Intercept of the Pomeron

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We show that in Reggeon field theory the intercept of the interacting Pomeron must be less than or equal to 1. The intercept is equal to 1 only when the bare intercept has a critical value. When the bare intercept exceeds the critical value a Pomeron field operator acquires a vacuum expectation value, and a discrete symmetry of the Lagrangian is spontaneously broken. It turns out that symmetry breaking is unacceptable even though $\alpha(0) < 1$. Therefore, Reggeon field theory appears to be valid only when the bare intercept does not exceed the critical value.

I. INTRODUCTION

Reggeon field theory (RFT) permits one to calculate Regge-cut corrections to Regge-pole exchange.[1] Such a procedure is essential in the case of the Pomeranchuk singularity which has an intercept, $\alpha(0)$, either at or very close to 1. The Pomeron has been studied extensively in RFT for $\alpha(0) = 1$.[1] However, the physical intercept of the Pomeron is exactly equal to 1 only when the bare intercept, α_0, takes on a certain critical value, α_{0c}. In this article we study the behavior of RFT for general values of α_0.

We are interested in studying the behavior of the theory as the bare intercept is varied for two reasons. First, because of the small magnitude of the triple-Pomeron coupling constant, we are not in the asymptotic domain at present accelerator energies.[2] As a result, we can conclude from the near constancy of cross sections at present energies that α_0 is near its critical value, but we cannot conclude that $\alpha_0 = \alpha_{0c}$. Second, we would like to explore the range of validity of the theory. The most obvious potential sources of difficulty are the constraints of direct-channel (s-channel) unitarity, such as the Froissart bound. Although cross-channel unitarity is built into RFT, s-channel unitarity is not, and one must verify *a posteriori* that it is not violated. For $\alpha_0 = \alpha_{0c}$, the theory has been put to a number of tests and it has passed all of them.[3] In particular, Cardy and Sugar have argued on general grounds that the Froissart bound is satisfied in this case.[4] A central result of this paper is that $\alpha(0) < 1$ for $\alpha_0 \neq \alpha_{0c}$, so there is no violation of the Froissart bound for any value of the input parameters. However, the situation is less satisfactory than this statement implies. As we shall see, considerations other than s-channel unitarity restrict the range of α_0 to $\alpha_0 \leq \alpha_{0c}$.[5]

The Lagrangian we study is that of a self-interacting Pomeron with only a triple-Pomeron coupling. The triple-Pomeron coupling is known to be sufficient for the study of the infrared ($J \approx 1, t \approx 0$) behavior of the theory when $\alpha(0)$ is near 1.[6] The Lagrangian density is

$$\mathcal{L} = -\tfrac{1}{2}\bar{\psi}\frac{\overleftrightarrow{\partial}}{\partial y}\psi - \alpha_0'\vec{\nabla}\bar{\psi}\cdot\vec{\nabla}\psi - \Delta_0\bar{\psi}\psi - \frac{ir_0}{2}(\bar{\psi}^2\psi + \bar{\psi}\psi^2). \tag{1}$$

The field $\psi(\vec{x}, y)$ destroys a Pomeron with impact parameter $\vec{x}$ and rapidity y. $\bar{\psi}(\vec{x}, y)$ is the field canonically conjugate to $\psi(\vec{x}, y)$. For the free theory $\bar{\psi} = \psi^\dagger$, but since the interaction term is anti-Hermitian, this property is not maintained in the presence of interactions. Later we will encounter some calculations in which the distinction between $\bar{\psi}$ and $\psi^\dagger$ is obvious, and we have set up our notation accordingly. The impact parameter, $\vec{x}$, is a two-dimensional vector which is conjugate to the transverse momentum of the Pomeron, $\vec{k}$. (The invariant momentum transfer is $t = -\vec{k}^2$.) The rapidity is conjugate to the Pomeron "energy", E, which is related to the angular momentum by $E = 1 - J$. $\Delta_0 = 1 - \alpha_0$ is the bare-intercept gap, α_0' is the slope of the bare Pomeron, and r_0 is the bare triple-Pomeron coupling constant.

We shall focus our attention on the complete inverse Pomeron propagator $\Gamma^{1,1}(E,k^2)$. It is a key Green's function because it exhibits the spectrum of the theory, including the renormalized Pomeron intercept $\alpha(0)=1-\Delta$. In addition, the sum of graphs exhibited in Fig. 1 gives the dominant contribution to the total cross section at high energies, and this contribution depends on the Pomeron propagator alone. Our objective is to calculate $\Gamma^{1,1}$ for all values of α_0 and determine the range of values taken on by $\alpha(0)$. For $\alpha_0 \ll \alpha_{0c}$ we can use perturbation theory and see directly that $\alpha(0) \ll 1$. As α_0 increases, so does $\alpha(0)$. By definition $\alpha(0)$ reaches 1 when $\alpha_0=\alpha_{0c}$. When α_0 is slightly less than α_{0c}, the renormalization group can be used to obtain $\Gamma^{1,1}$ in the infrared domain. In Sec. II we carry out this calculation and derive the scaling law

$$\Gamma^{1,1}(E,k^2,\eta_0) \underset{E,k^2,\eta_0\to 0}{\sim} C_0\eta_0^{(1-\gamma)/(1-\kappa)} \times F\left(C_1\frac{E}{\eta_0^{1/(1-\kappa)}}, C_2\frac{\alpha_0' k^2}{\eta_0^{z/(1-\kappa)}}\right), \tag{2}$$

where $\eta_0 \equiv \alpha_{0c}-\alpha_0=\Delta_0-\Delta_{0c}$. γ and z are the critical exponents which control the infrared behavior when $\alpha_0=\alpha_{0c}$.[7,8] κ is a new critical exponent which equals $\epsilon/12$ in the lowest order of the ϵ expansion. ($\epsilon=4-D$, where D is the number of transverse dimensions. Of course, physically $D=2$.) The parameters in the Lagrangian affect the constants C_0, C_1, and C_2, but not the form of F. F is a universal function which, like the critical exponents, depends only on ϵ. Equation (2) is a generalization of the scaling law previously obtained for $\alpha_0=\alpha_{0c}$,[7,8] and reduces to it for this value of α_0. That is,

$$\Gamma^{1,1}(E,k^2,\eta_0=0) \underset{E,k^2\to 0}{\sim} C_0'(-E)^{1-\gamma}F'\left(C_1'\frac{\alpha_0' k^2}{(-E)^z}\right). \tag{3}$$

In Sec. II we calculate F to first order in ϵ and find that $\Gamma^{1,1}$ has a singularity at $\alpha_0=\alpha_{0c}$ as one would expect from the form of Eq. (2). Since we have no instruction on how to pass this singularity, it is an obstacle in the way of our continuation to $\alpha_0>\alpha_{0c}$. Such a singularity is not an unusual occurrence, and it has a simple physical origin which

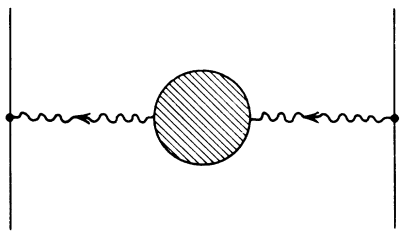

FIG. 1. Pomeron graphs giving the leading contribution to the total cross section at high energy.

is well known for Euclidean ϕ^4 field theory. This theory has been studied extensively in connection with the problem of second-order phase transitions,[9] and we pause to discuss its properties. The Lagrangian density is

$$\mathcal{L}=\tfrac{1}{2}\vec{\nabla}\phi\cdot\vec{\nabla}\phi+\tfrac{1}{2}m_0^2\phi^2+\frac{\lambda_0}{4!}\phi^4, \tag{4}$$

where $\phi(\vec{x})$ is a real scalar field, m_0 is the bare mass, and λ_0 is the bare coupling constant. m_0^2 is ordinarily taken to vary linearly with the temperature, T. The phase transition occurs when the renormalized mass, m, vanishes. This only happens for a certain critical value of the temperature, T_c, or of the bare mass, m_{0c}. For $T>T_c$ ($m_0^2>m_{0c}^2$) the system has a nondegenerate ground state and $m^2>0$. For $T<T_c$ there are two degenerate ground states. In one the field ϕ has a positive vacuum expectation value and in the other a negative vacuum expectation value. For example, if the system undergoing the phase transition is a ferromagnet in which spins can only point up or down, the vacuum expectation value of ϕ is the spontaneous magnetization. One cannot study the behavior of the theory for $T<T_c$ merely by continuing in the temperature. The indeterminancy in the ground state gives rise to singularities in the Green's functions at $T=T_c$ which prevent such a continuation. However, if an external magnetic field is present, the degeneracy of the ground state is removed, as are the singularities at $T=T_c$. One can then freely continue T below its critical value. If the external magnetic field is then turned off, the sign of the vacuum expectation value of ϕ (the spontaneous magnetization) depends upon the sign of the external magnetic field during the variation of T. This means that the external field performs the function of defining a direction in space and selecting one of the two possible ground states of the system. The external field is indispensable both physically and mathematically.

In this paper we shall verify that Reggeon field theory exhibits all the features found in Euclidean ϕ^4 field theory. This means, first of all, that we are dealing with a critical phenomenon when we study the problem of the Pomeron in RFT. The analogy with the phase-transition problem was not fully brought out in Refs. 7 and 8 because as long as $\alpha(0)=1$ one does not move off the critical point. However, Eq. (2) is an example of a generalization of the scaling laws to a neighborhood of the critical temperature. (η_0 is the analog of $T-T_c$.) In the present paper these extended scaling laws will be derived together with the full complement of critical exponents analogous to those found in the problem of the ferromagnet. We shall find that the

analogy with critical behavior is complete.

