Hadron production in photon-photon collisions*

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We analyze deep-inelastic photon-photon collisions via the two-photon mechanism in electron-positron (-electron) colliding beams in a form especially suitable for experimental analysis. It is shown that by a helicity analysis similar to that used in electroproduction experiments, we can separate five of the eight structure functions describing the process $\gamma^* + \gamma^* \to$ hadrons. The helicity cross sections for this process and for the process with one real photon (inelastic electron-photon scattering) are related to structure functions, and are evaluated using quark light-cone algebra. There are anomalous contributions to the structure functions for the inelastic electron-photon scattering which arise both in parton as well as generalized vector-meson-dominance models. This suggests a connection between these two types of models for photon-photon scattering. Further, we use vector-meson dominance to construct a sum rule for $\sigma_{\gamma\gamma \to \text{hadrons}}$ from which it is estimated that roughly 20% of the cross section should be built up from higher-mass vector states. Using a spectral representation for the total transverse cross section, and the "aligned-jet" vector-dominance model we achieve a connection, via a "correspondence principle," with the parton model for the hadron multiplicities in photon-photon collisions. We also comment on inclusive pion multiplicities and the approach to scaling for photon-photon processes in the light-cone algebra.

I. INTRODUCTION

Recently there has been a great deal of interest, both theoretical and experimental, in electron-positron (-electron) colliding-beam processes.[1] The apparent violation[2] of scaling[3] in the one-photon process $e^+e^- \to \gamma^* \to$ hadrons, followed by the discovery of new resonances,[4] has led to the belief that we are witnessing a threshold phenomenon in which new hadronic degrees of freedom are being excited.[5] However, the one-photon process is not the only interesting process that can be studied with electron-positron colliding-beam machines. It is well known that the two-photon process $e^\pm + e^- \to e^\pm + e^-$ + hadrons is the dominant process at higher energies.[6] Thus the two-photon process, which provides us information about the photon-photon annihilation reaction $\gamma^* + \gamma^* \to$ hadrons, is important not only for its intrinsic interest, but also as an important background to the one-photon process at high energies.

Most of the theoretical studies concerning the two-photon process have concentrated on the process $\gamma + \gamma \to$ hadrons, where both photons are nearly on the mass shell, and have used the equivalent-photon approach.[7] In this paper we study the two-photon reaction

$$e^\pm + e^- \to e^\pm + e^- + \text{hadrons}, \quad (1)$$

in the kinematic region where both photons are far off the mass shell. By two-photon reaction we shall mean the processes shown in Fig. 1, where hadrons with even charge conjugation are produced. There are additional diagrams shown in Fig. 2 which also contribute to the process (1), and where odd-charge-conjugation hadrons are produced. We shall call these diagrams background contribution. There is no interference between the diagrams of Fig. 1 and Fig. 2 because of the different transformation properties of the hadron states under charge conjugation. We shall be concerned only with the process described in Fig. 1(a), because we are interested in the photon-photon annihilation process $\gamma^* + \gamma^* \to$ hadrons. This process is described by eight invariant structure functions.[8] It is generally believed that detection of either or both of the scattered electrons (positrons) in addition to produced particles will be required in order to identify and separate the one-photon and two-photon processes. Once this is achieved, as has been done in a recent Frascati experiment,[9] one can take up the determination and study of the structure functions. However, before embarking on such a study, it is useful to know the order of magnitude of the cross section. It has been estimated[10] that for large invariant masses of two photons the effective cross section for the process shown in Fig. 1(a) is of the order of 10^{-36}–10^{-37} cm^2, which could be measured with colliding-beam facilities having luminosities of the order of 10^{33}–10^{34} cm^{-2}/sec. Figure 1(b) can be ignored in such a limit, and the background in the same kinematic region due to Fig. 2 is at most 17% of the total events.[10]

In Sec. II we review and extend the kinematics for the two-photon reaction (1) in a form which is more transparent from the viewpoint of the experimental determination of structure functions. We shall use the same helicity formalism[11] as has been used in electroproduction and neutrino ex-

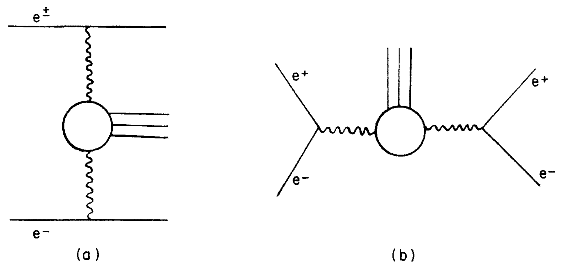

FIG. 1. Feynman diagrams contributing to the production of even-charge-conjugation hadronic states in process (1). There is another graph for e^-e^- scattering in which the final electron lines in (a) are interchanged. This contribution, which corresponds to backward scattering, is small and we shall ignore it. The contribution of (b) can also be ignored in the kinematic region in which we are interested.

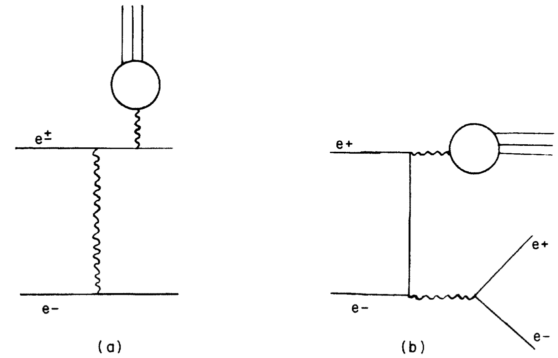

FIG. 2. Typical Feynman diagrams which constitute the background to process (1). Diagrams of type (a) and (b) contribute to $e^+e^- \to e^+e^-$ +hadrons, whereas diagrams of type (a) and the corresponding diagrams in which the final electron lines are interchanged contribute to $e^-e^- \to e^-e^-$ +hadrons. There are eight diagrams in each case. There is no interference between the diagrams of Fig. 1 and Fig. 2 because of charge conjugation invariance.

periments. By detecting the outgoing electrons (positrons) at a fixed value of the invariant mass and the energy of the corrdsponding photon we can separate five linear combinations of the eight invariant structure functions. This is analogous to the usual electroproduction and neutrino-induced production reactions. In Sec. III we evaluate the helicity cross sections or equivalently the structure functions for the process $\gamma^* + \gamma^* \to$ hadrons using quark-gluon light-cone algebra,[12] and find results similar to those obtained in the parton model.[13] We also discuss the structure functions for the case where one of the photons is real, the interesting case of inelastic electron-photon scattering,[14] and find anomalous contributions which are no longer scale-invariant.[15] If we were to assume that the real photon behaves merely as a light vector meson, we would expect the structure functions for inelastic electron-photon scattering to scale at least as rapidly as those for inelastic electron-nucleon scattering. The deviations from scaling can be attributed to the higher-mass vector-meson states or, in the parton model, to a possible quark-antiquark substructure of the real and virtual photon. This is further elaborated on in Sec. IV, where we use conventional vector dominance (ρ, ω, ϕ only) factorization of hadronic as well as photon cross sections and recent data on photoproduction of vector mesons to conclude that roughly 20% of the photon-photon cross section should come from higher-mass vector states. We further write a spectral representation for the total transverse cross section for $\gamma^* + \gamma^* \to$ hadrons and achieve, via the "aligned-jet" vector-dominance model, a connection with the parton model for multiplicities in photon-photon reactions. In the light-cone algebra the evolution of the final hadronic system in virtual photon-photon collisions is similar to that of the one-photon annihilation process $e^+e^- \to \gamma^* \to$ hadrons. Therefore, the inclusive pion distributions should be the same in the two reactions in this limit. This is discussed briefly in the concluding section, where we also comment on the question of the approach to scaling in the two-photon annihilation process.

