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Comment on classical derivations of Planck's spectrum

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Inconsistencies in a recent classical derivation of Planck's spectrum are shown not to obtain in an earlier version.

I. INTRODUCTION

Recently there appeared a critical reexamination[1] of Theimer's classical derivation of the Planck blackbody spectrum[2] which concluded that Theimer's classical theory is "either incomplete or inconsistent, and in any case its results are different from those of the quantum theory of blackbody radiation." It is the purpose of this note to draw attention to a different assumption employed in the reasoning used in the reexamination which appears to be both ill founded and not relevant to Theimer's classical derivation.

Both derivations are based on the assumption that there exists background electromagnetic radiation whose spectral energy density is a Lorentz invariant. This assumption has been exploited by several authors in recent years in various types of attempts to reformulate the foundations of quantum theory in order to resolve questions of interpretation[3-7] and to facilitate the formulation of a fully relativistic multiparticle quantum theory[8] (to be distinguished from quantum field theory). The essence of this assumption is that at every point in space in addition to any thermal radiation there is also background radiation which may be considered to be emitted by N sources which are incoherent with respect to each other and all other sources of radiation. If the effects of this radiation are not to violate the precepts of special relativity or the observed properties of space then it must be both isotropic and Lorentz invariant, otherwise it would make itself manifest in ways in which it does not. For example, the latter stipulation implies that an oscillator in equilibrium with the background would have the same average energy in every frame. That is, Lorentz invariance of a spectral energy density $\epsilon(\omega)$ means that the quantity of energy between two fixed values of ω (a and b, say) is the same in every equivalent frame, i.e.,

$$\int_a^b \epsilon(\omega)\,d^3k = \int_a^b \epsilon'(\omega')\,d^3k' = \int_a^b \epsilon'(\omega)\gamma(1-v/c)\,d^3k \quad [\gamma=(1-v^2/c^2)^{-1/2}]. \tag{1}$$

This expression is clearly invariant for the following function of ω:

$$\epsilon(\omega) = \text{const} \times \omega\,. \tag{2}$$

The constant scales the background and its effects and is empirically set to $\hbar/2$.

II. THEIMER'S DERIVATION

Theimer's derivation of the Planck blackbody spectrum exploits the following relationships among two incoherent classical radiation fields:

$$\langle(\delta\rho_i)^2\rangle = \langle\rho_i^2\rangle - \langle\rho_i\rangle^2 = \langle\rho_i\rangle^2\,, \tag{3}$$

$$\langle\rho\rangle_{\text{total}} = \langle\rho_1\rangle + \langle\rho_2\rangle\,, \tag{4}$$

$$\langle(\delta\rho)^2\rangle_{\text{total}} = \langle(\delta\rho_1)^2\rangle + \langle(\delta\rho_2)^2\rangle\,, \tag{5}$$

$$\langle\rho_1\rho_2\rangle = \langle\rho_1\rangle\langle\rho_2\rangle\,, \tag{6}$$

where ρ_i is the energy density of the ith field, $\langle\ \rangle$ indicates the expectation value, and $\delta\rho_i$ is the fluctuation of ρ_i. If now ρ_1 and ρ_2 are the energy densities of the background and thermal fields, respectively, then by manipulating (3)–(6) the following expression can be obtained:

$$\langle(\delta\rho_2)^2\rangle = \langle\rho_2\rangle^2 + 2\langle\rho_1\rangle\langle\rho_2\rangle\,. \tag{7}$$

Now, by invoking the fluctuation theorem for the thermal field, namely

$$\partial\langle\rho_2\rangle/\partial\beta = \langle(\delta\rho_2)^2\rangle \quad (\beta=-1/kT)\,, \tag{8}$$

one obtains the differential equation

$$\partial\langle\rho_2\rangle/\partial\beta = \langle\rho_2\rangle^2 + 2\langle\rho_1\rangle\langle\rho_2\rangle\,, \tag{9}$$

whose solution is, upon setting $\langle\rho_1\rangle = \hbar\omega/2$,

$$\langle\rho_2\rangle = \frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\,, \tag{10}$$

the Planck spectrum.

