Chou-Yang hypothesis: A critical assessment

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A critical assessment of the Chou-Yang hypothesis is carried out for high-energy proton-proton scattering. The proton opacity, obtained with allowance for the real part of the scattering amplitude, is compared with the electromagnetic (em) charge densities. It is shown that the relationship between the proton opacity and charge density is very sensitive both to the choice of em form factors (i.e., $F_1^{\,2}$ or $G_E^{\,2}$) and to the detailed parametrization of these form factors. A presentation of the proton opacity is suggested in which the charge density plays a modified but nevertheless important role. We speculate about this presentation in relation to inelastic diffraction scattering.

I. INTRODUCTION

According to the Chou-Yang model[1] the elastic scattering of hadrons at high energies is calculated using an eikonal formalism

$$S(b,s)=e^{i\chi(b,s)}\ , \tag{1}$$

where b is the (two-dimensional) impact parameter and s the c.m. energy squared. The main hypothesis is that at very high energies the elastic amplitude is purely absorptive and approaches a limiting, energy-independent, distribution. This is implemented by making χ purely imaginary and s independent, i.e.,

$$\chi(b,s)=i\Omega(b,s)\rightarrow i\Omega(b)\ , \tag{2}$$

where Ω is real. The most appealing physical assumption of the model is the identification of the opacity $\Omega(b)$ with the overlap of the matter densities of the colliding hadrons. This overlap is taken to be proportional to the electromagnetic charge distribution in b space:

$$\Omega(b)=\kappa\Omega_{\rm em}(b)\ . \tag{3}$$

$\Omega_{\rm em}(b)$ is known *a priori* from electron scattering experiments where the appropriate form factor $\Omega_{\rm em}(q^2)$ is measured, and

$$\Omega_{\rm em}(b)=\int\hat{\Omega}_{\rm em}(q^2)\,J_0(qb)\,q\,dq\ . \tag{4}$$

Aside from its intuitive physical appeal, the model accounts in a natural way[2-4] for the concave diffractive peak observed in high-energy p-p scattering and for the (diffractive) dip seen at $t\approx-1.4$ GeV/c^2. However, it is manifestly clear that the continued growth of the p-p total cross section through the Fermilab and CERN ISR energy range and the continued shrinkage of the diffraction peak make it impossible to contemplate the Chou-Yang model in an unadulterated form. This conclusion is strengthened by the experimental finding that the position of the dip in $d\sigma/dt$ is moving inward toward smaller $|t|$ with increasing energy.[5] It is thus concluded that $\Omega(b,s)$ is s-dependent all through the range $250\leq s\leq 3500$ GeV^2.

Attempts have been made[4] to salvage the intuitively appealing proportionality between matter and charge distributions by postulating that

$$\Omega(b,s)=\kappa(s)\,\Omega_{\rm em}(b) \tag{5}$$

in the hope that the desired energy dependence can be accommodated in the multiplicative factor $\kappa(s)$.

Unfortunately, such a factorized form for $\Omega(b,s)$ has two major failings:

1. It leads[6] to a very rapid growth with energy of the height of the second maximum seen in $d\sigma/dt$ at $t\approx-1.8$ GeV/c^2. This rapid growth is not compatible with the very moderate energy dependence observed experimentally over the ISR range.[5]

2. It incorporates a rapid growth with energy of the opacity at small b. Specifically, a factorizable Ω implies that the increment of $\Omega(b,s)$ is proportional to $\Omega(b,s)$ itself. This is in contradiction with the behavior of the opacity derived directly from the p-p elastic scattering data.[5,7]

An alternative possibility is to postulate that we have not yet reached high enough energies to be able to see the onset of the limiting behavior. Such a possibility implies that the growth of σ_T at Fermilab and ISR is a temporary feature; the truly asymptotic, constant cross section is approached from below. We shall discuss this possibility in some detail later.

In this paper we have attempted a critical analysis of the possible role of the Chou-Yang hypothesis in describing p-p elastic scattering. We have examined whether there is a natural framework for incorporating the physically appealing aspects of the hypothesis. We have been led to a picture in which the bulk of the p-p opacity is indeed controlled by the electromagnetic charge density, but in which a vital role is played by a second, energy-dependent component. We associate this second component with the forces responsible for the growth of the total cross sections. Some consequences of this association are studied.

In order to systematically study the Chou-Yang hypothesis we have to specify the input assumptions of the model precisely:

1. The model, in its standard formulation, concerns itself solely with helicity-nonflip amplitudes. It is tacitly assumed that all spin-flip effects have died out at high energies. Although this may be a very reasonable hypothesis for the small-t region, it is not as obviously reasonable at large t. Indeed, some interesting attempts have been made[2,8] to include spin effects. We believe that there is not yet sufficient data to pin down the additional freedom in such an approach. We shall, therefore, continue in this paper to assume dominance of the nonflip amplitude.

2. At present energies the elastic p-p amplitude is *not purely imaginary*. A study by Grein, Guigas, and Kroll,[7] using fixed-t dispersion relations, indicates that the real part can be significant for t values outside the diffraction peak, and in particular in the region of the diffraction dip. Using their (complex) amplitude, we have calculated the eikonal $\chi_{pp}(b,s)$ which is now complex:

$$\chi_{pp}(b,s)=\chi^R_{pp}(b,s)+i\Omega_{pp}(b,s)\ . \qquad (6)$$

It should be stressed that the opacity $\Omega_{pp}(b,s)$ obtained in the above analysis is somewhat different from the opacity usually obtained[5] under the assumption that the amplitude is purely imaginary.