II. KINEMATICS

The diagrams contributing to process (1) are shown in Figs. 1 and 2. We shall here be concerned primarily with the diagrams in Fig. 1, where the two photons are connected to the hadronic vertex. The diagrams in Fig. 2, whose contribution can be calculated in terms of the cross section for the one-photon process $e^+e^- \to \gamma^* \to$ hadrons, constitute the background to the "two-photon" process, Fig. 1. For Fig. 1(a) there is an exchange diagram for the case where both the colliding particles are electrons, which we ignore because the backward scattering involves much higher momentum transfer than the forward scattering. The process of Fig. 1(b) can be ignored in the kinematic limits in which we are interested, namely, when the masses of two photons are large but still small compared to the energies of the incoming and outgoing leptons (the light-cone limit). Therefore, we may restrict ourselves to the graph of Fig. 1(a). In this way the process (1) can be used to study the photon-photon annihilation process

$$\begin{aligned} &e^\pm + e^- \to e^\pm + e^- + \gamma^* + \gamma^*, \\ &\gamma^* + \gamma^* \to \text{hadrons} \end{aligned} \tag{2.1}$$

for spacelike photons.

The differential cross section for process (2.1) when the incoming particles are unpolarized, the polarizations of the outgoing leptons are not detected, and the final hadrons are summed over, can be written as[16]

$$\frac{d\sigma}{(d^3p_1'/E_1')(d^3p_2'/E_2')} = \frac{e^8}{64(2\pi)^{10}} \frac{1}{k_1^4 k_2^4 (p_1 \cdot p_2)} W_{\alpha\beta\mu\nu}(k_1^2, k_2^2, P^2) \times [(p'_{1\mu} p_{1\alpha} + p'_{1\alpha} p_{1\mu} - p_1 \cdot p_1' g_{\alpha\mu})(p'_{2\nu} p_{2\beta} + p'_{2\beta} p_{2\nu} - p_2 \cdot p_2' g_{\beta\nu})], \tag{2.2}$$

where p_1 (E_1) and p_2 (E_2) are the four-momenta (energies) of the incoming leptons (electrons or positrons), and the primed quantities denote the corresponding variables for the outgoing particles. $k_1(=p_1 - p_1')$ and $k_2(=p_2 - p_2')$ are the four-momenta of the spacelike photons ($k_1^2, k_2^2 < 0$), and $W_{\alpha\beta\mu\nu}$ is the unknown amplitude corresponding to the absorptive part of the forward elastic amplitude for the fundamental process $\gamma^*(k_1) + \gamma^*(k_2) \to \gamma^*(k_1) + \gamma^*(k_2)$. More explicitly

$$W_{\alpha\beta\mu\nu}(k_1^2, k_2^2, P^2) = \sum_P (2\pi)^4 \delta^4(k_1 + k_2 - P) \int d^4x\, d^4y\, d^4z \langle P | T^*(J_\alpha(x) J_\beta(y)) | 0\rangle^\dagger \langle P | T^*(J_\mu(z) J_\nu(0)) | 0\rangle, \tag{2.3}$$

where P is the four-momentum of the produced hadronic system.

It will be appropriate here to comment on the background contribution arising out of the "one-photon" process, Fig. 2. This contribution can be calculated in terms of the cross section for the one-photon process $e^+e^- \to$ hadrons. The estimated[10] cross section for this contribution is of the order of $0.59 \times 10^{-37}\,\text{cm}^2$ for $E_1 = E_2 = 3$ GeV, $E_1' = E_2' = 2.5$ GeV, $2q^2 = (k_1^2 + k_2^2 - \frac{1}{2}P^2) = -1.7\ \text{GeV}^2$, $w = q^2/P \cdot q = 1.75$, and $P^2 = 0.4\ \text{GeV}^2$. Assuming that scaling sets in rather early in the two-photon process (2.1) (see Sec. V), a fact motivated by the early scaling in electron-nucleon scattering, the effective cross section for process (2.1) can be calculated in the above kinematic region,[10] and its value is of the order of $2.9 \times 10^{-37}\,\text{cm}^2 - 1.3 \times 10^{-36}\,\text{cm}^2$ depending on the quark charge assignment. Thus the background contribution in the scaling limit (i.e., at the few-GeV level) is at most 17% of the total events.

Coming back to the cross-section formula (2.2), we write down the singularity-free invariant decomposition for $W_{\alpha\beta\mu\nu}$ as given by Brown and Muzinich,[8] namely,

$$W_{\alpha\beta\mu\nu}(k_1^2, k_2^2, P^2) = \sum_{i=1}^{8} I^i_{\alpha\beta\mu\nu} W_i(k_1^2, k_2^2, P^2), \tag{2.4}$$

where I^i ($i = 1, 2, \ldots, 8$) are Lorentz tensor factors which are given in the appendix of the paper of Brown and Muzinich. From (2.2) and (2.4) we can write the differential cross section as

$$\frac{d\sigma}{(d^3p_1'/E_1')(d^3p_2'/E_2')} = \frac{\alpha^4}{2(2\pi)^6} \frac{1}{k_1^4 k_2^4 (p_1 \cdot p_2)} \{k_1^4 k_2^4 W_1 + [2A(k_1 \cdot k_2)^2 - 2Bk_1 \cdot k_2 + CD](W_2 + W_3) + k_1^2 k_2^4 [C + (k_1 \cdot k_2)^2] W_4 + k_1^4 k_2^2 [D + (k_1 \cdot k_2)^2] W_5 + 2k_1^2 k_2^2 (2Ak_1 \cdot k_2 - B) W_6 + k_1^2 k_2^2 [2A + C + D + (k_1 \cdot k_2)^2](W_7 + W_8)\}, \tag{2.5a}$$

where

$$A = p_1' \cdot p_2'\, p_1 \cdot p_2 + p_1 \cdot p_2'\, p_1' \cdot p_2,$$

$$B = p_1' \cdot k_2 (p_1 \cdot p_2\, k_1 \cdot p_2' + p_1 \cdot p_2'\, k_1 \cdot p_2 - p_1 \cdot k_1\, p_2 \cdot p_2') + p_1 \cdot k_2 (p_1' \cdot p_2\, k_1 \cdot p_2' + p_1' \cdot p_2'\, k_1 \cdot p_2 - k_1 \cdot p_1'\, p_2 \cdot p_2') - p_1 \cdot p_1' (k_1 \cdot p_2'\, p_2 \cdot k_2 + k_1 \cdot p_2\, k_2 \cdot p_2' - k_1 \cdot k_2\, p_2 \cdot p_2'), \tag{2.5b}$$

$$C = 2p_1 \cdot k_2\, p_1' \cdot k_2 - p_1 \cdot p_1'\, k_2^2,$$

$$D = 2k_1 \cdot p_2\, k_1 \cdot p_2' - p_2 \cdot p_2'\, k_1^2.$$

It is obvious from the cross-section formula (2.5a) that by detecting the outgoing leptons only, one can determine in principle six linear combinations of eight independent structure functions. However, in practice it is difficult to do so from a complicated formula such as (2.5a). By using a helicity analysis similar to the one used in electroproduction, we show how to determine five linear combinations of these structure functions.

