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Possible test of the lepton number as the fourth color*

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We propose to test the hypothesis that the lepton number is a color in two-body lepton-hadron scattering, in particular neutrino-nucleon scattering. Detailed analysis shows that these tests can be done well at small momentum transfer (as small as possible) and high energies (say $E_\nu \geq 3$ GeV and $E_{\bar{\nu}} \geq 5$ GeV for $q^2 \leq -0.1$ GeV). Also, possible implications of this hypothesis for total cross sections are considered. Models and their implications for two-body reactions and vice versa are discussed.

I. INTRODUCTION

The discovery of the narrow resonances and dileptonic processes seems to confirm the existence of charm, a theoretical concept. There is another important hypothesis which still awaits experimental confirmation, namely, the lepton number as the fourth color invented by Pati and Salam.[1] In the meantime some theoretical arguments supporting this hypothesis have appeared.[2,3] A possibility has been shown in Ref. 3 of how the existence of magnetic monopoles in a color-confined theory of fractionally charged quarks might lead to the interpretation of the lepton number as the fourth color. Further, this hypothesis has been until now, in our opinion, the simplest and most appealing idea for the unification of quarks and leptons.

With these points to serve as motivation we want to propose some experimental tests which have direct bearing on the hypothesis and which might be possible at the presently available neutrino energies of about 200 GeV (Fermilab and possibly CERN-Gargamelle). To this end we pick out one of the characteristic features which are shared by all such models which treat the leptonic number as a color, namely, the existence of the exotic particles X carrying both baryonic as well as leptonic number, and we consider processes where X particles can participate in a characteristic way, that is, in a different way than all other "conventional" particles do. What is it? It is the ability of X particles to appear as a resonant state of an antilepton-baryon pair, lepton-antibaryon, etc., the manifestation of which will be most distinctive in appropriate two-body reactions. We are thus led to consider elastic and quasielastic lepton-nucleon scatterings, in particular neutrino-(anti-)nucleon scatterings.

We summarize our results:

In the conventional weak-interaction theory, that is, the W bosons are the only mediating particles, the differential cross sections

$$\frac{d\sigma_\nu}{dq^2} \text{ for } \nu + n \to e^- + p$$

and

$$\frac{d\sigma_{\bar{\nu}}}{dq^2} \text{ for } \nu + p \to e^+ + n$$

become energy independent for large E_ν (neutrino energy) and $E_{\bar{\nu}}$ (antineutrino energy).[4] Detailed analysis shows that, for example, for $q^2 = -0.1$ GeV^2 (q^2 is momentum transfer squared) the asymptotic region is reached already at $E_\nu \geqslant 3$ GeV or $E_{\bar{\nu}} \geqslant 5$ GeV. At these energies $d\sigma_\nu/dq^2$ and $d\sigma_{\bar{\nu}}/dq^2$ are about 95% of the asymptotic differential cross section, that is, they remain practically constant for increasing energies. On the other hand, the possible X contribution to appropriate two-body reactions, where the X particle can appear in the s channel, the differential cross section contains a pole at $s^2 = m_X{}^2$ (s^2 is the c.m. system energy, m_X is the mass of X), implying increasing cross section for increasing energies and revealing resonant structure, thus enabling one to detect the X contribution. Arguments are given that possible damping effects of the form factors of the lepton-hadronic currents are not strong enough to suppress the resonant structure.

Some models and other contributions suggest that possible resonant structure might appear already at an incident lepton energy of about $E = 80$ GeV. Then we discuss various models and their experimental implications. Further, we discuss two questions of a general nature and their experimental consequences:

(a) Which pair belongs to the same representation, quark-lepton or quark-antilepton?

(b) Do the electron and muon belong to the same color or not?

The second question is related to whether electron number and muon number are separately conserved or not.

This paper is organized as follows: In Sec. II we fix our notation and give an estimate of kinematical regions where one should start to look for the possible effects of X particles. For this end, we consider antineutrino-proton and neutrino-neutron quasielastic scatterings ($\bar{\nu}+p\to e^+ +n$ and $\nu+p\to e^- +p$) in the framework of the conventional theory, that is, using only W bosons. In Sec. III we give a qualitative treatment of the possible X particle effects to the above processes. In Sec. IV we consider the resulting cross section and try to obtain a vague idea at what energies one possibly would see the X contribution. In Sec. V models and their implications for experiments and vice versa are given.

