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New method for determining $\pi^0\pi^0$ correlation and its application in $\pi^{\pm}p$ reactions at 18.5 GeV/c

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For experiments in which the low detection efficiency of γ's severely limits the number of events associated with more than two photons, a method for determining $\pi^0\pi^0$ correlation utilizing only the single- and double-γ associated events is presented. This method is applied to data of π^-p and π^+p reactions at 18.5 GeV/c.

I. INTRODUCTION

In a multiparticle production process, the study of various moments of the multiplicity distributions of pions is of interest. For charged pions, the moments can be readily calculated from the experimentally known multiplicity distributions. For neutral pions, the information about multiplicity distribution is inferred indirectly via the 2γ decay of the π^0 meson. In many experiments, particularly in experiments using hydrogen bubble chambers, the γ-detection efficiency is low and hence, due to limited data on inclusive γ process, only the simplest moment of the π^0 distribution, i.e. $\langle n_0\rangle$, the average π^0 multiplicity can be obtained. The events with two and more γ's carry information about higher moments of the π^0 distribution. Thus, to determine the $\pi^0\pi^0$ correlation $C(00)$ it is necessary to have a reasonable sample of events associated with 2γ's as well as events with more than 2γ's.

We point out here that in experiments in which 1γ and 2γ events are observed with good statistics, information about $\pi^0\pi^0$ correlation can be extracted using these events only. In Sec. II, we define a new correlation function $C'(00)$, which is directly given in terms of the observed number of 1γ and 2γ events. In Sec. III, we relate $C'(00)$ to $C(00)$ using γ-inclusive data for π^-p and π^+p interactions[1] at 18.5 GeV/c. We determine $\pi^0\pi^0$ correlation here and obtain other $\pi\pi$ correlations from published data[2,3] for these reactions. In Sec. IV, we present the values of all $\pi\pi$ correlations for π^-p and π^+p reactions at 18.5 GeV/c. We summarize the results in Sec. V.

II. RELATION BETWEEN PHOTON AND π^0 CORRELATIONS

We denote $\sigma(n_0)$ as the cross section for the production of n_0 neutral pions and define $g(n_0)=\sigma(n_0)/\sigma_{\text{inel}}$ such that $\sum g(n_0)=1$, where σ_{inel} is the total inelastic cross section. We are interested in the $\pi^0\pi^0$ correlation function,

$$C(00)=\langle n_0(n_0-1)\rangle=\sum_{n_0=2}^{\infty} n_0(n_0-1)\,g(n_0)\,. \tag{1}$$

Note that $g(n_0)$'s are not known individually, and hence $C(00)$ cannot be readily evaluated. Thomas and Webber[4] have shown that $\pi^0\pi^0$ correlation is related to $\gamma\gamma$ correlation by

$$C(\gamma\gamma)=4C(00)+2\langle n_0\rangle\,,$$

where

$$C(\gamma\gamma)=\sum_{n_\gamma=2}^{\infty} n_\gamma(n_\gamma-1)\,g(n_\gamma)\,.$$

In determining $C(00)$ from $C(\gamma\gamma)$, the events associated with 2γ's as well as those associated with $>2\gamma$'s are important. If the γ conversion probability p is very low such that events with $n_\gamma>2$ are limited in statistics, the determination of $C(\gamma\gamma)$ becomes unreliable. However, it is possible to determine $C(00)$ using more statistically significant events associated with 1γ and 2γ events only.

Let σ_1 and σ_2 be the cross sections for the events with 1γ and 2γ respectively. Assuming that all γ's are due to $\pi^0\to 2\gamma$ decay, one can write

$$G(1)=\sigma_1/\sigma_{\text{inel}} =2p(1-p)\,g(1)+\sum_{n_0=2}^{\infty} 2n_0 p(1-p)^{2n_0-1} g(n_0)\,, \tag{2}$$

and

$$G(2)=\sigma_2/\sigma_{\text{inel}} =p^2 g(1)+\sum_{n_0=2}^{\infty} n_0(2n_0-1)p^2(1-p)^{2n_0-2} g(n_0)\,. \tag{3}$$

Here Eqs. (2) and (3) correspond to binomial distributions for observing 1γ and 2γ respectively; out of a total number of γ's, $n_\gamma=2n_0$, with a detection probability p. Eliminating $g(1)$ from these equations, we obtain

$$\sum_{n_0=2}^{\infty} n_0(n_0-1)(1-p)^{2n_0-2}g(n_0)=\frac{G(2)}{2p^2}-\frac{G(1)}{4p(1-p)}\,. \quad (4)$$

