Deep-inelastic lepton scattering in an SU(3) × U(1) gauge model

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(Received 9 February 1976)

Linear relations and sum rules for deep-inelastic lepton scattering are derived in the light-cone algebra approach from a set of weak, neutral, and electromagnetic currents based on an SU(3) × U(1) gauge model proposed by Schechter and Ueda.

Schechter and Ueda[1] have proposed a unified weak and electromagnetic gauge scheme based on the three-dimensional unitary group. Since a three-dimensional unitary group is taken to be fundamental, new leptons have to be introduced. As there seems to be some experimental evidence hinting at the existence of heavy leptons,[2] models of this type have an added appeal. In this note we derive some of the testable consequences of this model by analyzing the deep-inelastic structure functions due to the hadronic neutral currents. We briefly review the part of the model relevant for our calculation.

The electron and muon fields are combined into two column vectors:

$$\psi=\begin{pmatrix}\nu_e\\ e\\ e'\end{pmatrix},\quad \chi=\begin{pmatrix}\nu_\mu\\ \mu\\ \mu'\end{pmatrix}, \tag{1}$$

where e' and μ' are the new heavy negatively charged electron-type and muon-type leptons, respectively. Then the objects which transform as left-handed SU(3) triplets are $\frac{1}{2}(1+\gamma_5)\psi$ and $\frac{1}{2}(1+\gamma_5)\chi$. The lepton singlets are assigned to be the right-handed objects:

$$\begin{aligned}&\tfrac{1}{2}(1-\gamma_5)e\,,\quad \tfrac{1}{2}(1-\gamma_5)e'\,,\\ &\tfrac{1}{2}(1-\gamma_5)\mu\,,\quad \tfrac{1}{2}(1-\gamma_5)\mu'\,,\end{aligned} \tag{2}$$

and the left- and right-handed lepton currents are defined as

$$\begin{aligned}l^{b}_{a\alpha}&=i\bar\psi_b\gamma_\alpha(1+\gamma_5)\psi_a+i\bar\chi_b\gamma_\alpha(1+\gamma_5)\chi_a\,,\\ r^{b}_{a\alpha}&=i\bar\psi_b\gamma_\alpha(1-\gamma_5)\psi_a+i\bar\chi_b\gamma_\alpha(1-\gamma_5)\chi_a\,.\end{aligned} \tag{3}$$

The Yang-Mills gauge invariance under the group SU(3)×U(1) requires the introduction of nine vector gauge fields. The notation is as follows:

$$\begin{aligned}&\text{octet}\,,\quad (W^b_a)_\alpha \text{ with } (W^c_c)_\alpha=0\,,\\ &\text{singlet}\,,\quad D_\alpha\,.\end{aligned} \tag{4}$$

For the classification of gauge bosons according to the U-spin subgroup of SU(3) which singles out the 1 direction in unitary space, the abbreviation introduced is

$$\begin{aligned}F_\alpha&=-(\tfrac{3}{2})^{1/2}(W^1_1)_\alpha\,,\\ H_\alpha&=\frac{1}{\sqrt{2}}[(W^2_2)_\alpha-(W^3_3)_\alpha]\,,\end{aligned} \tag{5}$$

where F_α is a U-spin singlet while H_α is a member of a U-spin triplet. The photon field $\mathfrak{A}_\alpha$ cannot be identified as a pure member of the octet since the octet fields couple only to the left-handed currents while the photon must couple to the left plus right. Since D_α can couple to the right-handed currents, the simplest possibility is to let the fields D_α and F_α mix to give the photon and a massive neutral vector field Z_α:

$$\begin{pmatrix}F_\alpha\\ D_\alpha\end{pmatrix}=\begin{pmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}\mathfrak{A}_\alpha\\ Z_\alpha\end{pmatrix}, \tag{6}$$

where ϕ is the mixing angle. Since in the Lagrangian[1] H_α is the field which couples to the undesirable neutral currents, its mass must be extremely large.

The hadrons are constructed out of three fractionally charged quarks q_1, q_2, and q_3. The objects which transfer as a triplet under the left-handed SU(3) are the rotated quarks:

$$\tfrac{1}{2}(1+\gamma_5)U\begin{pmatrix}q_1\\ q_2\\ q_3\end{pmatrix}=\tfrac{1}{2}(1+\gamma_5)Q\,, \tag{7}$$

where U is the Cabibbo matrix

$$U=\begin{pmatrix}1&0&0\\ 0&\cos\theta&\sin\theta\\ 0&-\sin\theta&\cos\theta\end{pmatrix}. \tag{8}$$