Since the Chou-Yang hypothesis is motivated by the idea that absorption is proportional to matter density, we shall sssume that it is to the opacity (i.e., the imaginary part of χ_{pp}) alone that the hypothesis should apply. Throughout this paper, therefore, p-p opacity means Ω_{pp} obtained from the data allowing for a real part in the scattering amplitude.

3. The isoscalar electromagnetic charge density utilized in the Chou-Yang model calculations[1-4] is not uniquely defined. It is not clear, for example, whether one should use a charge distribution associated with the electric form factor $G_E(t)$ as recent studies have done,[2-4] or the Dirac form factor $F_1(t)$ as originally suggested.[1] We shall see that our results depend critically upon the particular choice of the form factor and also upon a detailed knowledge of its behavior. It is important to note that rough approximations to the form factor using dipole-type fits can lead to significantly different conclusions.

4. The model in its original form is used to describe a single channel, the elastic one, and the effects of all other channels are lumped into the absorption. Since we know that there is a significant amount of inelastic diffraction scattering at high energies, we feel that if the Chou-Yang hypothesis is relevant, it ought to be possible to generalize it so as to describe these channels explicitly.

In Sec. II we review the p-p data analysis by means of which the opacity $\Omega_{pp}(b,s)$ is obtained. In Sec. III we review the procedures employed to obtain the em charge densities from a study of the form factors We discuss in detail the necessary procedures for obtaining reliable distributions. In Sec. IV we compare the p-p opacity with the em charge densities and attempt to attach some physical significance to our findings. This is further elaborated upon in Sec. V, where we study the consequences of a particular extension of our ideas to a multichannel situation.

II. PROTON-PROTON DATA ANALYSIS

The standard implementation[3,4] of the Chou-Yang hypothesis, thus far, has been to attempt a fit to the relatively poorly determined p-p $d\sigma/dt$ distributions with an em form factor input. The availability of improved very-high-energy p-p data[5,7] offers the opportunity for a more direct comparison between the p-p opacity and the appropriate em form factors. This is essentially a comparison between two experimental distributions, where it is advisable to perform the study simultaneously in b space and t space so as to avoid systematic distortions. In particular we call attention to the fact that both σ_T growth and the diffraction $d\sigma/dt$ dip, which are gross t-space features, are associated with the fine structure of the b-space amplitude.

Our study thus depends on a careful evaluation of the high-energy p-p data. Such an analysis has been carried out recently.[7] The input assumptions for that analysis are as follows:

1. Spin effects are neglected. As we have noted in the Introduction, this is a fair approximation for $|t|\leq 1$ GeV/c^2, where lower-energy polarization data[9] indicate the diminishing importance of the flip amplitudes. For $|t|>1$ GeV/c^2 and in particular in the dip region this assumption is sensible

but has no direct experimental support.

2. Fixed-t dispersion relations are employed between the phase and modulus of the crossing-even amplitudes. A Regge phase is assumed for the crossing-odd amplitudes.

The result of these calculations is a numerical tabulation of the real and imaginary parts of $F_{pp}(t, s)$ and $F_{pp}(b, s)$, the spin-averaged p-p scattering amplitude in t and b spaces. *Quoted errors*[7] *on $F_{pp}(b, s)$ are 5%.* Adopting the notation

$$\chi_{pp}(b, s) = \chi_{pp}^R(b, s) + i\Omega_{pp}(b, s) \ , \tag{6}$$

$$2i(1 - e^{i\chi_{pp}}) = \mathrm{Re}F_{pp} + i\,\mathrm{Im}F_{pp} \ , \tag{7}$$

it is straightforward to obtain the opacity. One has

$$\Omega_{pp}(b, s) = -\tfrac{1}{2}\ln\{[1 - \tfrac{1}{2}\mathrm{Im}F_{pp}(b, s)]^2 + [\tfrac{1}{2}\mathrm{Re}F_{pp}(b, s)]^2\} \ . \tag{8}$$

Throughout this study we have utilized the opacity $\Omega_{pp}(b, s)$ obtained in this fashion. These $\Omega_{pp}(b, s)$ are slightly different from those obtained in the standard analysis[5] for small impact parameters $b < 0.5$ F. Our quantitative conclusions are sensitive to this modification.

Two basic features of the p-p amplitude emerge in b space. Both features have been known for some time,[5,7,10] and their implementation is crucial to our analysis:

1. σ_T growth through the ISR range is peripheral. Namely the growth is associated with higher b values. This is demonstrated in Fig. 1 where the increments of both $F_{pp}(b, s)$ and $\Omega_{pp}(b, s)$ over the ISR energy range are plotted. The intriguing problem of higher-s extrapolating is unfortunately unresolved. In particular it is an open question whether the radius of the proton b-space profile continues to increase indefinitely.

2. The very-small-b behavior of $F_{pp}(b, s)$ and $\Omega_{pp}(b, s)$ is consistent with energy independence; see Table I. This saturation (below the unitarity limit) is of considerable theoretical interest,[11] and, as we shall see, is of crucial importance in our analysis.