In terms of the experimentally measured quantities $G(1)$ and $G(2)$, we then have a new moment of π^0 distribution,

$$C'(00)=\sum_{n_0=2}^{\infty} n_0(n_0-1)(1-p)^{2n_0-2}g(n_0)\,. \quad (5)$$

Comparing Eq. (5) with Eq. (1), it is seen that $C'(00)$ involves an extra weight factor $w(n_0) = (1-p)^{2n_0-2}$ for the cross sections $g(n_0)$. Since $w(n_0)$ is less than unity, $C'(00)$ is, in general, less than $C(00)$; however, as $p\to 0$, $C'(00)$ tends to be equal to $C(00)$.

III. $\pi^0\pi^0$ CORRELATION IN $\pi^{\pm}p$ INTERACTION AT 18.5 GeV/c

In an inclusive γ study,[1] we have observed 4995 events with 1γ and 360 events with 2γ in π^-p reaction, and 6199 events with 1γ and 393 events with 2γ in π^+p reaction at 18.5 GeV/c. These data and the calculated values[5] of $C'(00)$ using Eq. (4) for these reactions are shown in Table I. As discussed in the previous section, the value of $C'(00)$ is somewhat less than that of $C(00)$ since the value of p is finite, although very small. To determine $C(00)$, we have to multiply $C'(00)$ by a factor which is slightly higher than unity. We estimate this factor in the following section.

Consider the charged-pion data[2,3] for these reactions. The multiplicity distributions $g(n_-)$ and $g(n_+)$ for π^- and π^+ are known, and hence one can calculate C' for a given value of p from these distributions from Eq. (5). This value of C' can then be compared to the value of C given by Eq. (1). We have calculated the values of C' and C using the values of p given in Table I for the $\pi^-\pi^-$ and $\pi^+\pi^+$ correlations for π^-p and π^+p reactions. These values and the ratio $r=C/C'$ are shown in Table II. It is seen from this table that r is insensitive to the value of the mean multiplicity $\langle n\rangle$ and the multiplicity distribution. A multiplicative factor $r=1.15$ for C' is therefore appropriate to yield C for π^-p and π^+p reactions at 18.5 GeV/c.

TABLE I. Data on inclusive γ production at 18.5 GeV/c for πp reactions.

n_γ	π^-p reaction: Cross section (mb)	$C'(00)$	π^+p reaction: Cross section (mb)	C'
1	2.10 ±0.03	1.78 ±0.20	1.82 ±0.02	1.66 ±0.18
2	0.151 ±0.011		0.116 ±0.016	
	$\sigma_{\rm inel}=21.17\pm0.49$ mb		$\sigma_{\rm inel}=19.68\pm0.77$ mb	
	$\langle p\rangle=0.0314$		$\langle p\rangle=0.0304$	
	$\langle p(1-p)\rangle=0.0299$		$\langle p(1-p)\rangle=0.0289$	
	$\langle p_1p_2\rangle=1.47\times10^{-3}$		$\langle p_1p_2\rangle=1.32\times10^{-3}$	

A consistency check for this estimate of r can be obtained by assuming that the π^0 multiplicity has a Poisson distribution with average value $\langle n_0\rangle$,

$$g(n_0)=\exp(-\langle n_0\rangle)\langle n_0\rangle^{n_0}/n_0!\,.$$

In this case $C(00)=\langle n_0\rangle^2$ and Eq. (5) can be explicitly evaluated to give

$$C'(00)=C(00)(1-p)^2\exp[-(2p-p^2)\langle n_0\rangle]\,,$$

and hence

$$r=(1-p)^{-2}\exp[(2p-p^2)\langle n_0\rangle]\,.$$

Using the values of $\langle n_0\rangle$ for our data[1] and the values of p shown in Table I, we obtain $r=1.20$ for π^-p data and $r=1.18$ for π^+p data. These values are consistent with those shown in Table II.

Using the values of r shown in Table II, we thus compute the values of $C(00)$. We obtain $C(00)=2.05\pm0.30$ for π^-p data and $C(00)=1.91\pm0.20$ for π^+p data at 18.5 GeV/c.