The electron current corresponding to photons k_1 and k_2 can be written as[11] (see Fig. 3)

$$j_\mu(k_1)=\bar{u}(p_1')\gamma_\mu u(p_1)$$
$$=a\epsilon_\mu^S+b\epsilon_\mu^R+c\epsilon_\mu^L\,,$$
$$j_\nu(k_2)=\bar{u}(p_2')\alpha_\nu u(p_2) \qquad (2.6\text{a})$$
$$=a'\epsilon_\nu'^S+b'\epsilon_\nu'^R+c'\epsilon_\nu'^L\,,$$

where

$$a=\left(\frac{E_1E_1'}{m^2}\right)^{1/2}\left(1+\frac{k_1^2}{4E_1E_1'}\right)^{1/2}\frac{(-k_1^2)^{1/2}}{|\vec{k}_1|}\,,$$
$$b=-c=\frac{1}{2m\sqrt{2}}(E_1+E_1')\frac{(-k_1^2)^{1/2}}{|\vec{k}_1|}\,,$$
$$a'=\left(\frac{E_2E_2'}{m^2}\right)^{1/2}\left(1+\frac{k_2^2}{4E_2E_2'}\right)^{1/2}\frac{(-k_2^2)^{1/2}}{|\vec{k}_2|}\,, \qquad (2.6\text{b})$$
$$b'=-c'=\frac{-1}{2m\sqrt{2}}(E_2+E_2')\frac{(-k_2^2)^{1/2}}{|\vec{k}_2|}\,,$$

and

$$|\vec{k}_1|^2=|\vec{k}_2|^2=\frac{(k_1\cdot k_2)^2-k_1^2k_2^2}{P^2}\,.$$

Similar results hold if the initial particle is a positron. We have used the center-of-mass frame of the colliding photons, with $\vec{k}_1$ along the z axis and neglected the lepton mass throughout. The polarization vectors are

$$\epsilon_S=\frac{1}{(-k_1^2)^{1/2}}(|\vec{k}_1|,0,0,k_1^0),$$
$$\epsilon_S'=\frac{1}{(-k_2^2)^{1/2}}(|\vec{k}_2|,0,0,-k_2^0),$$
$$\epsilon_R=\frac{1}{\sqrt{2}}(0,1,i,0),\quad \epsilon_R'=\frac{1}{\sqrt{2}}(0,-1,i,0), \qquad (2.6\text{c})$$
$$\epsilon_L=\frac{1}{\sqrt{2}}(0,1,-i,0),\quad \epsilon_L'=\frac{1}{\sqrt{2}}(0,1,i,0),$$

and

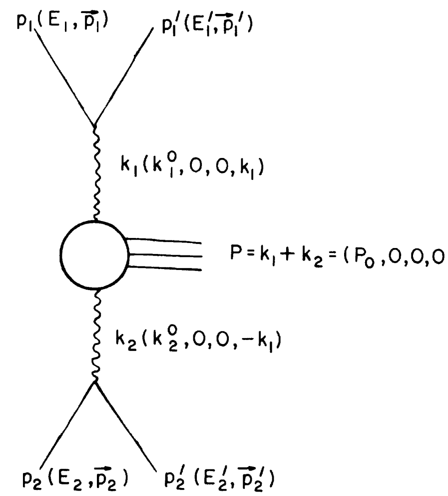

FIG. 3. Kinematics in the virtual-photon-photon center-of-mass system. The positive z direction is along $\vec{k}_1$.

$$(k_1^0)^2=\frac{(k_1^2+k_1\cdot k_2)^2}{P^2},\quad (k_2^0)^2=\frac{(k_2^2+k_1\cdot k_2)^2}{P^2}.$$

From Eqs. (2.6) it is clear that by varying E_1 and E_1' (for photon k_1 this means varying the angle of the outgoing electron p_1'), E_2 and E_2' (for photon k_2 this means varying the angle of outgoing electron p_2'), with fixed k_1^2, k_2^2, and P^2 one can determine the polarization state of the colliding photons. The corresponding helicity cross sections σ_{ij} $(i,j=S,R,L)$ can then be determined. These are related to the structure functions W_k $(k=1,2,\ldots,8)$, or equivalently the absorptive parts of the forward elastic helicity amplitudes $W_{lj;mn}$ for the process $\gamma^*(k_1,m)+\gamma^*(k_2,n)\to\gamma^*(k_1,l)+\gamma^*(k_2,j)$ by the following relations:

$$\sigma_{SS}=KW_{00;00}$$
$$=K(k_1^2k_2^2)(W_1+W_2+W_3+k_2^2W_4+k_1^2W_5+2k_1\cdot k_2W_6+2W_7+2W_8),$$
$$\sigma_{SR}=KW_{01;01}=\sigma_{SL}=KW_{0-1;0-1}$$
$$=Kk_1^2[-k_2^2(W_1+W_7+W_8)-k_2^4W_4-(k_1\cdot k_2)^2W_5],$$
$$\sigma_{RS}=KW_{10;10}=\sigma_{LS}=KW_{-10;-10}$$
$$=Kk_2^2[-k_1^2(W_1+W_7+W_8)-(k_1\cdot k_2)^2W_4-k_1^4W_5],$$
$$\sigma_{RR}=KW_{11;11}=\sigma_{LL}=KW_{-1-1;-1-1}$$
$$=K[k_1^2k_2^2(W_1+W_8)+(k_1\cdot k_2)^2(W_2+W_7+W_8)+(k_1\cdot k_2)^2(k_2^2W_4+k_1^2W_5)], \qquad (2.7)$$
$$\sigma_{RL}=KW_{1-1;1-1}=\sigma_{LR}=KW_{-11;-11}$$
$$=K[k_1^2k_2^2(W_1+W_7)+2k_1^2k_2^2(k_1\cdot k_2)W_6+(k_1\cdot k_2)^2(W_3+W_7+W_8)+(k_1\cdot k_2)^2(k_2^2W_4+k_1^2W_5)],$$
$$W_{11;00}=(k_1\cdot k_2)(k_1^2k_2^2)^{1/2}(W_2+W_8),$$
$$W_{11;-1-1}=(k_1\cdot k_2)^2(W_2+W_3)+2k_1^2k_2^2(k_1\cdot k_2)W_6+k_1^2k_2^2(W_7+W_8),$$
$$W_{01;-10}=-(k_1\cdot k_2)^2(k_1^2k_2^2)^{1/2}(W_3+W_7)-(k_1^2k_2^2)^{1/2}[(k_1\cdot k_2)^2+k_1^2k_2^2]W_6,$$

where

$$K=\left(\frac{\alpha}{2\pi}\right)^2\frac{1}{[(k_1\cdot k_2)^2-k_1^2k_2^2]^{1/2}}\,.$$