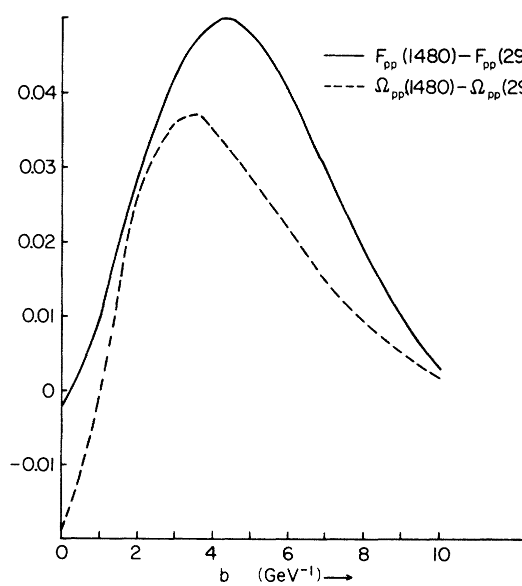

FIG. 1. Increment of the proton-proton profile function $F_{pp}(b, s)$ and the opacity $\Omega_{pp}(b, s)$ over the ISR energy range.

TABLE I. Momentum dependence of $\mathrm{Im}F_{pp}(b=0, s)$ and $\Omega_{pp}(b=0, s)$ over the ISR energy range.

P_L (GeV/c)	$\mathrm{Im}F_{pp}(b=0, s)$	$\Omega_{pp}(b=0, s)$
290	1.576	1.541
500	1.568	1.531
1070	1.568	1.517
1480	1.574	1.521

III. THE ELECTROMAGNETIC CHARGE DENSITY

A precise knowledge of the isoscalar form factors depends on an exceedingly good determination of the neutron form factors, since

$$F_i(I=0) = \tfrac{1}{2}(F_i^{\text{proton}} + F_i^{\text{neutron}}) \ . \tag{9}$$

The neutron's charge form factor is very small compared with that of the proton. In fact, in magnitude it is of the same order as the error bars on the proton data.[12,13] We have thus assumed that $F_i(I=0)$ is proportional to F_i^{proton}, and throughout this analysis the quoted form factors are actually proton form factors.

The comparison between the proton opacity and the em charge density is strongly dependent on the choice of form factor, i.e., the Dirac form factor F_1 or the charge form factor G_E. The two are related by the transformation

$$F_1(q^2) = G_E(q^2)\,\frac{4M^2 + \mu q^2}{4M^2 + q^2} \ , \tag{10}$$

where M is the proton mass, $\mu = 2.79$ is the proton total magnetic moment, and $q^2 = -t$ is the momentum transfer. (Scaling of the electric and magnetic form factors, i.e., $G_E = G_M/\mu$, is assumed.) Our studies also indicate a great sensitivity to the precise analytical formula used to present the q^2 dependence of these form factors.

We illustrate the sensitivity to the choice of form factor in Fig. 2 by comparing the ratio of Ω_{pp} to $\Omega_{\text{em}} = F_1^{\,2}$ and $\Omega_{\text{em}} = G_E^{\,2}$ using the standard dipole fit[13] for

$$G_E = \frac{1}{(1+q^2/0.71)^2} \tag{11}$$

and calculating F_1 by Eq. (10). The ratios are normalized to one at $b = 0$. It is clear that the relationship between Ω_{pp} and Ω_{em} crucially depends upon which form factor is chosen. This will be discussed in detail in the next section.

We examine the sensitivity to the actual form-factor parametrization by comparing two popular models:

1. The standard G_E dipole fit which gives a fair description of the data, especially for $q^2 < 1$, but fails in the range of large $q^2 > 1$.[13]

2. The Fried-Gaisser formula.[14] In a recently published study[13] of e-p scattering at $q^2 > 1$ it was found that this parametrization offered an excellent fit to the data. The quality of this fit is *not adequate* for the small $q^2 < 1$ range.

An indication of the sensitivity to the actual description of the form factors is given in Fig. 3 where we have plotted the ratio of Ω_{pp} to Ω_{em}, normalized to one at $b = 0$, utilizing the dipole and Fried-Gaisser G_E fits to evaluate Ω_{em}.

We have thus found it necessary to make new fits to the form-factor data over the entire q^2 range. Since we require the square of the form factors we have found it more convenient to fit directly the data on $F_1^2(q^2)$.

A good fit over the entire range[15] of q^2 can be achieved using a four-exponential fit

$$F_1^2(q^2) = \sum_{i=1}^{4} A_i e^{-B_i q^2} . \tag{12}$$

Taking all the available data points without any discrimination we obtain $\chi^2/n = 6.64$, which is rather disappointing. However, as can be seen from Fig. 4(a), which covers the region $q^2 \leq 2$, the large χ^2/n value arises from the incompatible data points with very small error bars, in the region $0.75 \leq q^2 \leq 1$. Assuming that the form factor is smooth in this region, it is perhaps more sensible to enlarge the error bars on the high-lying data of Bumiller *et al.*[15] when making the fit. If this is done (by a factor of 3) the χ^2/n improves to the acceptable value of 2.34 with the parameters