II. QUASIELASTIC NEUTRINO-(ANTI-)NUCLEON SCATTERING WITHOUT X PARTICLES AND ESTIMATE OF THE ASYMPTOTIC REGION

It is well known that in the conventional theory of weak interaction the differential cross sections for $\nu+n\to e^- +p$ and for $\bar{\nu}+p\to e^+ +n$ become independent of the incident neutrino energy E_ν and antineutrino energy $E_{\bar{\nu}}$, respectively, when E_ν and $E_{\bar{\nu}}$ become very large. Also the total cross sections of these quasielastic reactions approach an asymptotic value for $E_\nu, E_{\bar{\nu}}\to\infty$. In this section we will estimate the asymptotic region, that is, at what value of E_ν ($E_{\bar{\nu}}$) the asymptotic region is reached.

Let us fix the kinematics and notations:

$$l+n\to l'+n', \tag{1}$$

with momenta K_1, P_1, K_2, and P_2 for l, n, l', and n', respectively, and l,n (l',n') are incident (outgoing) lepton (or antilepton), nucleon.

The Lagrangian density for this process is

$$L_W(x)=g_W(J_\mu(x)+l_\mu(x))W_\mu(x)+\text{H.c.},$$

where $J_\mu(x)$ is the hadronic and $l_\mu(x)$ is the leptonic weak current and $W_\mu(x)$ is the intermediate W boson with mass m_W. The hadronic current $J_\mu(x)$ can be represented in quark fields, q_1 (proton quark) and q_2 (neutron quark), as $J(x)=iq_1(x)\gamma_\mu(1+\gamma_5)q_2(x)$ and has the following form factors:

$$\langle n'|J_\mu(0)|n\rangle=\frac{i}{(2\pi)^3}\left(\frac{M^2}{E_1E_2}\right)^{1/2}[\gamma_\mu F_V(q^2)-\sigma_{\mu\nu}q_\nu F_m(q^2)-iq_\mu H_V(q^2)+\gamma_\mu\gamma_5F_A(q^2)+iq_\mu\gamma_5F_T(q^2)-\gamma_5\sigma_{\mu\nu}q_\nu F_P(q^2)], \tag{2}$$

where M is the nucleon mass, E_1 (E_2) is the energy of the incident (outgoing) nucleon, and $q=p_1-p_2$.

The transition amplitude A_W is given by

$$\begin{aligned}A_W&=g_W{}^2\frac{g_{\mu\nu}-q_\mu q_\nu/m_W{}^2}{m_W{}^2-q^2}\langle n'|J_\mu(0)|n\rangle\langle l'|l_\nu(0)|l\rangle\\&\approx G\frac{1}{1-q^2/m_W{}^2}\langle n'|J_\mu(0)|n\rangle\langle l'|l_\mu(0)|l\rangle,\end{aligned} \tag{3}$$

where $G=g_W{}^2/m_W{}^2$ is the Fermi coupling constant and $q_\mu q_\nu/m_W{}^2$ is neglected. We can justify this neglect because $q_\mu q_\nu$ is of the order of lepton mass in the final result. The high-energy cross sections $d\sigma_\nu/dq^2$ and $d\sigma_{\bar{\nu}}/dq^2$ with definite q^2 are[4]

$$\frac{d\sigma_\nu}{dq^2}=\frac{d\sigma_{\bar{\nu}}}{dq^2}=\frac{G^2}{2\pi}\frac{1}{(1-q^2/m_W{}^2)^2}F^W(q^2),\quad\text{for } E_\nu, E_{\bar{\nu}}\to\infty \tag{4}$$

with

$$F^W(q^2)=|F_V(q^2)|^2+|F_A(q^2)|^2-q^2|F_m(q^2)|^2-q^2|F_T(q^2)|^2.$$

At $q^2=0$ this result is exact for every E_ν and $E_{\bar{\nu}}$. For $q^2\neq0$ it is an approximation, and the larger $|q^2|$ is the larger E_ν and $E_{\bar{\nu}}$ need be in order that the approximation be good. To estimate the rate of approximation as a function of q^2 one has to know the form factors. In our estimate we have used following form factors[5]:

$$\begin{aligned}F_V(q^2)&=\frac{1}{(1-q^2/m_V{}^2)^2},\\F_m(q^2)&=\frac{3.71}{2M}F_V(q^2),\\F_A(q^2)&=-1.248F_V(q^2),\\F_T(q^2)&=F_P(q^2)=H_V(q^2)=0,\\&m_V=0.84\text{ GeV},\quad M=\text{nucleon mass}.\end{aligned} \tag{5}$$

For the mass of the W, m_W, we have considered two cases, namely $m_W=\infty$ and $m_W=2M$.