Since the helicity cross sections σ_{ij} or equivalently the forward helicity amplitudes $W_{ij;\,mn}$ depend only on k_1^2, k_2^2, and P^2, it is clear from Eqs. (2.6) and (2.7) that by varying E_1 and E_1', E_2, and E_2' at fixed k_1^2, k_2^2, and P^2 one can experimentally separate five helicity cross sections. However, since for unpolarized electrons there is equal probability for the corresponding photon to be right-handed or left-handed [see Eq. (2.6b)], we cannot experimentally separate the right-handed and left-handed helicity cross sections. Thus only four helicity cross sections or four linear combinations of the structure functions for the process $\gamma^*+\gamma^*\to$ hadrons can be obtained in this manner. The remaining two linear combinations of the structure functions can then be obtained by determining the azimuthal variation of both the outgoing leptons, which is also clear from the differential-cross-section formula (2.5a) if we note that the kinematic factors such as $p_1'\cdot p_2'$ involve the azimuthal angles of the outgoing leptons via the relation (in the c.m. system of colliding leptons)

$$p_1'\cdot p_2'\simeq E_1'E_2'[1-\sin\Theta_1\sin\Theta_2\cos(\phi_1-\phi_2)+\cos\Theta_1\cos\Theta_2]. \qquad (2.8)$$

If the initial electrons are polarized, then the corresponding photon does have different probability to be right-handed or left-handed, and we can experimentally separate the right-handed and the left-handed helicity cross sections. For a left-handed initial electron the result is

$$\begin{aligned}j_\mu^L(k_1)&=\bar u(p_1')\gamma_\mu\frac{1-\gamma_5}{2}u(p_1)\\&=\frac{1}{2m}\frac{(-k_1^2)^{1/2}}{|\vec k_1|}\Big[(4E_1E_1'+k_1^2)^{1/2}\epsilon_\mu^S\\&\qquad+\frac{1}{\sqrt2}(E_1+E_1'+|\vec k_1|)\epsilon_\mu^R\\&\qquad-\frac{1}{\sqrt2}(E_1+E_1'-|\vec k_1|)\epsilon_\mu^L\Big],\end{aligned}\qquad (2.9)$$

and a similar relation for the k_2 photon. The only change in (2.9) when the initial particle is a positron is the interchange $R\leftrightarrow L$. One can then experimentally separate all five helicity cross sections in Eq. (2.7).

Finally, for the sake of completeness, we catalog some special cases. First, the case when one of the photons is nearly on the mass shell, $k_1^2\simeq 0$ corresponding to the interesting process of inelastic electron-photon scattering.[14] In this case

$$W_{\alpha\beta\alpha\nu}(0,k_2^2,P^2)=W_{\beta\nu}(k_2^2,P^2)=\left(g_{\beta\nu}-\frac{k_{2\beta}k_{2\nu}}{k_2^2}\right)V_1(k_2^2,P^2)+\left(k_{1\beta}-\frac{k_1\cdot k_2}{k_2^2}k_{2\beta}\right)\left(k_{1\nu}-\frac{k_1\cdot k_2}{k_2^2}k_{2\nu}\right)V_2(k_2^2,P^2), \qquad (2.10a)$$

where

$$\begin{aligned}V_1(k_2^2,P^2)&=(k_1\cdot k_2)^2(W_2+W_3+2k_2^2W_4+2W_7+2W_8),\\V_2(k_2^2,P^2)&=k_2^2(W_2+W_3+2W_7+2W_8),\end{aligned}\qquad (2.10b)$$

and

$$W_i=W_i\,(0,k_2^2,P^2),\quad i=1,2,\ldots,8.$$

The nonvanishing helicity cross sections are

$$\begin{aligned}\sigma_{RS}=\sigma_{LS}&=\frac{\alpha^2}{(2\pi)^2(k_1\cdot k_2)}[-k_2^2(k_1\cdot k_2)^2W_4],\\\sigma_{RR}=\sigma_{LL}&=\frac{\alpha^2}{(2\pi)^2(k_1\cdot k_2)}[(k_1\cdot k_2)^2(W_2+W_7+W_8)+k_2^2(k_1\cdot k_2)^2W_4],\\\sigma_{RL}=\sigma_{LR}&=\frac{\alpha^2}{(2\pi)^2(k_1\cdot k_2)}[(k_1\cdot k_2)^2(W_3+W_7+W_8)+k_2^2(k_1\cdot k_2)^2W_4].\end{aligned}\qquad (2.11a)$$

From these we can form the usual transverse and longitudinal cross sections, analogous to electroproduction cross sections

$$\begin{aligned}\sigma_T&=\tfrac12(\sigma_{RT}+\sigma_{LT})=\tfrac12[\tfrac12(\sigma_{RR}+\sigma_{RL})+\tfrac12(\sigma_{LL}+\sigma_{LR})]\\&=\frac{\alpha^2}{2(2\pi)^2(k_1\cdot k_2)}V_1,\\\sigma_S&=\tfrac12(\sigma_{RS}+\sigma_{LS})=\frac{\alpha^2}{2(2\pi)^2(k_1\cdot k_2)}\left[\frac{(k_1\cdot k_2)^2}{k_2^2}V_2-V_1\right].\end{aligned}\qquad (2.11b)$$

For the case when both the photons are real,[6] we have

$$W_{\alpha\beta\alpha\beta}(0,0,P^2)=2(k_1\cdot k_2)^2Z_2(0,0,P^2), \qquad (2.12a)$$

with

$$k_2^2Z_2(0,k_2^2,P^2)=V_2(0,k_2^2,P^2)$$

and (2.12b)

$$V_1(0,k_2^2,P^2)+\frac{(k_1\cdot k_2)^2}{k_2^2}V_2(0,k_2^2,P^2)=k_2^2Z_1(0,k_2^2,P^2),$$

where Z_1 and Z_2 are regular at $k_1^2, k_2^2=0$.

III. QUARK LIGHT-CONE ALGEBRA

We now evaluate the helicity cross sections for two-photon processes by using the interacting-quark algebra.[10,12] In this model the dominant contribution to the process $\gamma^* + \gamma^* \to$ hadrons comes from the box graph of Fig. 4(a). More precisely in the limit $q^2 \to -\infty$, with w and P^2 fixed, the integral in (2.3) is light-cone-dominated, and the tensor structure of $W_{\alpha\beta\mu\nu}$ is similar to that appearing in the single scalar and pseudoscalar productions by two virtual photons. Therefore, $W_{\alpha\beta\mu\nu}$ can be written in terms of two structure functions g_S and g_P as

$$\lim_{\substack{q^2\to-\infty \\ w \text{ and } P^2 \text{ fixed}}} (P\cdot q)^2 W_{\alpha\beta\mu\nu} = g_P(w,P^2)\epsilon_{\alpha\beta\gamma\delta}\epsilon_{\mu\nu\kappa\lambda}P_\gamma P_\kappa q_\delta q_\lambda + g_S(w,P^2)[P_\alpha P_\beta w - (P_\alpha q_\beta + q_\alpha P_\beta) + P\cdot q g_{\alpha\beta}][P_\mu P_\nu w - (P_\mu q_\nu + q_\mu P_\nu) + P\cdot q g_{\mu\nu}], \tag{3.1}$$

