Broken SU(4) symmetry and the new resonances

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Weinberg's spectral-function sum rules are modified to accommodate broken-symmetry effects of SU(4). With a simple choice of the symmetry-breaking term, the spectral-function sum rules yield the observed vector-meson mass spectrum as well as sum rules for the e^-e^+ decay rates of vector mesons. In particular, a new mass formula, which can be interpreted as the broken-symmetry version of the Schwinger formula, is derived. The agreement with experiment is excellent.

Among many theoretical attempts to explain recently discovered resonances,[1,2] the charm picture[3] of SU(4) seems the simplest. According to this picture[4-6] the ψ particle belongs to the $\underline{15+1}$ representation of SU(4), together with ρ, ω, and φ. Since the ψ particle is much heavier than the rest of the multiplet, the involved symmetry must be a badly broken one. The usual practice of evaluating various parameters in the exact SU(4) limit is a highly ambiguous procedure in this respect.[7]

In order to take into account the possibly large broken effects, a more reliable way may be to employ Weinberg's[8] first sum rule.[5,9] Unfortunately the first sum rule alone does not have much predictive power. In this connection we note that earlier Das, Mathur, and Okubo[10,11] modified Weinberg's second sum rule allowing for symmetry-breaking effects and obtained some interesting results.

In this note we extend the idea to SU(4) and use the modified second sum rule as well as the first. To be more specific, let us write Weinberg's first sum rule in the form[12]:

$$\int dm^2 \frac{\rho^{(ij)}(m^2)}{m^2} = A[\delta_{ij} + (X-1)\delta_{i0}\delta_{j0}] \quad (i,j=0,1,\ldots,15), \tag{1}$$

where $\rho^{(ij)}(m^2)$ are spectral functions of the Källén-Lehmann type appearing in the propagator for vector currents $V_\mu^{(i)}$. Here A and X are some constants independent of the SU(4) suffix i.

Concerning the modified second sum rule, let us assume the following:

$$\int dm^2 \rho^{(ij)}(m^2) = A[D\delta_{ij} + Y\delta_{i0}\delta_{j0} + F(d_{8ij} + \beta d_{15\,ij})] \tag{2}$$

where D, Y, F, and β are some constants. Owing to the existence of the F term, symmetry is now broken.

Saturating only the low-lying states ρ, K^*, ω, φ, and ψ we find from Eq. (1)

$$\begin{gathered} A = \frac{G_\rho{}^2}{m_\rho{}^2} = \frac{G_{K^*}{}^2}{m_{K^*}{}^2} = \sum_i \frac{(G_i^{(8)})^2}{m_i{}^2} \sum_i = \frac{(G_i^{(15)})^2}{m_i{}^2} = \frac{1}{X}\sum_i \frac{(G_i^{(0)})^2}{m_i{}^2}, \\ 0 = \sum_i \frac{G_i^{(8)}G_i^{(0)}}{m_i{}^2} = \sum_i \frac{G_i^{(8)}G_i^{(15)}}{m_i{}^2} = \sum_i \frac{G_i^{(15)}G_i^{(0)}}{m_i{}^2}, \\ (i=\omega, \varphi, \text{ and } \psi) \end{gathered} \tag{3}$$

where

$$(2k_0)^{1/2}\langle 0|V_\mu^i|K_j^*(k)\rangle = G_{K^*}\epsilon_\mu(k)\delta_{ij}, \qquad (2k_0)^{1/2}\langle 0|V_\mu^i|\omega(k)\rangle = G_\omega^{(i)}\epsilon_\mu(k) \text{ etc.} \tag{4}$$

and from Eq. (2)

$$\begin{gathered} G_\rho{}^2 = A\left[D + F\left(\frac{1}{\sqrt{3}} + \frac{\beta}{\sqrt{6}}\right)\right], \qquad (G_{K^*})^2 = A\left[D + F\left(-\frac{1}{2\sqrt{3}} + \frac{\beta}{\sqrt{6}}\right)\right], \\ \sum_i (G_i^{(8)})^2 = A\left[D + F\left(-\frac{1}{\sqrt{3}} + \frac{\beta}{\sqrt{6}}\right)\right], \qquad \sum_i (G_i^{(0)})^2 = A(D+Y), \qquad \sum_i (G_i^{(15)})^2 = A[D - (\tfrac{2}{3})^{1/2}\beta F], \\ \sum_i G_i^{(8)}G_i^{(0)} = \frac{AF}{\sqrt{2}}, \qquad \sum_i G_i^{(8)}G_i^{(15)} = \frac{AF}{\sqrt{6}}, \qquad \sum_i G_i^{(15)}G_i^{(0)} = \frac{AF}{\sqrt{2}}\beta. \end{gathered} \tag{5}$$