The hadron singlets in this model are the right-handed objects:

$$\tfrac{1}{2}(1-\gamma_5)q_1\,,\quad \tfrac{1}{2}(1-\gamma_5)q_2\,,\quad \tfrac{1}{2}(1-\gamma_5)q_3\,, \tag{9}$$

and the hadronic currents generating the usual chiral U(3)×U(3) are

$$(j^l)^b_{a\alpha} = i\bar{q}_b\gamma_\alpha(1+\gamma_5)q_a\,,$$
$$(j^r)^b_{a\alpha} = i\bar{q}_b\gamma_\alpha(1-\gamma_5)q_a\,. \tag{10}$$

In terms of the familiar vector and axial-vector currents,

$$V^i_\alpha = \tfrac{1}{2}i\bar{Q}\gamma_\alpha\lambda^i Q\,,$$
$$A^i_\alpha = \tfrac{1}{2}i\bar{Q}\gamma_\alpha\gamma_5\lambda^i Q\,, \tag{11}$$

the hadron part of the electromagnetic current is

$$h^{\text{em}}_\alpha = V^3_\alpha + \frac{1}{\sqrt{3}}V^8_\alpha\,. \tag{12}$$

The electromagnetic interaction is

$$\mathcal{L}^{\text{em}} = |e|\,J^{\text{em}}_\alpha \mathcal{a}_\alpha\,, \tag{13}$$

where the electromagnetic current of hadrons and leptons is

$$J^{\text{em}}_\alpha = -\tfrac{1}{2}(l^2_{2\alpha} + r^2_{2\alpha} + l^3_{3\alpha} + r^3_{3\alpha}) + h^{\text{em}}_\alpha\,, \tag{14}$$

and the usual phenomenological weak interaction is

$$\mathcal{L}^W = \frac{G}{\sqrt{2}}J^{(+)}_\alpha J^{(-)}_\alpha\,, \tag{15}$$

with

$$J^{(+)}_\alpha = l^2_{1\alpha} + (V+A)^{1-i2}_\alpha$$

and (16)

$$J^{(-)}_\alpha = l^1_{2\alpha} + (V+A)^{1+i2}_\alpha\,.$$

The SU(3)×U(1)-invariant unified weak-electromagnetic interaction of currents and gauge fields is given by the unique interaction Lagrangian

$$\begin{aligned}\mathcal{L} = {} & g\,\text{Tr}(l_\alpha W_\alpha) - g' D_\alpha[\text{Tr}(l_\alpha) + \tfrac{3}{2}(r^2_{2\alpha} + r^3_{3\alpha})] \\ & + g\,\text{Tr}(U j^l_\alpha U^{-1} W_\alpha) \\ & + g' D_\alpha[(j^r)^1_{1\alpha} - \tfrac{1}{2}(j^r)^2_{2\alpha} - \tfrac{1}{2}(j^r)^3_{3\alpha}]\,,\end{aligned} \tag{17}$$

where g and g' are real coupling constants related to the electric charge and mixing angle ϕ by

$$g = \frac{-|e|}{\sqrt{6}\cos\phi}\,, \quad g' = (\tfrac{2}{3})^{1/2} g \cot\phi\,. \tag{18}$$

Equations (17) and (18) follow from requiring the electromagnetic part of the interaction that emerges to be the usual one.

To analyze the structure functions of deep-inelastic lepton-hadron scatterings we consider the relevant part of the second-order semileptonic weak interaction which follows from Eq. (17) and is given in terms of V and A currents by

$$\mathcal{L}^W_{\text{semileptonic}} = \frac{1}{2}\frac{g^2}{m^2(H)}J^H_\alpha(l^2_{2\alpha} - l^3_{3\alpha}) + \frac{1}{\sin^2\phi}\frac{g^2}{m^2(Z)}J^Z_\alpha[3\sin^2\phi\, l^{\text{em}}_\alpha + \text{Tr}(l_\alpha) + \tfrac{3}{2}(r^2_{2\alpha} + r^3_{3\alpha})]\,, \tag{19}$$

where

$$J^H_\alpha = (V^3 + A^3) - \sqrt{3}\,(V^8 + A^8)\,, \tag{20}$$

$$J^Z_\alpha = -\left[V^3 - A^3 + \frac{1}{\sqrt{3}}(V^8 - A^8) - \tfrac{1}{2}\Omega\left(V^3 + \frac{1}{\sqrt{3}}V^8\right)\right], \tag{21}$$

and

$$\Omega = 4\sin^2\phi\,.$$

The structure functions of deep-inelastic lepton-hadron scattering have been analyzed in the light-cone algebra approach for the Weinberg-Salam model with different quark-quartet schemes by Riazuddin and Fayyazuddin,[3] and Budny and Scharbach.[4] Following Ref. 4 we give a parallel analysis for the Schechter-Ueda model.