where

$$q^2 = \tfrac{1}{4}(k_1 - k_2)^2, \quad w = \frac{q^2}{P\cdot q},$$

with

$$|w| > \left(1 + \frac{P^2}{4q^2}\right)^{-1} > 1.$$

Although this result is true in the quark-gluon model, a similar result holds in the free quark model[17] as well. If the above limit is followed by the limit $P^2 \to \infty$, with w fixed, then $W_{\alpha\beta\mu\nu}$ becomes identical to the tensor describing the pair production of massless quarks by two virtual photons, and the invariant functions $g_S(w,P^2)$ and $g_P(w,P^2)$ scale:

$$\lim_{\substack{P^2\to\infty \\ w \text{ fixed}}} g_S(w,P^2) = g_S(w) = \langle Q^4\rangle \int_{-1}^{1} \frac{dz}{2\pi}\frac{z^2(1-z^2)}{(w^2-z^2)^2},$$

$$\lim_{\substack{P^2\to\infty \\ w \text{ fixed}}} g_p(w,P^2) = g_P(w) = \langle Q^4\rangle \int_{-1}^{1} \frac{dz}{2\pi}\frac{z^2(1-z^2)}{(w^2-z^2)^2}, \tag{3.2}$$

where $\langle Q^4\rangle$ is an effective value of Q^4 over all the quarks. Note that the above ordering of limits, which is a peculiarity in the application of light-cone algebra to two-current processes, is important. However, this ordering is not important in the parton model.[13] In the parton model[18] one has, besides the graph in Fig. 4(a), additional terms shown in Fig. 4(b) in which a parton-parton scattering amplitude enters. Such amplitudes vanish rapidly as the virtual (mass)^2 of a parton line becomes large. One then recovers the light-cone-algebra result.

It is now straightforward to calculate the helicity cross sections in the quark-gluon model. The only nonvanishing cross sections are

$$\sigma_{SS} = KW_{00;00} = \frac{K}{k_1^2 k_2^2}[w((k_1\cdot k_2)^2 - k_1^2k_2^2) + P\cdot q k_1\cdot k_2]^2 \times \frac{g_S(w,P^2)}{(P\cdot q)^2}, \tag{3.3a}$$

$$\sigma_{RR} = KW_{11;11} = K[(k_1\cdot k_2)^2 - k_1^2k_2^2]\frac{g_P(w,P^2)}{(P\cdot q)^2} + Kg_S(w,P^2), \tag{3.3b}$$

and

$$W_{11;-1-1} = W_{11;11}. \tag{3.3c}$$

All these relations might eventually be tested; they may serve as crucial tests of the disconnected part of light-cone algebra in the spacelike region. Equations (3.3) offer an interesting possibility of experimentally determining the structure functions g_S and g_P, and test their scaling behavior as pre-

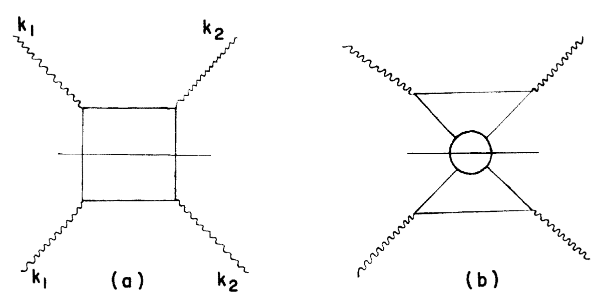

FIG. 4. Quark-parton-model diagrams contributing to the imaginary part of the forward $\gamma\gamma$ scattering amplitude. The graph (a) is the dominant contribution in the case of $\gamma^*\gamma^*$ scattering. In the case of inelastic electron-photon scattering ($k_1^2 \simeq 0$), these graphs give the non-scaling contribution to the structure functions V_1 and V_2. For this case there is another graph, similar to the graph in inelastic electron-nucleon scattering, which gives the scaling contribution.

dicted by Eq. (3.2). In view of the unexpected behavior of the one-photon cross section $e^+e^- \to \gamma^* \to$ hadrons, which involves a timelike photon, it is important to test these relations.

We now consider the case of photon-photon scattering, where one of the photons is nearly on the mass shell.[14] This case is analogous to inelastic electron-nucleon scattering, and is described by two structure functions V_1 and V_2 [see Eq. (2.10)]. If we assume that the real photon behaves merely as a light vector meson, then we would expect the photon structure functions V_1 and V_2 to scale at least as rapidly as those of the nucleon. However, parton models[15,19] predict the existence of anomalous contributions to the photon structure functions which are no longer scale-invariant. Such nonscaling contributions arise from disconnected pieces of the photon-photon scattering amplitude in which a parton coming from the real photon is directly connected to a parton in the virtual photon as in Fig. 4. Similar breakdown of scaling is expected to occur in generalized vector-dominance models as well. Because of the quantum-mechanical time-energy uncertainty principle the real photon can (for short times) transform into vector-meson states $V(n)$ of very high mass. If $m_{V(n)}{}^2 \gg k_1 \cdot k_2,\ k_2{}^2$, we cannot apply the impulse approximation to the inelastic electron-photon scattering. This is because the interaction time is too long compared with the lifetime of the virtual high-mass vector state constituents of the real photon. Only if the transitions of the photon into high-mass vector states are very rare should an approximate scaling law hold. It does not hold in generalized vector-meson-dominance models[20] which predict an s^{-1} scaling law for $\sigma(e^+e^- \to \text{hadrons})$. Indeed the parton model and the generalized vector-dominance model suggest a similar breakdown of scaling, and thus lead to the interesting possibility of dual relationships between resonance models and quark-parton models for photon-photon annihilation.

The anomalous contributions to the photon structure functions coming from graphs in Fig. 4 cannot be calculated unless we know the quark propagators and the off-shell quark scattering amplitudes. However, if we approximate the propagators of exchanged quarks by free-fermion propagators with effective mass m_0 (~0.3 GeV) we get, after a detailed calculation, the result[15] for the box diagram 4(a)

$$(k_1 \cdot k_2)V_2^B = \sum Q_i{}^4 \frac{\alpha x}{\pi} \left\{ [x^2 + (1-x^2)] \left(\ln \frac{P^2}{m_0{}^2} - 1 \right) + \tfrac{9}{2} x(1-x) \right\}, \qquad (3.4a)$$

$$(k_1 \cdot k_2)V_2^B + 2xV_1^B = \sum Q_i{}^4 \frac{4\alpha}{\pi} x^2(1-x), \qquad (3.4b)$$

where

$$x = \frac{-k_2{}^2}{2k_1 \cdot k_2}, \quad 0 \le x \le 1. \qquad (3.4c)$$

The contribution from Fig. 4(b) is more difficult to calculate. In the quark-parton model,[18] one could calculate its contribution by assuming that the quarks are kept near their effective mass shell. However, the extension of this assumption to large timelike parton momenta is not straightforward, but leaves us with a major uncertainty in the quark-parton analysis of photon-photon processes. This problem is intimately related to the coherence effects in the generalized vector-dominance models.