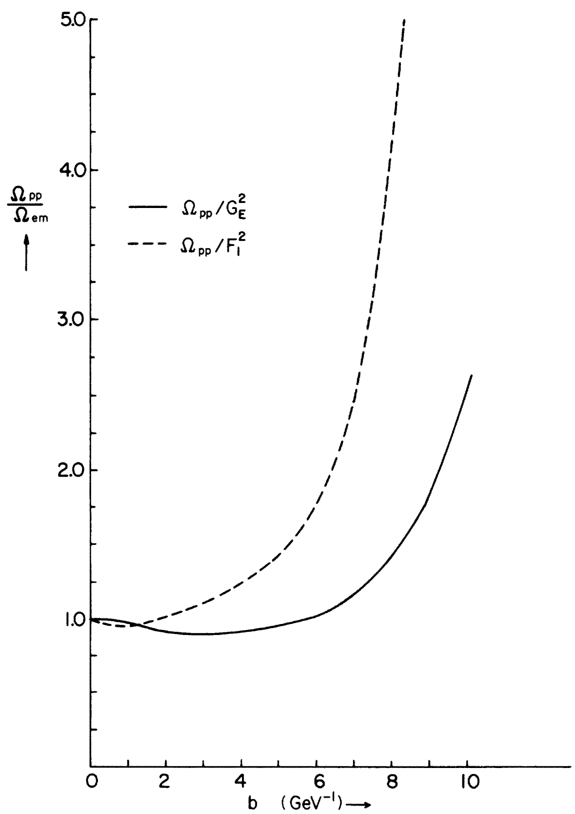

FIG. 2. The ratio of the proton opacity Ω_{pp} at $P_L = 1480$ GeV/c to the charge density Ω_{em}. The ratio is normalized to one at $b = 0$. Ω_{em} is calculated using both the standard dipole approximation to $G_E^2(q^2)$ and the $F_1^2(q^2)$ distribution that corresponds to this.

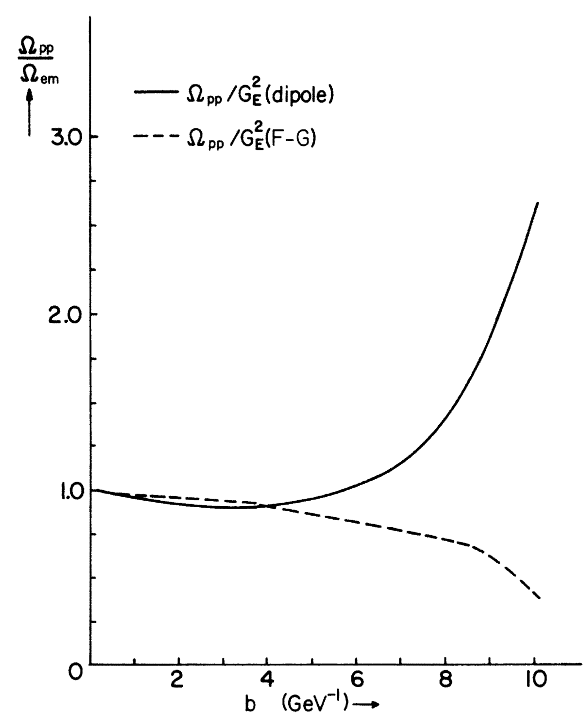

FIG. 3. The ratio of the proton opacity Ω_{pp} at $P_L = 1480$ GeV/c to the charge density Ω_{em}. The ratio is normalized to one at $b = 0$. Ω_{em} is calculated from G_E^2 using both the dipole and Fried-Gaisser approximations.

$$\begin{aligned} &A_1 = 0.758\times10^{-3}, \quad B_1 = 0.244\ \mathrm{GeV}/c^{-2},\\ &A_2 = 0.428\times10^{-1}, \quad B_2 = 0.794\ \mathrm{GeV}/c^{-2},\\ &A_3 = 0.3624, \quad B_3 = 2.202\ \mathrm{GeV}/c^{-2},\\ &A_4 = 0.5940, \quad B_4 = 6.308\ \mathrm{GeV}/c^{-2}\ . \end{aligned} \tag{13}$$

A five-exponential fit does not improve the fit significantly. An idea of the quality of the fit can be obtained from Fig. 4(a), and from Fig. 4(b) where the date on F_1^2 are compared with the fit for $q^2 > 1$. A more testing display of the same data is shown in Fig. 4(c) which shows the ratio of experimental F_1^2 to the fitted one. We believe that Eqs. (12) and (13) provide the best available fit to F_1^2 over the whole q^2 range.

The F_1^2 fit was transformed via Eq. (10) so as to provide us also with a G_E^2 distribution. This G_E^2 distribution coincides very closely with a direct fit to the measured values of G_E^2.

IV. COMPARISON BETWEEN PROTON OPACITY AND CHARGE DISTRIBUTION

The ratio between $\Omega_{pp}(b)$ and $\Omega_{em}(b)$, normalized to one at $b=0$, is shown in Fig. 5 at the two extreme ISR energies, $p_L = 290$ and 1480 GeV/c. Results are presented using both F_1^2 and G_E^2 as input distributions for $\Omega_{em}(b)$.