We now return to the problem of continuing $\Gamma^{1,1}$ to $\alpha_0 > \alpha_{0c}$. We see that a generalization of our program is required; we must introduce sources S and $\bar{S}$ coupled to $\bar{\psi}$ and ψ, respectively. These sources have no direct physical meaning in RFT, and all physical amplitudes are taken with S and $\bar{S}$ zero. On the other hand, they must be nonzero for the continuation through $\alpha_0 = \alpha_{0c}$. They serve the purpose of selecting one of two possible ground states for the system when $\alpha_0 > \alpha_{0c}$. In one of the ground states $\langle\psi\rangle \neq 0$ and $\langle\bar{\psi}\rangle = 0$, and in the other $\langle\bar{\psi}\rangle \neq 0$ and $\langle\psi\rangle = 0$. Here we must recall that $\bar{\psi} \neq \psi^\dagger$ ($\langle\psi^\dagger\rangle = \langle\psi\rangle^*$.) When ψ or $\bar{\psi}$ have nonzero vacuum expectation values we must distinguish between the Pomeron Green's functions, $G^{n,m}$, and correlation functions, $\tilde{G}^{n,m}$, which have all terms independent of $\vec{x}$ and y subtracted out. When $\langle\psi\rangle$ and $\langle\bar{\psi}\rangle$ are nonzero the Green's functions acquire additional terms which are independent of $\vec{x}$ and y. If the partial-wave amplitude were constructed by coupling the external particles to the Green's functions, then in momentum space the amplitude would have δ functions which violate its known analyticity properties. In order to obtain a satisfactory partial-wave amplitude it is necessary to couple the external particles to the correlation functions. For $S = \bar{S} = 0$ and $\alpha_0 \leq \alpha_{0c}$, $\langle\psi\rangle = \langle\bar{\psi}\rangle = 0$ and $G^{n,m} = \tilde{G}^{n,m}$ and the distinction disappears.

When $\alpha_0 > \alpha_{0c}$ we find a unique Pomeron correlation function $\tilde{\Gamma}^{1,1}$. The renormalized intercept is less than 1 so there is no violation of the Froissart bound. However, for $\alpha_0 > \alpha_{0c}$ the symmetry of the theory under the substitution $\psi(\vec{x}, y) \rightarrow \bar{\psi}(-\vec{x}, -y)$ is spontaneously broken. We call this rapidity reversal invariance, and a simple argument shows that its breaking is a disaster.

The action $\int d^D x\, dy\, \mathcal{L}$ is invariant under rapidity reversal with $\mathcal{L}$ given by Eq. (1). This immediately leads to a symmetry of the complete correlation functions in momentum space,

$$\tilde{G}^{n,m}(E_i, k_i, \langle\psi\rangle, \langle\bar{\psi}\rangle) = \tilde{G}^{m,n}(E_i, k_i \langle\bar{\psi}\rangle, \langle\psi\rangle), \qquad (5)$$

where n, m stands for n Pomerons in and m Pomerons out. For $\alpha_0 < \alpha_{0c}$, where $\langle\psi\rangle = \langle\bar{\psi}\rangle = 0$, the Green's functions are symmetric, and when we attach them to fast particles A and B, the partial-wave amplitude is independent of whether A is the source of the Pomerons and B the sink or vice versa. On the other hand, for $\alpha_0 > \alpha_{0c}$ $\langle\psi\rangle \neq \langle\bar{\psi}\rangle$, so Eq. (5) no longer predicts symmetry, and in Sec. III we shall exhibit diagrams which show that the Green's functions are in general asymmetric. We have been unable to find a way of producing a symmetric partial-wave amplitude when the vacuum is asymmetric as our calculation shows it to be. Of course, it is nonsense for a partial-wave amplitude to change when projectile and target are exchanged, so we apparently require that $\alpha_0 \leq \alpha_{0c}$ in order to have a sensible theory. Perhaps it is worth stressing that this restriction is not related to s-channel unitarity in any obvious way.

In Sec. II we carry out the calculations which we have just described. When $\eta_0 = \alpha_{0c} - \alpha_0$ is small we can use the renormalization group to obtain all quantities of interest. We also use the ϵ expansion, and while we think that the ϵ expansion is quantitatively dubious at $\epsilon = 2$, we also think it is qualitatively correct. In the text we give direct calculations of the Pomeron propagator and of the vacuum expectation values of the fields ψ and $\bar{\psi}$. In an appendix we give an alternative approach to the problem of spontaneous symmetry breaking by constructing the effective action to first order in ϵ.

We also discuss the case $\alpha_0 \ll \alpha_{0c}$. Here one can use perturbation theory directly because quantum corrections to the mean field approximation are small.

In Sec. III we summarize what we have learned and fill out the discussion of the analogy with a magnetic system near its critical point. We also discuss the energy scales which are present in Reggeon field. For $\eta_0 \neq 0$ the renormalized Pomeron pole will dominate the scattering amplitude at the very highest energies. However, for small values of η_0 there is a range of energies in which all of the cuts must be taken into account. The scattering amplitude then has a scaling form similar to that found for $\eta_0 = 0$. At even lower energies the bare Pomeron dominates.

Our results differ from those reported for RFT with a triple-Pomeron coupling and $\alpha_0 > \alpha_{0c}$.[10] In Ref. 10 a class of leading graphs was summed, while in the present paper the use of the renormalization group allows us to avoid such an uncontrolled approximation. We are confident of our results when ϵ is small. Abarbanel originally pointed out that RFT can acquire a vacuum expectation value for $\alpha_0 > \alpha_{0c}$, thereby ensuring that the gap in the expectation spectrum remains non-negative.[11]

The calculations presented in this paper were summarized in our recent report.[12] The present paper contains much more detail and reaches different conclusions. We still believe that spontaneous symmetry breaking imposes $\alpha(0) < 1$, but that it introduces the other problems we have mentioned.

II. POMERON CORRELATION FUNCTIONS

Our starting point is the path-integral representation of the Pomeron Green's functions. In the presence of sources

$$G^{n,m}(\vec{x}_1,y_1,\ldots,\vec{x}_n,y_n;\vec{x}_1',y_1',\ldots,\vec{x}_m',y_m';S,\bar{S})$$
$$=\langle 0|T[\bar{\psi}(\vec{x}_1,y_1)\cdots\bar{\psi}(\vec{x}_n,y_n)\psi(\vec{x}_1',y_1')\cdots\psi(\vec{x}_m',y_m')]|0\rangle$$
$$=Z(S,\bar{S})^{-1}\int\delta\psi\,\delta\bar{\psi}\,\bar{\psi}(\vec{x}_1,y_1)\cdots\bar{\psi}(\vec{x}_n,y_n)\psi(\vec{x}_1',y_1')\cdots\psi(\vec{x}_m',y_m')\exp\left[\int d^Dx\,dy\,(\mathfrak{L}+S\bar{\psi}+\bar{S}\psi)\right], \quad (6)$$

where $\mathfrak{L}$ is given in Eq. (1) and

$$Z(S,\bar{S})=\int\delta\psi\,\delta\bar{\psi}\exp\left[\int d^Dx\,dy\,(\mathfrak{L}+S\bar{\psi}+\bar{S}\psi)\right]. \quad (7)$$

We have chosen to work in terms of the rapidity variable, y, rather than the "time" variable, $t=iy$, introduced in Ref. 8 because with this choice the functional integrals in Eqs. (6) and (7) are well defined for $\Delta_0>0$. The free fields can be written in the form $\psi=\phi+i\chi$, $\bar{\psi}=\phi-i\chi$. Then for $\Delta_0>0$ the functional integrals are over all real values of ϕ and χ. In order to continue to $\Delta_0<0$ the contour of the ϕ integration must be distorted into the complex plane as is discussed in Ref. 4.

Our first task is to rewrite Eq. (6) in such a way that $G^{n,m}$ will have a well-behaved perturbation expansion in powers of r_0. To this end we introduce new field variables

$$\psi_c(\vec{x},y)=\psi(\vec{x},y)-v, \quad (8a)$$

$$\bar{\psi}_c(\vec{x},y)=\bar{\psi}(\vec{x},y)-\bar{v}, \quad (8b)$$

where v and $\bar{v}$ are values of ψ and $\bar{\psi}$ at which the classical potential

$$U_c(\psi,\bar{\psi})=\eta_0\bar{\psi}\psi+\tfrac{1}{2}ir_0\bar{\psi}\psi(\bar{\psi}+\psi)-S\bar{\psi}-\bar{S}\psi \quad (9)$$

is stationary. In defining the classical potential we have written the terms in the Lagrangian density proportional to $\bar{\psi}\psi$ in the form $\Delta_0\bar{\psi}\psi=\eta_0\bar{\psi}\psi+\Delta_{0c}\bar{\psi}\psi$. $\Delta_{0c}\bar{\psi}\psi$ does not appear in U_c because it plays the role of an intercept renormalization counterterm. Δ_{0c} is determined by the requirement that the inverse Pomeron propagator vanish at $E=k^2=0$ when $\eta_0=S=\bar{S}=0$. v and $\bar{v}$ are the classical approximations to $\langle\psi\rangle$ and $\langle\bar{\psi}\rangle$. Since we do not wish to consider the possibility of spontaneously breaking the invariance of the theory under translations in $\vec{x}$ and y, we shall assume that v and $\bar{v}$ are independent of these variables. It will therefore be sufficient to take S and $\bar{S}$ to be constants. v and $\bar{v}$ are solutions of the equations

$$\partial U_c/\partial\psi|_{\psi=v,\bar{\psi}=\bar{v}}=\eta_0\bar{v}+ir_0\bar{v}v+\tfrac{1}{2}ir_0\bar{v}^2-\bar{S}=0, \quad (10)$$

$$\partial U_c/\partial\bar{\psi}|_{\psi=v,\bar{\psi}=\bar{v}}=\eta_0 v+ir_0\bar{v}v+\tfrac{1}{2}ir_0v^2-S=0. \quad (11)$$

In terms of these new variables the classical potential becomes

$$U_c=\delta_0\bar{\psi}_c\psi_c+\tfrac{1}{2}ir_0\bar{v}\psi_c^2+\tfrac{1}{2}ir_0v\bar{\psi}_c^2+\tfrac{1}{2}ir_0(\bar{\psi}_c^2\psi_c+\bar{\psi}_c\psi_c^2), \quad (12)$$

where

$$\delta_0=\eta_0+ir_0(v+\bar{v}) \quad (13)$$

is the bare-intercept gap for the quanta of the field $\bar{\psi}_c$. In Eq. (11) we have omitted an irrelevant term which is independent of ψ_c and $\bar{\psi}_c$.