III. THE CRITICAL REEXAMINATION

The derivation of the Planck spectrum used in the critical reexamination proceeds as follows. First the differential equation for the first moment of the energy expectation is derived:

$$d\langle\rho\rangle/d\beta = \langle\rho^2\rangle - \langle\rho\rangle^2 = \langle\rho\rangle^2 \tag{11}$$

(the actual derivation was of a dimensionless quantity similar to a photon population and structurally equivalent to the energy density). By virtue of (3) this is the same as the fluctuation theorem. Then ρ is set equal to $\rho_1 + \rho_2$, to yield

$$d\langle\rho_2\rangle/d\beta = (\langle\rho_1\rangle^2 + 2\langle\rho_1\rangle\langle\rho_2\rangle + \langle\rho_2\rangle^2) \qquad (d\langle\rho_1\rangle/d\beta \equiv 0)\,, \quad (12)$$

where $\langle\rho_1\rangle^2$ is now to be purged from this equation by authority of a "renormalization postulate" which demands that divergent terms be disregarded. Thereafter, integrating (12) leads once again to the Planck spectrum.

This line of analysis is then continued by showing that the solutions to the "renormalized" differential equations for the higher-order moments are not consistent with the solution for the first moment. This inconsistency is the foundation for the conclusion regarding Theimer's classical derivation of Planck's spectrum to which the critical reexamination comes.

IV. COMMENTS AND CONCLUSION

Among the various differences between these two derivations particular attention is herein called to only one feature. In the first derivation the energy densities on both sides of the equation regarding fluctuations, Eq. (8), pertained to the thermal field, whereas in the second derivation the total field is intended in Eq. (11). This is tantamount to the assumption in each case that the subject field is described by the canonical ensemble formalism. This assumption seems to be on solid foundations with regard to the thermal field; however, with regard to the total field including the background it would seem not to be obviously correct, although it may be. The canonical ensemble formalism is derived on the basis of the assumption that the system under consideration is in constant thermal contact with a heat bath, a situation that obtains for radiation confined in a box. However, background radiation is not presumed to be so confined; therefore, it is not clear that the canonical ensemble formalism should describe it. In any case, the point here is only that these two derivations appear to be predicated on distinct and separate assumptions; therefore, inconsistency of either does not imply inconsistency of the other. Thus the flaw found in the critical reexamination pertains only to the derivation used therein. Theimer's classical derivation may also contain a flaw, perhaps even a fatal one, but it remains to be found.

Note added. The reply[9] to this comment both reveals that the rendering given in this comment of Theimer's derivation of the Planck spectrum is misleadingly imprecise and gives further insight into the differences between it and the derivation in the reexamination. In Theimer's derivation the renormalization is effected by defining a new term, the "renormalized" energy $\tilde{\rho}$, which *includes* the interference terms—nothing is left out. In contrast, in the other derivation renormalization is effected by *excluding* a term in the appropriate equation. Furthermore, in Theimer's derivation it is not assumed that $\langle\tilde{\rho}\rangle$, which satisfies Planck's spectrum, also satisfies classical statistics including the equation

$$\langle\rho^q\rangle = q!\,\langle\rho\rangle^q\,. \qquad \text{(N1)}$$

Thus, inconsistencies found in Ref. 1 using (N1) do not pertain to Theimer's derivation for this additional reason; in other words, the consistency of Theimer's derivation, although unverified, remains unchallenged.

It is the embodiment of inconsistency rather than of *ad hoc* assumptions that is the nemesis of theory. All theories are based upon hypotheses that are *ad hoc* from an internal perspective; it is by their results that they are judged. It is hoped that the ultimate value of Theimer's derivation of Planck's spectrum (or any self-consistent alternative) will be found in attempts to resolve questions on the interpretation of quantum mechanics rather than in the aesthetic appeal of its hypotheses.

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