We calculate $R(E)$ and $r(E)$, defined as

$$\begin{aligned}R(E)&=\frac{d\sigma(E)/dq^2}{d\sigma(E=\infty)/dq^2},\\r(E)&=\frac{\sigma(E)}{\sigma(E=\infty)},\end{aligned} \tag{6}$$

where E is the incident neutrino or antineutrino energy, as a function of E width q^2 as a parameter for $\nu+n\to p+l^-$ and $\bar{\nu}+p\to n+l^+$. $\sigma(E)$ is the quasielastic total cross section. The results are listed in Tables I and II (see Figs. 1 and 2).

From the tables we see that, for example, for $-q^2\leq0.3$ GeV $d\sigma_\nu/dq^2$ ($d\sigma_{\bar{\nu}}/dq^2$) reaches the asymptotic behavior at about $E_\nu\approx3$ GeV ($E_{\bar{\nu}}\approx5$ GeV). For the quasielastic total cross section σ_ν shows asymptotic behavior already at $E_\nu\approx1$ GeV, whereas

TABLE I. The numerical values of $R(E)$ defined by (6) are listed as a function of the incident neutrino (antineutrino) energy E in GeV, using the momentum transfer q^2 in GeV^2 and the mass of the intermediate vector boson m_W as parameters. The first (second) column is for $\nu+p\to e^-+p$ ($\bar\nu+p\to n+e^+$). The first (second) row of $R(E)$ is for $m_W=\infty$ ($m_W=2M$).

	$\nu+n\to e^-+p$	$\bar\nu+p\to n+e^+$
$q^2=0$	$R(E)=0$	$R(E)=0$
$-q^2=0.1$	$R(1)=\left\{\begin{matrix}1.19\\1.15\end{matrix}\right.$	$R(1)=\left\{\begin{matrix}0.76\\0.76\end{matrix}\right.$
	$R(3)=\left\{\begin{matrix}1.05\\1.05\end{matrix}\right.$	$R(3)=\left\{\begin{matrix}0.92\\0.92\end{matrix}\right.$
	$R(5)=\left\{\begin{matrix}1.04\\1.04\end{matrix}\right.$	$R(5)=\left\{\begin{matrix}0.95\\0.95\end{matrix}\right.$
	$R(10)=\left\{\begin{matrix}1.02\\1.02\end{matrix}\right.$	$R(10)=\left\{\begin{matrix}0.98\\0.98\end{matrix}\right.$
$-q^2=0.2$	$R(1)=\left\{\begin{matrix}1.38\\1.26\end{matrix}\right.$	...
	$R(3)=\left\{\begin{matrix}1.09\\1.09\end{matrix}\right.$	$R(3)=\left\{\begin{matrix}0.86\\0.86\end{matrix}\right.$
	$R(5)=\left\{\begin{matrix}1.05\\1.05\end{matrix}\right.$	$R(5)=\left\{\begin{matrix}0.91\\0.91\end{matrix}\right.$
	$R(10)=\left\{\begin{matrix}1.03\\1.03\end{matrix}\right.$	$R(10)=\left\{\begin{matrix}0.973\\0.973\end{matrix}\right.$
$-q^2=0.3$	$R(1)=\left\{\begin{matrix}1.37\\1.37\end{matrix}\right.$	...
	$R(3)=\left\{\begin{matrix}1.12\\1.12\end{matrix}\right.$	$R(3)=\left\{\begin{matrix}0.80\\0.80\end{matrix}\right.$
	$R(5)=\left\{\begin{matrix}1.09\\1.09\end{matrix}\right.$	$R(5)=\left\{\begin{matrix}0.88\\0.88\end{matrix}\right.$
	$R(10)=\left\{\begin{matrix}1.05\\1.05\end{matrix}\right.$	$R(10)=\left\{\begin{matrix}0.94\\0.94\end{matrix}\right.$
$-q^2=1$	$R(1)=\left\{\begin{matrix}1.8\\1.6\end{matrix}\right.$	$R(1)=\left\{\begin{matrix}0.15\\0.17\end{matrix}\right.$
	$R(3)=\left\{\begin{matrix}1.17\\1.17\end{matrix}\right.$	$R(3)=\left\{\begin{matrix}0.58\\0.58\end{matrix}\right.$
	$R(5)=\left\{\begin{matrix}1.10\\1.10\end{matrix}\right.$	$R(5)=\left\{\begin{matrix}0.75\\0.75\end{matrix}\right.$
	$R(10)=\left\{\begin{matrix}1.05\\1.05\end{matrix}\right.$	$R(10)=\left\{\begin{matrix}0.87\\0.87\end{matrix}\right.$

for $\sigma_{\bar\nu}$ one has to go beyond $E_{\bar\nu}=5$ GeV. Another interesting phenomena is that for $E_{\nu,\bar\nu}\geq 3$ GeV the difference between the results for $m_W=\infty$ and $m_W=2M$ vanishes.