The parameters F and D are not independent, but can be expressed as

$$F=\frac{2}{\sqrt{3}}\frac{1}{A}(G_\rho^2-G_{K^*}^2)$$
$$=-\frac{2}{\sqrt{3}}(m_{K^*}^2-m_\rho^2), \quad (6)$$
$$D=-\frac{F}{\sqrt{6}}\beta+\frac{1}{3A}(2G_{K^*}^2+G_\rho^2)$$
$$=\tfrac{1}{3}[m_\rho^2+2m_{K^*}^2+\sqrt{2}\,\beta(m_{K^*}^2-m_\rho^2)]\,. \quad (7)$$

After eliminating coupling constants altogether, we find the nontrivial constraint for vector-meson masses

$$\sigma^2(m_i^2-E^{(15)})+\epsilon^2(m_i^2X-E^{(0)})+\lambda^2(m_i^2-E^{(8)})+2\epsilon\lambda\sigma$$
$$=(m_i^2-E^{(15)})(m_i^2-E^{(8)})(m_i^2X-E^{(0)}), \quad (8)$$
$$(i=\omega,\ \varphi,\ \text{and}\ \psi)$$

where

$$E^{(8)}=D+F\left(-\frac{1}{\sqrt{3}}+\frac{\beta}{\sqrt{6}}\right),$$
$$E^{(15)}=D-(\tfrac{2}{3})^{1/2}\beta F,\quad E^{(0)}=D+Y, \quad (9)$$
$$\sigma=\frac{F}{\sqrt{2}},\quad \epsilon=\frac{F}{\sqrt{6}},\quad \lambda=\frac{F\beta}{\sqrt{2}}.$$

For given m_ρ^2 and $m_{K^*}^2$, Eq. (8) determines the ω, φ, and ψ masses in terms of β, X, and Y, or vice versa.[13]

Choosing

$$X=0.9322,\quad Y=-2605\ \text{MeV}^2,\quad \beta=21.49, \quad (10)$$

we can reproduce the physical masses of ω, φ, and ψ.[14] (This model will be referred to as model I.) Note that since the above value of Y gives $|Y/D|\le 0.000\,89$, Y may be neglected. Note also that the numerical value of β is identical to that needed to fit the quadratic mass formula in the mass mixing model.[5,15] It should be emphasized that our derivation is independent of details of the mixing model, and that Eqs. (1) and (2) specify mass dependence completely.

In order to understand the content of Eq. (8), we discuss two special cases of Eqs. (1) and (2) in detail:

(a) $Y=0$ and $X=1$ (referred to as model A),

(b) $Y=0$ and arbitrary X (referred to as model B).

Since the equations overdetermine the parameters in either case, we can derive sum rules.

In model A the mass constraint, Eq. (8), becomes

$$(m_i^2-m_\rho^2)[m_i^2-(2m_{K^*}^2-m_\rho^2)]\times\left[\beta-\frac{3m_i^2-2m_{K^*}^2-m_\rho^2}{4\sqrt{2}\,(m_{K^*}^2-m_\rho^2)}\right]=0. \quad (11)$$

Noticing that $m_\rho^2<2m_{K^*}^2-m_\rho^2$, we can identify

$$m_\omega^2=m_\rho^2, \quad (12a)$$
$$m_\varphi^2=2m_{K^*}^2-m_\rho^2, \quad (12b)$$
$$m_\psi^2=\tfrac{1}{3}[4\sqrt{2}\,\beta(m_{K^*}^2-m_\rho^2)+2m_{K^*}^2+m_\rho^2]\,. \quad (12c)$$

Equations (12a) and (12b) are known as nonet formulas,[16] and the agreement with experiment is very good. The last equation, (12c), relates m_ψ^2 to β. {Numerically β is 21.9 if the physical mass is substituted for m_ψ. This value is rather close to the actual value [see Eq. (10)].} Eq. (12c) is responsible for the vanishing of $G_\psi^{(8)}$ in this model.

