Current algebra, partial conservation of the axial-vector current, and $\bar{p}p \to K\bar{K}\pi\pi$

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Using the techniques of current algebra and the partially conserved axial-vector current (PCAC) hypothesis, we perform a soft-pion calculation of the amplitudes for four proton-antiproton annihilation processes: $\bar{p}p \to K^+K^-\pi^+\pi^-$, $\bar{p}p \to K^+K^-\pi^0\pi^0$, $\bar{p}p \to K^-K^0\pi^+\pi^0$, and $\bar{p}p \to K^+\bar{K}^0\pi^-\pi^0$. From these results we obtain predictions for the reaction rates of these annihilation processes at various center-of-mass energies normalized to the $\bar{p}p \to K^+K^-$ rate. The predictions are compared with the existing experimental data.

I. INTRODUCTION

Proton-antiproton annihilations occupy a special place in the general hierarchy of elementary-particle collision processes because of the richness of the allowed final states and the variety of possible observable phenomena. Despite the existence of a substantial amount of $\bar{p}p$ annihilation data, the ability of particle theory to explain these phenomena has left much to be desired. However, recently the formalism of current algebra[1] and the partially conserved axial-vector current (PCAC) hypothesis[2] have been applied to pion production from particle-antiparticle annihilation.[3-6] The class of annihilation processes $\bar{p}p \to K\bar{K}(m\pi)$ are particularly interesting to study in view of the sizable amount of data[7,8] available at various energies. The recent work of the authors[6] on the process $\bar{p}p \to K\bar{K}\pi$ suggests that the PCAC hypothesis is approximately valid and that valuable information might be obtained about other $\bar{p}p$ annihilation reactions involving multiple-pion production from a soft-pion study of these processes.

In this paper we extend our previous study to the annihilation processes $\bar{p}p \to K\bar{K}\pi\pi$. Soft-pion analyses of the reactions $\bar{p}p \to K^+K^-\pi^+\pi^-$ and $\bar{p}p \to K^+K^-\pi^0\pi^0$ have earlier been reported by Uritam.[4,5] However, the calculations which we present here are more precise and complete. To begin with, unlike Uritam's nonrelativistic calculations, we present a fully relativistic treatment of the $\bar{p}p$ annihilation process. This is necessary if one wishes to study these reactions for nonzero lab momentum where most of the experimental data are available. In addition, we extend Uritam's results for $\bar{p}p \to K^+K^-\pi^+\pi^-$ to include nonvanishing lab momentum. Finally, we also examine the reactions $\bar{p}p \to K^+\bar{K}^0\pi^-\pi^0$ and $\bar{p}p \to K^-K^0\pi^+\pi^0$ at various energies which have never previously been considered and where data are available.

We begin in Sec. II with a derivation of a general soft-pion theorem based on current algebra and PCAC. Such a theorem when applied to $\bar{p}p$ annihilation allows one to ultimately obtain a relation between the amplitudes of the processes $\bar{p}p \to K\bar{K}\pi\pi$ and $\bar{p}p \to K\bar{K}$. In Sec. III we study the process $\bar{p}p \to K^+K^-\pi^+\pi^-$. The soft-pion theorem in this case relates the amplitude for the emission of two soft pions to the matrix element for the emission of an isovector photon and to the pole terms of the matrix element of two axial-vector currents contracted with two-pion momenta. Upon evaluation of the photon-emission term and the pole terms, we calculate the ratio of the reaction rates $w(\bar{p}p \to K^+K^-\pi^+\pi^-)/w(\bar{p}p \to K^+K^-)$ as a function of lab momentum making use of the "gentleness" assumption of PCAC.

We present a similar analysis of the reaction $\bar{p}p \to K^+K^-\pi^0\pi^0$ in Sec. IV and of the reactions $\bar{p}p \to K\bar{K}\pi^\pm\pi^0$ in Sec. V. In Sec. VI we discuss the results and compare the theoretical predictions with the available experimental data.

II. SOFT-PION THEOREM

We begin by considering the reaction

$$i \to f + \pi^\alpha(k_1) + \pi^\beta(k_2)\,, \tag{2.1}$$

where i and f represent arbitrary multiparticle hadronic states, and the pions have four-momenta k_1 and k_2, and carry isospin α and β.

In the customary way we define the quantity

$$M_{\mu\nu}^{\alpha\beta} = i\int d^4x\, d^4y\, e^{ik_1x} e^{ik_2y} \langle f|\, T(A_\mu^\alpha(x), A_\nu^\beta(y))|i\rangle\,, \tag{2.2}$$

where $A_\mu^\alpha, A_\nu^\beta$ are the strangeness-conserving axial-vector currents and T denotes a time-ordered product.

Contracting $M_{\mu\nu}^{\alpha\beta}$ with k_1^μ and integrating by parts gives

$$ik_1^\mu M_{\mu\nu}^{\alpha\beta} = -i\int d^4x\,d^4y\,e^{ik_1x}e^{ik_2y}\langle f|T(\partial^\mu A_\mu^\alpha(x),A_\nu^\beta(y))|i\rangle - i\int d^4x\,d^4y\,e^{ik_1x}e^{ik_2y}\langle f|[A_0^\alpha(x),A_\nu^\beta(y)]|i\rangle\delta(x_0-y_0)\,, \tag{2.3}$$

where we have used the identity

$$\partial^\mu T(A_\mu(x),A_\nu(y)) = T(\partial^\mu A_\mu(x),A_\nu(y)) + [A_0(x),A_\nu(y)]\delta(x_0-y_0)\,. \tag{2.4}$$

The first term on the right-hand side of Eq. (2.3) involves a matrix element of the divergence of the axial-vector current. According to the PCAC hypothesis, we may replace this divergence by the pion field operator according to the relation

$$\partial^\mu A_\mu^\alpha(x) = c_\pi\phi_\pi^\alpha(x)\,, \tag{2.5}$$

where $c_\pi = M\mu^2 g_A/g_r K_{NN\pi}(0)$, and M is the nucleon mass; μ is the pion mass; g_r represents the renormalized pion-nucleon coupling constant ($g_r^2/4\pi = 14.6$); $K_{NN\pi}(0)$ is the pionic form factor of the nucleon $[K_{NN\pi}(\mu^2)=1]$; g_A is the renormalized axial-vector coupling constant ($g_A = 1.2$).

The second term appearing in Eq. (2.3) involves an equal-time commutation relation between two axial-vector currents. Such a term can be evaluated using the SU(3)× SU(3) current-algebra commutation relation

$$[A_0^\alpha(x),A_\nu^\beta(y)]\delta(x_0-y_0) = i\epsilon_{\alpha\beta\gamma}V_\nu^\gamma(x)\delta^4(x-y)\,. \tag{2.6}$$

Substituting Eqs. (2.5) and (2.6) into Eq. (2.3) yields

$$ik_1^\mu M_{\mu\nu}^{\alpha\beta} = -ic_\pi\int d^4x\,d^4y\,e^{ik_1x}e^{ik_2y}\langle f|T(\phi_\pi^\alpha(x),A_\nu^\beta(y))|i\rangle + \epsilon_{\alpha\beta\gamma}\int d^4x\,e^{i(k_1+k_2)x}\langle f|V_\nu^\gamma(x)|i\rangle\,. \tag{2.7}$$

Contracting Eq. (2.7) with k_2^ν and integrating once again by parts yields

$$\begin{aligned}-k_2^\nu k_1^\mu M_{\mu\nu}^{\alpha\beta} = {}& ic_\pi^2\int d^4x\,d^4y\,e^{ik_1x}e^{ik_2y}\langle f|T(\phi_\pi^\alpha(x),\phi_\pi^\beta(y))|i\rangle\\ &+ ic_\pi\int d^4x\,d^4y\,e^{ik_1x}e^{ik_2y}\langle f|[\phi_\pi^\alpha(x),A_0^\beta(y)]|i\rangle\delta(x_0-y_0)\\ &- i\epsilon_{\alpha\beta\gamma}k_1^\nu\int d^4x\,e^{i(k_1+k_2)x}\langle f|V_\nu^\gamma(x)|i\rangle\,,\end{aligned} \tag{2.8}$$

where we have again used PCAC and have also invoked the conserved vector-current hypothesis which requires that

$$\partial^\mu V_\mu^\alpha(x) = 0\,. \tag{2.9}$$

The second term appearing on the right-hand side of Eq. (2.8) involves a so-called σ term, which will be neglected in the usual way.[9] By repeating the contraction of $M_{\mu\nu}^{\alpha\beta}$ with the order of k_1 and k_2 reversed and combining the result with Eq. (2.8) gives the symmetric identity

$$k_1^\mu k_2^\nu M_{\mu\nu}^{\alpha\beta} = -ic_\pi^2\int d^4x\,d^4y\,e^{ik_1x}e^{ik_2y}\langle f|T(\phi_\pi^\alpha(x),\phi_\pi^\beta(y))|i\rangle - \tfrac{1}{2}i\epsilon_{\alpha\beta\gamma}(k_2-k_1)^\lambda\int d^4x\,e^{i(k_1+k_2)x}\langle f|V_\lambda^\gamma(x)|i\rangle\,. \tag{2.10}$$

If we now insert the Klein-Gordon operator and take the soft-pion limits, $k_1\to 0$ and $k_2\to 0$, of the expression we obtain the soft-pion theorem

$$\begin{aligned}k_1^\mu k_2^\nu M_{\mu\nu}^{\alpha\beta} = {}& -\frac{ic_\pi^2}{\mu^4}\int d^4x\,d^4y\,e^{ik_1x}e^{ik_2y}(\Box_x+\mu^2)(\Box_y+\mu^2)\langle f|T(\phi_\pi^\alpha(x),\phi_\pi^\beta(y))|i\rangle\\ &- \tfrac{1}{2}i\epsilon_{\alpha\beta\gamma}(k_2-k_1)^\lambda\int d^4x\,e^{i(k_1+k_2)x}\langle f|V_\lambda^\gamma(x)|i\rangle\,.\end{aligned} \tag{2.11}$$

In this result the first term on the right-hand side is, to within a numerical factor,[10] the amplitude for the process $i\to f+\pi^\alpha(k_1)+\pi^\beta(k_2)$; the second term is related to the amplitude for the process $i\to f+$ isovector photon; the left-hand side also does not vanish in the soft-pion limits since $M_{\mu\nu}^{\alpha\beta}$ has contributions from terms of order k_1^{-2} or k_2^{-2}. In fact, as we shall see when we apply this theorem to specific processes, it will be possible to express both the isovector photon term and the left-hand side in terms of the $i \to f$ amplitude. Since this soft-pion theorem has been derived in the unphysical region $k_1 = 0$, $k_2 = 0$ (off mass shell), we must extrapolate from zero-pion four-momentum to low three-momentum on the mass shell if we are to relate the physical amplitude for $i \to f + \pi + \pi$ to the amplitude for $i \to f$. Such an extrapolation is assumed to be "gentle" in the usual spirit of PCAC although, for the processes which are considered in the following sections, the pion three-momenta can be sufficiently large to make the extrapolation which is required increasingly demanding.

III. STUDY OF $\bar{p}p \to K^+K^-\pi^+\pi^-$

A. Application of soft-pion relations

In this section we present an analysis of the charged-pion reaction $\bar{p}p \to K^+K^-\pi^+\pi^-$. Although this $\bar{p}p$ annihilation process has been previously studied by Uritam,[5] we reanalyze this reaction, giving a fully relativistic treatment to the calculation and extending the results to include nonzero lab momentum.