The conclusion is that any experimental deviation from the asymptotic behavior at E larger than 5 GeV, say, must be due to other interactions than that mediated by the W boson. In the next section we give an estimate of what the deviation due to the X particle looks like.

TABLE II. Numerical values of $r(E)$ defined by (6) are listed as a function of incident neutrino (antineutrino) energy E in GeV. See caption of Table I.

$\nu+n\to e^-+p$	$\bar\nu+p\to e^++n$
$r(1)=\left\{\begin{matrix}1.12\\1.13\end{matrix}\right.$	$r(1)=\left\{\begin{matrix}0.43\\0.50\end{matrix}\right.$
$r(3)=\left\{\begin{matrix}1.07\\1.09\end{matrix}\right.$	$r(3)=\left\{\begin{matrix}0.73\\0.78\end{matrix}\right.$
$r(5)=\left\{\begin{matrix}1.04\\1.05\end{matrix}\right.$	$r(5)=\left\{\begin{matrix}0.82\\0.86\end{matrix}\right.$
$r(10)=\left\{\begin{matrix}1.02\\1.03\end{matrix}\right.$	$r(10)=\left\{\begin{matrix}0.91\\0.93\end{matrix}\right.$

III. X-PARTICLE CONTRIBUTION

The general form of the interaction of the X (Ref. 6) particle with leptons and quarks L_X is given by[1]

$$L_X=g_X(\bar l(y)\Gamma_\mu q(y))X_\mu(y)+\text{H.c.} \tag{7}$$

g_X is the coupling constant, $l(y)$ [$q(y)$] is the lepton field [quark field], and Γ_μ is an appropriate Dirac matrix. In the basic model of Pati and Salam $\Gamma_\mu=\gamma_\mu$, but as already mentioned by Pati and Salam there might be also an axial-vector X particle. In our calculation we will take $\Gamma_\mu=\gamma_\mu(1+\gamma_5)$ for simplicity. This choice of Γ_μ does not affect the main feature relevant to our consideration. From L_X we see the unconventional feature of X; it couples to a "lepton-quark current." In two-body reactions (1) X can appear in the u channel as well as in the s channel, depending on the leptons and quarks concerned. In this paper we concentrate on the s-channel effect of X [see Fig. 3(b)].

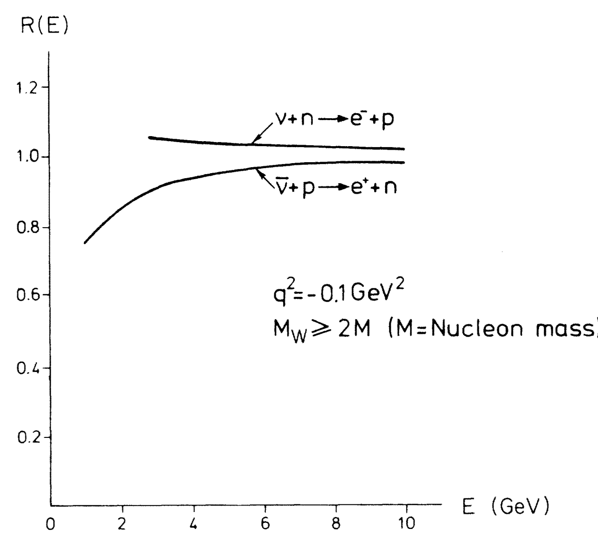

FIG. 1. $R(E)$ defined in Eq. (6) is plotted as a function of $E_{\nu,\bar\nu}$ for $q^2=-0.1$ GeV^2.

