right)^{1/2}\right\}^2, \tag{20}$$

$$\left(\frac{f_\rho}{f_\omega}\right)^2 = \frac{1}{3}\,\frac{m_\rho^2}{m_\omega^2}\left[\left(\frac{m_\omega^2\tan^2\theta}{m_0^2 + m_1^2\tan^2\theta}\right)^{1/2}(1 + a^{-2}\tan^2\psi)^{-1/2} + \sqrt{2}\,\frac{1 + ab\tan\psi\cot\varphi_C}{(\alpha - 2\beta\cot\varphi_C + \gamma\cot^2\varphi_C)^{1/2}} - \left(\frac{2}{3}\right)^{1/2}\frac{1 - ab\tan\psi\tan\varphi_B}{(\alpha + 2\beta\tan\varphi_B + \gamma\tan^2\varphi_B)^{1/2}}\right]^2, \tag{21}$$

$$\left(\frac{f_\rho}{f_\psi}\right)^2 = \frac{1}{3}\,\frac{m_\rho^2}{m_\psi^2}\left[\sqrt{2}\,\frac{a(a\tan\psi - b\cot\varphi_C)}{(\alpha - 2\beta\cot\varphi_C + \gamma\cot^2\varphi_C)^{1/2}} - \left(\frac{2}{3}\right)^{1/2}\frac{a(a\tan\psi + b\tan\varphi_B)}{(\alpha + 2\beta\tan\varphi_B + \gamma\tan^2\varphi_B)^{1/2}} + \frac{\tan\psi}{(1 + a^{-2}\tan^2\psi)^{1/2}}\left(\frac{m_\omega^2\tan^2\theta}{m_0^2 + m_1^2\tan^2\theta}\right)^{1/2}\right]^2. \tag{22}$$

III. NUMERICAL RESULTS

In all, we have seven relations, namely, (13), (15), (16), (17), (18), and (19) at our disposal. These seven relations are the constraints for the six unknowns $\theta, \psi, \varphi_B, \varphi_C, m_{F^*}^2, m_{D^*}^2$ when m_ρ, m_{K^*}, m_ϕ, m_ω, and m_ψ are taken from experiment.

Relation (15) alone restricts the values of $\tan\theta$ and $\tan\psi$ to the region where $|\tan\theta| \geq 0.82$ and $|\tan\psi| \leq 0.17$ (see Fig. 1).[9]

A systematic search for the four angles $\theta, \psi, \varphi_B, \varphi_C$ which must satisfy simultaneously relations

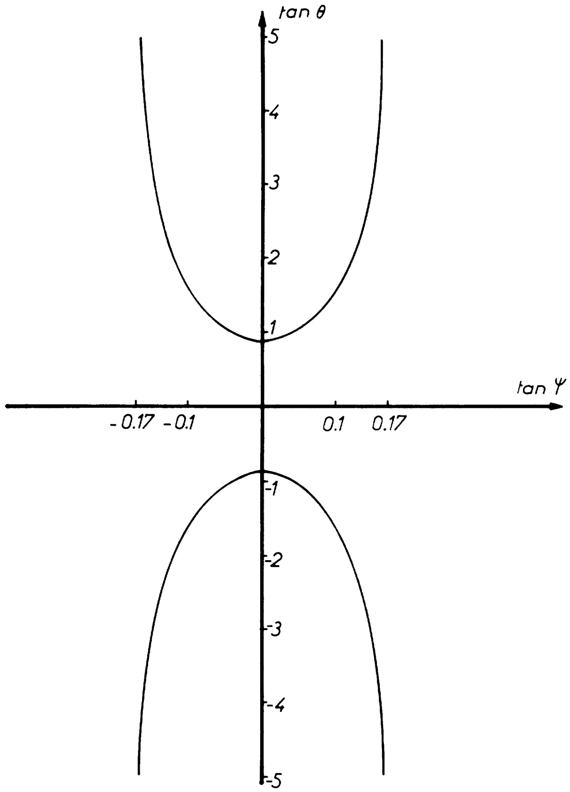

FIG. 1. Domain of variation for the (tan θ, tan ψ) constraint from Eq. (15).

(13), (15), (16), (18), and (19) reveals that (see Fig. 2)

$$|\tan\theta| = 0.83, \quad \tan\psi = \pm 0.001,$$
$$\tan\varphi_B = \mp 0.16, \quad \cot\varphi_C = -0.46,$$

or

$$|\theta| = 39°7', \quad |\theta_B| = |\theta_C| = 31°9', \quad |\theta_Y| = 47°3'$$
$$\varphi_B = \mp 9°10', \quad \varphi_C = -65°3' \tag{23}$$
$$\psi_B = \psi_C = \pm 0°16', \quad \psi_Y \simeq 0°,$$

from which (17) gives

$$m_{F^*} = 2.28 \text{ GeV and } m_{D^*} = 2.23 \text{ GeV}. \tag{24}$$

Notice that the seven relations are invariant under the interchange of (ψ, φ_B) and $(-\psi, -\varphi_B)$ and determine only $\tan^2\theta$. It is impressive that the angle ψ is so small and consistent with that of other work.[10] In addition, $|\tan\theta| = 0.83$ is remarkably close to the experimental mixing angle $\tan\theta_{\text{exp}} = 0.81$. Also, the masses of F^* and D^* are quite consistent with other theoretical estimates.[10]