In order to apply the soft-pion relation expressed by Eq. (2.11), we take $i = \bar{p}p$ and $f = K^+K^-$. Specializing to isospins corresponding to π^+ and π^- emission, the soft-pion relation becomes

$$k_1^\mu k_2^\nu M_{\mu\nu}^{+-} = -\frac{ic_\pi^{\,2}}{\mu^4} \int d^4x\, d^4y\, e^{ik_1x} e^{ik_2y} (\Box_x + \mu^2)(\Box_y + \mu^2) \langle K^+K^- | T(\phi_{\pi^+}(x)\phi_{\pi^-}(y)) | \bar{p}p\rangle$$
$$- \tfrac{1}{2} i (k_2 - k_1)^\lambda \int d^4x\, e^{i(k_1+k_2)x} \langle K^+K^- | V_\lambda^{\,3}(x) | \bar{p}p\rangle . \qquad (3.1)$$

The first term on the right-hand side of Eq. (3.1) is, to within a numerical factor, the amplitude for $\bar{p}p \to K^+K^-\pi^+\pi^-$; the second term is related to the amplitude for the process $\bar{p}p \to K^+K^- +$ isovector photon; the left-hand side does not vanish since $M_{\mu\nu}^{+-}$ has contributions from pole terms of order k^{-2}.

B. Evaluation of pole terms

The contributions to $M_{\mu\nu}^{\alpha\beta}$ to order k^{-2} can be calculated from the diagrams shown in Fig. 1 where the axial-vector currents are attached to the external $\bar{p}$ and p lines. Note that parity forbids any attachments of the currents to the $K^\pm$ lines. For $\pi^+\pi^-$ emission only diagrams (a), (b), and (c) contribute. These diagrams can be easily calculated using Feynman rules. The central interaction, which for diagrams (a) and (b) is $\bar{p}p \to K^+K^-$ and for diagram (c) is $\bar{n}n \to K^+K^-$, can be written in the relativistically invariant form $\mathfrak{M} = A + B\not{Q}$, where $Q = q_1 - q_2$ is the four-momentum difference of K^+ and K^-, and A and B are in general unknown functions of the kinematic variables. However, as it has been pointed out by both Uritam[4] and the authors,[6] upon evaluating the pole contribution, the contribution from the A term can be dropped. This can be explained as follows. At very low energies (nonrelativistic region) it is easy to show that the coefficient of the A term mixes large and small components of the Dirac spinors and can thus be ignored. In the relativistic region the preceding argument fails. However, we have pointed out[6] that there is a natural kinematic suppression of the $|A|^2$ term as compared to the $|B|^2$ term in the reaction rate[11] and that for lab momenta as large as 5 GeV/c the $|A|^2$ term can be safely ignored.[12]

The evaluation of diagrams (a), (b), and (c) in Fig. 1 gives

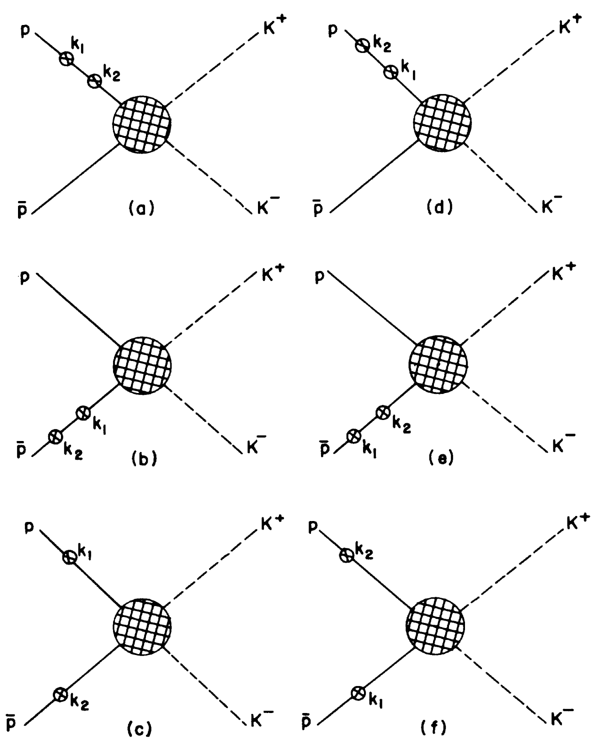

FIG. 1. Diagrams of order k^{-2} contributing to $M_{\mu\nu}^{\alpha\beta}$. The ⊗ denotes the axial-vector current vertex.

$$k_1^\mu k_2^\nu M_{\mu\nu}^{+-} = 2Bg_A{}^2\bar{v}^s(p_2)\bigg(\not{Q}\,\frac{1}{\not{p}_1-\not{k}_1-\not{k}_2-M}\,\gamma_5\not{k}_2\,\frac{1}{\not{p}_1-\not{k}_1-M}\,\gamma_5\not{k}_1+\gamma_5\not{k}_2\,\frac{1}{-\not{p}_2+\not{k}_2-M}\,\gamma_5\not{k}_1\,\frac{1}{-\not{p}_2+\not{k}_2+\not{k}_1-M}\,\not{Q}$$
$$+\gamma_5\not{k}_2\,\frac{1}{-\not{p}_2+\not{k}_2-M}\,\not{Q}\,\frac{1}{\not{p}_1-\not{k}_1-M}\,\gamma_5\not{k}_1\bigg)u^r(p_1)\,, \tag{3.2}$$

where p_1 and r are the proton momentum and spin state, and p_2 and s are the antiproton momentum and spin state.

Keeping only terms of order k^{-2} one finds after some manipulation

$$k_1^\mu k_2^\nu M_{\mu\nu}^{+-} = -2Bg_A{}^2\bar{v}^s(p_2)\tilde{N}_1 u^r(p_1)\,, \tag{3.3}$$

where

$$\tilde{N}_1 = \frac{\not{Q}(M\not{k}_2+p_1\cdot k_2)(M\not{k}_1-p_1\cdot k_1)}{(p_1\cdot k_1)p_1\cdot(k_1+k_2)} + \frac{(M\not{k}_2+p_2\cdot k_2)(M\not{k}_1-p_2\cdot k_1)\not{Q}}{(p_2\cdot k_2)p_2\cdot(k_1+k_2)} - \frac{(M\not{k}_2+p_2\cdot k_2)\not{Q}(M\not{k}_1-p_1\cdot k_1)}{(p_1\cdot k_1)(p_2\cdot k_2)}\,. \tag{3.4}$$

C. Evaluation of radiative term

We next evaluate the term appearing in Eq. (3.1) corresponding to the emission of an isovector photon: $\bar{p}p \to K^+K^-$ + isovector photon. Since in the soft-pion limit $k_1 \to 0$, $k_2 \to 0$, and the momentum of the photon $k = k_1 + k_2 \to 0$. Thus we can adopt the procedure developed by Low[13] for soft-photon emission. This procedure enables one to calculate the radiative amplitude to orders k^{-1} and k^0 in terms of the nonradiative amplitude.

The amplitude for the radiative process is given by

$$M_\mu = \int d^4x\, e^{ikx}\langle K^+K^-|V_\mu{}^3(x)|\bar{p}p\rangle\,. \tag{3.5}$$

This amplitude can be separated into two parts:

$$M_\mu = M_\mu^{(1)} + M_\mu^{(2)}\,. \tag{3.6}$$

$M_\mu^{(1)}$ is comprised of all diagrams (see Fig. 2) where the photon is emitted from either the charged K lines or from the proton or antiproton line.[14] $M_\mu^{(2)}$ consists of all diagrams (see Fig. 3) where the photon emerges from the interaction vertex rather than from an external line. Now, as $k \to 0$, it can be shown from Low's theorem that $M_\mu^{(1)} \sim k^{-1}$ and $M_\mu^{(2)} \sim$ constant. Thus, in the soft-pion limit only $(k_2-k_1)^\mu M_\mu^{(1)}$ contributes to the radiative term in Eq. (3.1). From the diagrams in Fig. 2 one finds as $k \to 0$,

$$M_\mu^{(1)} = \frac{1}{2}\left(\frac{q_{1\mu}}{q_1\cdot k} - \frac{q_{2\mu}}{q_2\cdot k} + \frac{p_{2\mu}}{p_2\cdot k} - \frac{p_{1\mu}}{p_1\cdot k}\right)$$
$$\times\, \bar{v}^s(p_2)B\not{Q}u^r(p_1)\,, \tag{3.7}$$

where the factor of $\frac{1}{2}$ represents the coupling constant of the isovector current to the isovector charges.

D. Amplitude and reaction rate

Substituting Eqs. (3.3) and (3.7) into Eq. (3.1) gives for the invariant amplitude for soft-pion emission in $\bar{p}p \to K^+K^-\pi^+\pi^-$

$$M_{sr}^{+-} = -\frac{2\mu^4}{c_\pi{}^2}Bg_A{}^2\bar{v}^s(p_2)\tilde{M}_1u^r(p_1)\,, \tag{3.8}$$

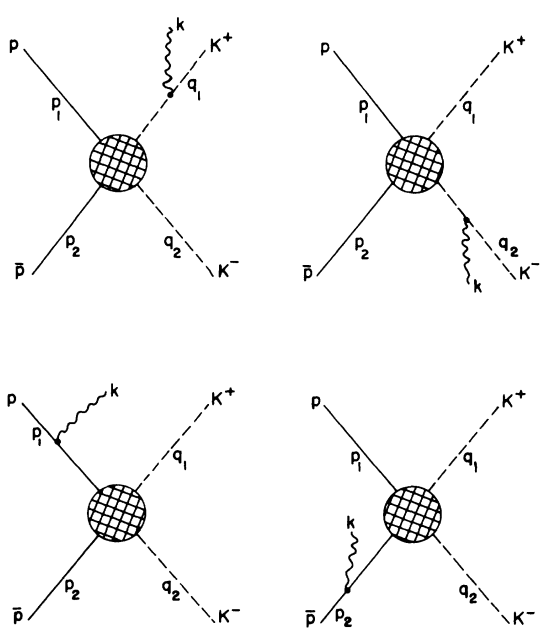

FIG. 2. Lowest-order diagrams contributing to $M_\mu^{(1)}$.

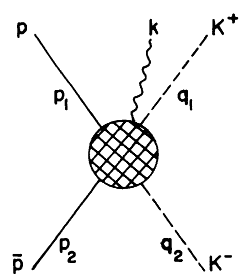

FIG. 3. Lowest-order diagram contributing to $M_\mu^{(2)}$.

where

$$\tilde{M}_1 = \tilde{N}_1 - \frac{1}{8g_A^2}\left[\frac{p_2\cdot(k_2-k_1)}{p_2\cdot(k_2+k_1)} - \frac{p_1\cdot(k_2-k_1)}{p_1\cdot(k_2+k_1)}\right]\not{Q}$$
$$-\frac{1}{8g_A^2}\left[\frac{q_1\cdot(k_2-k_1)}{q_1\cdot(k_2+k_1)} - \frac{q_2\cdot(k_2-k_1)}{q_2\cdot(k_2+k_1)}\right]\not{Q}, \quad (3.9)$$

where $\tilde{N}_1$ is defined in Eq. (3.4) and B is the quantity in the corresponding invariant amplitude for $\bar{p}p \to K^+K^-$, which is itself proportional to B:

$$M_{sr} = B\bar{v}^s(p_2)\not{Q}u^r(p_1). \quad (3.10)$$

Squaring the amplitude in Eq. (3.8), summing over final spin states, and averaging over initial states yields

$$\langle |M_{sr}^{+-}|^2\rangle = \frac{\mu^8 g_A^4|B|^2}{c_\pi^4}\sum^{+-}, \quad (3.11)$$

where $\sum^{+-}$ is a very lengthy expression given in Appendix B.