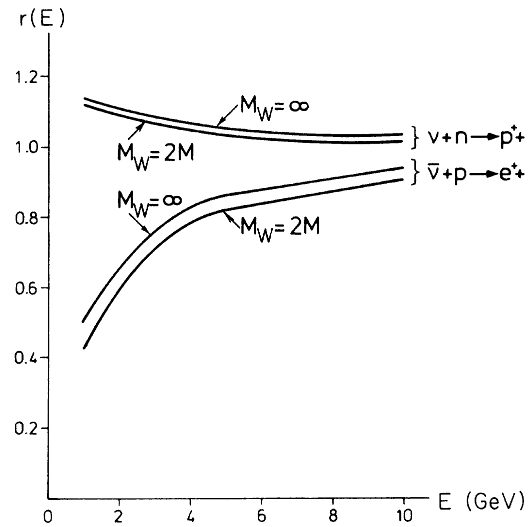

FIG. 2. $r(E)$ defined in Eq. (6) is plotted as a function of $E_{\nu,\bar{\nu}}$.

The amplitude A_X due to this s channel is

$$A_X = g_X^2 G^X_{\mu\nu} M^X_{\mu\nu}$$
$$= g_X^2 \frac{1}{m_X^2 - s^2} M^X_{\mu\mu}, \tag{8}$$

with

$$G^X_{\mu\nu} = \frac{g_{\mu\nu} - s_\mu s_\nu / m_X^2}{m_X^2 - s^2},$$
$$M^X = \langle l'n' | J^{LB\dagger}_\mu(0) | 0 \rangle \langle 0 | J^{LB}_\mu(0) | ln \rangle,$$
$$s = (P_1 + K_1), \quad s^2 > 0,$$

with

$$J^{LB}(0) \equiv \bar{l}(0) \Gamma_\mu q(0).$$

We have neglected $s_\mu s_\nu / m_X^2$, because this term

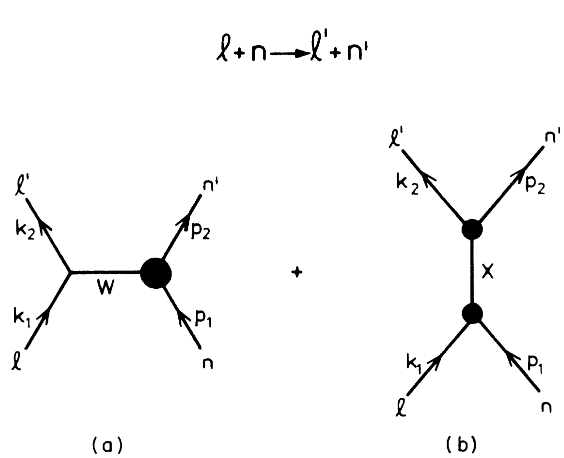

FIG. 3. (a) t-channel contribution of W boson. (b) s-channel contribution of X boson.

will be, in the final result, at most of the order of m^2/m_X^2 (m is the nucleon mass).

We parameterize the matrix element of the exotic current J^{LB} with form factors:

$$\langle 0 | J^{LB}_\mu(0) | ln \rangle \sim \gamma_\mu F^{LB}_1(s^2) + \sigma_{\mu\nu} s_\nu F^{LB}_2(s^2) + \cdots$$

Thus $M^X_{\mu\mu}$ is a function of these form factors:

$$M^X_{\mu\mu} = M^X_{\mu\mu}(F^{LB}_i(s^2), K_1, K_2, P_2). \tag{9}$$

$F^{LB}_i(s^2)$ are "exotic" form factors, of which nothing is known. We know, however, from (9) that $M^X_{\mu\mu}$ can have poles only if $F^{LB}_i(s^2)$ have poles. Therefore, the behavior of $F^{LB}_i(s^2)$ at high s^2, or correctly at those s^2 in which we are interested, is very relevant for the X contribution. $F^{LB}_i(s^2)$ might decrease strongly enough to compensate the effect of the X propagator. We have two arguments that this is not the case:

(a) We know that the electromagnetic form factors of the hadrons can be well represented by poles of the vector mesons, in particular the ρ meson, if the momentum transfer is not large, say $|q^2| \lesssim m_\rho^2$, where m_ρ is the ρ-meson mass. It is reasonable to assume that $F^{LB}(s^2)$ are produced via the same mechanism; a nucleon emits a virtual particle and becomes a lepton. This virtual particle must have leptonic and baryonic number. If the X mesons are only such particles, the form factors are constant and everything is all right. If there are other particles than X mesons, we would have form factors like the electromagnetic

$$F^{LB}_i(s^2) \sim \frac{1}{1 - s^2/m_a^2}, \tag{10}$$

where m_a is the mass of such a particle.

Now these particles are as exotic as X mesons and it is reasonable to assume that m_a is of the same order as m_X. Because we are interested in the energy region about $\frac{1}{2}m^2 \leq s^2 \leq m_X^2$, $F^{LB}_i(s^2)$ amplify the resonant effect of X.