We can improve the situation by not restricting X to be 1. In model B the mass constraint, Eq. (8), becomes

$$x_i^3+x_i^2\tfrac{1}{3}\left[2\alpha-3\sqrt{2}\,\beta\alpha+3\mu^2-\frac{1}{X}(3\mu^2-2\alpha+\sqrt{2}\,\beta\alpha)\right]$$
$$+x_i\frac{\alpha}{9}\left\{(2-3\sqrt{2}\,\beta)\left[3\mu^2-\frac{1}{X}(3\mu^2-2\alpha+\sqrt{2}\,\beta\alpha)\right]-\frac{2\alpha}{X}(3+X+3\beta^2)\right\}-\frac{2\alpha^2}{27X}[3\mu^2(X-1)+8\alpha(1-2\sqrt{2}\,\beta)]=0, \quad (13)$$

where $x_i=m_i^2-\mu^2$ and the abbreviation symbols $\mu^2\equiv\tfrac{1}{3}(4m_{K^*}^2-m_\rho^2)$ and $\alpha\equiv m_{K^*}^2-m_\rho^2$ are used. By eliminating β we can derive two expressions for X. When physical masses are substituted as inputs, one then gives $X=0.9317$ and the other gives $X=0.9333$. (Or, equivalently, $\beta=21.48$ or 21.49.) These values are very close, and also close to the value for β in Eq. (10).

We should like to express this fact in the form of sum rules. Note that in model A the parameter β is related only to m_ψ^2. Therefore, in model B let us assume that m_ω^2 and m_φ^2 can be determined independent of any choice of β. In other words, m_ω^2 and m_φ^2 should be determined from a coefficient of β in Eq. (13). Explicitly,

$$x_i^2\left(\frac{3X+1}{3X}\right)+x_i\left[\mu^2+\frac{1}{X}(\tfrac{2}{3}\alpha-\mu^2)+\frac{2\alpha}{9X}\right]-\frac{32}{27X}\alpha^2=0. \quad (14)$$

After eliminating X we obtain a sum rule,

$$\frac{x_\omega + x_\varphi + \frac{1}{3}(4m_{K^*}{}^2 - m_\rho{}^2)}{\frac{1}{3}(2m_{K^*}{}^2 + m_\rho{}^2)} = -\frac{9}{8}\frac{x_\omega x_\varphi}{m_{K^*}{}^2 - m_\rho{}^2)^2}. \quad (15)$$

$-\frac{9}{8}x_\omega x_\varphi/(m_{K^*}{}^2 - m_\rho{}^2)^2 = 1$ was obtained in model A and is known as the Schwinger formula.[16]

From the above sum rule we find

$$-\frac{9}{8}\frac{x_\omega x_\varphi}{(m_{K^*}{}^2 - m_\rho{}^2)^2} = 1.052 \quad \text{(model B)}.$$

This is in excellent agreement with experiment:

$$-\frac{9}{8}\frac{x_\omega x_\varphi}{(m_{K^*}{}^2 - m_\rho{}^2)^2} = 1.054 \quad \text{experiment}.$$

In order to be consistent with our assumption, $m_\omega{}^2$ and $m_\varphi{}^2$ should also be determined from the rest of Eq. (13).

After eliminating X, we can again construct the following sum rule:

$$\left(3\frac{\mu^2}{\alpha} - 8\right)\left(\frac{x_\omega + x_\varphi}{\alpha}\right)^2 - 3\left(1 + 3\frac{\mu^2}{\alpha}\right)\left(\frac{x_\omega x_\varphi}{\alpha^2}\right)\left(\frac{x_\omega + x_\varphi}{\alpha}\right) + 9\left(1 - \frac{3}{2}\frac{\mu^2}{\alpha}\right)\left(\frac{x_\omega x_\varphi}{\alpha^2}\right)^2 + \left(8 - 21\frac{\mu^2}{\alpha}\right)\left(\frac{x_\omega x_\varphi}{\alpha^2}\right) - 4\left(\frac{4}{3} + \frac{\mu^2}{\alpha}\right)\left(\frac{x_\omega + x_\varphi}{\alpha}\right) = -\frac{16}{9} + \frac{20}{3}\frac{\mu^2}{\alpha}.$$

Substituting an experimental value $(x_\omega + x_\varphi)/\alpha = -0.4867$, we find

$$-\frac{9}{8}\frac{x_\omega x_\varphi}{\alpha^2} = 1.067 \quad \text{(model B)}.$$

This agrees with experiment within 1.3%.

