Reply to "Comment on classical derivations of Planck's spectrum"

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The classical derivation of Planck's spectrum by Theimer and by Theimer and Peterson is analyzed, and it is shown that both papers invoke an *ad hoc* assumption, called the renormalization postulate. Kracklauer's suggestion that Theimer's first derivation is not based on the renormalization postulate is shown to be a consequence of an inadequate notation in Theimer's earlier paper. Some consequences of this result are discussed.

In the preceding note,[1] Kracklauer discusses two papers by Theimer[2] (paper I) and by Theimer and Peterson[3] (paper II) dealing with a classical derivation of Planck's blackbody-radiation spectrum. In paper II, Theimer and Peterson point out that the theory presented in paper I is afflicted by certain difficulties which invalidate Theimer's earlier claim that a classical derivation of Planck's spectrum can be given which is free of *ad hoc* assumptions and uses only well-established concepts of classical statistical physics. Some suggestions are made as to how the difficulties could be removed, and the possibility of a satisfactory classical blackbody-radiation theory is kept open. However, it is stated that Theimer's paper I does not yet present such a theory.

The main difficulty of the classical theory is the derivation of Kracklauer's Eq. (7), to be denoted Eq. (K7) in this paper. Once this equation is given, Planck's spectrum follows more or less automatically without conceptual difficulties. In paper II, Theimer and Peterson show that for deriving Eq. (K7) an *ad hoc* assumption has to be made, which is called the renormalization postulate. It states that observables that are functions of the energy density ρ should be renormalized such that only their finite temperature-dependent part has operational meaning and satisfies the principles of statistical mechanics. The constant part which is due only to the ground state and diverges after integration over all radiation frequencies should be omitted.

While the renormalization postulate is necessary for deriving Eq. (K7), it is not explicitly used in paper I. In fact, paper I generates the impression that Eq. (K7) is a unique and rigorous consequence of classical physics, and this is correctly noticed by Kracklauer. Consequently, he suggests that Theimer's earlier derivation of Planck's spectrum in paper I represents an acceptable classical theory of blackbody radiation, as long as no errors are discovered in Theimer's earlier formalism.

Following paper I, we consider one Fourier component of the radiation field with electrical field E. E is a sum of a ground-state contribution E_0 and a thermal contribution E_T. E_0 is produced by a large number N_0 of incoherent sources, each of which produces at some point $\vec{r}$ the field E_{0s} with a random phase angle θ_s $(s=1,2,\ldots,N_0)$. Similarly, E_T is produced by N_T random thermal sources which produce the fields $E_{T\sigma}$ with random phases θ_σ $(\sigma=1,2,\ldots,N_T)$. We have then

$$E=E_0+E_T=\sum_{s=1}^{N_0}E_{0s}e^{i\theta_s}+\sum_{\sigma=1}^{N_T}E_{T\sigma}e^{i\theta_\sigma}. \tag{1}$$

The energy density ρ is proportional to EE^*, and omitting the constant of proportionality we have

$$\rho=|E_0+E_T|^2 = |E_0|^2+|E_T|^2+E_0E_T^*+E_0^*E_T. \tag{2}$$

Let

$$\rho_0=|E_0|^2 = \text{energy density of the ground-state radiation}, \tag{3}$$

$$\rho_T=|E_T|^2 = \text{energy density of the thermal radiation}, \tag{4}$$

and

$$\tilde{\rho}_T=\rho_T+E_0E_T^*+E_0^*E_T = \text{renormalized temperature-dependent part of the total energy density } \rho; \tag{5}$$

then

$$\rho=\rho_0+\rho_T+E_0E_T^*+E_0^*E_T=\rho_0+\tilde{\rho}_T, \tag{6}$$

and we can see that the total energy density cannot be neatly separated into a sum of contributions from the ground state and the thermal radiation.

This is the result of the term $E_0E_T^*+E_0^*E_T$, which represents interference between the ground state and thermal radiation. The distinction between ρ_T and $\bar{\rho}_T$ has not been made explicitly in paper I, but it was clearly stated that only the temperature-dependent part of ρ satisfies classical statistical mechanics. In paper II, the use of $\bar{\rho}_T$ has been bypassed by a slightly different method of calculation.