The reactions where the structure functions are measured are

(a) $e + N \to e + H$,

(b) $\nu + N \to \mu + H$,

(c) $\nu + N \to \nu + H$,

where N is a nucleon, and the final hadron states H and the nucleon spins are summed. The deep-inelastic structure functions and kinematic definitions used are those of Gross and Llewellyn Smith[5] with an obvious extension to the neutral structure functions $F^Z_j(x)$ and $F^H_j(x)$, where x is defined as the Bjorken variable $-q^2/(2P\cdot q)$. The structure functions obey the Callan-Gross relation $F_2 = 2xF_1$.

We use the usual bilocal SU(3) light-cone algebra generated by generalizing the nonets of physical vector and axial-vector currents to the bilocal operators $V^a(x,y)$ and $A^a(x,y)$ and by assuming that these obey the commutators at lightlike separations found by substituting the free-quark-field expressions

$$V^a_\alpha(x,y) \sim \tfrac{1}{2}i\bar{Q}(x)\lambda^a\gamma_\alpha Q(y)\,,$$
$$A^a_\alpha(x,y) \sim \tfrac{1}{2}i\bar{Q}(x)\lambda^a\gamma_\alpha\gamma_5 Q(y)\,. \tag{22}$$

Following Fritzsch and Gell-Mann[6] we define the matrix elements $A^c(p\cdot z)$ and $S^c(p\cdot z)$:

$$\begin{aligned}\langle p|[V^c_\alpha(y,x) + V^c_\alpha(x,y)]|p\rangle &= 2\tilde{S}^c(p\cdot z)p_\alpha + \text{trace terms}\,, \\ \langle p|[V^c_\alpha(y,x) - V^c_\alpha(x,y)]|p\rangle &= 2\tilde{A}^c(p\cdot z)p_\alpha + \text{trace terms}\,,\end{aligned} \tag{23}$$

where $z^2 = (x-y)^2 = 0$. The deep-inelastic structure functions are linear combinations of the functions $S^c(x)$ and $A^c(x)$, where

$$\tilde{S}^c(p\cdot z)=\int_{-1}^{1}dx\,e^{-ix(p\cdot z)}S^c(x)\,,$$
$$\tilde{A}^c(p\cdot z)=\int_{-1}^{1}dx\,e^{-ix(p\cdot z)}A^c(x)\,,$$
$$F_2^{ab}(x)=-\tfrac{1}{2}x(if^{abc}S^c-d^{abc}A^c)\,,\tag{24}$$
$$F_3^{ab}(x)=\tfrac{1}{2}(if^{abc}A^c-d^{abc}S^c)\,.$$

The deep-inelastic structure functions associated with each current (14), (16), (20), and (21) are given by commuting the current with its adjoint and calculating F_2 in (24) from the VV and AA commutators and F_3 from the VA and AV commutators.[6] Six of the coefficients from the nucleon matrix elements are nonzero. They are A^c and S^c for $c=0$, 3, and 8. The structure functions are given below:

$$\frac{F_2^e}{x}=\tfrac{2}{3}(\tfrac{2}{3})^{1/2}A^0+\tfrac{1}{3}A^3+\frac{1}{3\sqrt{3}}A^8\,,$$

$$\frac{F_2^H}{x}=4(\tfrac{2}{3})^{1/2}A_0-2\left(A^3+\frac{1}{\sqrt{3}}A^8\right)\,,$$

$$\frac{F_2^Z}{x}=\frac{1}{3}\left[4(\tfrac{2}{3})^{1/2}A^0+2A^3+\frac{2}{\sqrt{3}}A^8\right]-\frac{\Omega}{3}\left[2(\tfrac{2}{3})^{1/2}A^0+A^3+\frac{1}{\sqrt{3}}A^8\right]+\frac{\Omega^2}{12}\left[2(\tfrac{2}{3})^{1/2}A^0+A^3+\frac{1}{\sqrt{3}}A^8\right]\,,$$

$$F_3^H=-\left[4(\tfrac{2}{3})^{1/2}S^0-2\left(S^3+\frac{1}{\sqrt{3}}S^8\right)\right]\,,$$

$$F_3^Z=\frac{1}{3}\left[4(\tfrac{2}{3})^{1/2}S^0+2S^3+\frac{2}{\sqrt{3}}S^8\right]-\frac{\Omega}{3}\left[2(\tfrac{2}{3})^{1/2}S^0+S^3+\frac{1}{\sqrt{3}}S^8\right]\,,$$

$$\frac{F_2^\nu}{x}=2\left[(\tfrac{2}{3})^{1/2}A^0+\frac{A^8}{\sqrt{3}}+S^3\right]\,,$$

$$\frac{F_2^{\bar\nu}}{x}=2\left[(\tfrac{2}{3})^{1/2}A^0+\frac{A^8}{\sqrt{3}}-S^3\right]\,,$$

$$F_3^\nu=2\left[(\tfrac{2}{3})^{1/2}S^0+\frac{S^8}{\sqrt{3}}-A^3\right]\,,$$

$$F_3^{\bar\nu}=2\left[(\tfrac{2}{3})^{1/2}S^0+\frac{S^8}{\sqrt{3}}+A^3\right]\,.$$