Before we conclude the discussion of masses, we should like to add two comments. One concerns a possible choice of the symmetry-breaking term. Since masses are more accurately known than coupling constants, the discussion of masses could be an important criterion with which to test the validity of available theoretical models. Earlier, for instance, a symmetry-breaking term of the type $\int dm^2 \rho(m^2)/m^4$ was introduced by Oakes and Sakurai[17] in connection with the current-mixing model.[18]

Extending to SU(4), we assume a symmetry-breaking term of the form

$$\int dm^2 \frac{\rho^{(ij)}(m^2)}{m^4} = a\delta_{ij} + b(d_{ij8} + \beta d_{ij15}), \quad (16)$$

where a, b, and β are some constants.

We repeated the calculation using Eq. (16) together with Eq. (1) and found the mass formula (13) again: however, the mass squares are now replaced by their inverse squares. Substituting physical masses for m_ω, m_φ, and m_ψ, we find that the two expressions for X do not give consistent numerical values. Thus the above symmetry-breaking term must be rejected from the discussions of masses alone. Of course our procedure is independent of details of the mixing model.

Let us now briefly discuss hitherto unobserved D^* and F^* particles.[19] After eliminating the coupling constants from the equations

$$A = \frac{G_\rho{}^2}{m_\rho{}^2} = \frac{(G_{D^*})^2}{m_{D^*}{}^2} = \frac{(G_{F^*})^2}{m_{F^*}{}^2},$$

$$(G_{D^*})^2 = D + F\left(\frac{1}{2\sqrt{3}} - \frac{\beta}{\sqrt{6}}\right),$$

$$(G_{F^*})^2 = D + F\left(-\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}}\right),$$

we can determine the masses.

When β of Eq. (12c) is substituted, the following masses are predicted[4,5]:

$$m_{F^*}{}^2 = \tfrac{1}{2}(m_\psi{}^2 + m_\varphi{}^2), \quad m_{D^*}{}^2 = \tfrac{1}{2}(m_\psi{}^2 + m_\rho{}^2) \quad \text{model A.} \quad (17)$$

More precisely, when β of Eq. (10) is substituted, the masses are predicted to be

$$m_{F^*} = 2283.6 \text{ MeV}, \quad m_{D^*} = 2236.3 \text{ MeV}.$$

These results agree with those of the quadratic mass formula.

In order to determine the coupling constants we must solve Eqs. (3) and (4). The ratios of the coupling constants, for instance, are found to be

$$\frac{G_i^{(0)}}{G_i^{(8)}} = a_i \quad \text{and} \quad \frac{G_i^{(15)}}{G_i^{(8)}} = b_i, \quad (18)$$

where

$$a_i=\frac{\sqrt{3}X[\sqrt{2}\alpha+3\beta(\mu^2-m_i^2)]}{4\sqrt{2}\alpha\beta+3(\mu^2-\frac{2}{3}\alpha+Y-Xm_i^2)}, \tag{19a}$$

$$b_i=\frac{\sqrt{2}\alpha+3\beta(\mu^2-m_i^2)}{4\sqrt{2}\alpha\beta+3(\mu^2-\frac{2}{3}\alpha-m_i^2)},\quad i=\omega,\ \varphi,\ \text{and}\ \psi. \tag{19b}$$

$G_i^{(8)}$ is expressible in the form

$$\frac{G_i^{(8)}}{m_i}=\left(\frac{(\vec{a}\times\vec{b})_i}{\sum_i(\vec{a}\times\vec{b})_i}A\right)^{1/2}. \tag{20}$$

Here, without loss of generality, we have chosen $G_i^{(8)}/m_i>0$. Numerically,

$$\begin{bmatrix} \frac{G_\omega^{(8)}}{m_\omega} & \frac{G_\omega^{(0)}}{m_\omega} & \frac{G_\omega^{(15)}}{m_\omega} \\ \frac{G_\varphi^{(8)}}{m_\varphi} & \frac{G_\varphi^{(0)}}{m_\varphi} & \frac{G_\varphi^{(15)}}{m_\varphi} \\ \frac{G_\psi^{(8)}}{m_\psi} & \frac{G_\psi^{(0)}}{m_\psi} & \frac{G_\psi^{(15)}}{m_\psi} \end{bmatrix}=\sqrt{A}\begin{bmatrix} 0.6149 & 0.6529 & 0.4061 \\ 0.7886 & -0.5085 & -0.3175 \\ 7.831\times10^{-4} & -0.4955 & 0.8608 \end{bmatrix}. \tag{21}$$