The differential reaction rate is given by

$$d^{12}w = \frac{(2\pi)^4}{(2\pi)^{12}}\frac{M^2}{p_1^0 p_2^0}\delta^4(p_1+p_2-k_1-k_2-q_1-q_2)\langle|M_{sr}^{+-}|^2\rangle\frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_2^0}\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}. \quad (3.12)$$

We evaluate the reaction rate in the center-of-mass frame. In this frame $p_1=(E,\vec{p})$, $p_2=(E,-\vec{p})$, $k_1=(\omega_1,\vec{k}_1)$, $k_2=(\omega_2,\vec{k}_2)$. We also introduce the new variables $y_1=\cos\theta_1$, $y_2=\cos\theta_2$, where θ_1 (θ_2) is the angle between $\vec{k}_1$ ($\vec{k}_2$) and the z axis (chosen to be along the $\vec{p}$ direction). Thus, $d^3k_1 = k_1^2dk_1d(\cos\theta_1)d\phi_1$, $d^3k_2 = k_2^2dk_2d(\cos\theta_2)d\phi_2$, where ϕ_1 (ϕ_2) is the azmuthal angle associated with $\vec{k}_1$ ($\vec{k}_2$). Since the only dependence on ϕ_1 and ϕ_2 in $\sum^{+-}$ are of the forms $\cos(\phi_1-\phi_2)$ and $\sin^2(\phi_1-\phi_2)$, the two integrations in the ϕ variables can be replaced by a single integration over the variable $x=\cos(\phi_1-\phi_2)$. Hence, we obtain for the differential rate in the center-of-mass frame

$$d^{11}w = \frac{2(g_r^2/4\pi)^2|B|^2K_{NN\pi}^4}{(2\pi)^5E^2M^2}\left[\frac{(\omega_1^2-\mu^2)(\omega_2^2-\mu^2)}{1-x^2}\right]^{1/2}\sum^{+-}\delta^4(p_1+p_2-k_1-k_2-q_1-q_2)$$
$$\times d\omega_1 d\omega_2 dy_1 dy_2 dx\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}. \quad (3.13)$$

We also find (see Appendix A) the reaction rate for $\bar{p}p\to K^+K^-$ in the center-of-mass frame to be

$$w(\bar{p}p\to K^+K^-) = \frac{|B|^2}{12\pi}(2E^2+M^2)\left(1-\frac{m_K^2}{E^2}\right)^{3/2}, \quad (3.14)$$

where m_K represents the kaon mass.

Integrating Eq. (3.13) and dividing by Eq. (3.14) yields for the ratio of the reaction rates

$$\frac{w(\bar{p}p\to K^+K^-\pi^+\pi^-)}{w(\bar{p}p\to K^+K^-)} = \frac{3(g_r^2/4\pi)^2EK_{NN\pi}^4}{4\pi^4M^2(E^2-m_K^2)^{3/2}(2E^2+M^2)}$$
$$\times\int_{-1}^1 dx\int_{-1}^1 dy_1\int_{-1}^1 dy_2\int_\mu^{\omega_{1\max}} d\omega_1\int_\mu^{\omega_{2\max}} d\omega_2\left[\frac{(\omega_1^2-\mu^2)(\omega_2^2-\mu^2)}{1-x^2}\right]^{1/2}$$
$$\times\int\sum^{+-}\delta^4(p_1+p_2-k_1-k_2-q_1-q_2)\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}, \quad (3.15)$$

where

$$\omega_{1\max} = \frac{E^2+(E-\mu)^2-2m_K^2}{2E-\mu}, \quad (3.16a)$$

$$\omega_{2\max} = \frac{(2E-\omega_1)\{2[E(E-\omega_1)-m_K^2]+\mu^2\}-A}{(2E-\omega_1)^2-(\omega_1^2-\mu^2)F^2}, \quad (3.16b)$$

$$A = ((\omega_1^2-\mu^2)F^2\{4[E(E-\omega_1)-m_K^2] - \mu^2(1-F^2)(\omega_1^2-\mu^2)-4\mu^2m_K^2\})^{1/2}, \quad (3.16c)$$

$$F = [(1-y_1^2)(1-y_2^2)]^{1/2}x + y_1y_2. \quad (3.16d)$$

The integration over the kaon variables (q_1, q_2) are carried out in Appendix C. The remaining 5-fold integration is calculated numerically. The results for the reaction-rate ratio as a function of center-of-mass energy as well as lab momentum are presented in Table I. It should be noted that in the final integration the pion's three-momentum takes on values as high as 1.6 GeV/c. Thus, for large lab momenta ($p_L \sim$few GeV), we are making a considerable extrapolation based on the PCAC assumption that the amplitudes involved are sufficiently "gentle" to permit the extrapolation. A comparison of the numerical results in Table I with the available experimental data[7,8] is made later in Sec. VI. It should be noted that our result for R at zero lab momentum is more than an order of magnitude lower than the result quoted by

TABLE I. Ratio of calculated reaction rates $R = w(\bar{p}p \to K^+K^-\pi^+\pi^-)/w(\bar{p}p \to K^+K^-)$ at various energies.

p lab momentum (GeV/c)	Center-of-mass energy (GeV)	R
0.0	1.87	0.065
0.5	1.94	0.095
1.0	2.08	0.198
1.5	2.25	0.394
2.0	2.43	0.677
2.5	2.60	1.063
3.0	2.77	1.518
3.5	2.93	2.050
4.0	3.08	2.671
4.5	3.22	3.292
5.0	3.36	3.998

Uritam.[4] Aside from relativistic corrections, this difference can be partially traced to an error of a factor of $\frac{1}{2}$ in Uritam's expression for the $\bar{p}p \to K^+K^-$ rate which has the effect of enhancing his value for R by a factor of 2.

IV. STUDY OF $\bar{p}p \to K^+K^-\pi^0\pi^0$

In this section we present a study of the neutral-pion reaction $\bar{p}p \to K^+K^-\pi^0\pi^0$. Although this $\bar{p}p$ annihilation process has been previously studied by Uritam,[5] we repeat the analysis here for the reasons cited earlier in the paper.

The soft-pion relation (2.11) becomes in this case

$$k_1^\mu k_2^\nu M_{\mu\nu}^{00} = -\frac{ic_\pi{}^2}{\mu^4}\int d^4x\, d^4y\, e^{ik_1x} e^{ik_2y}(\Box_x + \mu^2)(\Box_y + \mu^2) \times \langle K^+K^- | T(\phi_{\pi^0}(x)\phi_{\pi^0}(y)) | \bar{p}p\rangle , \tag{4.1}$$

where the isovector photon term is absent since $\epsilon_{\alpha\beta\gamma}$ vanishes for $\pi^0\pi^0$ emission. Once again the left-hand side of Eq. (4.1) does not vanish in the soft-pion limit due to pole terms in $M_{\mu\nu}^{00}$ of order k^{-2}. The contributions from these pole terms result from all six diagrams shown in Fig. 1. Evaluating these diagrams, keeping only terms of order k^{-2}, gives

$$k_1^\mu k_2^\nu M_{\mu\nu}^{00} = Bg_A{}^2 \bar{v}^s(p_2)\tilde{N}_2 u^r(p_1) , \tag{4.2}$$

where

$$\tilde{N}_2 = -\frac{\not{Q}(M\not{k}_1 - p_1\cdot k_1)(M\not{k}_2 + p_1\cdot k_2)}{(p_1\cdot k_1)p_1\cdot(k_1+k_2)} - \frac{(M\not{k}_2 + p_2\cdot k_2)(M\not{k}_1 - p_2\cdot k_1)}{(p_2\cdot k_2)p_2\cdot(k_1+k_2)}\not{Q} + \frac{(M\not{k}_2 + p_2\cdot k_2)\not{Q}(M\not{k}_1 - p_1\cdot k_1)}{(p_1\cdot k_1)(p_2\cdot k_2)} + (k_1 \leftrightarrow k_2) . \tag{4.3}$$

Combining Eqs. (4.2) and (4.3) with (4.1) gives for the invariant amplitude for soft-pion emission in $\bar{p}p \to K^+K^-\pi^0\pi^0$

$$M_{sr}^{00} = \frac{\mu^4}{c_\pi{}^2} Bg_A{}^2 \bar{v}^s(p_2)\tilde{N}_2 u^r(p_1) . \tag{4.4}$$

Squaring the amplitude, summing over final spin states, and averaging over initial spin states gives

$$\langle |M_{sr}^{00}|^2\rangle = \frac{\mu^8 g_A{}^4 |B|^2}{4c_\pi{}^4}\sum{}^{00} , \tag{4.5}$$

where $\sum^{00}$ is another lengthy expression exhibited in Appendix B.

The differential reaction rate is given by

$$d^{12}w = \frac{(2\pi)^4}{(2\pi)^{12}}\frac{M^2}{p_1^0 p_2^0}\delta^4(p_1+p_2-k_1-k_2-q_1-q_2)\langle |M_{sr}^{00}|^2\rangle \frac{d^3k_1}{2k_1^0}\frac{d^2k_2}{2k_2^0}\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0} , \tag{4.6}$$

which in the center-of-mass frame becomes

$$d^{11}w = \frac{(g_r{}^2/4\pi)^2 |B|^2 K_{NN\pi}{}^4}{2(2\pi)^5 E^2 M^2}\left[\frac{(\omega_1{}^2-\mu^2)(\omega_2{}^2-\mu^2)}{1-x^2}\right]^{1/2}\sum{}^{00}\delta^4(p_1+p_2-k_1-k_2-q_1-q_2) \times d\omega_1 d\omega_2 dy_1 dy_2 dx \frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0} . \tag{4.7}$$

Integrating this result and dividing by Eq. (3.14) gives the reaction rate ratio

$$\frac{w(\bar{p}p \to K^+K^-\pi^0\pi^0)}{w(\bar{p}p \to K^+K^-)} = \frac{3(g_r{}^2/4\pi)^2 E K_{NN\pi}{}^4}{16\pi^4 M^2(E^2 - m_K{}^2)^{3/2}(2E^2+M^2)} \times \int_{-1}^1 dx\int_{-1}^1 dy_1\int_{-1}^1 dy_2\int_\mu^{\omega_{1\max}} d\omega_1\int_\mu^{\omega_{2\max}} d\omega_2\left[\frac{(\omega_1{}^2-\mu^2)(\omega_2{}^2-\mu^2)}{1-x^2}\right]^{1/2} \times \int\sum{}^{00}\delta^4(p_1+p_2-k_1-k_2-q_1-q_2)\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0} . \tag{4.8}$$

The integrations over the kaon variables are performed in Appendix C and the remaining integrations are carried out numerically. The results for the rate ratio are presented in Table II and a comparison of our theoretical predictions for $\bar{p}p \to K^+K^-\pi^+\pi^-$ and $\bar{p}p \to K^+K^-\pi^0\pi^0$ are displayed in Fig. 4.

V. STUDY OF $\bar{p}p \to K^{\mp}K^0\pi^{\pm}\pi^0$

A. Soft-pion theorem and pole terms

We conclude our analysis of $\bar{p}p$ annihilation reactions with a study of the processes $\bar{p}p \to K^-K^0\pi^+\pi^0$ and $\bar{p}p \to K^+\bar{K}^0\pi^-\pi^0$. Since the results for these two reactions are identical, as they must be by the CPT theorem, we shall concentrate on the $\pi^+\pi^0$ emission process.