IV. $\pi\pi$ CORRELATIONS IN $\pi^{\pm}p$ REACTIONS AT 18.5 GeV/c

In the present paper, we have determined the $\pi^0\pi^0$ correlation in $\pi^{\pm}p$ reactions at 18.5 GeV/c. The $\pi\pi$ correlations in other charged states can be calculated from the data published elsewhere.[1-3]

TABLE II. Values of C' and C for charged-pion multiplicity distribution in $\pi^{\pm}p$ reactions at 18.5 GeV/c.

Reaction	$\langle n\rangle$	Pion system	C'	C	r	$\bar r$
π^-p	2.20 ±0.02	$\pi^-\pi^-$	3.06 ±0.05	3.55 ±0.06	1.16	1.154
	1.94 ±0.04	$\pi^+\pi^+$	2.49 ±0.10	2.87 ±0.11	1.15	
π^+p	1.34 ±0.05	$\pi^-\pi^-$	1.14 ±0.04	1.27 ±0.04	1.11	1.153
	3.12 ±0.07	$\pi^+\pi^+$	5.10 ±0.16	6.10 ±0.19	1.20	

For the sake of convenience we summarize these data in Table III. Here we show the values of f_2 given by

$$f_2 = \langle n(n-1)\rangle - \langle n\rangle^2 \text{ for like charges}$$

and

$$f_2 = \langle n_1 n_2\rangle - \langle n_1\rangle\langle n_2\rangle \text{ for unlike charges.}$$

The average multiplicity of π^-, π^+, and π^0 mesons are also shown in this table.

TABLE III. Average pion multiplicity and $\pi\pi$ correlations in $\pi^{\pm}p$ reactions at 18.5 GeV/c.

Pion system	Quantity	π^-p reaction	π^+p reaction
π^-	$\langle n_-\rangle$	2.20 ± 0.02	1.34 ± 0.05
π^+	$\langle n_+\rangle$	1.94 ± 0.04	3.12 ± 0.07
π^0	$\langle n_0\rangle$	1.80 ± 0.08	1.72 ± 0.12
$\pi^-\pi^-$	$f_2(--)$	-1.27 ± 0.30	-0.52 ± 0.17
$\pi^-\pi^+$	$f_2(-+)$	0.91 ± 0.21	0.58 ± 0.20
$\pi^-\pi^0$	$f_2(-0)$	0.09 ± 0.27	0.05 ± 0.20
$\pi^+\pi^+$	$f_2(++)$	-0.89 ± 0.25	-3.63 ± 0.28
$\pi^+\pi^0$	$f_2(+0)$	0.11 ± 0.27	-0.56 ± 0.27
$\pi^0\pi^0$	$f_2(00)$	-1.18 ± 0.38	-1.05 ± 0.46

V. SUMMARY AND DISCUSSION

Various aspects of inclusive pion production in $\pi^{\pm}p$ interactions at 18.5 GeV/c have been discussed in detail in previous publications.[1-3,6] Here we make few comments about the pion multiplicity distribution. For a pure Poisson distribution the value of $f_2 = 0$, while f_2 is less than (greater than) zero if the distribution is narrower (broader) than Poisson. From Table III, we observe that the largest deviation from a Poisson distribution occurs for π^- distribution in π^-p data and for π^+ distribution in π^+p data. The values of $f_2(++)$ and $f_2(00)$ are consistent with each other for π^-p data, and the values of $f_2(--)$ and $f_2(00)$ for π^+p data are comparable. Thus π^+-and π^0 mesons in π^-p reactions and π^--and π^0 mesons in π^+p reactions have similar distributions.

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[1]N. N. Biswas *et al.*, Phys. Rev. D 10, 3579 (1974).
[2]J. T. Powers *et al.*, Phys. Rev. D 8, 1947 (1973).
[3]P. T. Go *et al.*, Phys. Rev. D 11, 3092 (1975).
[4]G. H. Thomas and B. R. Webber, Phys. Rev. D 9, 3113 (1974).
[5]We note that the detection probability of a photon varies from event to event. In evaluating C' from Eq. (4), we have used the weight factor $p(1-p)$ for single-γ events and the weight factor p_1p_2 for double-γ events on an event-by-event basis. (p_i is the detection efficiency for photon $i = 1, 2$.) In Table I, the quantities $\langle p(1-p)\rangle$ and $\langle p_1p_2\rangle$ are presented.
[6]N. N. Biswas *et al.*, Phys. Rev. Lett. 26, 1589 (1971).