The result (3.4) shows that there are logarithmic scale-breaking terms present in the transverse amplitude V_1. Further, the cross section for virtual scalar photons, σ_S [defined in (2.11b)], does not vanish. The above contribution to the structure functions arises in addition to the scaling contribution from the simple vector-dominance (ρ, ω, ϕ) picture of the real photon. If the above parton contributions were really present, we could conclude that deep-inelastic scattering from a current is essentially different from deep-inelastic scattering on hadrons. Similar anomalous contributions to the structure functions arise in generalized vector-dominance models.[15,20] This suggests a close connection between the parton models and the generalized vector-dominance–type models for photon-photon reactions. This will be studied in more detail in the following section.

Finally we can even try to extend these considerations to on-shell photon-photon scattering. The parton contribution to the helicity amplitudes $W_{ij;mn}$ at $k_1{}^2 = k_2{}^2 = 0$ from the box diagram of Fig. 4(a) is[15]

$$\begin{aligned} W_{1-1;1-1} &\simeq \frac{1}{4\pi^2} Q_i{}^4 \ln \frac{P^2}{m_0{}^2}, \\ W_{11;11} &\simeq \text{constant}, \\ W_{-1-1;11} &\simeq \text{constant}. \end{aligned} \qquad (3.5)$$

There may of course be other contributions to $\gamma + \gamma \to$ hadrons besides (3.5).

IV. DIFFRACTIVE PHOTON BEHAVIOR AND THE VECTOR-DOMINANCE MODEL

It has been argued that at high energies the photon behaves qualitatively as a hadron[21]; the photon interacts with the hadronic objects only when it finds itself in a virtual hadronic state (virtual vector mesons), and thus interacts diffractively with hadrons at high energies. Photoproduction processes are reasonably well described by this picture, i.e., the photon virtually dissociates into a $\rho^0(\omega, \phi)$ which subsequently interacts either elastically or inelastically. This picture is taken to be still operative, and an integral part of the dynamics, in the high q^2 region. The scaling or pointlike behavior of photon-hadron interactions is then built up by an infinite series of vector mesons,[20,22] just as smooth Regge behavior is a result of summing up a series of s-channel resonances in dual models. There are many versions[23] of this "duality" or "correspondence principle." According to this argument the process $\gamma^*+\gamma^* \to$hadrons should be intimately related to $\gamma+\gamma \to$hadrons. The latter process should, at high energies, look like $\rho^0+\rho^0 \to$hadrons. In particular the final hadron distribution for $\gamma+\gamma \to$ hadrons should consist of two photon fragmentation regions and a hadron plateau in between (Fig. 5). Correspondence then demands that the process $\gamma^*+\gamma^* \to$hadrons should be dominated, at high energies, by $V_1+V_2 \to$hadrons (V_1, V_2 are vector mesons). We shall first assume a simple vector-dominance model (ρ, ω, ϕ only) and the factorization result $\sigma_{\gamma\gamma} \sim \sigma_{\gamma p}{}^2/\sigma_{pp}$ to estimate the contribution of contimuum or higher-mass vector states, and then go over to a spectral-function representation for the photon-photon cross section. It might be thought that there are "double" fixed-pole contributions in $\gamma^*\gamma^*$ scattering, which together with the Pomeron may give rise to a logarithmically rising total cross section rather than a constant cross section as predicted by the factorization property. Indeed, individual amplitudes in the covariant expansion of $\gamma^*\gamma^*$ scattering do show such a logarithmically rising behavior, but these logarithms cancel in the total cross section, giving the factorization result.[24]

We start by writing the total transverse cross section for $\gamma^*+\gamma^* \to$hadrons

$$\sigma_{TT}(k_1{}^2, k_2{}^2, P^2) = \sum_{\rho,\omega,\phi} \frac{r_{V_1V_2}}{[(1-k_1{}^2/m_{V_1}{}^2)(1-k_2{}^2/m_{V_2}{}^2)]^2}\,\sigma_{\gamma\gamma}(P^2), \tag{4.1}$$

where

$$r_{V_1V_2} = \frac{\alpha^2}{(f_{V_1}{}^2/4\pi)(f_{V_2}{}^2/4\pi)}\;\frac{\sigma_{V_1V_2}^{(\pm1,\pm1)}}{\sigma_{\gamma\gamma}}\,.$$

$\sigma_{\gamma\gamma}(P^2)$ is the total cross section for real photons and $\sigma_{V_1V_2}^{(\pm1,\pm1)}$ is the total cross section for two transverse vector mesons. At $k_1{}^2 = k_2{}^2 = 0$, we have the sum rule

$$\sigma_{\gamma\gamma}(P^2) = \sum_{\rho,\omega,\phi} r_{V_1V_2}\,\sigma_{\gamma\gamma}(P^2), \tag{4.2}$$

which is the generalization of the famous photoproduction sum rule.[25] To make an estimate of how much of this sum rule is saturated by ρ, ω, and ϕ we need the various cross sections $\sigma_{V_1V_2}$ which are not directly measurable. The use of vector dominance and the optical theorem gives

$$\frac{1}{16\pi}\,\sigma_{V_1V_2}{}^2 = \left[\frac{\alpha^2}{(f_{V_1}{}^2/4\pi)(f_{V_2}{}^2/4\pi)}\right]^{-2}\frac{1}{1+\eta^2}\,\frac{d\sigma^0}{dt}\,(\gamma\gamma\to\gamma\gamma), \tag{4.3a}$$

$$\begin{aligned}\frac{d\sigma^0}{dt}\,(\gamma\gamma\to\gamma\gamma) &= \frac{\alpha^2}{(f_V{}^2/4\pi)^2}\,\frac{d\sigma^0}{dt}\,(\gamma V\to\gamma V)\\ &= \frac{\alpha^2}{(f_V{}^2/4\pi)^2}\,\frac{d\sigma^0}{dt}\,(\gamma\gamma\to VV),\end{aligned} \tag{4.3b}$$

where $d\sigma^0/dt$ is the contribution of any vector meson V to the forward cross section, and η is the usual ratio of the real to the imaginary part of the corresponding amplitude. From these relations it is equally difficult to predict $\sigma_{V_1V_2}$. However, we can use factorization of hadronic cross sections to obtain

$$\sigma_{V_1V_2}{}^2 = \sigma_{V_1V_1}\,\sigma_{V_2V_2}, \tag{4.4a}$$

$$\sigma_{Vp}{}^2 = \sigma_{VV}\,\sigma_{pp}\,, \tag{4.4b}$$

which together with the familiar optical theorem and the VMD result

$$\sigma_{Vp}{}^2 = 16\pi(f_V{}^2/4\pi)\,\frac{1}{1+\eta^2}\,\frac{d\sigma^0}{dt}\,(\gamma p\to Vp) \tag{4.4c}$$

can be used to determine $r_{V_1V_2}$. We shall use the coupling constant values

$$f_\rho^2/4\pi=2.56, \quad f_\omega^2/4\pi=19.20. \quad f_\phi^2/4\pi=11.30, \tag{4.5a}$$

and a value for the total cross section $\sigma_{\gamma\gamma}$ given by[26]

$$\sigma_{\gamma\gamma} \simeq \frac{\sigma_{\gamma p}{}^2}{\sigma_{pp}} \simeq \frac{(100\ \mu\text{b})^2}{38.20\ \text{mb}} = 0.26\ \mu\text{b} \tag{4.5b}$$

to evaluate the contribution of ρ, ω, and ϕ to $r_{V_1V_2}$. The result is[27]

$$\sum_{V_1,V_2} r_{V_1V_2} = \frac{\alpha^2}{\sigma_{\gamma\gamma}}\left[(f_\rho^2/4\pi)^{-2}\sigma_{\rho\rho}+(f_\omega^2/4\pi)^{-2}\sigma_{\omega\omega}+(f_\phi^2/4\pi)^{-2}\sigma_{\phi\phi}+2(f_\rho f_\omega/4\pi)^{-2}\sigma_{\rho\omega}+\cdots\right]$$
$$\simeq 0.80. \tag{4.6}$$