(b) In our second argument we deal with the form factors as well as with the group structure of the hadronic currents J^X_μ which we obtain via a Fierz transformation. For this end we first consider two-body lepton-quark reactions, that is, n and n' of the reaction (1) are quarks (or antiquarks). We will then assume that the results are valid also for lepton-hadron processes, that is, n and n' are physical baryons.

As in the conventional case we treat the lepton fields as bare fields. For the quark field we use the relation $\langle n | \bar{q} | 0 \rangle \neq 0$ and $\langle n | \bar{q} | m \rangle = 0$, where n is a one-quark (a one-antiquark) state, and m is any state except the vacuum 0. With these assumptions $M^X_{\mu\mu}$ can be represented as follows:

$$M^X_{\mu\mu} = \langle l'n' | \bar{q}\Gamma_\mu l | 0\rangle\langle 0 | \bar{l}\Gamma_\mu q | ln\rangle$$
$$= \sum_{m=0}^{\infty} \langle l'n' | \bar{q}\Gamma_\mu l | m\rangle\langle m | \bar{l}\Gamma_\mu q | ln\rangle$$
$$= \langle l'n' | (\bar{q}\Gamma_\mu l)(\bar{l}\Gamma_\mu l) | ln\rangle, \tag{11}$$

with

$$\sum_{m=0}^{\infty} | m\rangle\langle m | = 1.$$

Using the Fierz transformation for $\Gamma_\mu = \gamma_\mu(1+\gamma_5)$, $(\bar{q}\Gamma_\mu l)(\bar{l}\Gamma_\mu q) = (\bar{q}\Gamma_\mu q)(\bar{l}\Gamma_\mu l)$, and the notation $J^X_\mu \equiv \bar{q}\Gamma_\mu q$ and $l^X_\mu \equiv \bar{l}\Gamma_\mu l$ (we omit the quark and lepton indices) we obtain

$$M^X_{\mu\mu} = \langle l' | l^X_\mu | l\rangle\langle n' | J^X_\mu | n\rangle. \tag{12}$$

Our main assumption is, as already mentioned, that this result is valid also for the case in which n and n' are real hadrons.

Because of the bare character of the leptonic fields $\langle l' | l^X_\mu | l\rangle$ can be calculated. For $\langle n' | J^X_\mu | n\rangle$ we parameterize with the form factors $F^X(q^2)$:

$$\langle n' | J^X_\mu(0) | n\rangle = \frac{i}{(2\pi)^3}\left(\frac{M^2}{E_1E_2}\right)^{1/2}\bar{U}(P_2)[\gamma_\mu F^X_V(q^2) - \sigma_{\mu\nu}q_\nu F^X_m(q^2) - iq_\mu H^X_V(q^2) + \gamma_\mu\gamma_5 F^X_A(q^2)$$
$$+ iq_\mu\gamma_5 F^X_T(q^2) - \gamma_5\sigma_{\mu\nu}q_\nu F^X_P(q^2)]U(P_1). \tag{13}$$

Combining (12) and (13) we obtain

$$M^X_{\mu\mu} = M^X_{\mu\mu}(F^X_i(q^2), K_1, K_2, P_2). \tag{14}$$

Now let us compare (9) and (14). In (9) M^X might have poles only in s^2, whereas in the form of (14) they occur only in q^2. The conclusion is that neither $F^X_i(q^2)$ nor $F^{LB}_i(s^2)$ have poles. This result is at first sight surprising, but it is related to the argument given in (a): If X is the only lepton-baryonic meson, $F^X_i(q^2)$ and $F^{LB}_i(s^2)$ are constants and the above result is obvious. If $F^{LB}_i(s^2)$ have form factors of the form given in (10), then the above result is compatible with $m_a \gg m_X$ so that $F^{LB}_i(s^2)$ do not become operative in the region $s^2 \le m_X{}^2$ which we are interested in.

In the following we will use $M^X_{\mu\mu}$ in the form of (14) in order to keep analogy to the W boson contribution given in (3). Then the asymptotic behavior of the differential cross section $d\sigma_l/dq^2|_X$ due to the X particle, when l is a (anti-) neutrino can be written as

$$\lim_{E_l\to\infty} \frac{d\sigma_l}{dq^2}\bigg|_X = \frac{q_X{}^4}{2\pi}\frac{1}{(m_X{}^2 - s^2)^2}F^X(q^2), \tag{15}$$

with

$$F^X(q^2) = |F^X_V(q^2)|^2 + |F^X_A(q^2)|^2 - q^2|F^X_m(q^2)|^2 - q^2|F^X_T(q^2)|^2.$$

In general $d\sigma_l/dq^2|_X$ would contain a function $F(q^2, s^2)$ instead of $F^X(q^2)$. But, as shown in (a), $F(q^2, s^2)$ depends on s^2 in such a way that in the region of s^2 around $m_X{}^2$ the resonant structure would not be suppressed.