The soft-pion theorem, Eq. (2.11), states in this case

$$k_1^\mu k_2^\nu M_{\mu\nu}^{+0} = -\frac{ic_\pi{}^2}{\mu^4}\int d^4x\, d^4y\, e^{ik_1x}e^{ik_2y}(\Box_x+\mu^2)(\Box_y+\mu^2)\langle K^0K^-|T(\phi_{\pi^+}(x)\phi_{\pi^0}(y))|\bar{p}p\rangle + \tfrac{1}{2}(k_1-k_2)^\lambda\int d^4x\, e^{i(k_1+k_2)x}\langle K^0K^-|V_\lambda^{1+i2}(x)|\bar{p}p\rangle\,, \tag{5.1}$$

where the second term on the right-hand side represents a radiative term which can be thought of as involving the emission of a "charged" isovector photon.

The pole terms of order k^{-2} appearing in $M_{\mu\nu}^{+0}$ are calculated from diagrams (a), (c), and (d) in Fig. 1. We find

$$k_1^\mu k_2^\nu M_{\mu\nu}^{+0} = \sqrt{2}\, g_A{}^2B^+\,\bar{v}^s(p_2)\tilde{N}_3u^r(p_1)\,, \tag{5.2}$$

where

$$\tilde{N}_3 = \frac{\not{Q}(M\not{k}_1+p_1\cdot k_1)(M\not{k}_2-p_1\cdot k_2)}{(p_1\cdot k_2)p_1\cdot(k_1+k_2)} - \frac{\not{Q}(M\not{k}_2+p_1\cdot k_2)(M\not{k}_1-p_1\cdot k_1)}{(p_1\cdot k_1)p_1\cdot(k_1+k_2)} - \frac{(M\not{k}_2+p_2\cdot k_2)\not{Q}(M\not{k}_1-p_1\cdot k_1)}{(p_1\cdot k_1)(p_2\cdot k_2)}\,. \tag{5.3}$$

B^+ is the kinematic function appearing in the interaction matrix for $\bar{p}n \to K^-K^0$. However, it can be argued[6] from isospin considerations and s-channel meson exchange that $B^+ \cong B$.

TABLE II. Ratio of calculated reaction rates $R = w(\bar{p}p \to K^+K^-\pi^0\pi^0)/w(\bar{p}p \to K^+K^-)$ at various energies.

p lab momentum (GeV/c)	Center-of- mass energy (GeV)	R
0.0	1.87	0.038
0.5	1.94	0.052
1.0	2.08	0.099
1.5	2.25	0.186
2.0	2.43	0.321
2.5	2.60	0.504
3.0	2.77	0.736
3.5	2.93	1.014
4.0	3.08	1.335
4.5	3.22	1.696
5.0	3.36	2.094

B. Evaluation of the radiative term

We next consider the evaluation of the radiative amplitude corresponding to the soft emission of a "charged" photon. The amplitude has the form

$$M_\mu^+ = \int d^4x\, e^{ikx}\langle K^+K^-|V_\mu^+|\bar{p}p\rangle. \tag{5.4}$$

Low's procedure, although originally proposed for a physical, uncharged photon, can again be employed here. As pointed out by Adler and Dothan,[15] Low's theorem can be easily generalized

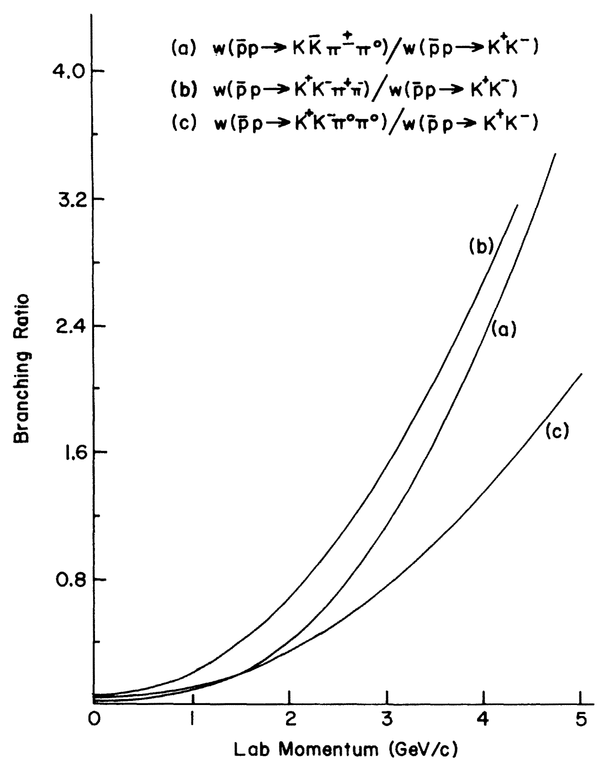

FIG. 4. Comparison of theoretical predictions for $\pi^+\pi^-$, $\pi^0\pi^0$, and $\pi^\pm\pi^0$ emission in $\bar{p}p$ annihilation.

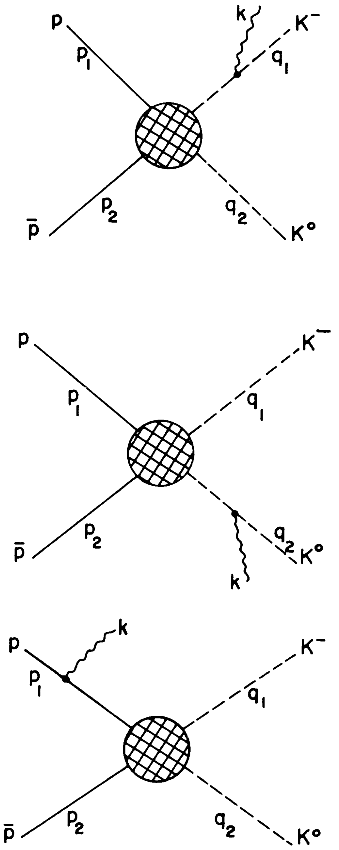

FIG. 5. Lowest-order diagrams contributing to $M_\mu^{+(1)}$.

to any case where the divergence of the particular current is known. In our case, we are dealing with the charged isospin current which is a conserved current according to Eq. (2.9) just as the electromagnetic current is conserved. Current conservation thus requires

$$k^\mu M_\mu^+ = 0. \tag{5.5}$$

Similar to the analysis performed in Sec. III, M_μ^+ can be divided into two parts:

$$M_\mu^+ = M_\mu^{+(1)} + M_\mu^{+(2)}, \tag{5.6}$$

where $M_\mu^{+(1)}$ consists of the sum of diagrams shown in Fig. 5 and $M_\mu^{+(2)}$ is represented by the diagram in Fig. 6. As $k \to 0$, Low's theorem requires that

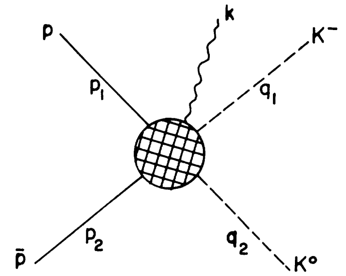

FIG. 6. Lowest-order diagram contributing to $M_\mu^{+(2)}$.

$M_\mu^{+(1)} \sim k^{-1}$ and $M_\mu^{+(2)} \sim$ constant so that only $(k_1 - k_2)^\mu M_\mu^{+(1)}$ contributes to Eq. (5.1) in the soft-pion limit. Upon evaluating the diagrams in Fig. 5 to order k^{-1} we find

$$M_\mu^{+(1)} = \left(\frac{q_{1\mu}}{q_1\cdot k} - \frac{q_{2\mu}}{q_2\cdot k} + \frac{p_{1\mu}}{p_1\cdot k}\right)\bar v^s(p_2) B \not{Q} u^r(p_1). \tag{5.7}$$

C. Amplitude and reaction rate

From Eqs. (5.1)–(5.3) and (5.7) we obtain for the invariant amplitude for soft-pion emission in $\bar p p \to K^- K^0 \pi^+ \pi^0$

$$M_{sr}^{+0} = \frac{\mu^4}{c_\pi^{\,2}} \sqrt{2}\, B g_A^{\,2} \bar v^s(p_2) \tilde M_3 u^r(p_1), \tag{5.8}$$

where

$$\tilde M_3 = \tilde N_3 + \frac{1}{2\sqrt{2}\, g_A^{\,2}}\left[\frac{p_1\cdot(k_1-k_2)}{p_1\cdot(k_1+k_2)} + \frac{q_1\cdot(k_1-k_2)}{q_1\cdot(k_1+k_2)} - \frac{q_2\cdot(k_1-k_2)}{q_2\cdot(k_1+k_2)}\right]\not{Q}. \tag{5.9}$$

Squaring the amplitude in Eq. (5.8) and performing the usual spin sums and averages gives

$$\langle |M_{sr}^{+0}|^2\rangle = \frac{\mu^8 g_A^{\,4} |B|^2}{2c_\pi^{\,4}} \sum{}^{+0}, \tag{5.10}$$

where the expression for $\sum^{+0}$ is presented in Appendix B.

Integrating Eq. (5.10) over the appropriate phase space leads to the following result for the reaction rate for $\bar p p \to K^- K^0 \pi^+ \pi^0$ relative to the rate for $\bar p p \to K^+ K^-$:

$$\frac{w(\bar p p \to K^- K^0 \pi^+ \pi^0)}{w(\bar p p \to K^+ K^-)} = \frac{3(g_r^{\,2}/4\pi)^2 E K_{NN\pi}^{\,4}}{8\pi^4 M^2 (E^2 - m_K^{\,2})^{3/2} (2E^2 + M^2)}$$
$$\times \int_{-1}^{1} dx \int_{-1}^{1} dy_1 \int_{-1}^{1} dy_2 \int_{\mu}^{\omega_{1\max}} d\omega_1 \int_{\mu}^{\omega_{2\max}} d\omega_2 \left[\frac{(\omega_1^{\,2} - \mu^2)(\omega_2^{\,2} - \mu^2)}{1 - x^2}\right]^{1/2}$$
$$\times \int \sum{}^{+0} \delta^4(p_1 + p_2 - k_1 - k_2 - q_1 - q_2) \frac{d^3 q_1}{2q_1^0} \frac{d^3 q_2}{2q_2^0}\,. \tag{5.11}$$

After performing the integrations over the kaon variables analytically, the remaining integrations are performed numerically with the results presented in Table III. It should be noted that the same results are valid for $\bar{p}p \to K^+\bar{K}^0\pi^-\pi^0$. Comparisons of these theoretical predictions with the existing experimental data are made in Sec. VI. In Fig. 4 we compare the momentum behavior of all three reaction ratios calculated in this paper. We observe that the momentum dependence of the reaction rates for $\bar{p}p \to K^+K^-\pi^+\pi^-$ and $\bar{p}p \to K\bar{K}\pi^\pm\pi^0$ are nearly the same, with the rates themselves being nearly equal for large lab momenta. In contrast, the reaction rate for $\bar{p}p \to K^+K^-\pi^0\pi^0$ grows less rapidly with lab momentum.

VI. COMPARISON OF SOFT-PION PREDICTIONS WITH EXPERIMENTAL DATA

In this paper we have made a soft-pion study of four $\bar{p}p$ annihilation reactions involving the production of a kaon pair and two pions. We have calculated the reaction rates for these processes at various lab momenta normalized to the $\bar{p}p \to K^+K^-$ rate. We now compare these theoretical results with the available experimental data.[7,8]

For the annihilation process involving $\pi^+\pi^-$ emission, a comparison of the soft-pion predictions for the ratio $R = w(\bar{p}p \to K^+K^-\pi^+\pi^-)/w(\bar{p}p \to K^+K^-)$ with the experimental data as a function of lab momentum is made in Fig. 7. In making this comparison we have subtracted out from the experimental data the "on-shell" contributions to the reaction rate from K^*-resonance production. Furthermore, we have assumed equal production rates for K^+K^- and $K^0\bar{K}^0$ as well as for $K^+K^-\pi^+\pi^-$ and $K^0\bar{K}^0\pi^+\pi^-$. Also, we have taken K^0 and $\bar{K}^0$ as having an equal mixture of K_S^0 and K_L^0 so that $w(\bar{p}p \to K^0\bar{K}^0\pi^+\pi^-) = 4w(\bar{p}p \to K_S^0K_S^0\pi^+\pi^-)$. As can be seen from Fig. 7, the comparison is very disappointing with the data being generally an order of magnitude larger than the soft-pion predictions.