Before considering the solutions of Eqs. (10) and (11) for general values of S and $\bar{S}$, let us discuss some special cases which are of particular importance. The Pomeron Green's functions which enter the physical scattering amplitude are to be evaluated with $S=\bar{S}=0$. In this case the four stationary points of U_c are as follows:

$$\begin{aligned}&\text{I.}\quad v=\bar{v}=0,\quad \delta_0=\eta_0;\\ &\text{II.}\quad v=\bar{v}=2i\eta_0/3r_0,\quad \delta_0=-\eta_0/3;\\ &\text{IIIa.}\quad v=0,\quad \bar{v}=2i\eta_0/r_0,\quad \delta_0=-\eta_0;\\ &\text{IIIb.}\quad v=2i\eta_0/r_0,\quad \bar{v}=0,\quad \delta_0=-\eta_0.\end{aligned} \quad (14)$$

For $\eta_0>0$ only solution I leads to an acceptable perturbation expansion of the Green's functions. In the other three cases the individual Feynman diagrams have unphysical singularities in the angular momentum plane: singularities which are arbitrarily far to the right of $J=1$. If one attempts to develop the perturbation series by expanding the integral of Eqs. (6) and (7) in power series in r_0, then the resulting functional integrals are not well defined.

For $\eta_0<0$ it is solutions IIIa and IIIb which lead to acceptable perturbation expansions. Solution IIIa will have a term in U_c proportional to ψ_c^2 and solution IIIb one proporational to $\bar{\psi}_c^2$. These terms can be included in the interaction Lagrangian since there can only be a finite number of ψ_c^2 or $\bar{\psi}_c^2$ vertices in any graph of finite order in r_0. (From Fig. 2 it is clear that each additional ψ_c^2 or $\bar{\psi}_c^2$ vertex in a diagram requires two additional triple-Pomeron vertices.) As a result, for both IIIa and IIIb the bare propagator for the ψ_c field will be

$$G_0^{1,1}(E,k^2)=i[E-\alpha_0'k^2-(-\eta_0)+i\epsilon]^{-1}. \quad (15)$$

For $\eta_0<0$ solution I leads to a perturbation expansion with unphysical singularities in the angular momentum plane and ill-defined functional integrals. The same is true for solutions II despite the fact that $\delta_0>0$. It gives rise to terms in U_c propor-

tional to both ψ_c^2 and $\bar{\psi}_c^2$, and it is clear from Fig. 3 that one must work to all orders in ψ_c^2 and $\bar{\psi}_c^2$ at any finite order in r_0. However, including the ψ_c^2 and $\bar{\psi}_c^2$ terms in the free Lagrangian leads to all of the difficulties found when $\delta_0 < 0$. This is most easily seen by working in terms of the real fields, ϕ_c and χ_c ($\psi_c = \phi_c + i\chi_c$, $\bar{\psi}_c = \phi_c - i\chi_c$). The term in the classical potential quadratic in the fields is $3\delta_0\phi_c^2 - \delta_0\chi_c^2$, so a series expansion of the Green's functions in powers of r_0 will again lead to ill-defined functional integrals.

In order to simplify the algebra in our later calculations it will be convenient to keep either $\bar{S}$ or S fixed at zero. Let us start with $\bar{S} = 0$. For $\eta_0 > 0$ we must choose the solution which goes over into I for $S \to 0$. That is,

$$v = [-\eta_0 + (\eta_0^2 + 2ir_0S)^{1/2}]/ir_0, \quad \bar{v} = 0,$$
$$\delta_0 = (\eta_0^2 + 2ir_0S)^{1/2}. \tag{16}$$

We start on the sheet of $(\eta_0^2 + 2ir_0S)^{1/2}$ such that at $S = 0$, $\delta_0 = \eta_0 > 0$. Then, taking S to be positive, the branch cuts can be drawn as in Fig. 4. If we now continue to negative values of η_0 without passing through the branch cuts, then upon setting $S = 0$ we arrive at solution IIIb with $\delta_0 = -\eta_0 > 0$. It is now clear why the continuation to negative η_0 must be made for a nonzero value of S. In order to have a well-defined perturbation expansion at each step we must pass between the two branch points of δ_0 which coalesce at $S = 0$.

For some purposes it is convenient to take δ_0 to be the independent variable and write

$$\eta_0 = (\delta_0^2 - 2ir_0S)^{1/2}. \tag{17}$$

Then starting on the sheet of the S plane for which $\eta_0 = \delta_0$ at $S = 0$, we can continue in S through the branch cut in η_0, and arrive back at $S = 0$ with $\eta_0 = -\delta_0$. From Eqs. (12), (16), and (17) we see that for $\bar{S} = 0$, the classical potential can be written in the form

$$U_c = \delta_0\bar{\psi}_c\psi_c + \tfrac{1}{2}[\delta_0 - (\delta_0^2 - 2ir_0S)^{1/2}]\bar{\psi}_c^2 + \tfrac{1}{2}ir_0(\bar{\psi}_c^2\psi_c + \bar{\psi}_c\psi_c^2). \tag{18}$$

For $S = 0$ one can proceed in exactly the same manner. In this case the appropriate solution is

$$v = 0, \quad \bar{v} = [-\eta_0 + (\eta_0^2 + 2ir_0\bar{S})^{1/2}]/ir_0,$$
$$\delta_0 = (\eta_0^2 + 2ir_0\bar{S})^{1/2} \tag{19}$$

and

$$U_c = \delta_0\bar{\psi}_c\psi_c + \tfrac{1}{2}[\delta_0 - (\delta_0^2 - 2ir_0\bar{S})^{1/2}]\psi_c^2 + \tfrac{1}{2}ir_0(\bar{\psi}_c^2\psi_c + \bar{\psi}_c\psi_c^2). \tag{20}$$

Starting with solution I for $\eta_0 > 0$, one arrives at IIIa for $\eta_0 < 0$.

In order to study the continuation in η_0 for general values of S and $\bar{S}$, we introduce the variables

$$v_\pm = \tfrac{1}{2}(v \pm \bar{v}), \quad S_\pm = \tfrac{1}{2}(S \pm \bar{S}) \tag{21}$$

and rewrite Eqs. (10) and (11) in the form

$$v_-(\eta_0 + ir_0v_+) = S_-, \tag{22}$$

$$v_+(\eta_0 + ir_0v_+)^3 + \tfrac{1}{2}ir_0v_+^2(\eta_0 + ir_0v_+)^2 - S_+(\eta_0 + ir_0v_+)^2 = \tfrac{1}{2}ir_0S_-^2. \tag{23}$$

The solution to Eq. (23) which reduces to I for $S = \bar{S} = 0$ and $\eta_0 > 0$ is[13]

$$ir_0v_+ = -\tfrac{2}{3}\eta_0 + \tfrac{1}{2}e + [(\tfrac{1}{3}\eta_0 + \tfrac{1}{2}e)^2 - \tfrac{1}{2}y + f]^{1/2}, \tag{24}$$

where

$$e = (\tfrac{1}{9}\eta_0^2 + \tfrac{2}{3}ir_0S_+ + y)^{1/2}, \tag{25}$$

$$f = -\tfrac{1}{3}\eta_0 y/e, \tag{26}$$

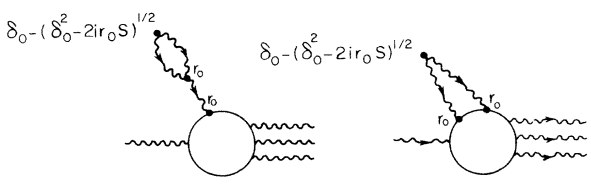

FIG. 2. Pomeron graphs with a $\bar{\psi}^2$ vertex, showing that it must be accompanied by two triple-Regge vertices.

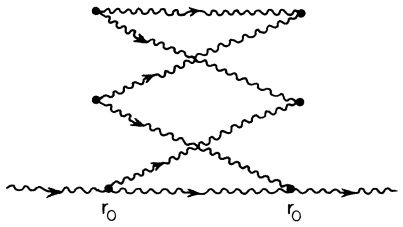

FIG. 3. Pomeron graph illustrating that one must work to all orders in ψ^2 and $\bar{\psi}^2$ when both are present.

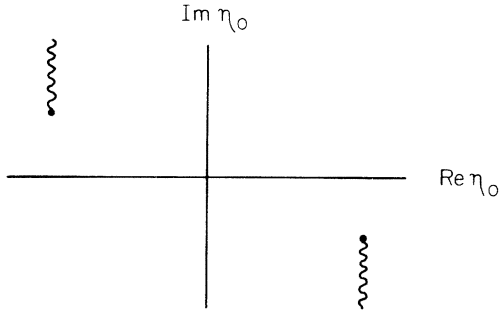

FIG. 4. Singularities of v and δ_0 in the η_0 plane when $S \neq 0$, $\bar{S} = 0$.

$$y=(-\tfrac{1}{2}q+\lambda)^{1/3}+(-\tfrac{1}{2}q-\lambda)^{1/3}+\tfrac{1}{9}(\eta_0^2-2ir_0S_+), \tag{27}$$

$$\lambda=(\tfrac{1}{4}q^2+\tfrac{1}{27}p^3)^{1/2}, \tag{28}$$

$$q=-\tfrac{8}{27}r_0^2S_-^2(\eta_0^2+2ir_0S_+)-\frac{2}{(27)^2}(\eta_0^2-2ir_0S_+)^3, \tag{29}$$

$$p=-\frac{4r_0^2}{3}S_-^2-\tfrac{1}{27}(\eta_0^2-2ir_0S_+)^2. \tag{30}$$

All of the branch cuts in η_0 are to be drawn so that they run parallel to the imaginary axis and do not intersect the real axis. (See, for example, Fig. 4.) With this convention e and y are even functions of η_0 for η_0 real, and f is an odd function. As a result, if we continue to negative values of η_0 without passing through any of the cuts, we find

$$\begin{aligned} v_+ &\underset{S_+,\,S_-\to 0}{\sim} i\eta_0/r_0-S_-/\eta_0 \quad (S_->0),\\ v_+ &\underset{S_+,\,S_-\to 0}{\sim} i\eta_0/r_0+S_-/\eta_0 \quad (S_-<0). \end{aligned} \tag{31}$$

Making use of Eq. (22) we see that we arrive at solution IIIa if the continuation is made with $S_-<0$ and at solution IIIb if it is made with $S_->0$. Since the result depends only on the sign of S_-, it will be sufficient to consider the two special cases $S=0$ and $\bar{S}=0$. This will lead to a considerable simplification in our subsequent calculations.