It is amusing to find the same 20% discrepancy here as in the photoproduction sum rule. There are two ways to interpret this result. One possibility is that this missing 20% is due to the effects of parton scattering (the seagull terms[28]) not dominated by the vector mesons. We shall consider the second possibility in that there are additional higher-mass vector states which provide this 20% contribution.

We now write the continuum version of (4.1), assuming the diagonal approximation,

$$\sigma_{TT}(k_1^2, k_2^2, P^2) = \int \frac{dm_1^2\, dm_2^2\, \rho_{TT}(m_1^2, m_2^2, P^2)}{(1-k_1^2/m_1^2)^2(1-k_2^2/m_2^2)^2}\,. \tag{4.7}$$

The double spectral function $\rho_{TT}(m_1^2, m_2^2, P^2)$ is proportional to the imaginary part of the forward amplitude for

$$V_1+V_2 \to V_1+V_2\,, \tag{4.8}$$

where V_1 and V_2 are vector states of masses m_1 and m_2. At sufficiently high energies, in analogy to electroproduction, the spectral function $\rho_{TT}(m_1^2, m_2^2, P^2)$ can be interpreted as the probability for the photons k_1 and k_2 to create respective hadron systems multiplied by the total cross section for these two hadron systems to produce the final hadrons. We can then write

$$\rho_{TT}(m_1^2, m_2^2, P^2) \sim \sigma_{e^+e^-}(m_1^2)\sigma_{e^+e^-}(m_2^2) \times \sigma_{\text{had}}(m_1^2, m_2^2, P^2)\,, \tag{4.9}$$

where $\sigma_{e^+e^-}(m^2)$ stands for the total cross section for production of hadrons in e^+e^- annihilation via one-photon exchange at a total c.m. energy m. Clearly the behavior of ρ_{TT}, and consequently that of σ_{TT}, depends on the behavior $\sigma_{e^+e^-}$ and σ_{had}. If we assume for large P^2, m_1^2, and m_2^2 that σ_{had} approaches a finite constant, and that $\sigma_{e^+e^-}(m^2)$ behaves as $1/m^2$ then σ_{TT} diverges logarithmically as $\ln(k_1^2k_2^2)$. There are two ways to avert this disastrous result. One is to suppose that $\sigma_{e^+e^-}(m^2) \propto m^{-4}$, for which there is no experimental justification at present. The second, and much more interesting possibility, is the "aligned-jet" version of the generalized vector-dominance model.[29] According to this model only those hadrons, the virtual vector mesons, from the photons k_1 and k_2 interact which have limited transverse momentum along the collision axis of the photon-photon system, and produce the final hadrons. It is generally believed that the final hadrons in $e^+e^- \to \gamma^* \to$ hadrons are jets of high momentum, which in general possess high transverse momentum. Only when these jets from the photons k_1 and k_2 are aligned along the photon-photon axis is there a limited transverse momentum. The probability for each jet to get aligned along this axis is the factor $1/m_1^2$ or $1/m_2^2$, respectively. When these factors are included in (4.7) we get

$$\sigma_{TT} \propto \frac{1}{k_1^2k_2^2}\,, \text{ for large } k_1^2,\ k_2^2,\ \frac{P^2}{k_1^2},\ \text{and } \frac{P^2}{k_2^2}. \tag{4.10}$$

Thus in the rapidity space the hadron distributions in the two photon fragmentation regions for the reaction $\gamma^*+\gamma^* \to$ hadrons (Fig. 5) should look characteristically the same as one would expect in a high-energy event of $e^+e^- \to \gamma^* \to$ hadrons (Fig. 6). This behavior is shown schematically in Fig. 7. Each photon fragmentation region now reveals its full structure. There are three fragmentation

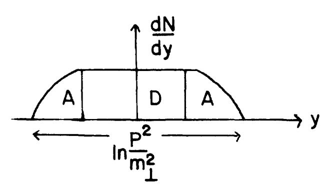

FIG. 5. Hadron distribution as a function of rapidity along the collision axis in photon-photon reactions. The distribution is similar to that of purely hadronic reactions, and consists of two photon fragmentation regions (*A*) and a hadron plateau (*D*) in between. *P* is the c.m. energy of the photon-photon system and *m* is the average transverse mass, typically of the order of 1 GeV.

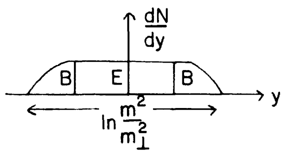

FIG. 6. Hadron distribution in the "jet" description of e^+e^- collisions (one-photon mechanism); m is the c.m. energy. B represents the parton fragmentation region and E the current plateau.

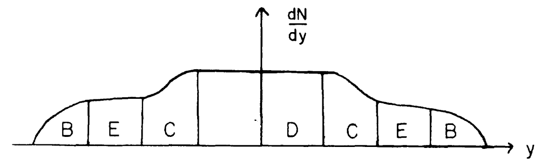

FIG. 7. Schematic illustration of hadron distribution as a function of rapidity along the photon-photon collision axis in the diffractive limit (4.11). The two photon fragmentation regions of Fig. 5 now reveal their full structure. C is the hole fragmentation region, whereas B, E, and D have the same meaning as in Figs. 5 and 6. In the case of $\gamma^*\gamma$ scattering, only the photon fragmentation region corresponding to photon k_2 should reveal its full structure.

regions in each fragmentation region, which are readily identified in the parton model[30] as a parton fragmentation region, a current plateau, and a hole fragmentation region, on each side of a hadron plateau. The average multiplicity is

$$\langle n\rangle = c_h \ln\left(\frac{P^2}{k_1^2 k_2^2}\right) + c_e \ln(k_1^2 k_2^2) + \text{constant}, \quad \text{for large } k_1^2, k_2^2, \frac{P^2}{k_1^2}, \frac{P^2}{k_2^2}, \tag{4.11}$$

where c_h and c_e are the heights of the hadron and the current plateaus, respectively. The connection with the parton model is now clear: if each hadron jet produced in e^+e^- annihilation corresponding to photons k_1 and k_2 is a descendant of a parton, then the configurations aligned along the photon-photon axis contain wee partons. According to the parton model these are the only types of partons which interact to produce final hadrons.[31] However, we emphasize that our arguments are based, not on the parton model, but on "correspondence" arguments (and of course on short-range correlations in rapidity[32]).