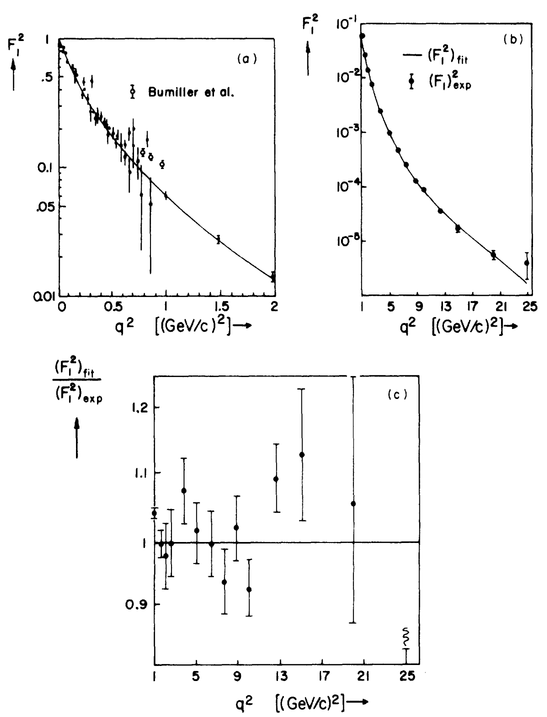

FIG. 4. Behavior of $F_1^2(q^2)$ compared with our parametrization. (a) The range $q^2<2$. (b) The range $1<q^2<25$. (c) $(F_1^2)_{fit}/(F_1^2)_{exp}$ for the high-q^2 data.

It is clear that the proton opacity differs significantly from $\Omega_{em}(b)$ irrespective of whether F_1^2 on G_E^2 is used to generate $\Omega_{em}(b)$. For the sake of consistency it is important to check that these differences in b space correspond to significant differences in t space. To study this we have taken the t-space Bessel transform of $\Omega_{pp}(b)$, denoted by $\hat{\Omega}_{pp}(t)$, and compared $\hat{\Omega}_{pp}(t)$ with the form-factor data as a function of $q^2 = -t$. The results of this comparison are presented in Fig. 6 for both $F_1^2(q^2)$ and $G_E^2(q^2)$. It is seen that there are indeed significant differences also in the t-space distributions.

Having confirmed the significance of our b-space comparisons, we shall, in the following, concentrate solely on the relationship between $\Omega_{pp}(b)$ and $\Omega_{em}(b)$.

Let us now consider whether $\Omega_{em}(b)$ can be given any significant role in the presentation of $\Omega_{pp}(b)$. Our results, as summarized in Fig. 5, rule out any factorized form of the type

$$\Omega_{pp}(b,s) = \kappa(s)\,\Omega_{em}(b)\ , \tag{5}$$

no matter which form factor is used to generate

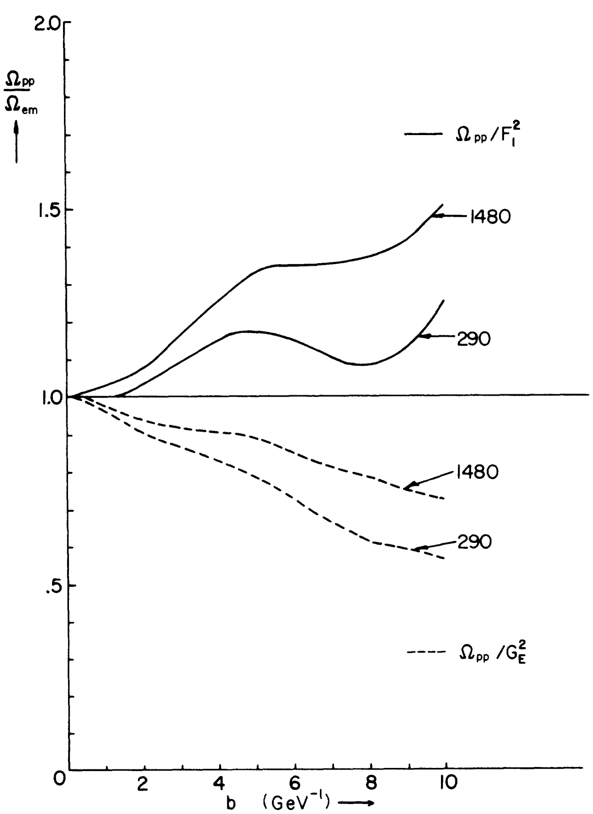

FIG. 5. Comparison of proton opacity and charge distribution at the two extreme ISR energies. Shown are the ratios Ω_{pp}/Ω_{em} normalized to one at $b=0$ using both F_1^2 and G_E^2 for Ω_{em}.

$\Omega_{em}(b)$. In fact, any factorized form

$$\Omega_{pp}(b,s)=\kappa(s)f(b)\ , \tag{14}$$

no matter what $f(b)$ is, is ruled out by the varying s dependence of Ω_{pp} in differenct b domains.

The next simplest possibility is to try to associate $\Omega_{em}(b)$ with at least a significant portion of $\Omega_{pp}(b)$. We could thus consider a relationship of the form

$$\Omega_{pp}(b,s)=C(s)\,\Omega_{em}(b)+\Delta(b,s)\ . \tag{15}$$

A study of the s behavior of $\Omega_{pp}(b,s)$ at $b=0$ (see Table I) together with the knowledge of the peripheral Ω_{pp} growth at $b\approx 0.5-1.0$ F (see Fig. 1) suggest that in fact an even simpler form than Eq. (15) is possible, namely one in which $C(s)$ is a constant independent of s. We are thus led to consider a very appealing substitute for the original Chou-Yang hypothesis, in which $\Omega_{em}(b)$ provides a fixed, energy-independent description of the bulk of $\Omega_{pp}(b,s)$ and in which all the energy dependence is carried by the additional peripheral term $\Delta(b,s)$. We thus consider a relationship of the type

$$\Omega_{pp}(b,s)=C\Omega_{em}(b)+\Delta(b,s)\ , \tag{16}$$

where $\Delta(b=0,s)=0$.