Some of the linear relations that follow are

$$F_2^{\nu p+\bar\nu p}=F_2^{\nu n+\bar\nu n}\,,$$
$$F_3^{\nu p+\bar\nu p}=F_3^{\nu n+\bar\nu n}\,,$$
$$xF_3^{\nu p-\nu n}=-6F_2^{ep-en}\,,$$
$$xF_3^{\nu p-\bar\nu p-\nu n+\bar\nu n}=-12F_2^{ep-en}\,,$$
$$F_2^Z=(2-\Omega+\tfrac{1}{4}\Omega^2)F_2^e\,,$$
$$F_2^{Hp-Hn}=-6F_2^{ep-en}\,,$$

where $F^{ab\pm cd}=F^{ab}\pm F^{cd}$, and F^a denotes F^{ap} or F^{an}. It may be noted that A^3, A^8, etc. would depend on the quantum numbers of the hadrons, e.g., A^3 would have opposite signs for the proton and neutron states.

If we expand $S^c(x)$ in a Taylor series of bilocal operators

$$S^c(x)=S_1^c\,\delta(x)-\frac{1}{2!}S_3^c\delta''(x)+\cdots\,,\tag{25}$$

the moments of the structure functions are given by the expansion coefficients of (25). From the definition of S_1^c, where $c=0$, 3, and 8,

$$S_1^c\langle p|\,p\rangle=2\langle p|\int d^3x\,V_0^c(x,x)|\,p\rangle\,.$$

The baryon current in this model can be written as

$$B_\alpha=(\tfrac{2}{3})^{1/2}V_\alpha^0\,.$$

Hence, e.g.,

$$S_1^0\langle p|\,p\rangle=2\langle p|\int d^3x\,V^0|\,p\rangle=\sqrt{6}\,B\,,$$

where B is the baryon number of the target. Then, since

$$F_3^{\nu+\bar\nu}=4\left[(\tfrac{2}{3})^{1/2}S^0+\frac{S^8}{\sqrt{3}}\right]\,,$$

one has the sum rules

$$\int_0^1 dx\,F_3^{\nu+\bar\nu}=4(2B+\tfrac{1}{3}Y)\,,$$
$$\int_0^1\frac{dx}{x}\,F_2^{\nu-\bar\nu}=4T_3\,,$$
$$\int_0^1 dx\,F_3^Z=\tfrac{1}{3}(8B+2T_3-\tfrac{2}{3}Y)-\tfrac{1}{3}\Omega(4B+T_3+\tfrac{1}{3}Y)\,,$$

where T and Y are the isotopic spin and hypercharge of the target, respectively.

Other conclusive tests result from Nachtmann's SU(2) positivity conditions.[7] They imply

$$(\tfrac{2}{3})^{1/2}A^0+\frac{1}{\sqrt{3}}A^8\pm A^3=u_\pm\geq 0\,,$$

$$(\tfrac{2}{3})^{1/2}A^0-\frac{2}{\sqrt{3}}A^8=v\geq 0\,.$$

Hence

$$F_2^{ep} = \frac{1}{9}(4u_+ + u_- + v),$$

$$F_2^{en} = \frac{1}{9}(u_+ + 4u_- + v),$$

$$F_2^{Zp} = \frac{1}{9}(2 - \Omega + \frac{1}{4}\Omega^2)(4u_+ + u_- + v),$$

$$F_2^{Hp} = 2(u_- + v).$$

Therefore, we obtain

$$\frac{1}{4} \leq F_2^{en}/F_2^{ep} \leq 4.$$

This coincides with the result obtained for the SU(2)× U(1) model with the fractionally charged quartet-quark scheme of Ref. 4.

The structure function $F_2^{\nu\nu p}$ is associated with the current J_α^Z only, and we get the bound

$$1 \leq F_2^{\nu\nu p}/F_2^{ep} \leq 2,$$

whereas another SU(3)× U(1)[8] model with integrally charged quarks predicts[9]

$$F_2^{\nu\nu p}/F_2^{ep} \geq \frac{4}{9}.$$

This may also be compared with the result obtained for the fractionally charged quartet-quark scheme in Refs. 3 and 4.

$$F_2^{\nu\nu p}/F_2^{ep} \geq \frac{9}{16}.$$

The bounds we obtain are interesting predictions in the Schechter-Ueda model in the light-cone algebra approach with SU(2) positivity conditions. It may be feasible to test them in the future.

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