IV. THE RESULTING DIFFERENTIAL CROSS SECTION AND RESONANT STRUCTURE DUE TO X PARTICLES

Let assume that l in the reaction (1) is an antineutrino. The resulting amplitude of the process (1) is then the sum of A_W of (3) and A_X of (8)

$$A = A_W + A_X. \tag{16}$$

The resulting differential cross section $d\sigma_{\bar{\nu}}/dq^2$ is, for large $E_{\bar{\nu}}$,

$$\frac{d\sigma_{\bar{\nu}}}{dq^2} = \frac{G^2}{2\pi}\frac{1}{(1 - q^2/M_W{}^2)^2}F^W(q^2)$$
$$+ \frac{g_X{}^4}{2\pi}\frac{1}{(M_X{}^2 - s^2)^2}F^X(q^2)$$
$$+ \text{interference term}.$$

From this relation we see that the X contribution becomes relevant only for s^2 around $m_X{}^2$. For $s^2 \ll m_X{}^2$ and $s^2 \gg m_X{}^2$ this contribution is negligible. For $m_X{}^2 = s^2$ we expect a resonant peak.

At what energy does one expect this resonant structure?

The contribution of the X particles depends on g_X, m_X, and $F^X(q^2)$, none of which is known experimentally up to now. We, therefore, cannot give any reliable model-independent estimate of this contribution. We want to consider some models proposed by Pati and Salam.[1] In the basic model $g_X{}^2/4\pi = 1$ and $m_X \ge 3\times10^4$ GeV. In this model an observation of X-particle effects at presently available energy is out of the question. In the economical model $m_X = 20$ GeV is assumed. For $m_X = 20$ GeV one would obtain a resonant peak $s^2 = m_X{}^2$ at a neutrino (antineutrino) energy $E \approx 200$ GeV (or at $E = 100$ GeV the cross section due to X will be 4 times larger than that of the low energy).

Important for us is the fact that there are models and considerations which suggest that the presently available neutrino and antineutrino energies are energetic enough to see possible resonant effects due to X particles.

V. MODELS AND TESTS

In this section we discuss models and their implications for resonant processes. Before considering specific models we want to make a general remark concerning the question of whether the quarks and leptons or quarks and antileptons belong to the same representation. All models discussed below contain quarks and leptons in the same representations. In this case any lepton-nucleon resonances are excluded (see Sec. V E below). In the following we use the same notation as Ref. 1, the first paper.

A. Basic model

One would expect resonant structure in the following reactions:

$$\begin{aligned}
&\bar{\nu}_e + p \to \bar{\nu}_e + p\,,\\
&\bar{\nu}_e + p \to e^+ + n\,,\\
&\nu_e + p \to \mu^+ + \begin{Bmatrix}\Sigma^0\\ \Lambda\end{Bmatrix},\\
&\bar{\nu}_e + p \to \bar{\nu}_\mu + \text{a charmed particle;}
\end{aligned} \tag{17}$$

$$\begin{aligned}
&\bar{\nu}_e + n \to \bar{\nu}_e + n\,,\\
&\bar{\nu}_e + n \to e^+ + \Delta^-,\\
&\bar{\nu}_e + n \to \mu^+ + \Sigma^-\,,\\
&\bar{\nu}_e + n \to \bar{\nu}_\mu + \text{a charmed particle;}
\end{aligned} \tag{18}$$

$$\begin{aligned}
&e^+ + p \to e^+ + p\,,\\
&e^+ + p \to \bar{\nu}_e + \Delta^{++},\\
&e^+ + p \to \mu^+ + \Sigma^+,\\
&e^+ + p \to \bar{\nu}_\mu + \text{a charmed particle;}
\end{aligned} \tag{19}$$

$$\begin{aligned}
&e^+ + n \to e^+ + n\,,\\
&e^+ + n \to \bar{\nu}_e + p\,,\\
&e^+ + n \to \mu^+ + \begin{Bmatrix}\Sigma^0\\ \Lambda\end{Bmatrix},\\
&e^+ + n \to \bar{\nu}_\mu + \text{a charmed particle}\,.
\end{aligned} \tag{20}$$

A specific feature of the basic model is that electron number and muon number are not separately conserved as the third and fourth reactions of (17)–(20) show. This is a consequence of the fact that the electron and muon belong to the same color.