Restricting ourselves to the quark model for simplicity, we suppose that $\mathcal{P}$, $\mathfrak{N}$, λ, and $\mathcal{P}'$ have charges of $\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$, and Z, respectively. The electromagnetic current V_μ^{em} now takes the form

$$V_\mu^{\text{em}}=V_\mu^{(3)}+\frac{1}{\sqrt{3}}V_\mu^{(8)}+\frac{Z}{\sqrt{2}}(V_\mu^{(0)}-\sqrt{3}V_\mu^{(15)}), \tag{22}$$

and the width for a vector meson V decaying into an e^-e^+ pair is

$$\Gamma_V=\frac{4\pi}{3}\left(\frac{e^2}{4\pi}\right)^2\frac{f_V^2}{m_V^3},\quad V=\rho,\ \omega,\ \varphi,\ \text{and}\ \psi \tag{23}$$

where the coupling constants f_V are defined by

$$\langle 0|V_\mu^{\text{em}}|V(k)\rangle=f_V\epsilon_\mu\frac{1}{(2k_0)^{1/2}}. \tag{24}$$

The relations between f_V and G_i are

$$f_\rho=G_\rho,$$
$$f_i=\frac{1}{\sqrt{3}}G_i^{(8)}+\frac{Z}{\sqrt{2}}(G_i^{(0)}-\sqrt{3}G_i^{(15)}),\quad i=\omega,\varphi,\ \text{and}\ \psi. \tag{25}$$

With these preliminaries, the final results are expressible in terms of widths.

It is well known that Weinberg's first sum rule alone yields a sum rule[9]

$$m_\omega\Gamma_\omega+m_\varphi\Gamma_\varphi+m_\psi\Gamma_\psi=[\tfrac{1}{3}+Z^2\tfrac{1}{2}(3+X)]m_\rho\Gamma_\rho. \tag{26}$$

Previously either a parameter X was assumed to be 1, or otherwise the value was not known. Since X is evaluated here, it is possible to test the above sum rule in a more quantitative manner.

In practice our models satisfy the individual sum rules:

$$m_\omega\Gamma_\omega+m_\varphi\Gamma_\varphi=\tfrac{1}{3}m_\rho\Gamma_\rho, \tag{27a}$$

$$m_\psi\Gamma_\psi=Z^2\tfrac{1}{2}(3+X)m_\rho\Gamma_\rho. \tag{27b}$$

Equation (27a) is a well-known SU(3) sum rule,[10] and Eq. (27b) can be obtained from the Weinberg sum rule under the additional assumption that ψ consists of a $\mathcal{P}'\bar{\mathcal{P}}'$ state only.

Furthermore, in model A we obtain the well-known results[20,5,11]

$$m_\omega\Gamma_\omega=\tfrac{1}{2}m_\varphi\Gamma_\varphi=\tfrac{1}{9}m_\rho\Gamma_\rho, \tag{28a}$$

$$m_\psi\Gamma_\psi=2Z^2m_\rho\Gamma_\rho, \tag{28b}$$

while in models I and B we have

$$m_\omega\Gamma_\omega=\tfrac{1}{9}m_\rho\Gamma_\rho(1.065-0.107Z)^2,$$
$$m_\varphi\Gamma_\varphi=\tfrac{2}{9}m_\rho\Gamma_\rho(0.966+0.062Z)^2, \tag{29a}$$

$$m_\psi\Gamma_\psi=(4.52\times10^{-4}-1.4047Z)^2m_\rho\Gamma_\rho$$
$$\approx1.97Z^2m_\rho\Gamma_\rho. \tag{29b}$$

For a range of Z of interest ($Z=\frac{2}{3}\sim\frac{4}{3}$), numerical results of model I and model B are close to those of model A. The only new feature is that the ω (or φ) width has a mild Z dependence due to the ψ-ω (or ψ-φ) mixing.