We have seen above how the ideas of "correspondence" relate the $\gamma\gamma$ scattering to $\gamma^*\gamma^*$ scattering. The picture that emerges is that as the four-momentum of each photon increases and becomes large but is still small compared to the c.m. energy of the photon-photon system, the corresponding photon fragmentation region reveals its full structure. The double real and double virtual photon-photon scattering should also be closely related to the case when one photon is real and the other virtual, the inelastic electron-photon scattering process. In this case only the photon fragmentation region corresponding to photon k_2 should reveal its full structure. The photon fragmentation region corresponding to k_1 ($k_1^2 \simeq 0$) should be characteristic of pure hadronic reactions. When the photon k_1 also becomes virtual with $-k_1^2 \ll P^2$, the size of photon (k_1) fragmentation region changes and starts revealing its full structure. This can also be seen from the spectral representation (4.7). For the $\gamma^*\gamma$ case we have

$$\sigma_{TT}(0, k_2^2, P^2) = \int \frac{dm_1^2 dm_2^2 \rho_{TT}(m_1^2, m_2^2, P^2)}{(1-k_2^2/m_2^2)^2} = \int \frac{dm_2^2 \rho'_{TT}(m_2^2, P^2)}{(1-k_2^2/m_2^2)^2}, \tag{4.12a}$$

where

$$\rho'_{TT}(m_2^2, P^2) = \int dm_1^2 \rho_{TT}(m_1^2, m_2^2, P^2) \sim (\text{probability real photon } k_1 \text{ is a hadron system}) \times (\text{probability virtual photon } k_2 \text{ is a hadron system}) \times (\text{probability that the two hadron systems interact}). \tag{4.12b}$$

The first factor is a constant. For the rest we can proceed as in the double virtual case. Inserting the "alignment" factor $1/m_2^2$, we conclude that the rapidity distribution consists of a "target" (real photon k_1) fragmentation region, a hadron plateau, and a "projectile" (virtual photon k_2) fragmentation region whose structure is typical of a hadronic event in e^+e^- annihilation, i.e., a parton fragmentation region, a current plateau, and a hole fragmentation region. The situation here is, thus, analogous to inelastic electron-nucleon scattering.[29]

V. CONCLUDING REMARKS

We have discussed the two-photon process $e^+e^- \to e^+e^-$ +hadrons in a form which might be suitable for experimental separation of the structure functions for the process $\gamma^* + \gamma^* \to$ hadrons. Specifically, by varying the energies of the leptons at a fixed value of the four-momentum and energy of the corresponding photon (this amounts to varying the angle of the outgoing leptons p_1' and p_2') we can separate five linear combinations of the eight invariant structure functions. These combinations of structure functions, which are related to the helicity cross sections for the photon-photon annihilation process, have been evaluated using quark algebra. For the case of one real photon the parton model predicts an anomalous nonscaling behavior of structure functions. This is attributed in the parton model to a possible quark-antiquark substructure of the real or virtual photon. Similar deviations occur in generalized VMD models. This suggests a close connection between parton models and generalized VMD models for photon-photon reactions. Indeed, the connection between the two types of models is obtained in Sec. IV for the final particle multiplicities in photon-photon reactions. The next question which arises naturally is: When are the predictions of the quark light-cone algebra expected to hold? In particular, when is the "scaling" [Eqs. (3.2) and (3.3)] expected to begin in the two-photon process? This question is particularly interesting because of the unexpected behavior of the one-photon annihilation process, and also because there have been speculations[33] regarding the difference between timelike and spacelike photons. In the case of one-photon annihilation, involving a timelike photon, it has been argued[34] that the approach to scaling is slower because (1) there are strong direct channel resonances (ρ, ω, ϕ, ρ', . . . , ψ, ψ'. . .) and presumably these must be well past before scaling sets in; (2) only one partial wave contributes to e^+e^- annihilation via the one-photon process. Since scaling is a cooperative phenomenon with a contribution from a large number of hadronic states, the approach to scaling should be slower in the one-photon annihilation process. The first of these arguments can be applied to the two-photon case as well, where we have even-charge-conjugation resonant states.[26] However, we can write the total photon-photon cross section as[35]

$$\sigma_{\gamma\gamma\to\text{had}} = \sigma_0 + \frac{\sigma_1}{\sqrt{P^2}},$$

where σ_0 corresponds to the Pomeranchuk trajectory and σ_1 is due to nonleading f and A_2 trajectories. Factorization of the Pomeron gives (Sec. IV)

$$\sigma_0 \simeq 0.26\ \mu\text{b},$$

whereas exchange degeneracy, the coupling of nonleading trajectories to p, n, and γ in other processes, leads to[26]

$$\sigma_1 \simeq 0.27\ \mu\text{b GeV}.$$

By duality, we can identify the non-Pomeranchuk component σ_1 with the average effects of direct-channel resonances. We can then compare the magnitude of σ_0 and σ_1, and thereby obtain the resonance to background ratio. For $\sqrt{P^2}$ above 6 GeV the resonance contribution to the total cross section is roughly 20%. We thus expect the scaling to build up rapidly for $\sqrt{P^2} \geq 6$ GeV in the two-photon-annihilation process. Further, all the $C=+$ resonant states such as $\pi^+\pi^-$, $\pi^0\pi^0$, K^+K^-, with $J^P = 0^+, 2^+, \ldots,$ $I^G = 0^+, 1^-$ are excited in the two-photon process. Thus many partial waves contribute to this process, and the predictions based on quark algebra should be fairly successful as in electroproduction, where the density of states is also large and all partial waves are excited.

We have also discussed the diffractive production of hadrons in photon-photon collisions using the conventional vector-dominance model. We find a 20% discrepancy which we attribute to higher-mass vector states. Assuming the "aligned-jet" version of the generalized vector-dominance model we predict the multiplicity distribution of produced hadrons, which is the same as obtained from parton-model arguments. In the diffractive region the full structure of the photon fragmentation regions should be revealed. In the light-cone-algebra limit (Sec. III) the production of final hadrons in virtual photon-photon collisions proceeds via a pair creation of quarks (Fig. 8) and is similar to that of the one-photon process. Therefore, the multiplicity distribution should be the same for one- and two-photon processes in this limit (Fig. 6). In particular the inclusive pion distribution[36] should be the same. We thus have for $\gamma^*\gamma^* \to \pi +$hadrons

$$\frac{dN^{\pi+}}{dz} = \frac{dN^{\pi-}}{dz} = \frac{dN^{\pi 0}}{dz},$$

where z is the fraction of parton momentum carried by the emerging pion. A similar relation for the one-photon process seems to be violated experimentally. However, it would be premature to

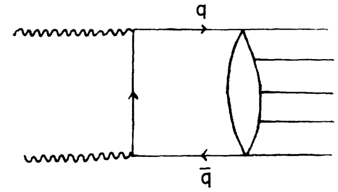

FIG. 8. Evolution of final hadrons via pair creation of quarks in the light-cone limit.

comment on this problem in the two-photon case, unless the effect of new particles ($\psi, \psi', \ldots$) on the inclusive pion production is well understood in the one-photon case.

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