Having determined the mixing angles, we can calculate the lepton-pair particle widths of the neutral vector mesons. The results are

$$\frac{m_\rho \Gamma(\rho \to e^+e^-)}{m_\phi \Gamma(\phi \to e^+e^-)} = \frac{m_\rho^2}{m_\phi^2}\left(\frac{f_\phi}{f_\rho}\right)^2 = 4.85, \tag{25}$$

$$\frac{m_\rho \Gamma(\rho \to e^+e^-)}{m_\omega \Gamma(\omega \to e^+e^-)} = \frac{m_\rho^2}{m_\omega^2}\left(\frac{f_\omega}{f_\rho}\right)^2 = 7.56, \tag{26}$$

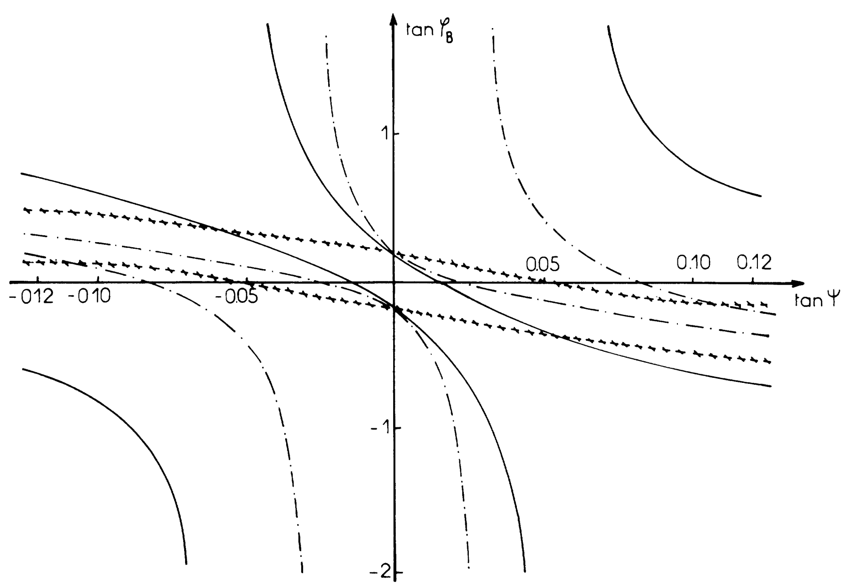

FIG. 2. Plots in the plane (tan ψ, tan φ_B) of the three relations (16), (18), and (19). The dotted-dashed curves are values of (tan ψ, tan φ_B) for which relation (16) is satisfied. The solid curves are values of (tanψ, tan φ_B) for which relation (18) is satisfied. The slashed lines are values of (tan ψ, tan φ_B) for which relation (19) is satisfied.

$$\frac{m_\rho\Gamma(\rho\to e^+e^-)}{m_\psi\Gamma(\psi\to e^+e^-)}=\frac{m_\rho{}^2}{m_\psi{}^2}\left(\frac{f_\psi}{f_\rho}\right)^2=1.03\,. \qquad (27)$$

The ratios (25) and (26) are in good agreement with the experimental values[11] 3.61 ± 0.8 and 8.50 ± 2.85, respectively. However, (27) is larger than the experimental value 0.35 ± 0.084 (see Ref. 12).

Thus we get[13] $m_\rho\Gamma(\rho\to e^+e^-):m_\omega\Gamma(\omega\to e^+e^-):m_\phi\Gamma(\phi\to e^+e^-):m_\psi\Gamma(\psi\to e^+e^-)=7.56:1:1.56:7.34$ instead of the SU(4)-symmetry limit $9:1:2:8$. Experimental ratios are $8.50\pm2.85:1:2.35\pm0.65:24.96\pm8.58$.

IV. CONCLUSIONS

We have determined the ω-ϕ-ψ mixing angles, the masses of F^* and D^* mesons, and the lepton-pair decay widths of the neutral vector mesons. The results are reasonable when compared to experiment or other theoretical work.

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[8]We use the convention that

$$\mathcal{J}_\mu^{D^{*+}}=\frac{1}{\sqrt2}\left(\mathcal{J}_\mu^{11}+i\mathcal{J}_\mu^{12}\right),\quad \mathcal{J}_\mu^{D^{*0}}=\frac{1}{\sqrt2}\left(\mathcal{J}_\mu^{9}+i\mathcal{J}_\mu^{10}\right),$$

$$\mathcal{J}_\mu^{F^{*+}}=\frac{1}{\sqrt2}\left(\mathcal{J}_\mu^{13}+i\mathcal{J}_\mu^{14}\right),\ \text{etc.}$$

[9]Equation (15) can be written in the form

$$\tan^2\theta=\frac{(3m_\phi{}^2+m_\rho{}^2-4m_{K^*}{}^2)\,m_0{}^2}{m_1{}^2\,(4m_{K^*}{}^2-m_\rho{}^2)-3m_0{}^4}$$

and the fact that $\tan^2\theta\ge0$ implies

$$\tan^2\psi\le\frac{4m_{K^*}{}^2-3m_\omega{}^2-m_\rho{}^2}{3m_\psi{}^2+m_\rho{}^2-4m_{K^*}{}^2}=(0.17)^2.$$

[10]For example, S. Borchardt *et al.* of Ref. 2, M. Gourdin of Ref. 2.

[11]The lepton-pair partial widths can be found, for example, in Particle Data Group, Rev. Mod. Phys. 47, 535 (1975), and in M. L. Perl, SLAC Report No. SLAC-Pub-1614, 1975 (unpublished).

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