Another interesting point is that in this model there is no resonant structure, for example, in $\bar{\nu}_\mu + p$ or $\bar{\nu}_\mu + n$ scattering, which is partly also due to the same color of electron and muon neutrinos. Thus, in the model the electron-muon symmetry is lost at high energies.

B. The economical model

In this model the electron and muon number are separately conserved. Therefore, one expects resonant structure only in the first and second reaction of (17)–(20).

C. The "prodigal" model and the "five-color" model

In these models the valence quantum number of the electron and its neutrinos are different from that of the basic model. There are also heavy leptons. As in the economical model, electron and muon number are conserved. However, owing to the valence structure one would expect no resonant structure in all of the above reactions in these models.

D. Implication of resonant structure

Let us assume that both $\bar{\nu}_e + p \to e^+ + n$ and $\bar{\nu}_\mu + p \to \mu^+ + n$ reveal resonant structure. What is the implication for models?

This result means that electron and muon do not belong to the same color, but they have the same valence structure. Thus the basic model as well as the other models discussed above are excluded.

A possible candidate is a "new five-color model" with the following representation of the fermions:

$$L,R = \begin{bmatrix} \mathcal{P}_a & \mathcal{P}_b & \mathcal{P}_c & \nu_e & \nu_\mu \\ \mathfrak{N}_a & \mathfrak{N}_b & \mathfrak{N}_c & e^- & \mu^- \\ \lambda_a & \lambda_b & \lambda_c & E^- & M^- \\ \chi_a & \chi_b & \chi_c & E^0 & M^0 \end{bmatrix}_{L,R} . \tag{21}$$

In this model one is able to introduce Higgs scalars in such a way that the observed muon-electron symmetry is conserved also in the high-energy region.

E. Lepton-nucleon resonances

In the case that quark-antilepton pairs belong to the same representation, which is in contrast to the models discussed so far, one would expect resonant structure in lepton-nucleon reactions like

$$\nu + n \to e^- + p\,, \tag{22}$$

provided the leptons have the same valence structure as in the basic model or the new five-color model. And one would not expect any resonant structure in the reactions of (17)–(20).

VI. DISCUSSION

We have discussed in the preceeding sections mainly differential cross sections of two-body reactions. As shown in Sec. II by $r(E)$ defined by (6) one can use also the total cross section of two-body reactions. In this case one should have higher energies than in the case of the differential cross section in order to be in the asymptotic region; $E_\nu \geq 5$ GeV and $E_{\bar{\nu}} \geq 10$ GeV.

Let us make a final remark about total cross sections of lepton (antilepton)-hadron scatterings, in particular neutrino- and antineutrino-nucleon scatterings. The present data show that $\sigma^{\nu}_{\rm tot}$ and $\sigma^{\bar{\nu}}_{\rm tot}$ rise linearly with the neutrino (antineutrino) energy. At energies where the W bosons become relevant, the linear rise will be slowed down. On the other hand, if there are s-channel contributions due to the X particles, the rise of the $\sigma_{\rm tot}$ will be stronger than linear at the resonant region. It might therefore happen that these two effects cancel each other, at least partly, in some energy region with the result that the linear rise remains.

The conclusion is that the total section might not be appropriate to test W or X effects. However, by comparing $\sigma^{\nu}_{\rm tot}$ and $\sigma^{\bar{\nu}}_{\rm tot}$ one might be able to obtain some information about their effects, because the s-channel contribution of X can occur only either in $\sigma^{\nu}_{\rm tot}$ or in $\sigma^{\bar{\nu}}_{\rm tot}$, but not simultaneously in both of them.

Finally, we have a remark about u-channel contributions of X particles. In most of the reactions considered so far, X mesons would become operative in the u channel, if not in the s channel. The u-channel contributions will result in a "second" asymptotic region (or second plateau) in the total cross section (the first asymptotic region is due to the W boson). Now we know that the ρ mass, or about 0.8 GeV, is a characteristic mass for the form factors of the conventional currents, resulting in the first asymptotic region starting at a lepton energy of about 1–3 GeV. From this analogy one might expect the second asymptotic region at an incident lepton energy E comparable to the characteristic mass of the form factors of the exotic currents J^{LB}, namely at $E = m_X - 3m_X$ (see Sec. III).

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[6]In the following we denote by X not only gauge particles, but also all other particles carrying lepton and baryon number.