We now form the ensemble average of ρ over all the random phases, and using the method outlined in paper I we get

$$\langle\rho\rangle=\langle\rho_0\rangle+\langle\rho_T\rangle,\quad \langle\rho_T\rangle=\langle\bar{\rho}_T\rangle, \tag{7}$$

which shows that the ensemble average of ρ separates clearly into a contribution from the ground state and the thermal field.

We come now to the second moments and find that

$$\langle\rho_x{}^2\rangle=2\langle\rho_x\rangle^2, \tag{8}$$

where ρ_x stands for ρ, or ρ_0 or ρ_T. But for $\langle\bar{\rho}_T{}^2\rangle$ we get

$$\begin{aligned}\langle\bar{\rho}_T{}^2\rangle&=2\langle\bar{\rho}_T\rangle^2+2\langle\rho_0\rangle\langle\rho_T\rangle\\&=2\langle\rho_T\rangle^2+2\langle\rho_0\rangle\langle\rho_T\rangle.\end{aligned} \tag{9}$$

Finally, we consider the mean-square fluctuations and find

$$\langle(\delta_{\rho x})^2\rangle\equiv\langle\rho_x{}^2\rangle-\langle\rho_x\rangle^2=\langle\rho_x\rangle^2; \tag{10}$$

but for $\langle(\delta\bar{\rho}_T)^2\rangle$ we get from Eq. (9) the more complicated relation

$$\langle(\delta\bar{\rho}_T)^2\rangle=\langle\rho_T\rangle^2+2\langle\rho_0\rangle\langle\rho_T\rangle. \tag{11}$$

This is the basic relation of the classical blackbody-radiation theory. The fluctuation $\langle(\delta\bar{\rho}_T)^2\rangle$ is also the renormalized part of the total fluctuation $\langle(\delta\rho)^2\rangle$. This can be seen from Eqs. (7) and (10), according to which

$$\langle(\delta\rho)^2\rangle=\langle\rho\rangle^2=\langle\rho_0\rangle^2+\langle\rho_T\rangle^2+2\langle\rho_0\rangle\langle\rho_T\rangle. \tag{12}$$

Removing the pure ground-state contribution $\langle\rho_0\rangle^2$ gives the renormalized fluctuation

$$\langle(\delta\rho)^2\rangle_T=\langle\rho_T\rangle^2+2\langle\rho_0\rangle\langle\rho_T\rangle. \tag{13}$$

It is now seen that Eqs. (11) and (13) are identical, so that

$$\langle(\delta\rho)^2\rangle_T=\langle(\delta\bar{\rho}_T)^2\rangle\neq\langle(\delta\rho_T)^2\rangle. \tag{14}$$

Since $\langle\rho_0\rangle^2=\langle(\delta\rho_0)^2\rangle$, Eqs. (12), (13), and (14) can be written in the form

$$\langle(\delta\rho)^2\rangle=\langle(\delta\bar{\rho}_T)^2\rangle+\langle(\delta\rho_0)^2\rangle, \tag{15}$$

which expresses the well-known statistical theorem that the mean-square fluctuations of two independent stochastic variables are additive. This fact has been emphasized by Theimer in paper I, who derived the basic Eq. (11) from this theorem. Unfortunately, in paper I, the differences between ρ_T and $\bar{\rho}_T$, and between the quantities appearing in Eq. (14), have not been indicated explicitly by the notation.

Summarizing, we have shown that the basic Eq. (11) can be derived only if the renormalization postulate is applied either to ρ of Eq. (6) or to $\langle(\delta\rho)^2\rangle$ of Eq. (12). While this postulate is quite plausible, we do not yet know how to apply it correctly to the canonical relations involving the moments of ρ of higher than second order. Thus, papers I and II represent a starting point for a classical theory of blackbody radiation which we consider promising but still afflicted by difficulties.

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[1]A. F. Kracklauer, preceding paper, Phys. Rev. D 14, 654 (1976).
[2]O. Theimer, Phys. Rev. D 4, 1597 (1971).
[3]O. Theimer and P. R. Peterson, Phys. Rev. D 10, 3962 (1974).