TABLE III. Ratios of calculated reaction rates $R = w(\bar{p}p \to K\bar{K}\pi^\pm\pi^0)/w(\bar{p}p \to K^+K^-)$ and $2R = [w(\bar{p}p \to K^-K^0\pi^+\pi^0) + w(\bar{p}p \to K^+\bar{K}^0\pi^-\pi^0)]/w(\bar{p}p \to K^+K^-)$ at various energies.

p lab momentum (GeV/c)	Center-of-mass energy (GeV)	R	$2R$
0.0	1.87	0.019	0.038
0.5	1.94	0.031	0.062
1.0	2.08	0.079	0.158
1.5	2.25	0.190	0.380
2.0	2.43	0.380	0.760
2.5	2.60	0.679	1.358
3.0	2.77	1.080	2.160
3.5	2.93	1.601	3.202
4.0	3.08	2.261	4.522
4.5	3.22	2.992	5.984
5.0	3.36	3.873	7.746

The soft-pion predictions for the ratio $w(\bar{p}p \to K^+K^-\pi^0\pi^0)/w(\bar{p}p \to K^+K^-)$ cannot be tested at this time because of the lack of any two-neutral-pion emission data. It should be noted that our results for $\pi^0\pi^0$ emission are in basically good agreement with the results of Uritam[5] except for large lab momentum where one expects our relativistic corrections to be important.

In the case of $\pi^\pm\pi^0$ emission a comparison of the soft-pion predictions for the ratios $w(\bar{p}p \to \bar{K}K\pi^\pm\pi^0)/w(\bar{p}p \to K^+K^-)$ and $[w(\bar{p}p \to \bar{K}K\pi^+\pi^0) + w(\bar{p}p \to \bar{K}K\pi^-\pi^0)]/w(\bar{p}p \to K^+K^-)$ with the experimental data is made in Figs. 8 and 9. Once again the discrepancy is very great with the soft-pion results being about an order of magnitude smaller than the data.

These large discrepancies between the soft-pion predictions and experiment are very serious indeed. One at first might attribute these discrepancies to a breakdown of PCAC, especially for large lab momentum, in view of the very demanding extrapolation which is required on the amplitudes in the application of this formalism. However, in view of the impressive number of suc-

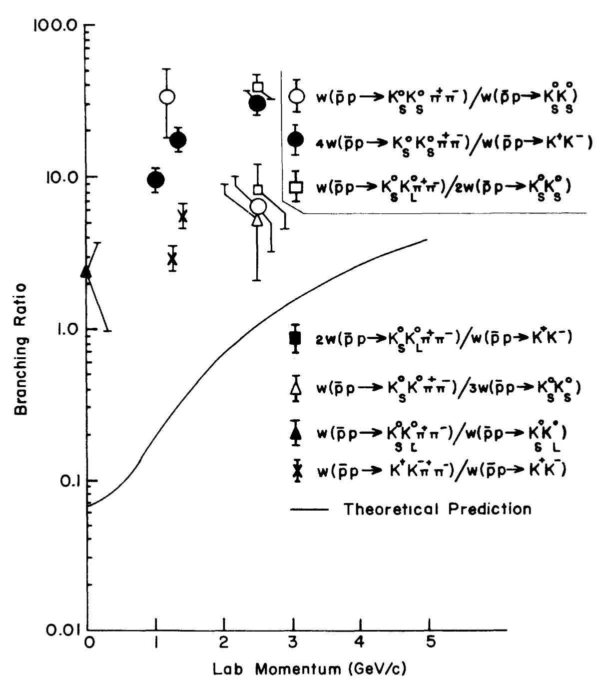

FIG. 7. Comparison of theoretical prediction of $w(\bar{p}p \to K^+K^-\pi^+\pi^-)/w(\bar{p}p \to K^+K^-)$ with experiment at various lab momenta. In presenting the data, K^* resonance production has been subtracted out.

cesses of the PCAC hypothesis, one is obliged to search for another source for the discrepancies.

Similar discrepancies, in fact, have been observed before[16,17] in the reactions $pp \to np\pi^+$ and $\bar{p}p \to \bar{\Lambda}\Lambda\pi^0$. For the process $pp \to np\pi^+$ the soft-pion predictions[18] for the cross sections are almost an order of magnitude smaller than the experimental cross sections.[19] Suspecting that "off-shell" resonance effects might be important, Schillaci and Silbar[16] examined the resonance contribution of the $\Delta(1236)$ isobar to the $pp \to np\pi^+$ amplitude and found that the isobar effects were substantial and could account for all of the observed discrepancies.

In view of the importance of resonance effects in reconciling soft-pion predictions and the experimental data for certain other strong interaction processes, it would appear that "off-shell" resonance effects may be decisively important in explaining the discrepancies which we are faced with in this paper between our soft-pion results for $\bar{p}p \to K\bar{K}\pi\pi$ and experiment. In Fig. 10 we display some of the "off-shell" resonance contributions to the $\bar{p}p \to K\bar{K}\pi\pi$ amplitude. Even though the "on-shell" contributions of the ordinary $K^*(890)$ have been subtracted out from the experimental data, the $K^*(890)$ can still give an "off-shell" contribution as shown in Fig. 10(a). In addition, the $K_A(1240)$ resonance can contribute to the ampli-

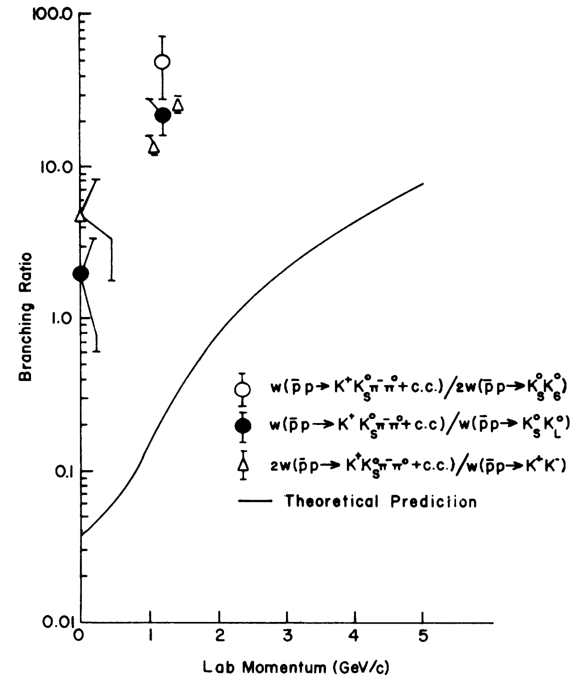

FIG. 9. Comparison of theoretical prediction of $[w(\bar{p}p \to K^+\bar{K}^0\pi^-\pi^0) + w(\bar{p}p \to K^-K^0\pi^+\pi^0)]/w(\bar{p}p \to K^+K^-)$ with experiment at various lab momenta.

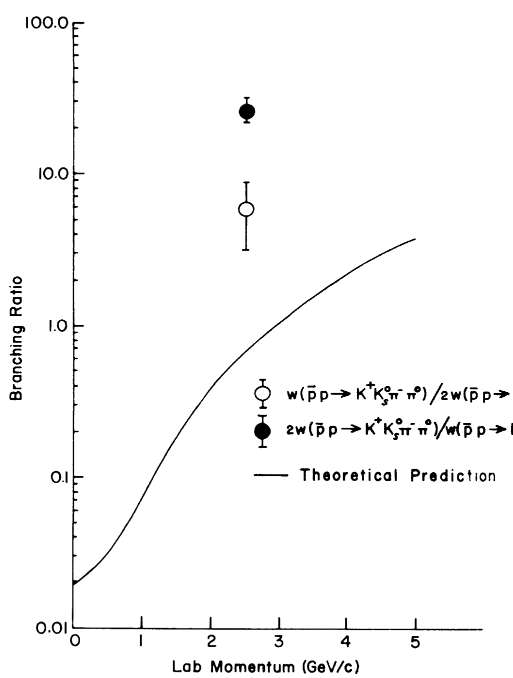

FIG. 8. Comparison of theoretical prediction of $w(\bar{p}p \to K^{\pm}K^0\pi^{\mp}\pi^0)/w(\bar{p}p \to K^+K^-)$ with experiment at various lab momenta.

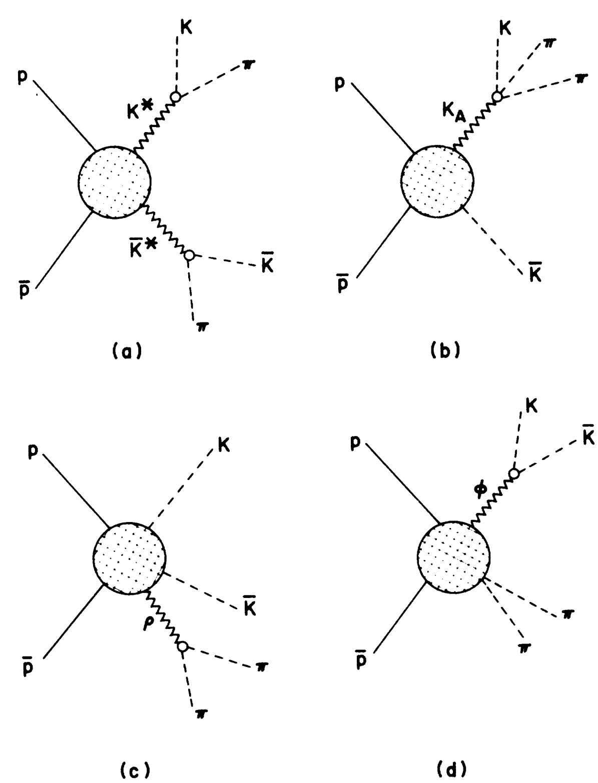

FIG. 10. Possible "off-shell" resonance contributions to the $\bar{p}p \to K\bar{K}\pi\pi$ amplitude.

tude [Fig. 10(b)] as well as the ordinary vector mesons ρ and ϕ [Figs. 10(c) and 10(d)]. Just how important these resonance effects are and whether they can successfully remove the presisting discrepancy between the predictions of PCAC and experiment must await further study.

We conclude that our soft-pion analysis of the class of $\bar{p}p$ annihilation processes, $\bar{p}p \to K\bar{K}\pi\pi$, has led to specific predictions as to the reaction rates for these processes relative to the reaction rate for $\bar{p}p \to K^+K^-$ as well as to the energy behavior of these reactions. The serious disagreement between the predictions of PCAC and the experimental data suggests that there may be important "off-shell" resonance effects on the amplitudes for these processes. The possible importance of such resonance contributions is now being investigated.

ACKNOWLEDGMENT

One of us (G. K. G.) would like to thank Professor T. W. B. Kibble for the hospitality extended to him at Imperial College, where much of this work was carried out.