For $S_-=0$ the branch points in λ coalesce, and one can no longer continue between them. If one insists on decreasing η_0 through zero for $S_-=0$, one arrives at solution II after setting $S_+=0$. This is just as unacceptable as attempting to perform the continuation with both S_+ and S_- equal to 0.

We are finally in a position to begin our study of the Pomeron correlation functions. We shall generally work with the single-particle irreducible proper vertex functions $\Gamma^{n,m}$ and $\tilde{\Gamma}^{n,m}$ constructed from $G^{n,m}$ and $\tilde{G}^{n,m}$, respectively. Notice that for the classical potential of Eq. (18) $\langle\bar{\psi}\rangle=\langle\bar{\psi}_c\rangle=0$ to all orders in perturbation theory, while for the potential of Eq. (20) $\langle\psi\rangle=\langle\psi_c\rangle=0$. In either case

$$\Gamma^{1,1}=\tilde{\Gamma}^{1,1}. \tag{32}$$

For general values of n,m the Green's functions differ from the correlation functions only in that the $G^{n,m}$ contain disconnected graphs in which one or more external particles disappear into the vacuum, while the $\tilde{G}^{n,m}$ do not. It is possible to calculate the correlation functions perturbatively without evaluating $\langle\psi\rangle$ or $\langle\bar{\psi}\rangle$. One need only calculate the Green's functions of the fields $\psi_c,\bar{\psi}_c$ omitting all disconnected graphs.

Let us imagine making a perturbation expansion of the correlation functions. For $S=\bar{S}=0$ and $\eta_0>0$ each successive term in the perturbation series will have an extra factor of r_0^2, three extra propagators, and one extra loop integral over E and $\vec{k}$. Thus each term will be down from the preceding one by a factor of $r_0^2/{\alpha_0'}^{D/2}\delta_0^{2-D/2}$. (For simplicity we have taken $E,\alpha_0'k^2\lesssim\delta_0$.) For $\eta_0<0$ there will be ψ_c^2 or $\bar{\psi}_c^2$ vertices which are proportional to δ_0. However, each of these vertices requires two additional triple-Pomeron vertices, four additional propagators, and an additional loop integration. (See Fig. 2.) As a result, the perturbation series will again be an expansion in powers of the dimensionless parameter $\lambda=r_0^2/{\alpha_0'}^{D/2}\delta_0^{2-D/2}$. When this parameter is small the leading J-plane singularity is a pole with intercept $1-\delta_0+O(\lambda)$ independent of the sign of η_0. So for small values of λ our continuation procedure does lead to $\alpha(0)<1$ for $\eta_0<0$. Notice that for $\eta_0<0$ the intercept shift, $\alpha(0)-\alpha_0=2\eta_0+O(\lambda)$, cannot be calculated in perturbation theory without first shifting the field. This is because the bare vacuum is unstable for $\eta_0<0$, and a perturbation expansion about an unstable vacuum is meaningless.

Experimentally we know that δ_0 is small, i.e., α_0 is close to its critical value. Furthermore, since we are interested in studying the large-s, small-t behavior of scattering amplitudes, we need to know the behavior of the correlation functions for small values E and k^2. It is clear from our power-counting argument that for δ_0, $E,\alpha_0'k^2\lesssim(r_0^2/{\alpha_0'}^{D/2})^{1/(2-D/2)}$ we cannot truncate the perturbation series. In this domain the renormalization group is the most convenient tool for studying the structure of the theory. We now turn to a discussion of it.

We introduce a renormalized field and a set of renormalized parameters through the relations

$$\psi_R(\vec{x},y)=Z_3^{-1/2}\psi(\vec{x},y), \quad S_R=Z_3^{1/2}S, \tag{33}$$

$$r=Z_1^{-1}Z_3^{3/2}r_0, \tag{34}$$

$$\alpha'=Z_2^{-1}Z_3\alpha_0', \tag{35}$$

$$\delta=Z_4^{-1}Z_3\delta_0. \tag{36}$$

The proper vertex functions for the renormalized field are related to those for the unrenormalized field by

$$\begin{aligned} &\Gamma_R^{n,m}(E_i,k_i,r,\alpha',\delta,S_R,\bar{S}_R,E_N)\\ &\qquad=Z_3^{(n+m)/2}\Gamma^{n,m}(E_i,k_i,r_0,\alpha_0',\delta_0,S,\bar{S}),\\ &\tilde{\Gamma}_R^{n,m}(E_i,k_i,r,\alpha',\delta,S_R,\bar{S}_R,E_N)\\ &\qquad=Z_3^{(n+m)/2}\tilde{\Gamma}^{n,m}(E_i,k_i,r_0,\alpha_0',\delta_0,S,\bar{S}). \end{aligned} \tag{37}$$

Here E_i and $\vec{k}_i$ are the "energy" and momentum of the Reggeons and E_N is an arbitrary renormalization energy. The Z_i are determined by the normalization conditions

$$\frac{\partial}{\partial E} i\Gamma_R^{1,1}(E, k^2, r, \alpha', \delta, S_R, \bar{S}_R, E_N)\Big|_{\substack{E=-E_N,\\ k^2=\delta=S_R=\bar{S}_R=0}} = 1\,, \tag{38}$$

$$\frac{\partial}{\partial k^2} i\Gamma_R^{1,1}(E, k^2, r, \alpha', \delta, S_R, \bar{S}_R, E_N)\Big|_{\substack{E=-E_N,\\ k^2=\delta=S_R=\bar{S}_R=0}} = -\alpha'\,, \tag{39}$$

$$\frac{\partial}{\partial \delta} i\Gamma_R^{1,1}(E, k^2, r, \alpha', \delta, S_R, \bar{S}_R, E_N)\Big|_{\substack{E=-E_N,\\ k^2=\delta=S_R=\bar{S}_R=0}} = -1\,, \tag{40}$$

$$\Gamma_R^{1,2}(E_i, k_i, r, \alpha', \delta, S_R, \bar{S}_R, E_N)\Big|_{\substack{E_1=2E_2=2E_3=-E_N,\\ k_i=\delta=S_R=\bar{S}_R=0}} = ir/(2\pi)^{(D+1)/2}\,, \tag{41}$$

and the parameter Δ_α by

$$\Gamma^{1,1}(E, k^2, r_0, \alpha_0', S, \bar{S})\big|_{E=k^2=\delta_0=S=\bar{S}=0} = 0\,. \tag{42}$$

In Eqs. (38)–(42) the vertex functions are to be evaluated on the sheet of S and $\bar{S}$ for which $\eta_0 = \delta_0$ when $S=\bar{S}=0$. With this normalization the Z_i are independent of δ, S, and $\bar{S}$. They depend only on the dimensionless coupling constant

$$g = r/(\alpha')^{D/4} E_N^{(4-D)/4}\,. \tag{43}$$

The renormalization-group equations for the $\tilde{\Gamma}^{n,m}$ follow from the fact that the $\tilde{\Gamma}^{n,m}$ are independent of E_N. Differentiating both sides of Eq. (37) with respect to E_N gives

$$\left[E_N \frac{\partial}{\partial E_N} + \beta(g)\frac{\partial}{\partial g} + \tau(g)\alpha' \frac{\partial}{\partial \alpha'} + \kappa(g)\delta\frac{\partial}{\partial \delta} - \tfrac{1}{2}(n+m)\gamma(g)\right]\tilde{\Gamma}_R^{n,m} = 0\,, \tag{44}$$

where

$$\beta(g) = E_N \frac{\partial g}{\partial E_N}\Big|_{r_0, \alpha_0', \delta_0, S, \bar{S}\ \text{fixed}}\,, \tag{45}$$

$$\tau(g) = E_N \frac{\partial}{\partial E_N} \ln(Z_2^{-1} Z_3)\Big|_{r_0, \alpha_0', \delta_0, S, \bar{S}\ \text{fixed}}\,, \tag{46}$$

$$\kappa(g) = E_N \frac{\partial}{\partial E_N} \ln(Z_4^{-1} Z_3)\Big|_{r_0, \alpha_0', \delta_0, S, \bar{S}\ \text{fixed}}\,, \tag{47}$$

$$\gamma(g) = E_N \frac{\partial}{\partial E_N} \ln Z_3\Big|_{r_0, \alpha_0', \delta_0, S, \bar{S}\ \text{fixed}}\,. \tag{48}$$

The renormalization-group equation can be written in a more useful form after some dimensional analysis. Using the fact that the action is dimensionless we see that (the square brackets denote the dimension of the quantity enclosed)

$$\begin{aligned} &[\psi] = [\bar{\psi}] = k^{D/2},\quad [S] = [\bar{S}] = Ek^{D/2},\quad [\alpha'] = Ek^{-2}\,,\\ &[r] = Ek^{-D/2},\quad [\delta] = [\tilde{\Gamma}_R^{1,1}] = E\,. \end{aligned} \tag{49}$$

Let us concentrate our attention on $\tilde{\Gamma}_R^{1,1}$. It can be written in the form

$$\tilde{\Gamma}_R^{1,1}(E, k^2, g, \alpha', \delta, S_R, \bar{S}_R, E_N) = E_N \Phi\left(E/E_N, \alpha' k^2/E_N, \delta/E_N, g, (S_R/E_N)(\alpha'/E_N)^{D/4}, (\bar{S}_R/E_N)(\alpha'/E_N)^{D/4}\right). \tag{50}$$

This scaling law tells us that

$$\tilde{\Gamma}_R^{1,1}(E, k^2, g, \alpha', \xi\delta, S_R, \bar{S}_R, E_N) = \xi \tilde{\Gamma}_R^{1,1}(E/\xi, k^2, g, \alpha'/\xi, \delta, S_R/\xi, \bar{S}_R/\xi, E_N/\xi)\,, \tag{51}$$

so

$$\xi\frac{\partial}{\partial\xi}\tilde{\Gamma}_R^{1,1}(E,k^2,g,\alpha',\xi\delta,S_R,\bar{S}_R,E_N)=\left(1-\alpha'\frac{\partial}{\partial\alpha'}-E_N\frac{\partial}{\partial E_N}-E\frac{\partial}{\partial E}-S_R\frac{\partial}{\partial S_R}-\bar{S}_R\frac{\partial}{\partial\bar{S}_R}\right)\tilde{\Gamma}_R^{1,1}. \tag{52}$$