As is obvious from Fig. 5, we will obtain totally different interpretations depending upon whether F_1^2 or G_E^2 is used to provide $\Omega_{em}(b)$. Let us therefore consider the two cases separately.

1. If $\Omega_{em}(b)$ is generated from G_E^2 we have a situation in which the peripheral term $\Delta(b,s)$ *is negative and its magnitude decreases with energy* (see Fig. 7). Thus $\Omega_{pp}(b,s)\to C\Omega_{em}(b)$ *from below* as $s\to\infty$. In this case the charge distribution is once again reinstated as the *limiting distribution* toward which the proton opacity tends at ultrahigh energies. Although formally identical to the original Chou-Yang hypothesis, it is very different in practical terms. The total cross section does approach a limiting value ($\approx$52 mb) from below, and $d\sigma/dt$ approaches a limiting distribution in t with a dip at $t=0.85$ GeV/c^2 but only at *exceedingly high energies*.

2. A completely different interpretation emerges if F_1^2 is chosen to generate $\Omega_{em}(b)$. In this case the peripheral term $\Delta(b,s)$ is *positive and increasing with energy* (see Fig. 7). Thus an approximate proportionality between matter and charge densities holds at small b, but the matter distribution evolves *away from* $C\Omega_{em}(b)$ as the energy increases. In this case the original Chou-Yang hypothesis is lost, but $\Omega_{em}(b)$ continues to play a significant role in the description of $\Omega_{pp}(b,s)$

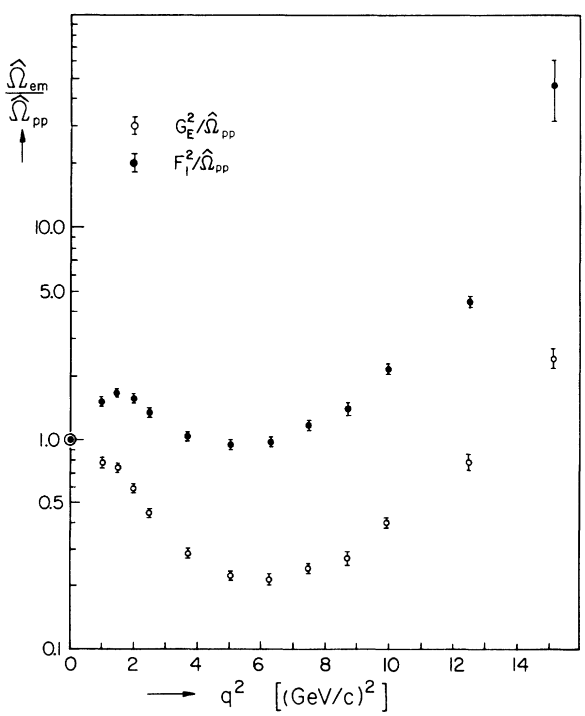

FIG. 6. Comparison of $F_1^2(q^2)/\hat{\Omega}_{pp}(q^2)$ and $G_E^2(q^2)/\hat{\Omega}_{pp}(q^2)$ normalized to one at $q^2=0$. Errors shown are due to the e-p input data.

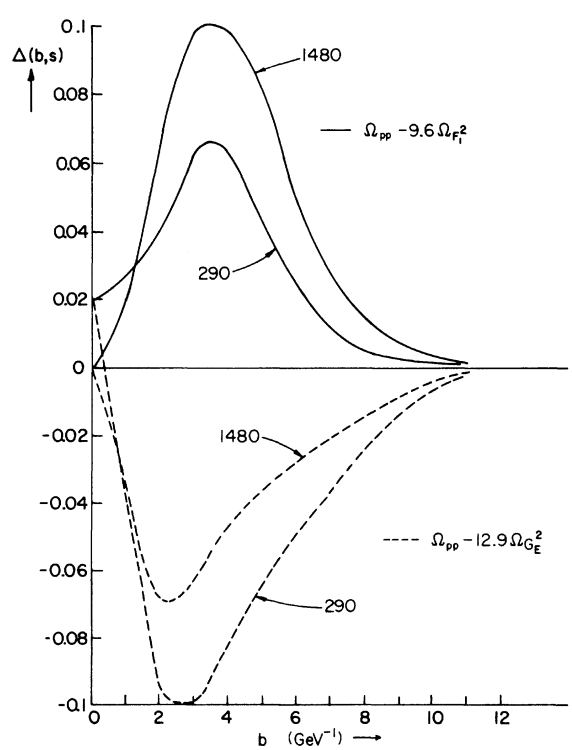

FIG. 7. b-space distributions of $\Delta(b,s)$ at the two extreme ISR energies. Ω_{em} is calculated from G_E^2 and F_1^2.

throughout the ISR energy range. In this picture it is not clear whether the total cross section will continue to grow indefinitely or whether it will ultimately saturate. The reason for the uncertainty lies in the limited knowledge we have about the growth of the peripheral term $\Delta(b, s)$. It is not clear whether the growth of $\Delta(b, s)$ is localized in b, which would lead to saturation, or whether there is also some movement toward larger values of b, which would lead to a continually increasing radius and thereby to a continually growing cross section.[16] Of course over the ISR region there is an effective growth of radius but if the saturation picture is correct then this effective growth of radius will eventually cease.