APPENDIX A

The differential reaction rate for $\bar{p}p \to K^+K^-$ is given by

$$d^6w = \frac{(2\pi)^4}{(2\pi)^6}\frac{M^2}{p_1^0 p_2^0}\,\delta^4(p_1+p_2-q_1-q_2)\langle|\mathfrak{M}|^2\rangle \times \frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}, \tag{A1}$$

where

$$\mathfrak{M} = B\bar{v}^s(p_2)\not{Q}u^r(p_1).$$

Now

$$\langle|\mathfrak{M}|^2\rangle = -\tfrac{1}{4}|B|^2\mathrm{Tr}\,\frac{M-\not{p}_2}{2M}\not{Q}\,\frac{\not{p}_1+M}{2M}\not{Q} = \frac{|B|^2}{4M^2}\{2(p_1\cdot Q)(p_2\cdot Q)-[(p_1\cdot p_2)+M^2]Q^2\}. \tag{A2}$$

Performing the integrations over the kaon variables (see Appendix C), we obtain in the center-of-mass frame

$$w(\bar{p}p \to K^+K^-) = \frac{|B|^2}{12\pi}(2E^2+M^2)\left(1-\frac{m_K{}^2}{E^2}\right)^{3/2}. \tag{A3}$$

APPENDIX B

In this appendix we explicitly write down the lengthy expressions for each $\sum$ calculated for the various annihilation processes. We introduce the notation

$$K=k_1+k_2,\quad R=q_1+q_2,\quad L=k_1-k_2,\quad Q=q_1-q_2,\quad P=p_1+p_2.$$

for $\bar{p}p \to K^+K^-\pi^+\pi^-$ we find

$$\begin{aligned}\sum{}^{+-} = \frac{1}{M^2}\Bigg(\Bigg\{ & M^4\mu^4\left(\frac{(P\cdot k_1)(P\cdot k_2)+(p_1\cdot k_1)(P\cdot k_1)+(p_2\cdot k_2)(P\cdot k_2)}{(p_1\cdot k_1)(p_2\cdot k_2)(p_1\cdot K)(p_2\cdot K)}\right)^2 + \left(\frac{(p_1\cdot k_1)(p_2\cdot k_2)-(p_2\cdot k_1)(p_1\cdot k_2)}{(p_1\cdot K)(p_2\cdot K)}\right)^2 \\ & -2\mu^2M^2\left(\frac{P\cdot K}{(p_1\cdot K)(p_2\cdot K)}\right)^2 + \frac{1}{64g_A{}^4}\left(\frac{p_2\cdot L}{p_2\cdot K}-\frac{p_1\cdot L}{p_1\cdot K}\right)^2 \\ & -\frac{1}{4g_A{}^2}\left(\frac{p_2\cdot L}{p_2\cdot K}-\frac{p_1\cdot L}{p_1\cdot K}\right)\Bigg[M^2(k_1\cdot k_2)\left(\frac{1}{(p_1\cdot k_1)(p_1\cdot K)}+\frac{1}{(p_2\cdot k_2)(p_2\cdot K)}\right) \\ & +\frac{(p_1\cdot k_1)(p_2\cdot k_2)-(p_1\cdot k_2)(p_2\cdot k_1)}{(p_1\cdot K)(p_2\cdot K)}\Bigg]\Bigg\}[2(p_1\cdot Q)(p_2\cdot Q)-\tfrac{1}{2}P^2Q^2] \\ & +\frac{1}{16g_A{}^4}\left(\frac{(R\cdot L)(Q\cdot K)-(R\cdot K)(Q\cdot L)}{(R\cdot K)^2-(Q\cdot K)^2}\right)^2[2(p_1\cdot Q)(p_2\cdot Q)-\tfrac{1}{2}P^2Q^2]+\frac{2M^4G}{(p_1\cdot k_1)(p_2\cdot k_2)(p_1\cdot K)(p_2\cdot K)} \\ & +\frac{2M^2[M^2\mu^2-(p_1\cdot k_1)^2]}{(p_1\cdot k_1)^2(p_2\cdot k_2)(p_1\cdot K)}[(P\cdot k_2)^2Q^2+P^2(k_2\cdot Q)^2-2(P\cdot k_2)(P\cdot Q)(k_2\cdot Q)]+(p_1,k_1 \leftrightarrow p_2,k_2) \\ & +\frac{1}{4g_A{}^2}\left(\frac{p_2\cdot L}{p_2\cdot K}-\frac{p_1\cdot L}{p_1\cdot K}\right)\frac{M^2}{(p_1\cdot k_1)(p_2\cdot k_2)}\{2(P\cdot k_1)(p_1\cdot Q)(k_2\cdot Q)+2(P\cdot k_2)(p_2\cdot Q)(k_1\cdot Q) \\ & -\tfrac{1}{2}[2(P\cdot k_1)(P\cdot k_2)-P^2(k_1\cdot k_2)]Q^2-P^2(k_1\cdot Q)(k_2\cdot Q) \\ & -2(k_1\cdot k_2)(p_1\cdot Q)(p_2\cdot Q)\}\Bigg),\end{aligned} \tag{B1}$$

where

$$G=Q^2\Big\{\mu^2(P\cdot k_1)^2+\mu^2(P\cdot k_2)^2-\frac{1}{2M^2}(p_1-p_2)^2(k_1\cdot k_2)(P\cdot k_1)(P\cdot k_2)-\frac{1}{M^2}[(p_1\cdot k_1)(p_2\cdot k_2)-(p_1\cdot k_2)(p_2\cdot k_1)]^2\Big\}$$
$$+(p_1\cdot Q)(k_1\cdot Q)\Big\{2(P\cdot k_2)(k_1\cdot k_2)-2\mu^2(P\cdot k_1)+\frac{2}{M^2}(p_2\cdot k_2)[(p_1\cdot k_1)(p_2\cdot k_2)-(p_2\cdot k_1)(p_1\cdot k_2)]\Big\}$$
$$+(p_1\cdot Q)(k_2\cdot Q)\Big\{2(P\cdot k_1)(k_1\cdot k_2)-2\mu^2(P\cdot k_2)+\frac{2}{M^2}(p_2\cdot k_1)[(p_2\cdot k_1)(p_1\cdot k_2)-(p_1\cdot k_1)(p_2\cdot k_2)]\Big\}$$
$$-\frac{2}{M^2}[(k_1\cdot k_2)(P\cdot k_1)(P\cdot k_2)](p_1\cdot Q)(p_2\cdot Q)$$
$$+(p_2\cdot Q)(k_1\cdot Q)\Big\{2(P\cdot k_2)(k_1\cdot k_2)-2\mu^2(P\cdot k_1)+\frac{2}{M^2}(p_1\cdot k_2)[(p_2\cdot k_1)(p_1\cdot k_2)-(p_1\cdot k_1)(p_2\cdot k_2)]\Big\}$$
$$+(p_2\cdot Q)(k_2\cdot Q)\Big\{2(P\cdot k_1)(k_1\cdot k_2)-2\mu^2(P\cdot k_2)+\frac{2}{M^2}(p_1\cdot k_1)[(p_1\cdot k_1)(p_2\cdot k_2)-(p_1\cdot k_2)(p_2\cdot k_1)]\Big\}$$
$$+\Big[\mu^2P^2+\frac{P^2}{M^2}(p_1\cdot k_2)(p_2\cdot k_2)-2(P\cdot k_2)^2\Big](k_1\cdot Q)^2+\Big[\mu^2P^2+\frac{P^2}{M^2}(p_1\cdot k_1)(p_2\cdot k_1)-2(P\cdot k_1)^2\Big](k_2\cdot Q)^2$$
$$+(k_1\cdot Q)(k_2\cdot Q)\Big\{\frac{(p_1-p_2)^2}{M^2}[(p_1\cdot k_1)(p_2\cdot k_2)+(p_1\cdot k_2)(p_2\cdot k_1)]-2P^2(k_1\cdot k_2)$$
$$+4[(p_1\cdot k_1)(p_2\cdot k_2)+(p_2\cdot k_1)(p_1\cdot k_2)]\Big\},\qquad\text{(B2)}$$

For $\bar{p}p\to K^+K^-\pi^0\pi^0$ we find

$$\sum^{00}=\frac{1}{M^2}\Big\{\Big[\sum\nolimits_1^{00}+\sum\nolimits_2^{00}\Big][2(p_1\cdot Q)(p_2\cdot Q)-\tfrac{1}{2}P^2Q^2]+\sum\nolimits_3^{00}+\sum\nolimits_4^{00}\Big\},\qquad\text{(B3)}$$

where

$$\sum\nolimits_1^{00}=\frac{(p_1\cdot L)^2[M^2\mu^2-(p_1\cdot k_2)^2][M^2\mu^2-(p_1\cdot k_1)^2]}{(p_1\cdot k_1)^2(p_1\cdot k_2)^2(p_1\cdot K)^2}+(p_1\leftrightarrow p_2)$$
$$+\frac{[M^2\mu^2-(p_2\cdot k_2)^2][M^2\mu^2-(p_1\cdot k_1)^2]}{(p_1\cdot k_1)^2(p_2\cdot k_2)^2}+(k_1\leftrightarrow k_2)$$
$$+4\Big[\frac{M^2(k_1\cdot k_2)-(p_1\cdot k_1)(p_1\cdot k_2)}{(p_1\cdot k_2)(p_1\cdot K)}+\frac{M^2(k_1\cdot k_2)-(p_2\cdot k_1)(p_2\cdot k_2)}{(p_2\cdot k_1)(p_2\cdot K)}\Big]^2$$
$$-2\,\frac{(p_1\cdot L)(p_2\cdot L)[M^2\mu^2+(p_1\cdot k_1)(p_2\cdot k_1)][M^2\mu^2+(p_1\cdot k_2)(p_2\cdot k_2)]}{(p_1\cdot k_1)(p_2\cdot k_1)(p_1\cdot k_2)(p_2\cdot k_2)(p_1\cdot K)(p_2\cdot K)}$$
$$-2\,\frac{(p_1\cdot L)[M^2\mu^2-(p_1\cdot k_1)^2][M^2\mu^2+(p_1\cdot k_2)(p_2\cdot k_2)]}{(p_1\cdot k_1)^2(p_1\cdot k_2)(p_2\cdot k_2)(p_1\cdot K)}+(k_1\leftrightarrow k_2)$$
$$+4\Big[\frac{(p_2\cdot L)[M^2(k_1\cdot k_2)-(p_2\cdot k_1)(p_2\cdot k_2)]}{(p_2\cdot k_1)(p_2\cdot k_2)(p_2\cdot K)}-\frac{(p_1\cdot L)[M^2(k_1\cdot k_2)-(p_1\cdot k_1)(p_1\cdot k_2)]}{(p_1\cdot k_1)(p_1\cdot k_2)(p_1\cdot K)}\Big]$$
$$\times\Big[1+\frac{M^2(k_1\cdot k_2)-(p_1\cdot k_1)(p_1\cdot k_2)}{(p_1\cdot k_2)(p_1\cdot K)}+\frac{M^2(k_1\cdot k_2)-(p_2\cdot k_1)(p_2\cdot k_2)}{(p_2\cdot k_1)(p_2\cdot K)}\Big]$$
$$+2\,\frac{(p_2\cdot L)[M^2\mu^2-(p_2\cdot k_2)^2][M^2\mu^2+(p_1\cdot k_1)(p_2\cdot k_1)]}{(p_1\cdot k_1)(p_2\cdot k_1)(p_2\cdot k_2)^2(p_2\cdot K)}+(k_1\leftrightarrow k_2),\qquad\text{(B4)}$$

$$\sum\nolimits_2^{00}=4\,\frac{[M^2(k_1\cdot k_2)-(p_1\cdot k_1)(p_1\cdot k_2)][M^2(k_1\cdot k_2)-(p_2\cdot k_1)(p_2\cdot k_2)]}{(p_1\cdot k_1)(p_2\cdot k_1)(p_1\cdot k_2)(p_2\cdot k_2)}$$
$$-2\,\frac{[M^2\mu^2+(p_1\cdot k_1)(p_2\cdot k_1)][M^2\mu^2+(p_1\cdot k_2)(p_2\cdot k_2)]}{(p_1\cdot k_1)(p_2\cdot k_1)(p_1\cdot k_2)(p_2\cdot k_2)}$$
$$+8\Big[\frac{M^2(k_1\cdot k_2)-(p_1\cdot k_1)(p_1\cdot k_2)}{(p_1\cdot k_2)(p_1\cdot K)}+\frac{M^2(k_1\cdot k_2)-(p_2\cdot k_1)(p_2\cdot k_2)}{(p_2\cdot k_1)(p_2\cdot K)}\Big],\qquad\text{(B5)}$$