Combining this result with Eq. (44) and writing $t=\ln\xi$ gives

$$\left([(1-\kappa(g)]\frac{\partial}{\partial t}-\beta(g)\frac{\partial}{\partial g}+[1-\tau(g)]\alpha'\frac{\partial}{\partial\alpha'}+E\frac{\partial}{\partial E}+S_R\frac{\partial}{\partial S_R}+\bar{S}_R\frac{\partial}{\partial\bar{S}_R}-[1-\gamma(g)]\right)\tilde{\Gamma}_R^{1,1}(E,k^2,g,\alpha',\xi\delta,S_R,\bar{S}_R,E_N)=0. \tag{53}$$

The solution to this equation is standard and is given by

$$\tilde{\Gamma}_R^{1,1}(E,k^2,g,\alpha',\delta,S_R,\bar{S}_R,E_N)=\tilde{\Gamma}_R^{1,1}(E_t(-t),k^2,g_t(-t),\alpha_t'(-t),e^{-t}\delta,S_t(-t),\bar{S}_t(-t),E_N)\times\exp\left\{\int_{-t}^{0}dt'[1-\gamma(g_t(-t))]/[1-\kappa(g_t(-t))]\right\}, \tag{54}$$

where

$$\frac{dg_t(t)}{dt}=-\beta(g_t)/[1-\kappa(g_t)], \tag{55}$$

$$\frac{d\ln\alpha_t'(t)}{dt}=[1-\tau(g_t)]/[1-\kappa(g_t)], \tag{56}$$

$$\frac{d\ln E_t(t)}{dt}=\frac{d\ln S_t(t)}{dt}=\frac{d\ln\bar{S}_t(t)}{dt}=[1-\kappa(g_t)]^{-1}, \tag{57}$$

with boundary conditions

$$g_t(0)=g,\quad \alpha_t'(0)=\alpha',\quad E_t(0)=E,\quad S_t(0)=S_R,\quad \bar{S}_t(0)=\bar{S}_R. \tag{58}$$

Equation (54) will be useful for the study of the small E, k^2, and δ behvaior of the theory provided $\beta(g)$ has a zero for which $d\beta/dg>0$. We know that such a zero exists for D near four[7,8] and we believe that one also exists in the physical number of dimensions, $D=2$. Assuming this to be the case,

$$g_t(t)\underset{t\to\infty}{\sim}g_1, \tag{59}$$

where g_1 is the position of the zero of $\beta(g)$. Similarly,

$$\alpha_t'(t)\underset{t\to\infty}{\sim}C_\alpha\alpha' e^{tz/(1-\kappa)},\qquad E_t(t)\underset{t\to\infty}{\sim}C_E E e^{t/(1-\kappa)}, \tag{60}$$

with analogous expressions for $S_t(t)$ and $\bar{S}_t(t)$. Here C_α and C_E are constants and

$$z=1-\tau(g_1)=1-\tau,\qquad \kappa=\kappa(g_1). \tag{61}$$

Setting $t=\ln(\delta/E_N)$ we find

$$i\tilde{\Gamma}_R^{1,1}(E,k^2,g,\alpha',S_R,\bar{S}_R,E_N)\underset{\delta\to0}{\sim}E_N C_0\left(\frac{\delta}{E_N}\right)^{(1-\gamma)/(1-\kappa)}\Phi(\rho_1,\rho_2,\rho_3,\rho_4), \tag{62}$$

with

$$\rho_1=C_1\frac{E}{E_N}\left(\frac{\delta}{E_N}\right)^{-1/(1-\kappa)},\qquad \rho_2=C_2\frac{\alpha' k^2}{E_N}\left(\frac{\delta}{E_N}\right)^{-z/(1-\kappa)},$$
$$\rho_3=C_3\frac{S_R}{E_N}\left(\frac{\alpha'}{E_N}\right)^{D/4}\left(\frac{\delta}{E_N}\right)^{-(1+zD/4)/(1-\kappa)},\qquad \rho_4=C_4\frac{\bar{S}_R}{E_N}\left(\frac{\alpha'}{E_N}\right)^{D/4}\left(\frac{\delta}{E_N}\right)^{-(1+zD/4)/(1-\kappa)}. \tag{63}$$

The C_i are constants and $\gamma=\gamma(g_1)$. For the physically interesting cases, $S=\bar{S}=0$, we shall write Eq. (62) in the form

$$i\tilde{\Gamma}_R^{1,1}(E,k^2,g,\alpha',\delta,E_N)\underset{\delta\to0}{\sim}E_N C_0\left(\frac{\delta}{E_N}\right)^{(1-\gamma)/(1-\kappa)}\Phi(\rho_1,\rho_2). \tag{64}$$

The same scaling law holds independent of the sheet of the $S,\bar{S}$ plane one is on, i.e., independent of whether $\eta_0=\pm\delta_0$. However, in general Φ will have a different functional form for the two cases.

In Eq. (64) the limit $\delta\to0$ is to be taken for fixed values of the scaling variables ρ_1 and ρ_2. It is equivalent to

$$i\tilde{\Gamma}_R^{1,1}(E,k^2,g,\alpha',\delta,E_N)\underset{E_N\to0}{\sim}E_N C_0'\left(\frac{-E}{E_N}\right)^{1-\gamma}\Phi'(\rho_1',\rho_2'), \tag{65}$$

with

$$\rho_1'=C_1'\frac{\delta}{E_N}\left(\frac{-E}{E_N}\right)^{-(1-\kappa)},\qquad \rho_2'=C_2'\frac{\alpha' k^2}{E_N}\left(\frac{-E}{E_N}\right)^{-z}. \tag{66}$$

This result can also be obtained directly from Eqs. (44) and (50) by scaling E rather than δ. Equations (64) and (65) are the natural generaliza-

tions of the scaling low of Refs. 7 and 8 and reduce to it in the limit $\delta \to 0$ for fixed E, k^2.

The behavior of the physical Pomeron trajectory function, $\alpha_R(-k^2)$, for small values of δ can be read off directly from Eq. (64). The pole arises from a zero of $\Phi(\rho_1, \rho_2)$. For $k^2 = 0 = \rho_2$ the zero will occur at $\rho_1 = \rho_{10}$ where ρ_{10} depends only on D. Expanding about this point for small k^2, the trajectory is given by

$$\left.\frac{\partial\Phi}{\partial\rho_1}\right|_{\substack{\rho_1=\rho_{10},\\ \rho_2=0}} (\rho_1-\rho_{10}) + \left.\frac{\partial\Phi}{\partial\rho_2}\right|_{\substack{\rho_1=\rho_{10},\\ \rho_2=0}} \rho_2 = 0\,, \tag{67}$$

from which we see that

$$\Delta = 1-\alpha_R(0) \underset{\delta\to 0}{\sim} \left(\frac{\delta}{E_N}\right)^{1/(1-\kappa)} E_N\rho_{10}/C_1, \tag{68}$$

$$\alpha_R'(0) \underset{\delta\to 0}{\sim} \left(\frac{\delta}{E_N}\right)^{\tau/(1-\kappa)} \alpha'\left(-C_2\frac{\partial\Phi}{\partial\rho_2}\Big/C_1\frac{\partial\Phi}{\partial\rho_1}\right)_{\substack{\rho_1=\rho_{10},\\ \rho_2=0}}, \tag{69}$$

For small values of $\epsilon = 4-D$, g_1 is of order $\epsilon^{1/2}$, so one can use perturbation theroy to calculate the critical exponents and scaling functions. For definiteness let us set $\bar{S}=0$ so that the classical potential is given by Eq. (18). The diagrams which contribute Σ to order r_0^2 are shown in Fig. 5. Writing the unrenormalized correlation function in the form

$$i\tilde{\Gamma}^{1,1}(E, k^2, r_0, \alpha_0', S) = E - \alpha_0' k^2 - \delta_0 - \Sigma\,, \tag{70}$$

we see that

$$\Sigma_a = \frac{r_0^2}{2(8\pi\alpha_0')^{D/2}}\,\Gamma(1-D/2)(\tfrac{1}{2}\alpha_0' k^2 + 2\delta_0 - E)^{D/2-1}, \tag{71}$$

$$\Sigma_b = -\frac{r_0^2}{(8\pi\alpha_0')^{D/2}}\,\Gamma(1-D/2) \times [\delta_0 - (\delta_0^2 - 2ir_0S)^{1/2}](2\delta_0)^{D/2-2}\,, \tag{72}$$

$$\Sigma_c = -\frac{r_0^2}{(8\pi\alpha_0')^{D/2}}\,\Gamma(2-D/2)[\delta_0 - (\delta_0^2 - 2ir_0S)^{1/2}] \times \int_0^1 dx[2\delta_0 + \alpha_0' k^2 x(1-x/2) - Ex]^{D/2-2}\,, \tag{73}$$

where the subscripts refer to the diagrams in Fig. 5. For $S=0$ and $\eta_0 = +\delta_0$ only Σ_a is nonzero. Using Eqs. (35)–(41) we find that to order g^2

$$Z_2 = 1 + \frac{g^2}{4(8\pi)^{D/2}}\,\Gamma(2-D/2)\,, \tag{74}$$

$$Z_3 = 1 + \frac{g^2}{2(8\pi)^{D/2}}\,\Gamma(2-D/2)\,, \tag{75}$$

$$Z_4 = 1 + \frac{g^2}{(8\pi)^{D/2}}\,\Gamma(2-D/2)\,, \tag{76}$$

and

$$\gamma(g) = -\frac{\epsilon}{12}(g^2/g_1^2)\,, \tag{77}$$

$$\tau(g) = -\frac{\epsilon}{24}(g^2/g_1^2)\,, \tag{78}$$

$$\kappa(g) = \frac{\epsilon}{12}(g^2/g_1^2)\,, \tag{79}$$

where

$$\frac{g_1^2}{(8\pi)^{D/2}} = \frac{\epsilon}{6} + O(\epsilon^2)\,. \tag{80}$$

Furthermore, from the study of the theory with $\eta_0 = 0$ (see Refs. 7 and 8) we know that to order g^3

$$\beta(g) = -\frac{\epsilon}{4}g(1 - g^2/g_1^2)\,, \tag{81}$$

so to leading order in ϵ

$$\kappa = -\gamma = -2\tau = \epsilon/12\,. \tag{82}$$

We can now explicitly integrate Eqs. (55)–(57) and find

$$g_t^2(t) \underset{t\to\infty}{\sim} g_1^2 + (g^2 - g_1^2)(g_1^2/g^2)^{1+\epsilon/12} \times e^{-\epsilon/2(1+\epsilon/12)t}\,, \tag{83}$$

$$C_0 = (g_1^2/g^2)^{1/3},\quad C_1 = (g_1^2/g^2)^{-1/6},$$
$$C_2 = (g_1^2/g^2)^{-1/4},\quad C_3 = C_4 = (g_1^2/g^2)^{1/4}. \tag{84}$$

In obtaining Eqs. (83) and (84) we have taken the limit $t \to \infty$ for fixed g, which cannot be interchanged with the limit $g \to 0$. In fact setting $t = \ln(\delta/E_N)$ we see that our expressions for the C_i are valid only for $(\delta/E_N)(g_1^2/g^2)2/\epsilon \ll 1$. In the limit $g \to 0$, for fixed δ, perturbation theory becomes applicable.