In both cases (1) and (2) above, our interpretation of the data depends crucially on our normalization of the size of the electromagnetic contribution at $b=0$. This, we believe, is necessitated by the experimental fact that for small b the proton opacity shows *no energy dependence*. Accordingly, it is not surprising that both interpretations have as a built-in property a peripheral cross-section growth. Our picture is radically different from a recent interpretation[17] of the proton opacity in which a proportionality between $\Omega_{pp}(b)$ and $\Omega_{\rm em}(b)$ is enforced for large $b > 1.5$ F. In that case an energy-dependent proportionality factor is needed and there is no resemblance between $\Omega_{pp}(b)$ and $\Omega_{\rm em}(b)$ for small b. In fact, $\Omega_{\rm em}(b)$ then plays a negligible role in describing $\Omega_{pp}(b)$ for most of the b range, and the Chou-Yang hypothesis is thus practically abandoned.

It is intriguing, but probably very unreliable, to try to estimate the energy dependence of $\Omega_{pp}(b, s)$ beyond the ISR energy range. In the first picture ($G_E{}^2$) we assume convergence to a limiting value and that the rate of change, at fixed b, is roughly s-independent. We obtain a limit of 52 mb reached at $s \simeq 10^5$ GeV^2. The second picture ($F_1{}^2$) is, as described above, more complicated, since both magnitude and position of $\Delta(b, s)$ could change with energy (see Fig. 7). If no change occurs in the position, one would ultimately approach a limiting value $\sigma_T \simeq 100$ mb, but the energy at which this happens would be impractically large.

V. INELASTIC DIFFRACTION

In this section we attempt to speculate on the link between elastic scattering and inelastic diffraction scattering utilizing the results of Sec. IV.

We follow the usual procedure[11, 17] and split all physical states into two sets: those that scatter diffractively $|D_1\rangle$, $|D_2\rangle, \ldots,$ where $|D_1\rangle = |pp\rangle$, and the rest. For simplicity we keep only two diffractive states $|D_1\rangle$, $|D_2\rangle$ and ignore the nondiffractive states (clearly a drastic approximation). The 2×2 S matrix that remains describes the independent processes

$$|D_1\rangle \to |D_1\rangle,\quad |D_2\rangle \to |D_2\rangle,\quad \text{and}\quad |D_1\rangle \to |D_2\rangle. \tag{17}$$

One could of course diagonalize this S matrix and consider its eigenstates, which are then unphysical linear superpositions of $|D_1\rangle$ and $|D_2\rangle$. In our opinion there is very little to gain by this. We ask whether the particular modified Chou-Yang description offered in Sec. IV is of any relevance to the problem of inelastic diffraction.

In particular there is the following intriguing possibility. Consider the 2×2 matrix of scattering amplitudes in the eikonal form

$$\underline{F}(b, s) = 2i(\underline{I} - e^{i\underline{\chi}}), \tag{18}$$

where $\underline{\chi}$ is now a 2×2 eikonal matrix. The elastic-scattering amplitude $F_{11}(b, s)$ will not simply depend on χ_{11} but will be a function of all the $\underline{\chi}$ elements. If we write, as we did earlier, for the pp elastic scattering

$$F_{pp} = F_{11}(b, s) = 2i(1 - e^{i\chi_{pp}}), \tag{7b$'$}$$

then the elastic-scattering eikonal χ_{pp} is a function of χ_{11}, χ_{22}, and χ_{12}. In Sec. IV we found that $\Omega_{pp} = \mathrm{Im}\chi_{pp}$ differed from $\Omega_{\rm em}$ by a small energy-dependent term $\Delta(b, s)$. Could it be that the diagonal elements of the opacity matrix, i.e., $\Omega_{ii} = \mathrm{Im}\chi_{ii}$, are *purely electromagnetic in shape and energy-independent,* and that Ω_{pp} differs from $\Omega_{\rm em}$ only because of the existence of the off-diagonal element χ_{12}? Physically, this would mean that the diagonal parts of the matter overlap for both $|D_1\rangle \to |D_1\rangle$ and $|D_2\rangle \to |D_2\rangle$ are proportional to each other and to $\Omega_{\rm em}$.

To examine this possibility let us write

$$\underline{\chi} = \begin{pmatrix} i\lambda\Omega_{\rm em} & \epsilon \\ \epsilon & i\lambda'\Omega_{\rm em} \end{pmatrix} \tag{19}$$

with λ, λ' constants and $\epsilon(b, s)$ an unknown function at this stage. Since experimentally we can only study the scattering $|D_1\rangle \to |D_1\rangle$ and $|D_1\rangle \to |D_2\rangle$ it is clear that we have too little information to fix λ, λ', and ϵ uniquely. However, we shall show that under the assumptions that

$$\lambda \approx \lambda',\quad |\epsilon| \ll \lambda\Omega_{\rm em}, \tag{20}$$

we can solve for λ and ϵ from the p-p elastic data alone. We can then calculate the inelastic diffraction and, as will be seen, obtain a reasonable value for this cross section.