$$\sum_3^{00} = -4M^4 \frac{[(p_1\cdot k_1)(p_2\cdot k_1)+(p_1\cdot k_2)(p_2\cdot k_2)]}{(p_1\cdot k_1)(p_2\cdot k_1)(p_1\cdot k_2)(p_2\cdot k_2)(p_1\cdot K)(p_2\cdot K)}$$
$$\times\Bigg(\frac{M^2\mu^2+(p_1\cdot k_1)(p_2\cdot k_1)}{M^2}[(P\cdot k_2)^2Q^2+P^2(k_2\cdot Q)^2-2(P\cdot k_2)(P\cdot Q)(k_2\cdot Q)]+(k_1\to k_2)$$
$$-\frac{1}{2M^2}(P\cdot k_1)(P\cdot k_2)\{(p_1-p_2)^2(k_1\cdot k_2)+2[(p_1\cdot k_1)(p_2\cdot k_2)+(p_1\cdot k_2)(p_2\cdot k_1)]\}Q^2$$
$$-\frac{2}{M^2}(P\cdot k_1)(P\cdot k_2)(k_1\cdot k_2)(p_1\cdot Q)(p_2\cdot Q)-2(P\cdot k_2)^2(k_1\cdot Q)^2-2(P\cdot k_1)^2(k_2\cdot Q)^2$$
$$+(k_1\cdot Q)(k_2\cdot Q)\Bigg\{4[(p_1\cdot k_1)(p_1\cdot k_2)+(p_2\cdot k_1)(p_2\cdot k_2)]$$
$$+\frac{(p_1-p_2)^2}{M^2}[(p_1\cdot k_1)(p_2\cdot k_2)+(p_2\cdot k_1)(p_1\cdot k_2)]-2P^2(k_1\cdot k_2)\Bigg\}$$
$$+2(P\cdot k_2)\left[(k_1\cdot k_2)+\frac{(p_1\cdot k_1)(p_2\cdot k_2)}{M^2}\right](p_1\cdot Q)(k_1\cdot Q)+(p_1\to p_2)$$
$$+2(P\cdot k_1)\left[(k_1\cdot k_2)+\frac{(p_2\cdot k_1)(p_1\cdot k_2)}{M^2}\right](p_1\cdot Q)(k_2\cdot Q)+(p_1\to p_2)\Bigg), \tag{B6}$$

$$\sum_4^{00} = \frac{2M^2(p_1\cdot L)[M^2\mu^2-(p_1\cdot k_1)^2]}{(p_1\cdot k_1)^2(p_1\cdot k_2)(p_2\cdot k_2)(p_1\cdot K)}[2(P\cdot k_2)(P\cdot Q)(k_2\cdot Q)-P^2(k_2\cdot Q)^2-(P\cdot k_2)^2Q^2]-(p_1,k_1\to p_2,k_2)$$
$$+\frac{2M^4(p_1\cdot L)}{(p_1\cdot k_1)(p_2\cdot k_1)(p_1\cdot k_2)^2(p_1\cdot K)}\Bigg(\frac{(M^2\mu^2-(p_1\cdot k_2)^2)}{M^2}[P^2(k_1\cdot Q)^2+(P\cdot k_1)^2Q^2-2(P\cdot k_1)(P\cdot Q)(k_1\cdot Q)]$$
$$+\frac{2}{M^2}[M^2(k_1\cdot k_2)-(p_1\cdot k_1)(p_1\cdot k_2)]$$
$$\times\{[\tfrac{1}{2}P^2(k_1\cdot k_2)-(P\cdot k_1)(P\cdot k_2)]Q^2+2(P\cdot k_2)(p_1\cdot Q)(k_1\cdot Q)$$
$$+2(P\cdot k_1)(p_2\cdot Q)(k_2\cdot Q)-2(k_1\cdot k_2)(p_1\cdot Q)(p_2\cdot Q)-P^2(k_1\cdot Q)(k_2\cdot Q)\}\Bigg)$$
$$-(p_1,k_1\to p_2,k_2)$$
$$-4M^2\left[\frac{M^2(k_1\cdot k_2)-(p_1\cdot k_1)(p_1\cdot k_2)}{(p_1\cdot k_2)(p_1\cdot K)}+\frac{M^2(k_1\cdot k_2)-(p_2\cdot k_1)(p_2\cdot k_2)}{(p_2\cdot k_1)(p_2\cdot K)}\right]$$
$$\times(\{[\tfrac{1}{2}P^2(k_1\cdot k_2)-(P\cdot k_1)(P\cdot k_2)]Q^2+2(P\cdot k_1)(p_1\cdot Q)(k_2\cdot Q)+2(P\cdot k_2)(p_2\cdot Q)(k_1\cdot Q)$$
$$-2(k_1\cdot k_2)(p_1\cdot Q)(p_2\cdot Q)-P^2(k_1\cdot Q)(k_2\cdot Q)\}+\{p_1\to p_2\}). \tag{B7}$$

For $\bar{p}p\to K^-K^0\pi^+\pi^0$ we find

$$\sum^{+0} = \frac{1}{M^2}\Bigg(F[2(p_1\cdot Q)(p_2\cdot Q)-\tfrac{1}{2}P^2Q^2]$$
$$+\frac{1}{2g_A{}^4}\frac{[(R\cdot L)^2(Q\cdot K)^2+(Q\cdot L)^2(R\cdot K)^2-2(R\cdot L)(R\cdot K)(Q\cdot L)(Q\cdot K)]}{[(R\cdot K)^2-(Q\cdot K)^2]^2}[2(p_1\cdot Q)(p_2\cdot Q)-\tfrac{1}{2}P^2Q^2]$$
$$-\frac{2M^2[M^2\mu^2-(p_1\cdot k_1)^2]}{(p_1\cdot k_1)^2(p_1\cdot k_2)(p_2\cdot k_2)}[(P\cdot k_2)^2Q^2+P^2(k_2\cdot Q)^2-2(P\cdot k_2)(P\cdot Q)(k_2\cdot Q)]$$
$$+\left\{\frac{4M^2[M^2(k_1\cdot k_2)-(p_1\cdot k_1)(p_1\cdot k_2)]}{(p_1\cdot k_1)(p_1\cdot k_2)(p_2\cdot k_2)(p_1\cdot K)}+\frac{1}{\sqrt{2}g_A{}^2}\frac{M^2(p_1\cdot L)}{(p_1\cdot k_1)(p_2\cdot k_2)(p_1\cdot K)}\right\}$$
$$\times[(P\cdot k_1)(P\cdot k_2)Q^2+P^2(k_1\cdot Q)(k_2\cdot Q)-2(P\cdot k_1)(p_1\cdot Q)(k_2\cdot Q)-2(P\cdot k_2)(p_2\cdot Q)(k_1\cdot Q)]\Bigg) \tag{B8}$$

where

$$F=\frac{[M^2\mu^2-(p_1\cdot k_1)^2][M^2\mu^2-(p_1\cdot k_2)^2]}{(p_1\cdot k_1)^2(p_1\cdot k_2)^2}+\frac{[M^2\mu^2-(p_1\cdot k_1)^2][M^2\mu^2-(p_2\cdot k_2)^2]}{(p_1\cdot k_1)^2(p_2\cdot k_2)^2}$$
$$-2\frac{[M^2\mu^2-(p_1\cdot k_1)^2][M^2\mu^2+(p_1\cdot k_2)(p_2\cdot k_2)]}{(p_1\cdot k_1)^2(p_1\cdot k_2)(p_2\cdot k_2)}-4\frac{[M^2(k_1\cdot k_2)-(p_1\cdot k_1)(p_1\cdot k_2)]^2}{(p_1\cdot k_1)(p_1\cdot k_2)(p_1\cdot K)^2}$$
$$+4\frac{[M^2(k_1\cdot k_2)-(p_1\cdot k_1)(p_1\cdot k_2)][M^2(k_1\cdot k_2)+(p_1\cdot k_1)(p_2\cdot k_2)]}{(p_1\cdot k_1)(p_1\cdot k_2)(p_2\cdot k_2)(p_1\cdot K)}$$
$$+\frac{1}{8g_A{}^4}\frac{(p_1\cdot L)^2}{(p_1\cdot K)^2}+\frac{1}{\sqrt{2}\,g_A{}^2}\frac{(p_1\cdot L)^2[M^2(k_1\cdot k_2)-(p_1\cdot k_1)(p_1\cdot k_2)]}{(p_1\cdot k_1)(p_1\cdot k_2)(p_1\cdot K)^2}$$
$$+\frac{1}{\sqrt{2}\,g_A{}^2}(p_1\cdot L)\frac{[M^2(k_1\cdot k_2)+(p_1\cdot k_1)(p_2\cdot k_2)]}{(p_1\cdot k_1)(p_2\cdot k_2)(p_1\cdot K)}\,. \tag{B9}$$

APPENDIX C

In calculating the totally integrated reaction rates for $\bar{p}p\to K\bar{K}\pi\pi$ the following types of integrals are encountered:

$$T=\int\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}\,Q^2\delta^4(P-K-R)\,, \tag{C1}$$

$$T(A,B)=\int\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}\,(Q\cdot A)(Q\cdot B)\delta^4(P-K-R)\,, \tag{C2}$$

$$T_{KK}=\int\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}\,Q^2\frac{(Q\cdot K)^2(R\cdot L)^2}{[(R\cdot K)^2-(Q\cdot K)^2]^2}\,\delta^4(P-K-R)\,, \tag{C3}$$

$$T_{KL}=\int\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}\,Q^2\frac{(Q\cdot K)(Q\cdot L)(R\cdot K)(R\cdot L)}{[(R\cdot K)^2-(Q\cdot K)^2]^2}\delta^4(P-K-R)\,, \tag{C4}$$

$$T_{LL}=\int\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}\,Q^2\frac{(Q\cdot L)^2(R\cdot K)^2}{[(R\cdot K)^2-(Q\cdot K)^2]^2}\,\delta^4(P-K-R)\,, \tag{C5}$$

$$T_{KK}(A,B)=\int\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}\,\frac{(Q\cdot A)(Q\cdot B)(Q\cdot K)^2(R\cdot L)^2}{[(R\cdot K)^2-(Q\cdot K)^2]^2}\,\delta^4(P-K-R)\,, \tag{C6}$$

$$T_{KL}(A,B)=\int\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}\,\frac{(Q\cdot A)(Q\cdot B)(Q\cdot K)(Q\cdot L)(R\cdot K)(R\cdot L)}{[(R\cdot K)^2-(Q\cdot K)^2]^2}\,\delta^4(P-K-R)\,, \tag{C7}$$

$$T_{LL}(A,B)=\int\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}\,\frac{(Q\cdot A)(Q\cdot B)(Q\cdot L)^2(R\cdot K)^2}{[(R\cdot K)^2-(Q\cdot K)^2]^2}\,\delta^4(P-K-R)\,. \tag{C8}$$