For $g = g_1$, the scaling law is exact for all values of δ. As a result, for $\bar{S}=0$ the scaling function,

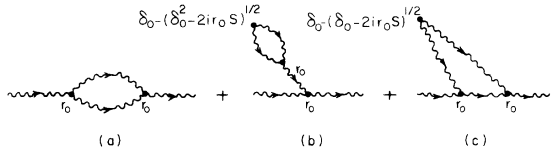

FIG. 5. Lowest-order graphs contributing to the Pomeron self-energy when $S \neq 0$, $\bar{S}=0$.

Φ, can be obtained from the relation

$$i\tilde{\Gamma}_R^{1,1}(E,k^2,g_1,\alpha',E_N,S_R,E_N) = E_N\Phi(\rho_1,\rho_2,\rho_3)\Big|_{\substack{\rho_1=E/E_N,\\ \rho_2=\alpha' k^2/E_N,\\ \rho_3=(S_R/E_N)(\alpha'/E_N)^{D/4}}} \tag{85}$$

Writing

$$\Phi = \Phi_0 + \epsilon\Phi_1 + O(\epsilon^2) \tag{86}$$

we find after a little algebra that[14]

$$\Phi_0(\rho_1,\rho_2,\rho_3) = \rho_1 - \rho_2 - 1, \tag{87}$$

$$\Phi_1(\rho_1,\rho_2,\rho_3) = \tfrac{1}{12}(2-\rho_1+\tfrac{1}{2}\rho_2)[1-\ln(2-\rho_1+\tfrac{1}{2}\rho_2)] - \tfrac{1}{6}[1-(1-2ig_1\rho_3)^{1/2}]\left\{1-\ln 2+\int_0^1 dx\,\ln[2-\rho_1 x+\rho_2 x(1-\tfrac{1}{2}x)]\right\}. \tag{88}$$

The square root is defined so that $(1-2ig_1\rho_3)^{1/2}=1$ for $S=0$ and $\eta_0=\delta_0$.

We can read off most of the results quoted in the Introduction from Eqs. (62), (64), and (86)–(88). Let us start with $S=0$ and $\eta_0=\delta_0>0$. Then $\delta = Z_4^{-1}Z_3\eta_0$, and Eq. (64) is clearly equivalent to Eq. (2). (Recall that with our renormalization procedure the Z's are independent of η_0, S, and $\bar{S}$.) $\tilde{\Gamma}_R^{1,1}=Z_3\Gamma^{1,1}$ has a branch point at $\eta_0=0$. If we now take S to be different from zero, $\delta = Z_4^{-1}Z_3(\eta_0^2+2ir_0S)^{1/2}$, so there is no longer a singularity at $\eta_0=0$, and we are free to continue to negative values of η_0. Then setting $S=0$ we find that for $\eta_0<0$, $\eta_0=-\delta_0=-Z_4Z_3^{-1}\delta$ and $(1-2ig_1\rho_3)^{1/2}|_{S=\rho_3=0}=-1$. [The simplest way to keep track of the phase of $(1-2ig_1\rho_3)^{1/2}$ is to perform the continuation before going through the renormalization-group analysis.] Alternatively we can continue in S for fixed values of δ_0 and δ. Starting at $S=0$, $\eta_0=\delta_0$ we move through the cut in $(1-2ig_1\rho_3)^{1/2}$ and arrive back at $S=0$ with $\eta_0=-\delta_0$. Although the critical indices are the same for either sign of η_0, the scaling function $\Phi(\rho_1,\rho_2)$ is not. For the different signs of η_0 we evaluate the generalized scaling function $\Phi(\rho_1,\rho_2,\rho_3)$ at $\rho_3=0$ on different sheets of the ρ_3 plane.

It is left as an exercise for the reader to show that if one takes $\bar{S}$ rather than S to be different from zero one obtains the same expression for $\tilde{\Gamma}_R^{1,1}$ with ρ_3 replaced by ρ_4. Thus the physical correlation frunction ($S=\bar{S}=0$) is independent of the path of continuation. Our final result for it is that to leading order in ϵ

$$i\tilde{\Gamma}_R^{1,1}(E,k^2,g,\alpha',\delta,E_N) \underset{\delta\to 0}{\sim} \delta\left(\frac{\delta}{E_N}\left(\frac{g^2}{g_1^2}\right)^{2/\epsilon}\right)^{\epsilon/6} \times\left(\rho_1-\rho_2-1+\frac{\epsilon}{12}(2-\rho_1+\tfrac{1}{2}\rho_2)[1-\ln(2-\rho_1+\tfrac{1}{2}\rho_2)] - \frac{\epsilon}{6}(1-d)\left\{1-\ln 2+\int_0^1 dx\,\ln[2-\rho_1 x+\rho_2 x(1-\tfrac{1}{2}x)]\right\}\right), \tag{89}$$

with

$$\rho_1 = \frac{E}{\delta}\left[\frac{\delta}{E_N}\left(\frac{g_1^2}{g^2}\right)^{2/\epsilon}\right]^{-\epsilon/12}, \tag{90}$$

$$\rho_2 = \frac{\alpha' k^2}{\delta}\left[\frac{\delta}{E_N}\left(\frac{g_1^2}{g^2}\right)^{2/\epsilon}\right]^{-\epsilon/8}, \tag{91}$$

$$d = \mathrm{sgn}(\eta_0). \tag{92}$$

From Eqs. (68), (69), and (89) we can read off the behavior of the physical intercept gap and slope parameter for small δ:

$$\Delta = 1-\alpha(0) \underset{\delta\to 0}{\sim} \delta\left(\frac{\delta}{E_N}\left(\frac{g_1^2}{g^2}\right)^{2/\epsilon}\right)^{\epsilon/12}\left\{1-\frac{\epsilon}{6}[\tfrac{1}{2}-(1-d)\ln 2]\right\}, \tag{93}$$

$$\alpha_R' \underset{\delta\to 0}{\sim} \alpha'\left[\frac{\delta}{E_N}\left(\frac{g_1^2}{g^2}\right)^{2/\epsilon}\right]^{-\epsilon/24}\left[1+\frac{\epsilon}{6}(1-d)(\tfrac{5}{4}-2\ln 2)\right]. \tag{94}$$

Although one does not need to know $\langle\psi_R\rangle$ or $\langle\bar{\psi}_R\rangle$ in order to calculate the correlation functions, they are still of some interest. Their behavior for small δ can also be obtained from the renormalization-group analysis. Let us again start by taking $\bar{S}=0$. Then $\langle\bar{\psi}_R\rangle=0$, and we may write

$$\langle\psi_R\rangle = Z_3^{-1/2}\langle\psi\rangle = \psi_R(g,\alpha',\delta,S_R,E_N)\,. \tag{95}$$

Using the fact that $\langle\psi\rangle$ is independent of E_N we obtain

$$\left[E_N\frac{\partial}{\partial E_N}+\beta(g)\frac{\partial}{\partial g}+\tau(g)\,\alpha'\frac{\partial}{\partial\alpha'}+\kappa(g)\,\delta\frac{\partial}{\partial\delta}+\tfrac{1}{2}\gamma(g)\right]\psi_R(g,\alpha',\delta,S_R,E_N)=0\,. \tag{96}$$

Combining this equation with the scaling law

$$\psi_R(g,\alpha',\delta,S_R,E_N)=\psi_R\left(g,\frac{\alpha'}{\xi},\delta,\frac{S_R}{\xi},\frac{E_N}{\xi}\right) \tag{97}$$

gives

$$\left([1-\kappa(g)]\frac{\partial}{\partial t}-\beta(g)\frac{\partial}{\partial g}+[1-\tau(g)]\,\alpha'\frac{\partial}{\partial\alpha'}+S_R\frac{\partial}{\partial S_R}-\tfrac{1}{2}\gamma(g)\right)\psi_R(g,\alpha',e^t\delta,S_R,E_N)=0\,, \tag{98}$$

which has the standard solution

$$\psi_R(g,\alpha',\delta,S_R,E_N)=\psi_R(g_t(-t),\alpha_t'(-t),e^{-t}\delta,S_t(-t),E_N)\exp\left\{\int_{-t}^{0}dt'\tfrac{1}{2}\gamma(g_t(t'))/[1-\kappa(g_t(t'))]\right\}. \tag{99}$$

Again setting $t=\ln(\delta/E_N)$ we find

$$\psi_R(g,\alpha',\delta,S_R,E_N)\underset{\delta\to 0}{\sim}\left(\frac{\delta}{E_N}\right)^{(\gamma/2+D\epsilon/4)/(1-\kappa)}C_0'\left(\frac{E_N}{\alpha'}\right)^{D/4}\chi(g_1,1,\rho_3)\,, \tag{100}$$

where to leading order is ϵ, $C_0'=(g_1^2/g^2)^{-1/3}$.