The experimental results are[14]

$$m_\omega\Gamma_\omega=(0.59\pm0.13)\ \text{MeV}^2,$$
$$\tfrac{1}{2}m_\varphi\Gamma_\varphi=(0.68\pm0.06)\ \text{MeV}^2,$$
$$\tfrac{1}{9}m_\rho\Gamma_\rho=(0.55\pm0.07)\ \text{MeV}^2,$$
$$m_\psi\Gamma_\psi=(14.86\pm1.86)\ \text{MeV}^2.$$

Both Eqs. (28a) and (29a) agree with the data within the accuracy of experiments.

Equation (28b) does not agree with the data if $Z=\frac{2}{3}$. In order to fit the data we must have $Z=1 \sim\frac{4}{3}$.[5] Exactly the same situation holds for Eq. (29b).

In order to obtain a better agreement with observed masses, it was important to allow X to deviate from 1. However, present experiments on widths do not distinguish the small difference between the $X=1$ and the $X\neq 1$ cases. So far we have not included contributions of higher excited states such as ψ', ρ', and ω'. According to SU(4), the ψ' particle should belong to a member of another $\underline{15}\oplus\underline{1}$ representation of SU(4), together with ρ', ω', and φ'.

Following the idea of Ref. 5, let us suppose that these excited states alone satisfy the sum rules

$$\int dm^2 \frac{\rho^{(ij)}(m^2)}{m^2} = A'[\delta_{ij} + (X-1)\delta_{i0}\delta_{j0}], \tag{1'}$$

$$\int dm^2 \rho^{(ij)}(m^2) = A'[D\delta_{ij} + Y\delta_{i0}\delta_{j0} + F(d_{8ij} + \beta d_{15\,ij})], \tag{2'}$$

where only A' is a new parameter, which may or may not be equal to A. It then follows that these excited states should obey exactly the same mass formulas as those of the lowest states. The observed particles seem to fit roughly the mass formulas of Eq. (12a) and (12c).[5] Here we only mention the results for widths.

Independent of Z (or X) we should have

$$\frac{A}{A'} = \frac{m_\rho \Gamma_\rho}{m_{\rho'}\Gamma_{\rho'}} = \frac{m_\psi \Gamma_\psi}{m_{\psi'}\Gamma_{\psi'}}, \tag{30}$$

where $A = G_\rho{}^2/m_\rho{}^2$ and $A' = G_{\rho'}{}^2/m_{\rho'}{}^2$. From the experimental width[14] we find $m_\psi\Gamma_\psi/m_{\psi'}\Gamma_{\psi'} = 1.83$ and from the ρ' experiments we obtain $0.93 \lesssim A/A' \lesssim 1.85$.[21] Thus the above sum rule seems compatible with the present experiments.

Furthermore, model A predicts

$$\frac{A}{A'} = \frac{m_\rho \Gamma_\rho}{m_{\rho'}\Gamma_{\rho'}} = \frac{m_\psi \Gamma_\psi}{m_{\psi'}\Gamma_{\psi'}} = \frac{m_\omega \Gamma_\omega}{m_{\omega'}\Gamma_{\omega'}} = \frac{m_\varphi \Gamma_\varphi}{m_{\varphi'}\Gamma_{\varphi'}}. \tag{31}$$

Experimental data are not available at present.

In conclusion we should like to stress that the spectral-function approach is a powerful technique to handle a broken symmetry such as SU(4).

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[13]A parameter A does not enter into the discussion on masses.

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$$m_\rho = 770\text{ MeV},\quad m_\varphi = 1019.7\text{ MeV},\quad m_\omega = 782.7\text{ MeV},$$
$$m_\psi = 3095\text{ MeV},\quad m^0_{K^*} = 898.3\text{ MeV},$$

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[19]We follow the notation of Ref. 4 ($D^*_+ = \mathcal{P}'\mathfrak{N}$ and $F^*_+ = \mathcal{P}'\lambda$).

[20]Exact SU(4) symmetry with ideal mixing predicts

$$m^7_\omega \Gamma_\omega = \tfrac{1}{2} m^7_\varphi \Gamma_\varphi = \tfrac{1}{9} m^7_\rho \Gamma_\rho = \frac{1}{18 Z^2}\, m^7_\psi \Gamma_\psi .$$

Here γ is an arbitrary parameter and cannot be fixed from group-theoretical arguments alone. We find that the value of γ chosen in the literature ranges from 1 to -1. Our Eqs. (1) and (2) uniquely yield that γ should be 1.

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