By evaluating these Lorentz-invariant integrals in the K^+K^- center-of-mass system, we find

$$T=-\frac{\pi}{2}R^2\left(\frac{R^2-4m_K{}^2}{R^2}\right)^{3/2}\,, \tag{C9}$$

$$T(A,B)=\frac{\pi}{6}[(R\cdot A)(R\cdot B)-R^2(A\cdot B)]\left(\frac{R^2-4m_K{}^2}{R^2}\right)^{3/2}\,, \tag{C10}$$

$$T_{KK}=-\frac{\pi}{4}R^2(R\cdot L)^2\left(\frac{R^2-4m_K{}^2}{R^2}\right)^{3/2}\left\{\frac{1}{(R\cdot K)^2-\dfrac{R^2-4m_K{}^2}{R^2}\,R(K,K)}-\frac{1}{(R\cdot K)\left(\dfrac{R^2-4m_K{}^2}{R^2}\right)^{1/2}R^{1/2}(K,K)}\tanh^{-1}\left[\left(\frac{R^2-4m_K{}^2}{R^2}\right)^{1/2}\frac{R^{1/2}(K,K)}{(R\cdot K)}\right]\right\}\,, \tag{C11}$$

where $R(A,B)=(R\cdot A)(R\cdot B)-R^2(A\cdot B)$,

$$T_{KL}=\frac{(R\cdot K)^2}{R(K,K)}\,T_{KK}\,, \tag{C12}$$

$$T_{LL}=-\frac{\pi}{8}\frac{(R^2)^3}{R^2(K,K)}\left(\frac{R^2-4m_K{}^2}{R^2}\right)^{3/2}$$
$$\times\Bigg\{\frac{(R^2)^{1/2}}{(R\cdot K)[(R^2-4m_K{}^2)R(K,K)]^{1/2}}\Bigg[\frac{(R^2-4m_K{}^2)R(K,K)}{(R^2)^3}\,[R(K,K)R(L,L)-(R\cdot K)^2(R\cdot L)^2]$$
$$+\frac{(R\cdot K)^2}{(R^2)^2}\,[R(K,K)R(L,L)-3(R\cdot K)^2(R\cdot L)^2]\Bigg]$$
$$\times\tanh^{-1}\left[\left(\frac{(R^2-4m_K{}^2)R(K,K)}{(R\cdot K)^2R^2}\right)^{1/2}\right]$$
$$+\frac{R^2}{R^2(R\cdot K)^2-(R^2-4m_K{}^2)R(K,K)}\Bigg[\frac{(R^2-4m_K{}^2)R(K,K)}{(R^2)^3}\,[R(K,K)R(L,L)-(R\cdot K)^2(R\cdot L)^2]$$
$$-\frac{(R\cdot K)^2}{(R^2)^2}\,[R(K,K)R(L,L)-3(R\cdot K)^2(R\cdot L)^2]\Bigg]\Bigg\}\,, \qquad \text{(C13)}$$

$$T_{KK}(A,B)=\frac{\pi}{8}\,(R\cdot L)^2\left(\frac{R^2-4m_K{}^2}{R^2}\right)^{1/2}\frac{(R^2)^2}{R^2(K,K)}$$
$$\times\Bigg\{\frac{(R^2)^{1/2}}{(R\cdot K)[(R^2-4m_K{}^2)R(K,K)]^{1/2}}\Bigg[\left(\frac{R(A,K)R(B,K)-R(A,B)R(K,K)}{(R^2)^2}\right)\left(\frac{(R^2-4m_K{}^2)R(K,K)}{R^2}\right)$$
$$-3(R\cdot K)^2\left(\frac{3R(A,K)R(B,K)-R(K,K)R(A,B)}{(R^2)^2}\right)\Bigg]$$
$$\times\tanh^{-1}\left[\left(\frac{(R^2-4m_K{}^2)R(K,K)}{(R\cdot K)^2(R^2)^2}\right)^{1/2}\right]$$
$$+\frac{R^2}{R^2(R\cdot K)^2-(R^2-4m_K{}^2)R(K,K)}\Bigg[\left(3(R\cdot K)^2-\frac{2(R^2-4m_K{}^2)R(K,K)}{R^2}\right)\left(\frac{3R(A,K)R(B,K)-R(K,K)R(A,B)}{(R^2)^2}\right)$$
$$+\left(\frac{(R^2-4m_K{}^2)R(K,K)}{R^2}\right)\left(\frac{R(K,K)R(A,B)-R(A,K)R(B,K)}{(R^2)^2}\right)\Bigg]\Bigg\}, \qquad \text{(C14)}$$

$$T_{KL}(A,B)=\frac{\pi}{8}\,(R\cdot K)(R\cdot L)\left(\frac{R^2-4m_K{}^2}{R^2}\right)^{1/2}\left(\frac{R^2}{R(K,K)}\right)^3$$
$$\times\Bigg\{\frac{(R^2)^{1/2}}{(R\cdot K)[R^2-4m_K{}^2)R(K,K)]^{1/2}}\left[\left(3(R\cdot K)^2-\frac{(R^2-4m_K{}^2)}{R^2}R(K,K)\right)\alpha_1-3(R\cdot K)^2\beta_1\right]$$
$$\times\tanh^{-1}\left[\left(\frac{(R^2-4m_K{}^2)R(K,K)}{R^2(R\cdot K)^2}\right)^{1/2}\right]$$
$$+\frac{R^2}{R^2(R\cdot K)^2-(R^2-4m_K{}^2)R(K,K)}\Bigg[\left(3(R\cdot K)^2-2\,\frac{(R^2-4m_K{}^2)R(K,K)}{R^2}\right)\beta_1$$
$$-3\left((R\cdot K)^2-\frac{(R^2-4m_K{}^2)R(K,K)}{R^2}\right)\alpha_1\Bigg]\Bigg\}\,, \qquad \text{(C15)}$$

where

$$\alpha_1=\frac{1}{(R^2)^3}\,\{R(B,K)[R(K,K)R(A,L)-(R\cdot K)(R\cdot L)R(A,K)]+R(A,K)[R(K,K)R(B,L)-(R\cdot K)(R\cdot L)R(B,K)]$$
$$+(R\cdot K)(R\cdot L)[R(K,K)R(A,B)-R(A,K)R(B,K)]\}, \qquad \text{(C16)}$$

$$\beta_1=\frac{2}{(R^2)^3}\,(R\cdot K)(R\cdot L)R(A,K)R(B,K), \qquad \text{(C17)}$$

$$T_{LL}(A,B)=\frac{\pi}{32}\frac{(R^2)^4}{R^4(K,K)}\left(\frac{R^2-4m_K^2}{R^2}\right)^{1/2}$$
$$\times\left(\frac{(R^2)^{1/2}}{(R\cdot K)[(R^2-4m_K^2)R(K,K)]^{1/2}}\right.$$
$$\times\left\{\left[\left(\frac{(R^2-4m_K^2)R(K,K)}{R^2}\right)^2+\frac{2(R\cdot K)^2(R^2-4m_K^2)R(K,K)}{R^2}-3(R\cdot K)^4\right]\alpha_2\right.$$
$$\left.-3(R\cdot K)^4\beta_2+(R\cdot K)^2\left(3(R\cdot K)^2-\frac{(R^2-4m_K^2)R(K,K)}{R^2}\right)\gamma_2\right\}\tanh^{-1}\left[\left(\frac{(R^2-4m_K^2)R(K,K)}{(R\cdot K)^2R^2}\right)^{1/2}\right]$$
$$+\frac{R^2}{R^2(R\cdot K)^2-(R^2-4m_K^2)R(K,K)}\left\{\left[\left(\frac{(R^2-4m_K^2)R(K,K)}{R^2}\right)^2\right.\right.$$
$$\left.-4\frac{(R\cdot K)^2(R^2-4m_K^2)R(K,K)}{R^2}+3(R\cdot K)^4\right]\alpha_2$$
$$+(R\cdot K)^2\left(3(R\cdot K)^2-2\frac{(R^2-4m_K^2)R(K,K)}{R^2}\right)\beta_2$$
$$\left.\left.-3(R\cdot K)^2\left((R\cdot K)^2-\frac{(R^2-4m_K^2)R(K,K)}{R^2}\right)\gamma_2\right\}\right),\qquad\text{(C18)}$$

where

$$\alpha_2=\frac{3}{(R^2)^4}[R(K,K)R(A,L)-(R\cdot K)(R\cdot L)R(A,K)][R(K,K)R(B,L)-(R\cdot K)(R\cdot L)R(B,K)]$$
$$+\frac{R(K,K)}{(R^2)^2}\epsilon_{\mu\nu\lambda\sigma}R^\mu A^\nu K^\lambda L^\sigma\epsilon_{abcd}R^aB^bK^cL^d,\qquad\text{(C19)}$$

$$\beta_2=\frac{8}{(R^2)^4}(R\cdot K)^2(R\cdot L)^2R(A,K)R(B,K),\qquad\text{(C20)}$$

$$\gamma_2=\frac{4}{(R^2)^4}[R(K,K)R(L,L)-(R\cdot K)^2(R\cdot L)^2]R(A,K)R(B,K)$$
$$+\frac{4}{(R^2)^4}\left[\frac{(R\cdot K)^2(R\cdot L)^2}{R(K,K)R(L,L)-(R\cdot K)^2(R\cdot L)^2}\right][R(K,K)R(A,L)-(R\cdot K)(R\cdot L)R(A,K)]$$
$$\times[R(K,K)R(B,L)-(R\cdot K)(R\cdot L)R(B,K)]$$
$$+\frac{4}{(R^2)^2}R(K,K)\left[\frac{(R\cdot K)^2(R\cdot L)^2}{R(K,K)R(L,L)-(R\cdot K)^2(R\cdot L)^2}\right]\epsilon_{\mu\nu\lambda\sigma}R^\mu A^\nu K^\lambda L^\sigma\epsilon_{abcd}R^aB^bK^cL^d$$
$$+\frac{8}{(R^2)^4}(R\cdot K)(R\cdot L)R(B,K)[R(K,K)R(A,L)-(R\cdot K)(R\cdot L)R(A,K)]$$
$$+\frac{8}{(R^2)^4}(R\cdot K)(R\cdot L)R(A,K)[R(K,K)R(B,L)-(R\cdot K)(R\cdot L)R(B,K)],\qquad\text{(C21)}$$

with

$$\epsilon_{abcd}R^ap_1^bK^cL^d=-\epsilon_{abcd}R^ap_2^bK^cL^d$$
$$=4E(E^2-M^2)^{1/2}[(\omega_1^2-\mu^2)(\omega_2^2-\mu^2)(1-y_1^2)(1-y_2^2)(1-x^2)]^{1/2}.\qquad\text{(C22)}$$

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[10]In our notation, the Lorentz-invariant amplitude, M_{fi}, is defined by

$$S_{fi} = \delta_{fi} + i(2\pi)^4\delta(P_i - P_f)M_{fi}/N,$$

where $N^2 = \Pi N_i^2$; $N_i^2 = 2E$ for bosons, $N_i^2 = E/m$ for fermions. The integrals in Eq. (2.11) are related to S_{fi} through the Lehmann-Symanzik-Zimmermann (LSZ) reduction formulas. In this way Eq. (2.11) can be expressed in terms of the Lorentz-invariant amplitudes, which will be utilized from now on.

[11]The cross terms A^*B and B^*A which appear in the square of the amplitude give no contribution to the totally integrated reaction rate.

[12]One might also argue on the basis of the "gentleness" assumption of PCAC that the annihilation amplitude for $\bar{p}p \to K\bar{K}\pi\pi$ should be slowly varying and that an extrapolation to nonvanishing lab momentum is allowable.

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[14]If one restricts $\bar{p}p$ annihilation to occur *at rest*, photon emission cannot take place from the proton-antiproton state, since this is a neutral state of two antiparticles at rest, with no net charge and no higher moments of the charge. However, for annihilation at nonzero lab momentum, photon emission can occur from the $\bar{p}p$ state.

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