In order to obtain the ϵ expansion for the scaling function, χ, we must compute the perturbation series for

$$\langle\psi\rangle = v+\langle\psi_c\rangle\,. \tag{101}$$

The leading contribution to $\langle\psi_c\rangle$ comes from the diagram of Fig. 6. It gives

$$\langle\psi_c\rangle=\frac{i\delta_0}{r_0}\frac{r_0^2}{(8\pi\alpha_0')^{D/2}}\left[1-\left(1-\frac{2ir_0S}{\delta_0^2}\right)^{1/2}\right]\Gamma(1-D/2)(2\delta_0)^{D/2-2}. \tag{102}$$

Putting all this together we find that to leading order in ϵ

$$\langle\psi_R\rangle\underset{\delta\to 0}{\sim}\delta\left(\frac{\delta}{E_N}\left(\frac{g_1^2}{g^2}\right)^{2/\epsilon}\right)^{-\epsilon/6}\left(\frac{E_N}{\alpha'}\right)^{D/4}\left(-\frac{2}{g_1}\right)[1-(1-2ig_1\rho_3)^{1/2}]\left[1-\frac{\epsilon}{6}(1-\ln 2)\right]. \tag{103}$$

If we take $\bar{S}$ rather than S different from zero, then we find $\langle\psi_R\rangle=0$ and $\langle\bar{\psi}_R\rangle$ is given by the right-hand side of Eq. (103) with ρ_3 replaced by ρ_4.

III. SUMMARY AND CONCLUSIONS

The calculations of Sec. II show that for η_0 near zero the inverse correlation function $\tilde{\Gamma}^{(1,1)}$ exhibits all the features of the correlation function of a statistical system near a second-order phase transition. There is a scaling law, with the scaling function evaluated in the ϵ expansion in Eqs. (89)–(92). For $\eta_0\neq 0$ there are only short-range correlations in rapidity. By this we mean that the Pomeron correlation functions and the correlation functions for particles produced in the central region fall off exponentially for large rapidity separations.[15] $\Delta=1-\alpha(0)$ is the reciprocal of the correlation length. It never becomes negative, and it vanishes only at $\eta_0=0$. Just as in the statistical mechanics problem, one finds long-range correlations only at the critical point. For small values of η_0 the behavior of Δ is given by Eq. (93), where δ is proportional to η_0. The dependence of the order ϵ correction on the sign of η_0 is typical of phase

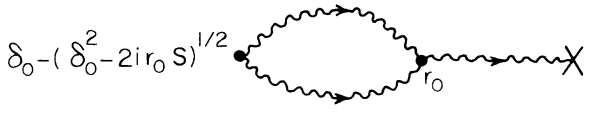

FIG. 6. Lowest-order contribution to $\langle\psi_c\rangle$ when $S\neq 0$, $\bar{S}=0$.

transitions; the plot of Δ versus η_0 is not symmetric about $\eta_0 = 0$. A similar asymmetry appears in the scaling function. For large $|\eta_0|$ our power-counting discussions of perturbation graphs indicate $\Delta \approx 1 - |\eta_0|$, so the asymmetry dies away when the renormalized Pomeron intercept is well below 1. In fact power counting merely shows that quantum corrections to the classical potential prediction die away for large $|\eta_0|$. If we were to cut off our field theory and take $D > 4$, the classical potential prediction would be correct for any $|\eta_0|$. This contention is again verified by power counting in perturbation theory. Therefore, mean field or classical results break down only for $D \leq 4$ and small $|\eta_0|$—the physically relevant situation. In this regime the renormalization group is indispensable for the analysis.

For $\eta_0 < 0$, one of the fields ψ or $\bar{\psi}$ acquires a vacuum expectation value. This vacuum expectation value vanishes as $\eta_0 \to 0$ in the manner stated in Eq. (103); such a continuous variation at $\eta_0 = 0$ is what makes the phase transition second order rather than first order. Assuming that it is $\langle\psi\rangle$ which is nonzero, the nonderivative terms in the Lagrangian density are given by Eq. (18), with $S = 0$ and $(\delta_0^2 - 2ir_0 S)^{1/2} = -\delta_0$. The term $\delta_0 \bar{\psi}_c^2$, treated as part of the interaction, destroys the symmetry of the correlation functions. This is illustrated for the partial-wave amplitude in Fig. 7, where the lowest-order diagrams are shown for a process where particle A interacts with two Pomerons, and particle B interacts with one Pomeron. When A is the sink and B the source there is an extra diagram which is not there when A is the source and B the sink. The breakdown in time-reversal invariance for the partial-wave amplitude leads to a breakdown in Lorentz invariance for the scattering amplitude, since the amplitude with A as the target will be different from the amplitude with B as the target. There are two possible conclusions which one can draw from this disaster. One is that Reggeon field theory is simply not physically acceptable for $\eta_0 < 0$. Unfortunately it is unclear what mechanism in the underlying dynamics forces $\eta_0 \geq 0$. The other possibility is that there is an alternative way of continuing to $\eta_0 < 0$ which leads to physically acceptable results. Our procedure of performing the continuation in the presence of constant external sources is open to criticism. We argued in Sec. II that it was sufficient to take S and $\bar{S}$ to be independent of $\vec{x}$ and y since we expected $\langle\psi\rangle$ and $\langle\bar{\psi}\rangle$ to be in order that the translational invariance in these variables not be broken. This is what is done in statistical mechanics, and it certainly makes sense to study the behavior of a ferromagnet in a constant external field. However, it is the hadrons which are the sources of the Reggeons, and these are well localized in $\vec{x}$ and y. Therefore, it might be argued that one should really study the continuation to $\eta_0 < 0$ in the presence of localized sources. This is a problem which deserves further consideration.[16] We merely note that it is far from trivial to simultaneously maintain time-reversal invariance, translation invariance, and Reggeon unitarity.

We are now in a position to enumerate the possible high-energy behaviors which can be obtained in Reggeon field theory. The two quantities which set the energy scale are η_0 and r_0^2/α_0' (for $D = 2$). For $\eta_0 = 0$ and $(r_0^2/\alpha_0')\ln s \gtrsim 1$ we will see the scaling behavior of Refs. 7 and 8. For $r_0^2/\alpha_0' > |\eta_0| > 0$ we will see approximate scaling behavior for $(r_0^2/\alpha_0')\ln s \gtrsim 1 > |\eta_0| \ln s$, but at higher energies where $|\eta_0| \ln s \gtrsim 1$ the high-energy behavior will be dominated by the renormalized Pomeron pole, which is below 1. For $|\eta_0| > r_0^2/\alpha_0'$ we will not see scaling behavior at all. In this limit one should use perturbation theory, using experiment as a guide to the selection of an appropriate finite set of couplings.

Since total cross sections are approximately constant at high energies and r_0 is small, η_0 must be small. The question of why nature chooses $\Delta_0 \approx \Delta_{0c}$ remains elusive. In principle it could have been answered within Reggeon field theory, as in Ref. 10, and the explanation of $\Delta_0 \approx \Delta_{0c}$ has slipped away from us.

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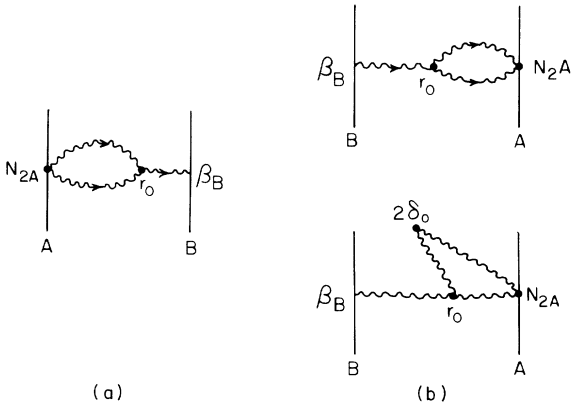

FIG. 7. Lowest-order graphs proportional to $N_{2A}\beta_B$: (a) when particle A emits two Pomerons and particle B absorbs one: (b) when particle A absorbs two Pomerons and particle B emits one.

APPENDIX

An alternative method of studying the spontaneous symmetry breaking is to construct the effective action[17]

$$\Gamma(\Psi,\overline{\Psi}) = \sum_{n,m=1}^{\infty} \frac{1}{n!m!}(\Psi - \langle\psi\rangle)^n(\overline{\Psi} - \langle\overline{\psi}\rangle)^m \times i\tilde{\Gamma}^{n,m}(0,0). \tag{A1}$$

$\tilde{\Gamma}^{n,m}(0,0)$ are the proper vertex functions of the shifted fields, $\psi - \langle\psi\rangle$, $\overline{\psi} - \langle\overline{\psi}\rangle$, evaluated at $E_i = \vec{k}_i = 0$, and Ψ and $\overline{\Psi}$ are constants. In order to write Eq. (A1) we have had to redefine $i\tilde{\Gamma}^{1,1}$ to be minus the expression given in Eq. (70).

The zero-loop (or classical) approximation to Γ is

$$\Gamma_0(\Psi,\overline{\Psi}) = -\eta_0\overline{\Psi}\Psi - \tfrac{1}{2}ir_0(\overline{\Psi}^2\Psi + \overline{\Psi}\Psi^2). \tag{A2}$$

A typical one-loop diagram is shown in Fig. 8. In evaluating these diagrams one may think in terms of a two-channel problem in which channel one contains a Pomeron propagating clockwise with a Green's function $G_0^+ = E - \alpha_0' k^2 - \eta_0 + i\epsilon$ and channel two contains a Pomeron propagating counterclockwise with a Green's function $G_0^- = -E - \alpha_0' k^2 - \eta_0 + i\epsilon$. The interactions, denoted by x's in Fig. 8 are given by the matrix

$$V = \begin{pmatrix} \frac{\partial^2 U_c}{\partial\psi\partial\overline{\psi}} & \frac{\partial^2 U_c}{\partial\overline{\psi}^2} \\ \frac{\partial^2 U_c}{\partial\psi^2} & \frac{\partial^2 U_c}{\partial\psi\partial\overline{\psi}} \end{pmatrix}_{\substack{\psi=\Psi \\ \overline{\psi}=\overline{\Psi}}}$$

$$= \begin{pmatrix} \eta_0 + ir_0(\Psi+\overline{\Psi}) & ir_0\Psi \\ ir_0\overline{\Psi} & \eta_0 + ir_0(\Psi+\overline{\Psi}) \end{pmatrix}. \tag{A3}$$

Writing

$$G_0 = \begin{pmatrix} G_0^+ & 0 \\ 0 & G_0^- \end{pmatrix} \tag{A4}$$

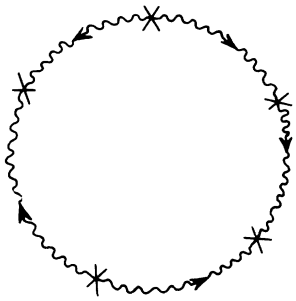

FIG. 8. One-loop graph for the effective action.

the one-loop contribution to Γ is[17,18]

$$\Gamma_1 = \tfrac{1}{2}\sum_{n=2}^{\infty}\frac{1}{n}\int\frac{dE}{2\pi i}\int\frac{d^Dk}{(2\pi)^D}\,\mathrm{tr}(G_0V)^n$$

$$= \tfrac{1}{2}\,\mathrm{tr}\int_0^1 dz\int\frac{dE}{2\pi i}\int\frac{d^Dk}{(2\pi)^D}[(1-zG_0V)^{-1}G_0V - G_0V]$$

$$= \tfrac{1}{2}\int\frac{d^Dk}{(2\pi)^D}(\alpha_0'k^2 + \eta_0 + ir_0(\Psi+\overline{\Psi}) - \{[\alpha_0'k^2 + \eta_0 + ir_0(\Psi+\overline{\Psi})]^2 + r_0^2\overline{\Psi}\Psi\}^{1/2}). \tag{A5}$$

The $\frac{1}{2}$ is the usual closed-loop factor and the factor of $1/n$ arises because the graphs are invariant under a simultaneous rotation of all vertices.