Define the complex vector $\vec{n}$ by

$$\vec{n} = [\epsilon, 0, \tfrac{1}{2}i(\lambda - \lambda')\Omega_{em}], \tag{21}$$

with complex length

$$n = [\epsilon^2 - \tfrac{1}{4}(\lambda - \lambda')^2 \Omega_{em}{}^2]^{1/2} . \tag{22}$$

Then the matrix scattering amplitude can be written as

$$\underline{F}(b, s) = 2i\left[\underline{I} - e^{-1/2(\lambda+\lambda')\Omega_{em}}\left(\cos n + i\vec{n}\cdot\underline{\vec{\sigma}}\,\frac{\sin n}{n}\right)\right], \tag{23}$$

where $\underline{\sigma}$ are the usual Pauli matrices. Thus elastic scattering is given by

$$F_{pp}(b, s) = F_{11} = 2i\left\{1 - e^{-1/2(\lambda-\lambda')\Omega_{em}} \times\left[\cos n - \tfrac{1}{2}(\lambda - \lambda')\Omega_{em}\frac{\sin n}{n}\right]\right\}, \tag{24}$$

and inelastic diffraction by

$$F_{diff}(b, s) = F_{12} = 2\, e^{-1/2(\lambda+\lambda')\Omega_{em}}\epsilon(b, s)\frac{\sin n}{n} . \tag{25}$$

Making use of approximation (20) we can rewrite (24) and (25) as

$$F_{11} \approx 2i(1 - e^{-(\lambda\Omega_{em} + 1/2\epsilon^2)}), \tag{24$'$}$$

$$F_{12} \approx 2e^{-\lambda\Omega_{em}}\epsilon(b, s) . \tag{25$'$}$$

Comparing with Eq. (16) of Sec. IV, we have

$$\Omega_{pp} = C\Omega_{em} + \Delta \approx \lambda\Omega_{em} + \tfrac{1}{2}\epsilon^2 . \tag{26}$$

Thus $\lambda \approx C$, and we indeed see that the energy-dependent term Δ can be generated by the off-diagonal element ϵ of $\underline{\chi}$. We thus have

$$\epsilon^2 \approx 2\Delta . \tag{27}$$

At this point we recall that the structure of Δ depended upon whether we took Ω_{em} from $F_1{}^2$ or $G_E{}^2$. In the $F_1{}^2$ case Δ is positive so $\epsilon = \sqrt{2D}$ *is real*. In the $G_E{}^2$ case, on the other hand, Δ is negative and we have $\epsilon = i(-2\Delta)^{1/2}$ *purely imaginary*. Looking at Eq. (25′) we see that the two cases correspond to having either a real or an imaginary amplitude for F_{diff}. The emergence of a real amplitude for a diffractive channel in unconventional though quite permissible.

Actually, our result is just a manifestation of a more general property, common to many models.[18] In particular, in multieikonal models with imaginary off-diagonal elements, the net correction to the output elastic amplitude is negative. Accordingly, with growing total and elastic cross sections, a diffractive cross section which increases with energy necessitates a real amplitude (this is, our $F_1{}^2$ case). On the other hand, a decreasing diffractive cross section is consistent with an imaginary amplitude (this is, our $G_E{}^2$ case).

We note two very interesting conclusions which emerge from such a description:

1. We obtain a peripheral output inelastic profile function [see Eq. (25′)] not solely because of the eikonalization, but because of our identification of the off-diagonal element with the energy-dependent portion of the p-p opacity. In this we differ from recent treatments of this problem,[11,17] in particular from models which take every element of $\underline{\chi}$ proportional to Ω_{em}.

2. Our calculations demonstrate the possibility that a peripheral growth of the total cross section can be associated with a decreasing diffractive cross section. Namely, the peripherality of the increasing σ_T as such is no proof of an increasing diffractive cross section.

Although our two-channel approximation is clearly unrealistic, it is interesting to calculate the elastic and inelastic diffraction cross sections quantitatively. Let us define

$$\sigma_{el} = \frac{\pi}{2}\int |F_{11}(b, s)|^2 b\, db , \tag{28}$$

$$\sigma_{diff} = \frac{\pi}{2}\int |F_{12}(b, s)|^2 b\, db . \tag{29}$$

We obtain the cross sections summarized in Table II. Thus the extension of our modified Chou-Yang model leads to cross sections of the right order of magnitude and a suitably peripheral impact-parameter profile for the inelastic diffractive amplitude.

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TABLE II. Calculated elastic and diffractive cross sections.

P_L (GeV/c)	σ_{el} (mb)	σ_{diff} (mb) $F_1{}^2$ case	σ_{diff} (mb) $G_E{}^2$ case
290	7.48	3.1	5.9
1480	8, 35	6.0	3.5

where this work was completed. This work was begun while one of us (U. M.) was visiting Westfield College and Saclay. He wishes to thank both Theory Groups for their warm hospitality, and the U. K. Science Research Council for a Senior Visiting Fellowship. E. L. wishes to thank Professor R. Vinh Mau for his hospitality at the Laboratoire de Physique Théorique des Particules Élêmentaires, University Pierre et Marie Curie, Paris. J. K. wishes to thank members of the Theoretical Physics Department at Westfield College for their kind hospitality and the U. K. Science Research Council for a post doctoral fellowship.

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