In order to carry out the renormalization of Γ it is convenient to isolate the term in Γ_1 which is singular at $D=4$. We therefore write $\Gamma_1 = \Gamma_{1a} + \Gamma_{1b}$ with

$$\Gamma_{1a} = -\tfrac{1}{2}\int\frac{d^Dk}{(2\pi)^D}\frac{\frac{1}{2}r_0^2\overline{\Psi}\Psi}{\alpha_0'k^2 + \eta_0 + ir_0(\Psi+\overline{\Psi})}$$

$$= -\frac{r_0^2\overline{\Psi}\Psi}{4(4\pi\alpha_0')^{D/2}}\Gamma(1-D/2)[\eta_0 + ir_0(\overline{\Psi}+\Psi)]^{D/2-1}. \tag{A6}$$

Then introducing the renormalized variables

$$\Psi_R = Z_3^{-1/2}\Psi, \qquad \overline{\Psi}_R = Z_3^{-1/2}\overline{\Psi}, \tag{A7}$$

and making use of Eqs. (34)–(36), (74)–(76), we see that to leading order in $\epsilon = 4 - D$

$$\Gamma' = \Gamma_0 + \Gamma_{1a} = -\eta\overline{\Psi}_R\Psi_R\left\{1 + \frac{g^2}{(8\pi)^2}\left[\ln\left(\frac{2\eta + 2ir(\Psi_R+\overline{\Psi}_R)}{E_N}\right) - 1\right]\right\} - \tfrac{1}{2}ir\overline{\Psi}_R\Psi_R(\overline{\Psi}_R+\Psi_R)\left[1 + 2\frac{g^2}{(8\pi)^2}\ln\left(\frac{\eta + ir(\Psi_R+\overline{\Psi}_R)}{E_N}\right)\right]. \tag{A8}$$

In evaluating Γ_{1b} to leading order in ϵ we can set $D=4$ before performing the k integration and obtain

$$\Gamma_{1b} = \frac{2}{(8\pi\alpha')^2}[\eta + ir(\Psi_R+\overline{\Psi}_R)]^3\{-\tfrac{1}{6} + \tfrac{1}{4}C - \tfrac{1}{3}(1+C)^{3/2} + \tfrac{1}{2}(1+C)^{1/2} + \tfrac{1}{2}C\ln[\tfrac{1}{2} + \tfrac{1}{2}(1+C)^{1/2}]\}, \tag{A9}$$

where

$$C = r^2 \Psi_R \Psi_R [\eta + ir(\Psi_R + \bar{\Psi}_R)]^{-2}. \tag{A10}$$

Notice that Γ_{1b} vanishes like $(\bar{\Psi}_R \Psi_R)^2$ when either Ψ_R or $\bar{\Psi}_R$ goes to zero.

The next step is to write down a renormalization-group equation for Γ. From Eqs. (37), (96), and (A1) we see that

$$\left[E_N \frac{\partial}{\partial E_N} + \beta(g)\frac{\partial}{\partial g} + \tau(g)\alpha' \frac{\partial}{\partial \alpha'} + \kappa(g)\delta \frac{\partial}{\partial \delta} - \frac{\gamma(g)}{2}\left(\Psi_R \frac{\partial}{\partial \Psi_R} + \bar{\Psi}_R \frac{\partial}{\partial \bar{\Psi}_R}\right)\right]\Gamma = 0 . \tag{A11}$$

Since $S = \bar{S} = 0$, $\delta = |\eta|$. Now $[\Gamma] = E k^0$, so Γ can be written in the form

$$\Gamma(g, \alpha', \delta, \Psi_R, \bar{\Psi}_R, E_N) = E_N (E_N/\alpha')^{D/2} A\left(g, \frac{\delta}{E_N}, \Psi_R \left(\frac{\alpha'}{E_N}\right)^{D/4}, \bar{\Psi}_R \left(\frac{\alpha'}{E_N}\right)^{D/4}\right), \tag{A12}$$

which leads to the scaling law

$$\Gamma(g, \alpha', \xi\delta, \Psi_R, \bar{\Psi}_R, E_N) = \xi \Gamma(g, \alpha'/\xi, \delta, \Psi_R, \bar{\Psi}_R, E_N/\xi) . \tag{A13}$$

Combining this with Eq. (A11) gives

$$\left[(1-\kappa(g))\frac{\partial}{\partial t} - \beta(g)\frac{\partial}{\partial g} + (1-\tau(g))\alpha' \frac{\partial}{\partial \alpha'} - 1 + \tfrac{1}{2}\gamma(g)\left(\Psi_R \frac{\partial}{\partial \Psi_R} + \bar{\Psi}_R \frac{\partial}{\partial \bar{\Psi}_R}\right)\right]\Gamma(g, \alpha', e^t\delta, \Psi_R, \bar{\Psi}_R, E_N) = 0 , \tag{A14}$$

which has the standard solution

$$\Gamma(g, \alpha', \delta, \Psi_R, \bar{\Psi}_R, E_N) = \Gamma(g_t(-t), \alpha_t'(-t), e^{-t}\delta, \Psi_{Rt}(-t), \bar{\Psi}_{Rt}(-t), E_N) \exp\left\{\int_{-t}^{0} dt' [1 - \kappa(g_t(t'))]^{-1}\right\} . \tag{A15}$$

Here

$$\frac{d}{dt} \ln \Psi_{Rt}(t) = \frac{d}{dt} \ln \bar{\Psi}_{Rt}(t) = \tfrac{1}{2}\gamma(g_t)[1 - \kappa(g_t)]^{-1} , \tag{A16}$$

with boundary conditions $\Psi_{Rt}(0) = \Psi_R$, $\bar{\Psi}_{Rt}(0) = \bar{\Psi}_R$. Setting $t = \ln(\delta/E_N)$ we find that

$$\Gamma(g, \alpha', \delta, \Psi_R, \bar{\Psi}_R, E_N) \underset{\delta \to 0}{\sim} (\delta/E_N)^{[1+(D/2)(1-\tau)](1-\kappa)^{-1}} C_0'' E_N (E_N/\alpha')^{D/2} A(g_1, 1, x, y) , \tag{A17}$$

where, to leading order in ϵ,

$$\begin{aligned} x &\equiv \Psi_{Rt}(-t)[\alpha_t'(-t)/E_N]^{D/4} = \Psi_R (\alpha'/E_N)^{D/4} (E_N/\delta)[(\delta/E_N)(g_1^2/g^2)^{2/\epsilon}]^{\epsilon/6} , \\ y &\equiv \bar{\Psi}_{Rt}(-t)[\alpha_t'(-t)/E_N]^{D/4} = \bar{\Psi}_R (\alpha'/E_N)^{D/4} (E_N/\delta)[(\delta/E_N)(g_1^2/g^2)^{2/\epsilon}]^{\epsilon/6} , \\ C_0'' &= (g_1^2/g^2)^{-1/3} . \end{aligned} \tag{A18}$$

From Eqs. (A8), (A9), and (A17) we see that

$$A(g_1, 1, x, y) = -dxy\left[1 + \frac{\epsilon}{6}(\ln 2 - 1)\right]\left[-\tfrac{1}{2} g_1 xy(x+y) - xy\frac{\epsilon}{6}\; d + i g_1 (x+y)\right] \ln[d + i g_1 (x+y)] + O(\epsilon^2 \text{ or } (xy)^2) , \tag{A19}$$

with $d = \mathrm{sgn}(\eta)$.

The vacuum expectation values of the fields ψ and $\bar{\psi}$ are given by

$$\left.\frac{\partial \Gamma}{\partial \Psi_R}\right|_{\substack{\Psi_R = \langle\psi\rangle \\ \bar{\Psi}_R = \langle\bar{\psi}\rangle}} = 0 , \qquad \left.\frac{\partial \Gamma}{\partial \bar{\Psi}_R}\right|_{\substack{\Psi_R = \langle\psi\rangle \\ \bar{\Psi}_R = \langle\bar{\psi}\rangle}} = 0 . \tag{A20}$$

Clearly solving these equations is equivalent to finding the stationary points of $A(g_1, 1, x, y)$ as a function of x and y. For $\eta > 0$ the only stable solution is $x = y = 0$, i.e., $\langle\psi\rangle = \langle\bar{\psi}\rangle = 0$. For $\eta < 0$ there are two stable solutions:

$$x = -\frac{2i}{g_1}\left[1 + \frac{\epsilon}{6}(\ln 2 - 1)\right], \quad y = 0$$

and

$$x = 0, \quad y = -\frac{2i}{g_1}\left[1 + \frac{\epsilon}{6}(\ln 2 - 1)\right], \tag{A21}$$

which lead to the same values of $\langle\psi\rangle$ and $\langle\bar{\psi}\rangle$ as found in Sec. II.

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